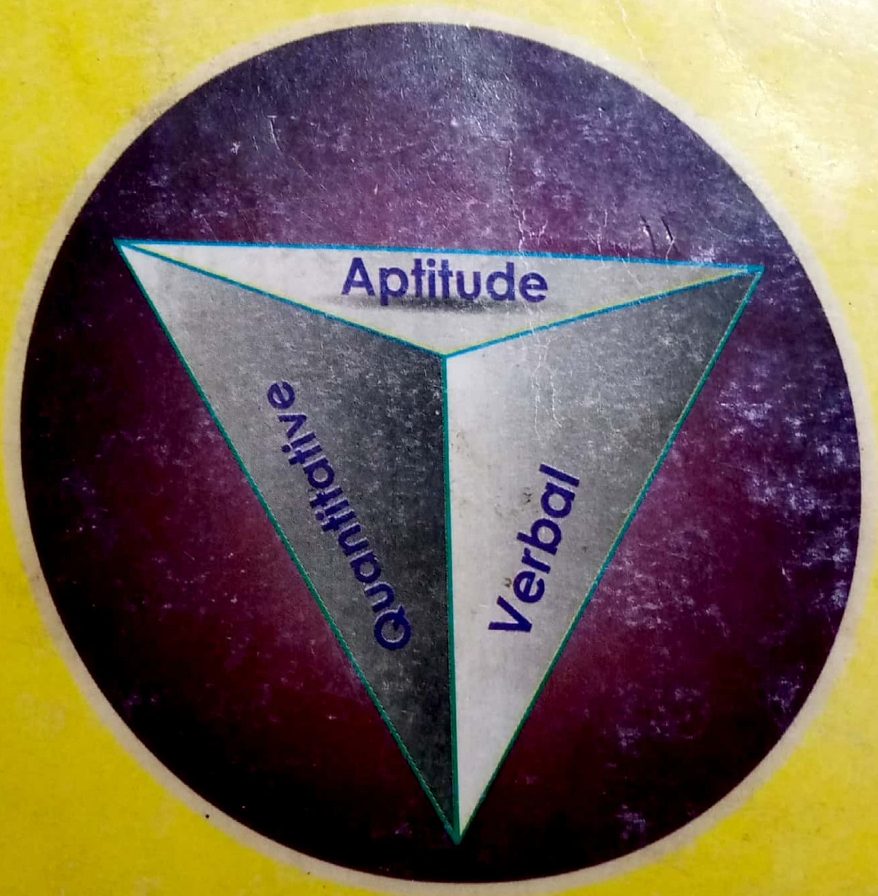
Extreme Series

Scholastic Aptitude Test

University Entrance Exam



Extreme Series

New Edition

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UNIT – ONE

1. NUMBER - SYSTEM

Prime Numbers (P):
A natural number, which is greater than 1 and divisible by 1 and by itself only, is called a **prime number**

Prime numbers (P) = 2, 3, 5, 7, 11, 13, 17, …

i. The smallest prime number is 2
ii. Except 2; all other prime numbers are odd.
iii. There are infinite prime numbers

1.1 COMPOSITE NUMBERS (C):

A natural numbers, which is greater than 1 and is not prime, is called **composite number**.

∴ Composite number (C) = 4, 6, 8, 9, 10, 12, 14, 15, …

i. The smallest composite number is 4.
ii. A composite number can be even or odd
iii. There are infinite composite number.

Note: 1 and 0 are neither prime nor composite.

1.2 DIVISIBILITY OF A NUMBER

An integer is divisible:

- by 2 if it is even
- by 3 if the sum of its digits is divisible by 3

Example:2,145 is divisible by 3 because 2+1+4+5 =12 is divisible by 3.

Divisibility test for 4.
An integer is divisible by 4 if and only if, the last two digits of the integer is divisible by 4

Example:

a. 57,658 is divisible by 4, because 28 is divisible by 4.
b. 1,172 is divisible by 4, because 72 is divisible by 4.

Divisibility test for 5

An integer is divisible by 5; if and only if, its unit digit is divisible by 5, that is, if and only if the unit digit is 0 or 5.

Example:

a. 2,365 is divisible by 5, because the unit digit is 5.

b. 1270 is divisible by 5, because the unit digit is 0.

Divisibility test for 6

An integer is divisible by 6; if and only if the integer is divisible by both 2 and 3.

Example: 7,134 is divisible by 6, because 4 is divisible by 2 also
$7 + 1 + 3 + 4 = 15$ and 15 is divisible by 3
$\therefore$ 7, 134 is divisible by 6.

Divisibility Test for 7.

To determine whether a given number is divisible by 7, double the unit digit of the given number. Find the difference between this number and the number formed by omitting the unit digit from the given number. If necessary, repeat this procedure until you obtain a small difference. If the final difference is divisible by 7, then the given number is also divisible by 7. If the final difference is not divisible by 7, then the given number is not divisible by 7.

Illustrative Example

1. Determine whether each of the following number is divisible by 7

a. 182 b. 1001 c. 273

Solution:

(a) To test divisibility of 182 for 7:

- Double of the unit digit: $2\times 2 = 4$
- The number formed by omitting the unit digit is 18
- Subtract 4 from 18: $18 - 4 = 14$

Because 14 is divisible by 7, thus the original number 182 is divisible by 7.

(b) To test divisibility of 1001 for 7:

- Double of the unit digit yields $1\times 2 = 2$
- Subtract 2 from 100 yields $100 - 2 = 98$
- Again, double the unit digit of 98. Yields $8\times 2 = 16$

- Again subtract 9 from 16, yields 16 – 9 = 7, because 7 is divisible by 7, therefore, 1001 is divisible by 7.

(c) To test divisibility of 273 for 7:

- Double of the unit digit, yields $2 \times 3 = 6$

The number formed by omitting the unit digit is 27

- Subtract 6 from 27, yields $27 - 6 = 19$

Because the final result 19 is not divisible by 7, thus 273 is not divisible by 7.

Divisibility Test for 8

A number is divisible by 8 if the last three digits of the number form a number that is divisible by 8

Illustrative Example

2. Determine whether each of the following numbers are divisible by 8

a. 97,136 b. 19,168 c. 16,278

Solution:

a. The last three digit of 97,136 form the number 136 which is divisible by 8.Thus, 97, 136 is divisible by 8.

b. The last three digit of 19, 168 form the number 168 which is divisible by 8.Thus, 19,168 is divisible by 8

c. The last three digit of 16,278 form the number 278, which is not divisible by 8. Thus 16,278 is not divisible by 8

Divisibility Test for 9

A number is divisible by 9 if the sum of all the digits of the number is divisible by 9

Illustrative Example

3. Determine whether each of the following number is divisible by 9.

a. 621,513 b. 732,624 c. 833,805

Solution:

a. The sum of the digit, $6 + 2 + 4 + 5 + 1 + 3 = 18$ and 18 is divisible by 9. Thus, 621, 513 is divisible by 9

b. The sum of the digit, $7 + 3 + 2 + 6 + 2 + 4 = 24$ and 24is not divisible by 9. Thus 732624 is not divisible by 9.

c. The sum of the digit $8 + 3 + 3 + 8 + 0 + 5 = 27$ and 27 is divisible by 9. Thus, 833, 805 is divisible by 9

A divisibility test for 13.

To determine whether a given number is divisible by 13, multiply the unit digit of the given number by 4. Find the sum of this multiple of 4 and the number formed by omitting the unit digit from the given number. If necessary, repeat this procedure until you obtain a final sum. If the final sum is divisible by 13, then given number is divisible by 13.

Illustrative Example

4. Determine whether each of the following number is divisible by 13.
   a. 1079 b. 1885 c. 14,507

To test divisibility by 13:

a. 1079: Four times the unit digit, yield $4 \times 9 = 36$. The number formed by omitting the unit digit is 107.

- The sum of $36 + 107 = 143$

Now repeat the procedure on 143.

- Four times the unit digit yield $4 \times 3 = 12$
- The sum of 14 and $12 = 26$

The final sum 26 is divisible by 13.
Thus 1079 is divisible by 13.

b. 1885 ⟵ Exercise left for you

c. 14,507 ⟵ Exercise left for you.

Divisibility Test for 11.

A natural number is divisible by 11, if and only if, the sum of the digit in the places that are even power of 10 **minus** the sum of the digits in the places that are odd powers of 10 is divisible by 11.

In short: To test divisibility for 11:
start at one end of the number and compute:
(The sum of every other digit) – (the sum of the remaining digits)

Illustrative Example

5. Determine whether each of the following number is divisible by 11.
   a. 57,729,364,583. b. 8, 471, 986 c. 4807

Solution:

a. To test divisibility for 11:
(Sum of every other digit) – (Sum of the remaining digit)
$= (5 + 7 + 9 + 6 + 5 + 3) - (7 + 2 + 3 + 4 + 8)$
$= 35 - 24 = 11$ ⟵ divisible by 11
Thus 57, 729, 364, 583 is divisible by 11.

d. False, for example 12 divisible by 2 and 4 but 12 is not divisible by $\longrightarrow 8$

e. True

10. If $x = 14 \times 22 \times 39$, which of the following is NOT an integer?

A. $x \div 21$ C. $x \div 26$

B. $x \div 24$ D. $x \div 77$

Solution: The easiest way to answer is break x into prime factor.

Thus $x = 2 \times 7 \times 2 \times 11 \times 3 \times 13$

A. $\frac{x}{21} = \frac{2\times7\times2\times11\times3\times13}{3\times7} = 2\times2\times11\times13$

B. $\frac{x}{24} = \frac{2\times7\times2\times11\times3\times13}{2\times2\times2\times3} = \frac{7\times11\times13}{2}$

C. $\frac{x}{26} = \frac{2\times7\times2\times11\times3\times13}{2\times13} = 7\times2\times11\times3$

D. $\frac{x}{77} = \frac{2\times7\times2\times11\times3\times13}{7\times11} = 2\times2\times3\times13.$

Answer: B

1.3 DIVISION ALGORITHM

If x and y are positive integers, when y is divided by x, there exist unique integers q and r, called the quotient and remainder respectively, such that $\mathbf{y = xq + r}$ and $0 \le r < x$

Illustrative Example

11. Suppose $P = 173 \times 34 + 40$. **Find** the remainder when

a. P is divided by 173.

b. P is divided by 34.

c. P is divided by 17.

Solution: $P = 173 \times 34 + 40$

a. $\frac{P}{173} = \frac{173\times34+40}{173} = 34 + \frac{40}{173}$

$\therefore$ **Remainder r = 40.**

b. $\frac{P}{34} = \frac{173\times34+40}{34} = 173 + \frac{40}{34}$

$= 173 + \frac{34}{34} + \frac{6}{34} = 173 + 1 + \frac{6}{34} = 174 + \frac{6}{34}$

Thus $5y = 5(2k) = 10k$. In other words, $5x$ and $5y$ are divisible by 10

Answer: C

21. If p and q are primes greater than 2. Which of the following must be true?

I. $p + q$ is even II. pq is odd III. $p^2 - q^2$ is even.

A. I, II and III
B. I and II only
C. I and III only
D. I only

Solution:

All primes greater than 2 are odd, so p and q are odd

- $P + q =$ even, example $3 + 7 = 10$, even
- $P \cdot q =$ odd, example, $(3)(7) = 21$, odd.
- $P^2 - q^2 = (q - q)(p + q) =$ (even) (even) = even

Answer: A

1.4 NUMBER OF DIVISORS

How many divisors does 24 have?

The divisors of 24 are 1, 2, 3, 4, 6, 8, 12, 24

Thus 24 have 8 divisors.

If P is any prime and n is any natural number, then the divisors of $P^n = P^0 \cdot P^1 \cdot P^2 \cdot P^3 \ldots P^n$. Therefore there are $(n + 1)$ divisors of P^n.

If p and q are different primes then $P^n q^m$ will have $(n + 1)(m + 1)$ divisors.

Illustrative Example

22. Find the number of divisors of each of the following.

a. 36 b. 1000 c. 6^5

Solution

a. prime factor of $36 = 2 \times 2 \times 3 \times 3 = 2^2 \cdot 3^2$

Because 2^2 has $2+1 = 3$ divisors and 3^2 has $2+1 = 3$ divisors. Thus $36 = 2^2 \cdot 3^2$ has $(2+1)(2+1)$ or 9 divisors

b. $1000 = 10^3 = (2 \times 5)^3 = 2^3 \cdot 5^3$

Thus $1000 = 2^3 \cdot 5^3$ has $(3+1)(3+1)$ or 16 divisors

c. $6^5 = (2 \times 3)^5 = 2^5 \cdot 3^5$

Thus $65 = 2^5 \cdot 3^5$ has $(5+1)(5+1)$ or 36 divisors

Example 22 and 23 refer to the following **definition.**

For any positive integer **n**, $\tau(n)$ **represents the number of positive divisors of n.**

23. Which of the following is (are) true?

I $\tau(5) = \tau(7)$

$$\Rightarrow \frac{x}{y} = q + \frac{r}{y} = 63 + 0.04$$

$$\frac{x}{y} = 63 + \frac{6}{y}, \text{ thus } \frac{6}{y} = 0.04$$

$$\Rightarrow 6 = 0.04y, \text{ therfore, } y = \frac{6}{0.04} = 150$$

Answer: B

1.5 INTEGERS

The integers are $\{\ldots -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$
The positive integers are $\{1, 2, 3, 4, 5, \ldots\}$
The negative integers are $\{\ldots, -5, -4, -3, -2, -1\}$
Note: The integer 0 is neither positive nor negative.

Consecutive integers are two or more integers, written in sequence, each of which is 1 more than the preceding integer.
$-3, -2, -1, 0, 1, n, n+1, n+2, n+3\ldots$

Consecutive integers can be represented by, $n, n+1, n+2, n+3 \ldots$ where n is an integer.

Consecutive even integers Can be represented by $2n, 2n+2, 2n+4 \ldots$ where n is an integer

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Consecutive odd integers can be represented by $2n+1, 2n+3, 2n+5, \ldots$ where n is an integer.

HOW TO COUNT CONSECUTIVE NUMBERS.
- The number of integers from A to B inclusive is $B - A + 1$.

Example: How many integers are there from 83 through 429 inclusive
Set up: $429 - 83 + 1 = 347$.

A. 3 B. 4 C. 5 D. 6

Solution: Let the two digit integer be represented by

$M = 10t + u$ and when reversed $N = 10u + t$

$\Rightarrow N - M = (10u + t) - (10t + u) = 27 \Rightarrow 9u - 9t = 27$

$\Leftrightarrow 9(u - t) = 27$

$\therefore u - t = \frac{27}{9} = 3,$ **Answer: A**

37. If the sum of five consecutive odd integers is 735, what is the largest of these integers?

A. 155 B. 151 C. 145 D. 143

Solution:

The set of odd integers are described by {n/2n + 1, where n is integer}.

- Sum of five consecutive odd integers will be:

$(2n + 1) + (2n + 3) + (2n + 5) + (2n + 7) + (2n + 9) = 735$

$\Rightarrow 10n + 25 = 735$

$\Rightarrow 10n = 735 - 25 = 710$

$\Rightarrow n = \frac{710}{10} = 71$

Thus, largest of these: $2n + 9 = 2(71) + 9 = 151$ **Answer: B**

1.6 LEAST COMMON MULTIPLE (LCM) OF INTEGERS

The ***least common multiple*** **(LCM)** of two or more integers is the smallest positive integer that is a multiple of each of them.

Example: The LCM of 3 and 4 is 12

- Multiple of 3 = 3, 6, 9, 12, 15, … 3n
- Multiple of 4 = 4, 8, 12, 16, 20, … 4n.

Infinitely many positive integers are multiples of both 3 and 4 including 12, 24, 36, 48, … but 12 is the smallest one.

There are several methods for finding least common multiples.

a. **The intersection of set method,** we first find the set of all positive multiple of both numbers

Example: To find the LCM of 8 and 12 denote by M_8 and M_{12}

$M_8 = \{8, 16, 24, 32, \ldots\}$

$M_{12} = \{12, 24, 36, 48, \ldots\}$

The set of common multiples is $M_8 \cap M_{12} = \{24, 48, 72, \ldots\}$

Because the least number in $M_8 \cap M_{12}$ is 24, the LCM of 8 and 12 is 24 written LCM (8, 12) = 24.

b. The prime factorization method.

To find the LCM:

- List prime factorization of each number
- Multiply these primes, each raised to the highest power of the prime that occurs either of the prime factorization.

1.7 GREATEST COMMON FACTOR (GCF) (OR GREATEST COMMON DIVISOR)

The greatest common factor (divisor) of two or more integers is the largest integer that is a factor of each of them (or divides each of them).

Example: The greatest common divisor of 24 and 30 is 6.

$F_{24} = \{1, 2, 3, 4, 6, 8, 12, 24\}$

$F_{30} = \{1, 2, 3, 5, 6, 10, 15, 30\}$

$F_{24} \cap F_{30} = \{1, 2, 3, 6\}$

Because the greatest number in the set of common positive divisor is 6

$\therefore$ GCF (24, 30) = 6

Method of finding GCF.

- **The prime factorization method.**

To final GCF of two or more positive integers, first find the prime factorizations of the given numbers and then identify each common prime factor of the given numbers.

The GCD is the product the common factors, each raised to the lowest power of each common prime factor

Illustrative Example

38. Find the GCD and the LCM for each of the following using the prime factorization.

a. 40 and 60 b. 30 and 45

Solution:

a. $40 = 2 \times 2 \times 2 \times 5 = 2^3 \times 5$

$60 = 2 \times 2 \times 5 \times 5 = 2^2 \times 5^2$

$\therefore$ LCM $(40, 60) = 2^3 \times 5^2 = 200$

$\therefore$ GCD $(40, 60) = 2^2 \times 5 = 20.$

b. $30 = 2 \times 3 \times 5 = 2^1 \times 3^1 \times 5^1$

$45 = 3 \times 3 \times 5 = 3^2 \times 5^1$

$\therefore$ LCM $(30, 45) = 2^1 \times 3^2 \times 5^1 = 90$

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$\therefore$ GCD (30, 45) = $3^1 \times 5^1 = 15$

Extreme fact

The product of the GCF and LCM of two numbers is equal to the product of the two numbers.

- That is GCD **(a, b)** × LCM **(a, b) = ab.**

$$\text{GCD (a,b)} = \frac{ab}{\text{LCM (a,b)}}$$

Illustrative Example

39. If the LCM (a, b) = 144 and a × b = 1728, then what is the GCD (a, b)?

Solution: use the extreme fact.

$$\text{GCD (a,b)} = \frac{ab}{\text{LCM (a,b)}} = \frac{1728}{144} = 12.$$

40. What is the smallest number that is divisible by

a. both 34 and 35

b. both 36 and 48.

Solution: We are being asked for the LCM of the given number.

a. $\text{LCM(34,35)} = \frac{34 \times 35}{\text{GCF (34,35)}} = \frac{1190}{1} = 1190$

$\therefore$ 1,190 is the smallest number that divides evenly into both 34and 35

b. $\text{LCM (36,48)} = \frac{36 \times 48}{\text{GCF (36,48)}} = \frac{36 \times 48}{12} = 144$

$\therefore$ 144 is the smallest number that is divisible by both 36 and 48.

41. Suppose $x = 2^5 \times 7^2 \times 11$

$y = 2^2 \times 7^4 \times 13$

$z = 2^3 \times 7 \times 17$

a. What is the GCD (x, y, z)?

b. What is the LCM (x, y, z)?

Solution:

a. GCD $(x, y, z) = 2^2 \times 7 = 28$

b. LCM $(x, y, z) = 2^5 \times 7^4 \times 11 \times 13 \times 17.$

1.8 EXPONENTS

Repeated multiplication of the same number is indicated by an exponent:

Example:

a. $5 \times 5 \times 5 = 5^3$

b. $3 \times 3 \times 3 \times 3 = 3^4$

c. $a \times a \times a \times \ldots \times a = a^n$, where **a** is used as a factor n – times.

Exponent Fact

For any numbers **b** and **c** positive integers m and n:

i. $b^m b^n = b^{m+n}$

ii. $\frac{b^m}{b^n} = b^{m-n}$

iii. $(b^m)^n = b^{mn}$

iv. $b^m c^m = (bc)^m$

Illustrative Example

42. If $3^x \times 3^y = 3^{100}$, what is the arithmetic mean of x and y.

Solution:

Since $3^x \times 3^y = 3^{x+y}$, we see that $x + y = 100$

$\Rightarrow$ Arithmetic means of x and y is

$$\frac{x+y}{2} = \frac{100}{2} = 50$$

43. If $50^{100} = k(100^{50})$, what is the value of k

A. 2^{50} B. $\left(\frac{1}{2}\right)^{50}$ C. 25^{50} D. 50^{50}

Solution:

$$50^{100} = k(100^{50}) \Rightarrow (50^{50})(50^{50}) = k(2^{50})(50^{50})$$

$$\Rightarrow k = \frac{50^{50}}{2^{50}} = \left(\frac{50}{2}\right)^{50} = 25^{50}$$

Answer: C

44. If $\frac{0.0015 \times 10^m}{0.03 \times 10^k} = 5 \times 10^7$, then $m - k =$

A. 9 B. 8 C. 6 D. 5

Solution:

$$\Rightarrow \frac{0.0015 \times 10^m}{0.03 \times 10^k} = \frac{15 \times 10^{-4} \times 10^m}{3 \times 10^{-2} \times 10^k} = \frac{15 \times 10^{m-4}}{3 \times 10^{k-2}} = 5 \times 10^7$$

$\Rightarrow 5 \times 10^{m-4-k+2} = 5 \times 10^7$

$\Rightarrow 10^{m-k-2} = 10^7$

$\Rightarrow m - k - 2 = 7$

$m - k = 7 + 2 = 9$

Answer: A

45. The value of $\frac{2^{-11} + 2^{-12} + 2^{-13} + 2^{-14}}{5}$ is how many times the value of $x\ 2^{-14}$.

A. 3 B. 4 C. $\frac{3}{2}$ D. $\frac{5}{2}$

Solution: The value of $\frac{2^{-11} + 2^{-12} + 2^{-13} + 2^{-14}}{5}$ is x times the value of 2^{-14}, then

$\Rightarrow (x)(2^{-14}) = 2^{-14}\left(\frac{2^3 + 2^2 + 2^1 + 2^0}{5}\right) \Rightarrow x = \frac{8+4+2+1}{5} = \frac{15}{5} =$

Answer: A

46. If $3^x = 81$, then $(2^{x+1})(5^{x-3})$

A. 50 B. 160 C. 40 D. 80

Solution:

$3^x = 81 \Rightarrow 3^x = 3^4 \Leftrightarrow x = 4$

Then $(2^{x+1})(5^{x-3}) = (2^{4+1})(5^{4-3}) = (2^5)(5) = (32)(5) = 160$

Answer: B

47. If $m = 2^{n-1}$ and $3^{2n-3} = 27$, what is the value of $\frac{m}{n}$?

A. $\frac{4}{3}$ B. $\frac{3}{4}$ C. 4 D. 3

Solution: $3^{2n-3} = 3^3 \Leftrightarrow 2n - 3 = 3$

$\Rightarrow 2n = 6$, therefore, $n = 3$

Then $\frac{m}{n} = \frac{2^{3-1}}{3} = \frac{2^2}{3} = \frac{4}{3}$

Answer: A

> **Square Root of A number**
> If $\mathbf{b}^2 = \mathbf{a}$, then **b** is a square root of **a**, denoted by $b = \sqrt{a}$.

Solution: $5x + 13 = 31 \Rightarrow 5x = 31 - 13 = 18$

$\Rightarrow 5x + 31 = 18 + 31 = 49 \longleftarrow$ Add 31 to both side

Therefore $\sqrt{5x+31} = \sqrt{49} = 7$ **Answer: C**

1.9 THE ARITHMETIC OF INEQUALITIES

For any number **a** and **b** exactly one of the following is true;

$a > b$ or $a = b$ or $a < b$.

i) **If a < b, then a + c < b + c and a – c < b – c**

Example: Let c = 100

$5 < 8 \Rightarrow 5 + 100 < 8 + 100 \Rightarrow 105 < 108$

$5 < 8 \Rightarrow 5 - 100 < 8 - 100 \Rightarrow -95 < -92$

ii) **If a < b, and c < d then a + c < b + c and a – c < b – c**

Example: Let C = 100

$5 < 8$ and $7 < 10 \Rightarrow 5 + 7 < 8 + 10 \ldots (12 < 18)$

iii) **Multiplying or dividing an inequality by a positive number preserves it.**

If a < b, and c is positive, then ac < bc and $\frac{a}{c} < \frac{b}{c}$.

Example, let c = 100

$5 < 8 \Rightarrow 5 \times 100 < 8 \times 100 \ldots (500 < 800)$

$$5 < 8 \Rightarrow \frac{5}{100} < \frac{8}{100}$$

iv) **Multiplying or dividing an inequality by a negative number reverses it.**

If a < b and c is negative, then ac > bc and $\frac{a}{c} > \frac{b}{c}$

Example: Let c = –100

$5 < 8 \Rightarrow 5 \times (-100) > 8 \times (-100) \Rightarrow -500 > -800$

$$5 < 8 \Rightarrow \frac{5}{-100} > \frac{8}{-100} \Rightarrow -0.05 > -0.08$$

it means by -1

v) **Taking negative reverses an inequality.**

If a < b, then –a > –b and if a > b, –a < – b.

Example: $5 < 8 \Rightarrow -5 > -8$ and $8 > 5 \Rightarrow -8 < -5$

vi) **If a and b are both positives or both negatives and $a < b$ then $\frac{1}{a} > \frac{1}{b}$.**

Example: $5 < 8 \Rightarrow \frac{1}{5} > \frac{1}{8}$

$-8 < -5 \Rightarrow -\frac{1}{8} > -\frac{1}{5}$

Extreme fact: Important inequalities for Numbers Between 0 and 1.

A. **If $0 < x < 1$,** and a is positive, then $(x)(a) < a$

Example: $0.8 \times 4 < 4 \Rightarrow 3.2 < 4$

B. **If $0 < x < 1$,** and m and n are integers with $m > n > 1$ then $x^m < x^n < x$

Example, let $m = 6$, $n = 4$, $x = \frac{1}{2}$

$\Rightarrow \left(\frac{1}{2}\right)^6 < \left(\frac{1}{2}\right)^4 < \frac{1}{2}$.

C. **If $0 < x < 1$,** then $\sqrt{x} > x$ and $x^2 < x$

Example a. let $x = \frac{1}{4}$, then $\sqrt{\frac{1}{4}} > \frac{1}{4} \Rightarrow \frac{1}{2} > \frac{1}{4}$

b. let $x = \frac{4}{9}$, then $\sqrt{\frac{4}{9}} > \frac{4}{9} \Rightarrow \frac{2}{3} > \frac{4}{9}$

D. **If $0 < x < 1$,** then $\frac{1}{x} > x$. In fact $\frac{1}{x} > 1$

Example, let $x = 0.4$

$\Rightarrow \frac{1}{0.4} > 0.4 \Rightarrow \frac{10}{4} > \frac{4}{10}$

Illustrative Example

52. If $0 < a < b < 1$, which of the following is (are) true.

I $a - b$ is negative II $\frac{1}{ab}$ is positive III $\frac{1}{b} - \frac{1}{a}$ is positive

A. I and II only
B. II only
C. I, II, and III
D. III only

Solution:

$$r^{\frac{1}{2}} > r > r^2 \Rightarrow r^2 < r < r^{\frac{1}{2}} \Rightarrow s < r < t$$

Answer: B

60. If **m** is number between 0 and 1, which of the following is NOT more than **m**?

A. m^2 B. $\frac{1}{m}$ C. 2m D. $\sqrt{m}$

Solution: Not more than m means **"less than m"**

- If $0 < m < 1$, then $m^2 < m$, $\frac{1}{m} > m$, and $\sqrt{m} > m$
- If $0 < m < 1$, and $a > 1$, then $ma > m$

Answer: A

Quantitative Comparison

Quantitative comparison question can be treated as an equation or inequality.

Either: Quantity A is greater **all the time no matter what;**

Quantity B is greater **all the time no matter what;**

Quantity A = Quantity B, **all the time**

Quantitative Comparison Problem

Illustrative Example

61. Compare the following two quantities.

Quantity A	Quantity B
$11 \times \sqrt{2}$	$9 \times \sqrt{3}$

A. The two quantities are equal.

B. Quantity A is greater than quantity B.

C. Quantity B is greater than A.

D. The two quantities cannot be compared.

Solution:

Quantity: $A = 11 \times \sqrt{2} = \sqrt{121} \times \sqrt{2} = \sqrt{242}$

Quantity: $B = 9 \times \sqrt{3} = \sqrt{81} \times \sqrt{3} = \sqrt{243}$

$\Rightarrow B > A$

Answer: C

When n is divided by 11, there is quotient, q and a reminder, r such that $n = 11q + r$,

$\Rightarrow 100n = 1100q + 100r = 1100q + 99r + r = 11(100q + qr) + r$

Therefore the reminder is the same in both quantity.

∴ The two quantity are equal

67. Consider the following two quantities.

Quantity A	Quantity B
$3^{48} + 3^{48} + 3^{48} + 3^{48}$	3^{49}

Which quantity is greater?

Solution:

Quantity A $= 3^{48} + 3^{48} + 3^{48} + 3^{48} = 3^{48}(1 + 1 + 1 + 1) = 4 \times 3^{48}$

Quantity B $= 3^{49} = 3^{1} \times 3 = 3 \times 3^{48}$

∴ Quantity A is greater.

68. Suppose x and y are positive integers and $x > y$.

Quantity A	Quantity B
$\frac{x}{y}$	$\frac{x+1}{y+1}$

Which quantity is greater?

Solution: Let $x = 3$, $y = 2$

$$\frac{3}{2} > \frac{3+1}{2+1} \Rightarrow \frac{3}{2} > \frac{4}{3}$$

∴ Quantity A is greater.

Important - Fact

- (Divisor × Quotient) + Remainder = Dividend
- A number (DIVDEND) can be made completely divisible with the help of either of the following rules:

Rule I By *subtracting remainder from dividend.*

Rule II By *adding (divisor – remainder) to the dividend*

- **Remainder Rule.** This rule is applicable when the same number (dividend) is divided by two different divisors which are multiples of each other.

Suppose, the smaller divisor $= x$, then the larger divisor $= kx$, where K is integer greater than 1.

Now, when the number is divided by x the remainder r (say)
But, when the same number is divided by kx, remainder = R (say),
then by the remainder rule $\boxed{2x+r=R}$

Conceptual Example

69. Find the least number, that must be subtracted from 4,785, to get a number exactly divisible by 354

A. 183 B. 181 C. 201 D. 366

Solution:

- The least number to be subtracted is the remainder from dividend.

✓ On dividing 4,785 by 354, the remainder is 183.

Answer: A

70. What least number must be added to 38243 to get a number exactly divisible by 261

A. 137 B. 237 C. 124 D. 24

Solution:

- On dividing 38,243 by 261, the remainder is 137. By rule II: the least number to be added to the dividend = divisor – remainder = 261 – 137 = 124

∴ The least number to be added = 124 **Answer: C**

71. Find the greatest number of 3 digits, which is exactly divisible by 47.

A. 977 B. 987 C. 997 D. 957

Solution:

- The greatest number of 3 digit = 999

✓ On dividing 999 by 47, remainder is 12. Now by applying rule I. the required number = dividend – remainder = 999 – 12 = 987

Answer: B

72. Find the least number of 4- digits, which is exactly divisible by 29.

A. 986 B. 1014 C. 1015 d. 1029

Solution:

- The least number of 4 – digit = 1000.
- On dividing 1000 by 29, remainder = 14

∴ By rule II, the required number = dividend + (divisor – remainder).
= 1000 + (29 – 14) = 1015

Answer: C

UNIT – TWO

2. FRACTIONS

A fraction consists of two parts: a numerator and a denominator.

In a fraction $\frac{a}{b}$;

a is the numerator and

b is the denominator.

- If the numerator is less than the denominator, the fraction is called **proper** fraction.

Example: $\frac{1}{2}, \frac{5}{24}, \frac{3}{7}$... etc are proper fraction.

- If the numerator is more than denominator, the fraction is called an improper fraction.

Example: $\frac{4}{3}, \frac{2}{1}, \frac{5}{1}, \frac{7}{5}$,... are improper fraction.

2.1 Equivalent fraction

Two fractions are equivalent if multiplying or dividing both the numerator and denominator of the first fraction by the same number gives the same second fraction.

Example:

a. $\frac{1}{2}$ and $\frac{4}{8}$ are equivalent fraction because $\frac{1}{2} = \frac{1\times4}{2\times4} = \frac{4}{8}$

b. $\frac{3}{8}$ and $\frac{15}{40}$ are equivalent fraction because $\frac{3}{8} = \frac{3\times5}{8\times5} = \frac{15}{40}$.

Comparing fractions.

To compare two fractions, cross multiply. The larger number will be on the same side as the larger fraction.

Example: Which quantity is greater

Quantity A	Quantity B
$\frac{7}{8}$	$\frac{8}{9}$

Solution: Cross multiplying gives

7×9 Versus 8×8, which gives

63 Versus 64

Hence $\frac{8}{9}$ is greater than $\frac{7}{8}$ $\therefore$ **Answer: Quantity B.**

- Always reduce a fraction to its lowerst terms.

Example: Which quantity is greater, smaller or equal

Quantity A	Quantity B
$\frac{2x^2+12x+18}{(x+3)^2}, x \neq -3$	2

Solution:

$$\text{Quantity A} = \frac{2(x^2+6x+9)}{(x+3)^2} = \frac{2(x+3)^2}{(x+3)^2} = 2$$

Quantity B = 2

Hence the two quantities are equal.

Illustrative Example

1. Which of the following is NOT equivalent to $\frac{15}{24}$

A. $\frac{45}{72}$ B. $\frac{75}{120}$ C. $\frac{195}{312}$ D. $\frac{3}{8}$

Solution: Short cut method.

To determine whether two fractions are equivalent convert them to decimal by dividing. For the fraction to be equivalent the two quotients must be the same.

UNIT – THREE

3. PERCENT'S, RATIO AND PROPORTION

3.1 PERCENTS

Percent means *per hundred* or number out of 100.

- A percent can be represented as a fraction with a denominator of 100, or as a decimal.

Example: $56\% = \frac{56}{100} = 0.56$

Common fractional equivalents of percents and decimal

Percent	Fraction	Decimal
50%	$\frac{50}{100} = \frac{1}{2}$	0.5
25%	$\frac{25}{100} = \frac{1}{4}$	0.25
75%	$\frac{75}{100} = \frac{3}{4}$	0.75
20%	$\frac{20}{100} = \frac{1}{5}$	0.2

Solving percent Problems

Percent problems often require you to translate a sentence into a mathematical equation.

Example: (a) What is 40% of 60.

(b) 24 is 40% of what number?

(c) 24 is what percent of 60?

Translate the sentence into a mathematical equation as follows:

Let x = the unknown number.

(a) What is 40% of 60.

$$x = 40\% \text{ of } 60 = \frac{40}{100} \times 60 = 24$$

(b) 24 is 40% what number

$$24 = 40\% \text{ of } x \Rightarrow \frac{40}{100}x = 24$$

$$\therefore x = \frac{100}{40}(24) = 60$$

(c) 24 is what percent of 60?

$$24 = (x\%) \text{ of } 60 \Rightarrow 24 = \frac{x}{100}(60)$$

$$x = (24)\left(\frac{100}{60}\right) = 40$$

Illustrative Example

1. What percent of 30 is 6?
A. 10% B. 20% C. 30% D. 35%

Solution:

$$x\% \text{ of } 30 = 6 \Rightarrow \left(\frac{x}{100}\right)(30) = 6$$

$$x = \left(\frac{10}{3}\right)(6) = 20$$

Answer: B

2. 7 is 28% of what number
A. 20 B. 21 C. 25 D. 35

Solution:

$$7 = 28\% \text{ of } x \Rightarrow \frac{28}{100} \times x = 7$$

$$\therefore x = \frac{100 \times 7}{28} = 25$$

Answer: C

3. If 15% of a number is 4.5, then 45% of the same number is
A. 1.5 B. 3.5 C. 13.5 D. 15

Solution:

$$15\% \text{ of } x = 4.5 \Rightarrow \frac{15}{100}x = 4.5$$

$$\Rightarrow x = \frac{4.5}{15}(100) = 30, \text{ then } 45\% \text{ of } 30 = \frac{45}{100}(30) = 13.5,$$

Answer: C

3.2 Percent Increase and Decrease

Often you will need to find the percent of increase (or decrease). To find it, calculate the increase (or decrease) and divide it by the original amount:

$$\text{Percent of change} = \frac{\text{Amount of change}}{\text{original amount}} \times 100\%$$

The percent increase of quantity is $\frac{\text{actual increase}}{\text{orignial amount}} \times 100\%$

The percent decrease of quantity is $\frac{\text{actual decrease}}{\text{orginal amount}} \times 100\%$

Illustrative Example

14. A price decreased from birr 60 to birr 45. What was the percent decrease in the price?

A. 15% B. $33\frac{1}{3}\%$ C. 25% D. 40%

Solution:

$$\text{Percent decrease} = \frac{\text{actual decrease}}{\text{original amount}}$$

$$= \frac{(60-45)}{60} = \frac{15}{60} \times 100\% = 25\%$$ **Answer: C**

15. A price increased from birr 50 to birr 80. What was the percent increase in the price?

A. 60% B. 30% C. 50% D. 40%

Solution:

$$\text{Percent increase} = \frac{\text{actual increase}}{\text{original amount}} \times 100\%$$

$$= \frac{80-50}{50} \times 100\% = \frac{30}{50} \times 100\% = 60\%$$

Answer: A

16. What is 40% more than 40?

A. 44 B. 48 C. 56 D. 16

Solution: 40 + 40% of 40

A. 30,000 birr C. 35,000 birr
B. 31,250 birr D. 32,500 birr

Solution:

$$\text{Percent increase} = \frac{25,000-20,000}{20,000}\times 100\% = 25\%$$

Her raise next year will be 25% × 25,000 birr = 6,250 birr.
Her salary next year will be 25,000 + 6,250 = 31,250 birr

3.3 RATIO

A **ratio** is a comparison of two numbers or quantities by division.
Ratio can be expressed in the following ways:

a to b **a:b** $\frac{a}{b}$

Ratio must expressed in the same units.

Example: What is the ratio of 2m to 400cm?

Solution: The units must be the same, so change meter to centimeter 2m = 200cm.

$$\text{The ratio is } \frac{200cm}{400cm} = \frac{2}{4} = \frac{1}{2}$$

Note: If a set of objects is divided into two groups in the ratio of **a:b**, then:

a/b → a/(a+b) , b/(a+b)

i) First group contains: $\frac{a}{a+b}$ of the object.

ii) Second group contains $\frac{b}{a+b}$ of the object.

Illustrative Example

43. Last year, the ratio of the number of mathematics tests Daniel passed to the number of mathematics test he failed was 3: 2. What percent of his mathematics tests did Daniel pass?

A. 150% B. $66\frac{2}{3}\%$ C. 60% D. 40%

Solution:

$$\text{Daniel pass } \frac{3}{3+2} = \frac{3}{5} = 60\% \text{ of his mathematics tests.}$$

Answer: C

3.4 PROPORTION

A proportion is an equation that states the two ratios are equivalent.
Example: The ratio of x to y is equal to the ratio of 5 to 4 is translated as $\frac{x}{y} = \frac{5}{4}$.

Example: What is the fourth proportion to 3, 4, 5?

Set up: $\frac{3}{4} = \frac{5}{x} \Rightarrow 3x = 20 \Rightarrow x = \frac{20}{3}$

$\therefore$ The 4^{th} proportion is $x = \frac{20}{3}$.

3.5 DIRECT AND INVERSE PROPORTIONALITY

Direct Proportionality

As one quantity increase (decreases) another quantity also increases (decreases).

This type of problem can be solved by setting up a direct proportion.

Two variables are direct proportional if one is a constant multiple of the other.

$\frac{y}{x} = k \Rightarrow y = kx$ where k is a constant

Illustrative Example

51. A machine makes 18 articles in 5 hours. How many articles will it make in 15 hours?
A. 45 B. 54 C. 6 D. 9

Solution: Use direct proportionality $\frac{18}{5} = \frac{x}{15} \Rightarrow x = 15\left(\frac{18}{5}\right) = 54$

Answer: B

52. A certain machine fills a bag with 7 kg of a potato chips in 3.5 seconds. At this rate, how many seconds will it take the machine to fill a bag with 15 kg of potato chips? (UEE 2005)
A. 6.5 B. 7.0 C. 7.5 D. 8

Solution:

$\frac{7\text{ kg}}{3.5\text{ second}} = \frac{15\text{ kg}}{x} \Rightarrow x = \left(\frac{3.5}{7}\right)(15) = 7.5$ **Answer: C**

53. If **a** apples cost **c** cents, how many apples can be bought for **d** birr?

A. $\frac{100d}{ac}$ B. $\frac{ad}{100}$ C. $\frac{c}{100ad}$ D. $\frac{100ad}{c}$

Solution: $\frac{\text{apple}}{\text{cents}} = \frac{a}{c} = \frac{x}{100\,d} \Rightarrow x = \frac{100\,ad}{c}$ **Answer: D**

54. A car travels 240 km on 15 liters of petrol. How much petrol will be needed for a journey of 408 km?

A. 17ℓt B. 34ℓt C. 28.9ℓt D. 32ℓt

Solution:

Set up a proportion $\frac{240\text{ km}}{15\ell t} = \frac{408\text{km}}{y}$

$\Rightarrow y = \frac{15}{240}(408) = 28.9\ell t$ **Answer: C**

55. At a certain school, 45% of the students purchased a year book. If 540 students purchased year books, how many students did not busy a year book?

A. 243 B. 540 C. 575 D. 660

Solution:

Set up a proportion

Since 45% bought a year book, 55% did not.

$\Rightarrow \frac{45\%}{55\%} = \frac{540}{x} \Rightarrow x = 540\left(\frac{55}{45}\right) = 660$ **Answer: D**

Inverse Proportionality

If an increase in one quantity produce a decrease in a second quantity or if a decrease in one quantity produce an increase in a second quantity, the two quantities are in **inverse proportional.**

The statement "y is inversely proportional to x" is written as $y = \frac{k}{x}$, where k is a constant $xy = k$.

UNIT – FOUR

4. AVERAGE

How to find the Average

- $\text{Average} = \dfrac{\text{sum of terms}}{\text{number of terms}}$
- Sum of terms = (Average) (number of terms)

How to find combined average.

If the average of **n** item is A and the average of **m** item is B then the average of **n + m** item is equal to $\dfrac{\mathbf{nA + mB}}{\mathbf{n + m}}$

Illustrative Example

1. If the average of 38, 41, 44, 46, and x is 40, what is x

A. 28 B. 30 C. 31 D. 34

Solution: Using the general idea of average:

$$\frac{38+41+44+46+x}{5} = 40$$

$$\Rightarrow 169 + x = 5x40 \Rightarrow x = 200 - 169 = 31$$ **Answer: C**

2. For a certain student, the average of ten test scores is 70. If the high and low scores are dropped, the average is 74. What is the average of the high and low scores?

A. 72 B. 73 C. 54 D. 58

Solution:

- The sum of the ten scores is $70 \times 10 = 700$
- The sum of the eight scores after the two scores have been dropped is $8 \times 74 = 592$. So the two scores that were dropped total $700 - 592 = 108$. Then the average of the two dropped is $\dfrac{108}{2} = 54$. **Answer: C**

3. In a certain shipment, the average weight of six packages is 750 kg. If another package is added to the shipment, the average weight of seven packages is 764 kg. What is the weight of the additional package?

Solution:

Set up: Weighted Avg $= \frac{1 \times 60 + 2 \times 57}{3} = \frac{174}{3} = 58$

Answer: B

4.1 Average Speed

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time}}$$

Illustrative Example

8. For the first 3 hours of her trip, Heran drove at 50km per hour. Then, because of construction delays, she drove at only 40 km per hour for the next 2 hours.
What was her average speed, in kilometer per hour, for the entire trip?

Solution:

$$\text{Average speed} = \frac{(3)(50)+2(40)}{5} = \frac{230}{5} = 46 \text{ km/h}$$

9. Roza drove 120 km one way at an average speed of 40 km/hr and returned by the same 120 km route at an average speed of 60 km/hr. what was Roza's average speed for the entire 240 km round trip?

A. 48 km/hr C. 50 km/hr
B. 100 km/ hr D. 44 km/hr

Solution:

Set up: To drive 120 km at 40 km/hr takes 3 hours. To return at 60km/hr takes 2 hours. The total time, then is 5 hours.

$$\text{Average speed} = \frac{240 \text{ km}}{5 \text{ hours}} = 48 \text{ km/hr}$$

Answer: A

How to find the AVERAGE of CONSECUTIVE NUMBERS.

- The average of consecutive numbers is **the average of the smallest number and the largest number.**

4.2 DATA INTERPRETATION

Occasionally a question or set of questions will be based on data provided in a table or graph. Some examples of tables and graphs are given below.

Illustrative Example

18. Refer to the following table.

Population by Age Group (in thousands)	
Age	Population
15 years and under	52, 464
16 – 40 years	74, 593
41 – 60 years	38, 720
60 years and over	21, 084

How many people are 40 years old or younger?

A. 74,593,000 C. 127,057,000
B. 59,804,000 D. 134,397,000

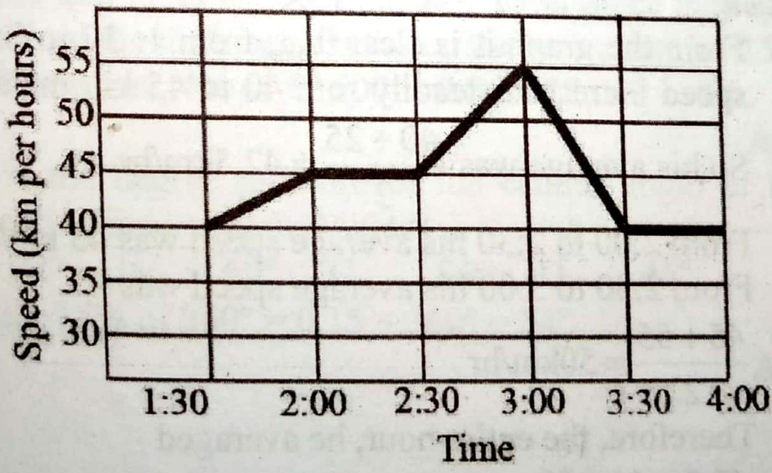
Solution: The figures in the table are given in thousands. Therefore, 52,464 + 74,593 = 127,057 thousand

⇒The result is 127,057 thousand, which is 127,057,000.

Answer: C

From question 19 – 20 refer to the following graph.

Speeds at which Robel Drove on Saturday Morning



19. For what percent of the time was Robel diriving at 45 km/hr or faster?

A. 40% B. 50% C. 25% D. 20%

UNIT – FIVE

5. WORD PROBLEMS

Translating words into Mathematical Symbols.

Before we begin solving word problems we need to be very comfortable with translating words into Mathematical symbols.

Following is a partial list of words and their mathematical equivalents.

English words	Mathematical Meaning
Is, was, will be, had, has, will have, is equal to, is the same as	Equals (=)
Plus, more than, sum, increased by, added to, exceeds,	Addition (+)
Fewer, less than, difference, decreased by, younger than, gave, lost.	Subtraction (–)
Times, of, product, twice, double, half	Multiplication (×)
Divided by, ratio quotient, per, for	Division (÷)
More than, greater than	>
At least	≥
Fewer than, less than	<
At most	≤

5.1 Age problems

In problems involving ages, remember that "years ago" means you need to subtract. "years from now" means you need to add.

Illustrative Example

1. Abebe is 20 years older than Towfik In 10 years, Towfik's age will be half that of Abebe's. What is Towfik's age?
A. 10 years B. 8 years C. 25 years D. 5 years

Solution: Let x = Towfik's age, and then $x + 20$ is Abebe's age.
Ten years from now, Towfik's age will be $x + 10$ and Abebe's age will be $x + 30$
Summarizing this information in a table:

$$\Rightarrow B = \frac{A+S}{2} = \frac{(B+2)+32}{2}$$

$$\Rightarrow 2B = B + 34 \Rightarrow B = 34 \text{ years}$$

Answer: D

7. The ages of three people are such that the age of one person is twice the age of the second person and three times the age of the third person. If the sum of the ages of the three people is 33, then the age of the youngest person is

A. 18 B. 6 C. 9 D. 3

Solution: Let x = the age of the oldest, y = the age of the second person, and z = the age of the youngest person.

$$\Rightarrow x = 2y \text{ and } x = 3z \Leftrightarrow y = \frac{x}{2}, z = \frac{x}{3}$$

The sum of the ages of the three people is $x + y + z = 33$

$$\Rightarrow x + \frac{x}{2} + \frac{x}{3} = 33 \Leftrightarrow 6x + 3x + 2x = 198$$

$$\Rightarrow 11x = 198 \Rightarrow x = \frac{198}{11} = 18$$

$$\therefore y = \frac{x}{2} = \frac{18}{2} = 9 \text{ and } z = \frac{x}{3} = \frac{18}{3} = 6$$

$\therefore z = 6$

Answer: B

5.2 Distance - problems

All distance problems involve one of three variations of the same formula:

Distance = rate × time ⇒ d = rt

$$\Rightarrow \text{rate} = \frac{\text{distance}}{\text{time}} \Rightarrow r = \frac{d}{t}$$

$$\Rightarrow \text{time} = \frac{\text{distance}}{\text{rate}} \Rightarrow t = \frac{d}{r}$$

$$\Rightarrow R = \frac{20}{\frac{2}{3}} = 20 \times \frac{3}{2} = 30 \text{ mph,}$$

Answer: A

Motion in Opposite Directions.

11. Two students start jogging at the same point and time but in opposite directions. If the rate of one jogger is 2 mi/ hr faster than the other and after 3 hours they are 30 miles apart, what is the rate of the faster jogger?

A. 3 B. 4 C. 5 D. 6

Solution: Let r = the rate of slower jogger.

then r + 2 = the rate of faster jogger.

since they are jogging for 3 hour the distance traveled by the slower jogger is d = rt = 3r and distance traveled by faster jogger is 3 (r + 2) = 3r + 6 since they are 30 mile apart, the total distance traveled is the sum of each.

$$\Rightarrow 3r + 3(r + 2) = 30 \Rightarrow 6r = 24, \text{ hence } r = \frac{24}{6} = 4$$

$\therefore$ the rate of faster jogger is **r + 2 = 4 + 2 = 6.**

Answer: D

5.3 WORK PROBLEMS.

In work problem, the rates at which certain persons or machines work alone are usually given, and it is necessary to compute the rate at which they work together (or vice versa).

The basic formula for solving work problems is $\frac{1}{r} + \frac{1}{t} = \frac{1}{h}$, where **r** and **t** are, the number of hour it takes to complete a job when working alone, and **h** is the number of hours it takes to do the job when working together.

A. $3\frac{1}{3}$ hr B. 4 hr C. 5 hr D. $6\frac{1}{4}$ hr

Solution: Let x = number of hours press B would take working alone.

$$\frac{1}{2.5} = \frac{1}{10} + \frac{1}{x} \Rightarrow \frac{1}{x} = \frac{1}{2.5} - \frac{1}{10}$$

$$\Rightarrow \frac{1}{x} = \frac{10-2.5}{25} = \frac{7.5}{25}$$

$$\therefore x = \frac{25}{7.5}\text{hr} = 3\frac{1}{3}\text{hr}$$

Answer: A

15. A tank is being drained at constant. If it takes 3 hours to $\frac{6}{7}$ of its capacity, how much longer will it take to drain the tank completely

A. $\frac{1}{2}$ hr B. $\frac{3}{4}$ hr C. 1 hr D. $\frac{3}{2}$ hr

Solution: Rate $= \frac{w}{t} = \frac{\frac{6}{7}}{3} = \frac{6}{21} = \frac{2}{7}$

Now, since $\frac{6}{7}$ of the work has been completed, $\frac{1}{7}$ of the work remains.

$\Rightarrow$ W = R x t gives $\frac{1}{7} = \frac{2}{7} x\ t \Rightarrow t = \frac{1}{2}$hr.

Answer: A

5.4 Mixture Problems

In mixture problems, substances with different characteristics are combined, and it is necessary to determine the characteristics of the resulting mixture.

Illustrative Example

16. How many liters of a solution that is 15% salt must be added to 5 liters of a solution that is 8% salt so that the resulting solution is 10% salt?

A. 3 liters B. 2 liters C. 4 liters D. 5 liters

Solution: Let x = the number of liters of the 15% solution.

The amount of salt in the 15% solution $[0.15x]$ plus the amount of salt in the 8% solution $[(0.08)(5)]$ must be equal to the amount of salt in the 10% mixture $[0.10(x+5)]$. Therefore

$$0.15x + 0.08(5) = 0.10(x+5)$$

$$\Rightarrow 15x + 40 = 10x + 50$$

$$\Rightarrow 15x - 10x = 50 - 40 = 10$$

$$\therefore 5x = 10 \Leftrightarrow x = \frac{10}{5} = 2 \text{ liters.}$$

That is two liters of the 15% salt solution must be added to the 8% solution to obtain the 10% solution.

Answer: B

5.5 Interest Problems

A. Simple Interest Problems

Interest = Amount × Time × Rate

I = PRT, where rate (R) is in percent and the time (T) is given in year.

Illustrative Example

17. If 8000 birr is invested at 6% simple annual interest, how much interest is earned after 3 months?

Solution: P = 8000 birr, rate = 6% $T = \frac{3}{12} = \frac{1}{4}$.

$$I = PRT = (8000)(0.06)\left(\frac{3}{12}\right) = 120\text{birr.}$$

$$\Rightarrow A = 20{,}000\left(1+\frac{0.1}{2}\right)^{1(2)} = 20{,}000(1.05)^2 = 22{,}050 \text{ birr}$$

5.6 Miscellaneous collection of word problems.

21. Two students appeared at an examination. One of them secured 9 marks more than the other and his mark was 56% of the sum of their mark. The marks obtained by them are:

A. 39, 30 B. 43, 34 C. 42, 33 D. 41, 32

Solution: Let x an $x + 9$ was their mark.

$x + (x + 9) = 2x + 9$

$x + 9$ is 56% of $2x + 9$

$x + 9 = 0.56\ (2x + 9) = 1.12\,x + 5.04$

$1.12\,x - x = 9 - 5.04 = 3.96$

$$\therefore x = \frac{3.96}{0.12} = 33 \quad \text{and } 33 + 9 = 42$$

Answer: C

22. The product of 3 and 6 more than a certain number is 5 times that number. What is the number ?

A. 18 B. 9 C. 12 D. 45

Solution: Let x = the number

$\Rightarrow 3\ (6 + x) = 5x \Leftrightarrow 18 + 3x = 5x$

$\Rightarrow 2x = 18$

$\therefore x = 9$

Answer: B

23. In one month, Hana used $\frac{1}{6}$ of her monthly salary for a car payment and $\frac{1}{4}$ more than the car payment for rent. What fraction of her monthly salary did Hana use that month for the car payment and rent combined

A. $\frac{7}{12}$ B. $\frac{3}{8}$ C. $\frac{5}{12}$ D. $\frac{5}{24}$

$$\therefore 16+\frac{15}{4}(16)=16+60=76\text{kg}$$

Answer: B

26. In the afternoon, sara read 100 pages at the rate of 60 pages per hour, in the evening, when she was tired. She read another 100 pages at the rate of 40 pages per hour. In pages per hour, what was her average rate of reading for the day?

A. 52 B. 50 C. 45 D. 48

Solution:

- In the afternoon she read for $\frac{100}{60}=\frac{5}{3\text{ hr}}$
- In the evening, she read for $\frac{100}{40}=\frac{5}{2}$ hr.

$\Rightarrow$ Total time $=\frac{5}{3}\text{hr}+\frac{5}{2}\text{ hr}=\frac{25}{6}\text{hr.}$

$\therefore$ Average rate $=\frac{100+100}{\frac{25}{6}}=\frac{200\times 6}{25}=48$ pages per hour

Answer: D

5.7 DISCOUNT

If a price is discounted by x%, then the price becomes $(100-x)\%$ of the original price.

Illustrative Example

27. A certain customer paid 240 birr for a dress. If that price represented a 25% discount on the original price of the dress, what was the original price of the dress?

A. 180 birr B. 300 birr C. 320 birr D. 200 birr

Solution: Let x = the original price

$x-0.25x=240\Rightarrow 0.75x=240$

$\therefore x=\frac{240}{0.75}=320$ birr

Answer: C

28. The price of an item is discounted by 10% and then this reduced price is discounted by an additional 20%. These two discounts are equal to an overall discount of what percent?

A. 28% B. 72% C. 70% D. 30%

Solution: Let x = the original price of the item.

- Then after the first 10% discount will be $x - 10\% x = 0.9x$.
- The price after the 2nd 20% discount will be $0.9x - (0.9x)(0.2) = 0.72x$

$\therefore$ The overall discount is 100% −72% = 28%

Answer: A

5.8 PROFIT

Gross profit = (Revenue) – (Expense) or
= (Selling price) – (Cost price).

Illustrative Example

29. A certain appliance costs a marchant 600 birr. At what price should the marchant sell the appliance in order to make a gross profit of 40% of the cost of the appliance?

A. 360 birr B. 240 birr
C. 840 birr D. 960 birr

Solution: If x is the selling price of the appliance, then

$$x - 600 = (0.4)(600) \Rightarrow x = 600 + 240 = 840$$

Answer: C

6. COUNTING AND PROBABILITY

6.1 COUNTING PRINCIPLE

How to solve a group problem involving Neither/Both.

Some SAT problems involve two groups with overlapping elements, and possibly elements that belong to neither group. It is easy to identify this type of question because the words "both" and/ or "neither" appear in the question. These problems are quite easy if you just memorize the following formula:

$$\boxed{\text{Total} = \text{Group}_1 + \text{Group}_2 + \text{Neither} - \text{Both}}.$$

Illustrative Example

1. Of the 120 students at a certain school, 54 students are taking mathematics and 38 are taking chemistry and 43 are taking neither mathematics nor chemistry. How many are taking both mathematics and chemistry?

   A. 28 B. 15 C. 14 D. 30

   Solution: use, $\text{group}_1 + \text{group}_2 + \text{neither} - \text{both} = \text{Total}$

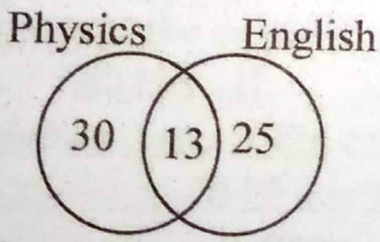
   $54 + 38 + 43 - \text{both} = 120$

   $135 - \text{Both} = 120 \Rightarrow \text{Both} = 135 - 120 = 15$ **Answer: B**

2. A school has a total enrollment of 90 students. There are 30 students taking physics, 25 taking English, and 13 taking both. What percentage of the students are taking either physics or English?

   A. 30% B. 58% C. 36% D. 47%

   Solution:



The number of students enrolled in either physics or English or both is $30 + 25 - 13 = 42$

$$\Rightarrow \frac{\text{Physics or English enrollment}}{\text{Total enrollment}} = \frac{42}{90} = 0.47$$

$\Rightarrow 47\%$ **Answer: D**

A. $\frac{5}{4}$ B. $\frac{4}{5}$ C. $\frac{3}{4}$ D. $\frac{3}{5}$

Solution:

She read $313 - 309 + 1 = 5$ pages

She read for 2: 14 – 2: 1 = 4 minutes

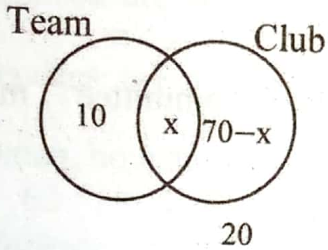
So she read at the rate of $\frac{5}{4}$ pages per minute.

Answer: A

11. In a group of 100 students, more students are on a team than are members of a club. If 70 are in clubs and 20 are neither on a team nor in a club, what is the minimum number of students who could be both on a team and in a club?

A. 60 B. 61 C. 80 D. 40

Solution: $100 - 20 = 80$ are either.



Since 70 are in club, so that 10 are in team.

$\Rightarrow 10 + x > 70 \Rightarrow x > 60$ since x must be integer, the least it can be 61.

Answer: B

6.2 FUNDAMENTAL PRINCIPLE OF COUNTING

If an event occurs **m** times, and each of the **m** event s is followed by a second event which occurs **k** times, then the first event follows the second event **m** × **k** times.

Illustrative Example

12. How many four – digit numbers have only even digits?

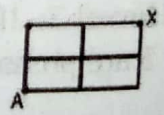
A. 480 B. 500 C. 400 D. 320

Solution:

- The first digit can be 2, 4, 6 or 8 ⇒ 4 ways.
- The 2^{nd}, 3^{rd} and fourth digits can be chosen in any of 5 ways (0, 2, 4, 6, 8).

Therefore, the total number of four digits numbers with only even digits is $4\times5\times5\times5 = 500$ **Answer: B**

13. In the figure at the right how many paths are there form A to x if the only ways to move are up and to the right.

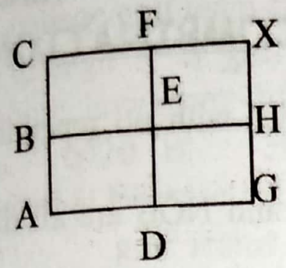


A. 9 B. 8 C. 6 D. 5

Solution: It is better labeling all the vertices and systematically:

→ ABCFX, ABEFX, ABEHX

→ ADEFX, ADEHX, ADGHX



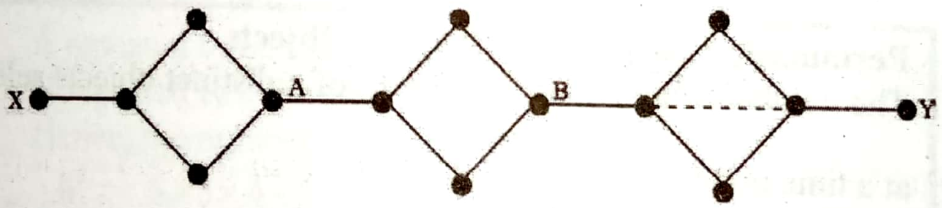
In all, there are 6 paths from A to X. **Answer: C**

14. How many three – digit numbers can be formed with the digits 1,3 and 5?

A. 6 B. 4 C. 12 D. 8

Solution: 135, 153, 315, 351, 513, 531 **Answer: A**

15. The diagram below shows the various paths along which a cat can travel from point X, where it is released to point Y, where it is rewarded with a food pellet.



How many different paths from X to Y can the cat take if it goes directly from X to Y without retracing any point along a path?

A. 6 B. 7 C. 12 D. 14

Solution:

The total number of ways to:

→ get from X to A is 2 path = 2 ways

→ get from A to B is 2 path = 2 ways

→ get from B to y is 3 path = 3 ways

Thus, the total number of different path is 2×2×3 = 12.

Answer: C

16. Let A be the set of primes less than 7, and B be the set of positive odd numbers less than 7. How many different sums of the form a + b are possible if a in A and b is in B?

A. 6 B. 7 C. 8 D. 9

Solution: A = {2, 3, 5}, B = {1, 3, 5}

List the sums systematically; First add 2 to each number in B, then 3, and then 5: 3, 5, 7, 4, 8, 6, 8, 10 = {3, 4, 5, 6, 7, 8, 10}.

There are 7 different sums. **Answer: B**

6.3 PERMUTATION AND COMBINATION

An arrangement of things in a definite order with no repetition is a **permutation.**

Example: BON, BNO, OBN, ONB, NBO, and NOB are all different arrangements of the three letters, B, O, and N.

> **Definition of n Factorial**
>
> **n factorial (n!)** is the product of the natural numbers **n** through **1**. That is $n! = n\times(n-1)\times(n-2)\ldots 3\times2\times1$

Here are some examples

$S! = 5 \times 4 \times 3 \times 2 \times 1$

$8! = 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$0! = 1$

> **Permutation Formula for Distinct Objects**
>
> The number of permutations P(n, k) of n distinct objects selected k at a time is $P(n, k) = \dfrac{n!}{(n-k)!}$

Permutation of N objects, k of which are identical

The number of arrangements of N objects, in a straight line, where k_1 objects of one kind are the same, k_2 objects of antoehr kind are the same and the rest are different, is given by $\dfrac{N!}{k_1!k_2!}$ where $N = k_1 + k_2$.

Illustrative Example

17. A 20 km Marathon has 10 people entered. In how many different ways can the first, second, and third place prizes be awarded?

A. 720 B. 360 C. 90 D. 1080

Solution:

The order in which the runners finish is important, so the number of ways to place first, second, and third is $P(10,3)=\frac{10!}{(10-3)!}$ $=10\times9\times8=720$ ways. **Answer: A**

18. A college golf team consists of five players who are ranked form 1 through 5. If golf coach has seven players from which to choose, how many different golf teams can the coach select?

A. 6720 B. 5040 C. 42 D. 2520

Solution: Because the players are ranked, the number of different golf teams possible is the number of permutation of seven players selected five at a time.

$$P(7,5)=\frac{7!}{(7-5)!}=\frac{7!}{2!}=\frac{7\times6\times5\times4\times3\times2!}{2!}=5040$$ **Answer: B**

Illustrative Example

19. Find the number of rearrangements of the letters in each of the following words:

A. BANANA B. STATISTICS

Solution:

a) There are six letters in the word BANANA with:
A repeated three times
N repeated two times
Hence, the number of arrangements is

$$\frac{6!}{3!2!}=\frac{6\times5\times4\times3\times2\times1}{3\times2\times1\times2\times1}=60 \text{ way}$$

b) There are ten letters in the word STATISTICS, with three Ss, and three Ts, and two I's duplicated in the word. Hence, the number of arrangement is

$$\frac{10!}{3!3!2!}=\frac{10.9.8.7.6.5.4.3.2.1}{3.2.1.3.2.1.2.1}=50400$$

20. From the letters a, b, c, d and e how many four letter groups can be formed if each letter can be formed if each letter can be used exactly once?

A. 625 B. 120 C. 125 D. 240

Solution: $5\times4\times4\times2=120$ **Answer: B**

21. Three women and four men are to be seated in a row of seven chairs. How many different seating arrangements are possible if there are no restrictions on the seating arrangements?

A. 480 B. 5040 C. 720 D. 840

Solution:

Since there are no restriction on seating arrangement then the number of seating arrangement is $P(7,7) = \frac{7!}{(7-7)!} = 7! = 5040$.

Answer: B

22. Three women and four men are to be seated in a row of seven chairs. How many different seating arrangements are possible if the women sit together and the men sit together?

A. 144 B. 30 C. 288 D. 72

Solution:

There are 3! Ways to arrange the women and 4! Ways to arrange the men. We must also consider that either the women or the men could be seated at the beginning of the row. There are two ways to do this. There are $2(3!\times4!)$ ways to seat the women together and the men together $2(3!\times4!) = 2(3\times2\times1\times4\times3\times2\times1) = 288$.

Answer: C

6.4 COMBINATIONS

A **combination** is a collection of objects for which the order is not important.

The number of combinations of n objects chosen k at a time is

$$C(n,k) = \frac{n!}{k!(n-k)!}.$$

Illustrative Example

23. A basket ball team consists of 11 players. In how many different ways can a coach choose the five starting players, assuming the position of a player is not considered?

A. 55440 B. 462 C. 11088 D. 55

Solution:

$$C(11,5) = \frac{11!}{5!(11-5)!} = \frac{11\times10\times9\times8\times7\times6!}{5\times4\times3\times2\times1\times6!} = 462$$

Answer: B

24. At the beginning of semester (I) of a mathematics class for preparatory school teachers, each of the class's 27 students shook

hands with each of the other students exactly once. How many handshakes took place?

A. 54 B. 108 C. 216 D. 351

Solution:

Since the handshake between persons A and B is the same as that between persons B and A, so that

$$C(27,2)=\frac{27!}{2!(27-2)}=\frac{27\times 26}{2}=351$$ **Answer: D**

6.5 PRO ABILITY OF AN EVENT

For an experiment with sample space S of equally likely out comes, the probability P (E)

of an event E is given by: $P(E)=\frac{n(E)}{n(S)}$ where n(E) is the number of elements in the event and n(s) is the number of elements in the sample space.

Probability of Mutually Exclusive Events

Two events A and B are **mutually exclusive** if $A\cap B=\phi$

Properties of Probability

1. **Impossible event:** if P (E) = 0
2. **Certain even** if P(E) = 1
3. For any event E, $0\le P(E)\le 1$
   - **Likely event** if $\frac{1}{2}<P(E)<1$
   - **Unlikely event** if $0<P(E)<\frac{1}{2}$
4. **Addition Rule for probability.**
   If A and B are events, then $P(A\cup B)=P(A)+P(B)-P(A\cap B)$
5. **Condition Probability**
   Let A and B be two events in a sample space S. Then the conditional probability of B given that A has occurred is
   $$P(B|A)=\frac{P(A\cap B)}{P(A)}$$
6. **Product Rule for probability**
   - $P(A \text{ and } B)=P(A\cap B)=P(A).\,P(B|A)$

- If A and B are independent event then
  $P(A \text{ and } B) = P(A \cap B) = P(A).P(B)$

7. **Odd in favor and odd against**

   a) The **odds in favor** of an event A are given by:

   $$\frac{P(A)}{P(A')} = \frac{P(A)}{1-P(A)}$$

   b) The **odds against** an event **A** are given by

   $$\frac{P(A')}{P(A)} = \frac{1-P(A)}{P(A)}$$

Illustrative Example

A bag contains 2 red balls, 4 blue balls, and 3 white balls. (for 25-27) questions

25. What is the probability of the event R that a ball drawn at random is red?

A. $\frac{2}{9}$ B. $\frac{1}{5}$ C. $\frac{4}{9}$ D. $\frac{1}{3}$

Solution:

The bag contains a total of $2+4+3=9$

$\therefore P(R)=\frac{2}{9}$ **Answer: A**

26. What is the probability of the event "**not red**"?

A. $\frac{4}{9}$ B. $\frac{4}{5}$ C. $\frac{7}{9}$ D. $\frac{5}{9}$

Solution: $P(\text{not } R) = 1-P(R) = 1-\frac{2}{9}=\frac{7}{9}$ **Answer: C**

27. What is the probability of the event that a ball drawn at random is either red **(R)** or blue **(B)**?

A. $\frac{2}{3}$ B. $\frac{5}{9}$ C. $\frac{4}{9}$ D. $\frac{1}{5}$

Solution: Since $R \cap B = \phi$, so $P(R \cup B) = P(R) + P(B)$

UNIT – SEVEN

7. ALGEBRAIC EXPRESSIONS.

A mathematical expression that contains a variable is called an algebraic expression. Some examples of **algebraic expressions** are $x^2,\ 4x+3y,\ 2\left(y^3-\frac{1}{z^4}\right)$. Two algebraic expressions are called **like terms** if both the variable parts and the exponents are identical.

7.1 OPERATIONS OF ALGEBRAIC EXPRESSIONS.

When simplifying algebraic expressions; Perform operations within parentheses first and then exponents and then multiplication and then division and then addition and lastly subtraction. This can be recall as **PEMDAS. (Please Excuse My Dear Aunt Selam)**

Illustrative Example

1. $7-(9-2^3[6\div3+1])=$

   A. -8 B. 8 C. 22 D. 10

   Solution:

   $7-(9-2^3[6\div3+1])=$

   $\Rightarrow 7-(9-8[2+1])$ ⟵ By performing the exponential and the division within the inner most parentheses

   $\Rightarrow 7-(9-8[3])$ ⟵ By performing the addition within the inner most parentheses.

   $\Rightarrow 7-(9-24)$ ⟵ By performing the multiplication

   $\Rightarrow 7-(-15)$ ⟵ By performing the subtraction within parentheses.

   $\Rightarrow 7+15=22$ **Answer: C**

2. If $x=-4$ and $y=3$, then $x^2-\left(y-\left[x+\frac{1}{2}\right]\right)-12$

   A. $\frac{3}{2}$ B. $\frac{-3}{2}$ C. $\frac{21}{2}$ D. $\frac{-5}{2}$

Solution:

$$x^2-\left(y-\left[x+\frac{1}{2}\right]\right)-12=$$

$$\Rightarrow(-4)^2-\left(3-\left[-4+\frac{1}{2}\right]\right)-12=16=\left(3-\left[\frac{-7}{2}\right]\right)-12$$

$$=16-\left(3+\frac{7}{2}\right)-12$$

$$=4-\frac{13}{2}=\frac{-5}{2},$$

Answer: D

The three most important binomial products on SAT course are these:

- $(x-y)(x+y)=x^2+xy-yx-y^2=x^2-y^2$
- $(x-y)^2=(x-y)(x-y)=x^2-xy-xy+y^2$
  $=x^2-2xy+y^2$
- $(x+y)^2=(x+y)(x+y)=x^2+xy+xy+y^2$
  $=x^2+2xy+y^2$

Illustrative Example

3. If $a-b=18.4$ and $a+b=15$, what is the value of a^2-b^2?

A. 276 B. 33.4 C. 184 D. 270.4

Solution:

$$a^2-b^2=(a-b)(a+b)=(18.4)(15)=276$$

Answer: A

4. If $x=y+\frac{1}{y}$ and $z=y-\frac{1}{y}$, where $z\neq 0$ then $(x-z)(x+z)$ is equal to

A. $\frac{1}{y^2}$ B. y^2 C. 2 D. 4

Solution:

$$x-z=\left(y+\frac{1}{y}\right)-\left(y-\frac{1}{y}\right)=\frac{1}{y}+\frac{1}{y}=\frac{2}{y}$$

$$x+z=\left(y+\frac{1}{y}\right)+\left(y-\frac{1}{y}\right)=y+y=2y$$

Then the product of * and **

Solution: let $x = 5, y = 3$

$$x^2 - y^2 = (x - y)(x + y),$$
$$= (5 - 3)(5 + 3) = (2)(8) = 16$$
$$\Rightarrow 16 = (4)(4)$$

$\Rightarrow$ It must multiple of 4 **Answer: B**

14. If $a^2 - b^2 = 21$ and $a^2 + b^2 = 29$, which of the following could be the value of ab

I -10 II $5\sqrt{2}$ III 10

A. I only B. II only
C. III only D. I and III only

Solution:

Adding the two equation

$$+\begin{cases} a^2 - b^2 = 21 \\ a^2 + b^2 = 29 \end{cases}$$

$$2a^2 = 50 \Rightarrow a^2 = 25 \text{ and } b^2 = 4$$
$$\therefore a = \pm 5 \text{ and } b = \pm 2$$
Thus, $ab = (\pm 5)(\pm 2) = \pm 10$

Answer: D

7.2 SOLVING EQUATIONS

The basic principle to which you must adhere in solving any equation is that you can manipulate the equation in any way, as long as you do the same thing to both sides. For example, you may always add the same number to each side, subtract the same number from each side, multiply or divide each side by the same number (except 0), square each side, take the square root of each side.

Illustrative Example

15. If $x - 8 = 7$, what is the value of $x - 12$?

A. -15 B. -7 C. -1 D. 3

Solution:

$$x - 8 = 7 \Rightarrow x = 8 + 7 = 15$$
$$\therefore x - 12 = 15 - 12 = 3$$

Answer: D

16. If $4x + 6 = 30$, what is the value of $3x + 1$?

A. 9 B. 19 C. 15 D. 18

UNIT – EIGHT

8. GEOMETRIC APTITUDE

8.1 TRIANGLES

A closed and plane figure, bounded by three lines segments is called **a triangle**. A triangle is divided in to 3 based on their sides

- **Scalene;** if all its sides are different lengths and all its angles are of different measures.

Isosceles Triangle

A triangle is **an isosceles;** if its any two sides are equal and the angels opposite to these two sides are also equal.

Equilateral triangle

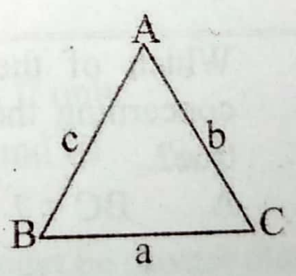
A triangle is an equilateral triangle; if all its sides are equal and all its angles are also equal in such away that each angle is 60°.

A triangle is divided into 3 based on their angles;

- **acute angles triangle:** if each of its angles is **less than 90°**
- **right angles triangle:** if only one of its angle is 90°.
- **obtuse angles triangle**: if only one of its angles is greater than 90°.

Triangle Fact (A):

Let a, b, and c be the sides of ΔABC, with $a \le b \le c$.



- $a^2 + b^2 = c^2$ if and only if $\angle c = 90°$.
- $a^2 + b^2 < c^2$ if and only if angle C is obtuse.
- $a^2 + b^2 > c^2$ if and only if angle ($\angle c$) is acute.

$\Rightarrow m(\angle A) = 180° - 120° = 60°$

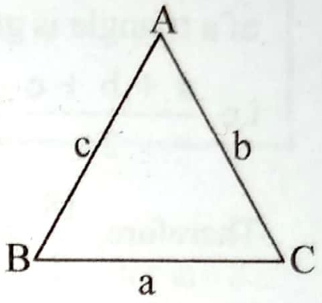
Then B is the largest angle, C is the smallest, and A is in between.

Therefore $AB < BC < AC \Rightarrow 7 < BC < 8$

Answer: C

Triangle Fact (C) (Triangle Inequality)

- The sum of the lengths of any two sides of a triangle is greater than the length of the third side. That is $a + b > c$, $b + c > a$. and $c + a > b$.



The difference between the length of any two sides of a triangle is less than the length of the third side.

Illustrative Example

4. Is it possible to have a triangle with sides:

A.	3 cm, 5 cm and 9 cm?	C.	4cm, 7cm, and 11cm?
B.	6 cm, 4cm and 10cm?	D.	8cm, 7cm and 12cm?

Solution:

A. No; $3 + 5 = 8 < 9$

B. No; $6 + 4 = 10$

C. No; $4 + 7 = 11$

D. Yes; $8 + 7 = 15 > 12$, $7 + 12 = 19 > 8$

$8 + 12 = 20 > 7$

Answer: D

5. If the lengths of two sides of a triangle are 5 and 7, which of the following could be the length of the third side?

(I) 2 (II) 4 (III) 13

A.	I only	C.	I and II only
B.	II only	D.	I, II and III

Solution:

Let x be the third side. The third side must be greater than $7 - 5 = 2$ and must be less than $7 + 5 = 12$. That is $2 < x < 12$.

Answer: B

Triangle fact C (Heron's Formula)

The area of a triangle is given by $A = \frac{1}{2}bh$, where b = base and h = height or $A = \sqrt{S(S-a)(S-b)(S-c)}$, where

$$S = \frac{a+b+c}{2}$$

Illustrative Example

9. What is the area of an equilateral triangle whose sides are 8?

A. $16\sqrt{3}$ B. 32 C. $32\sqrt{3}$ D. 16

Solution:

If A represents the area of an equilateral triangle with side length x, then $A = \frac{x^2\sqrt{3}}{4}$.

$$\therefore\ A = \frac{8^2\sqrt{3}}{4} = \frac{64\sqrt{3}}{4} = 16\sqrt{3}$$

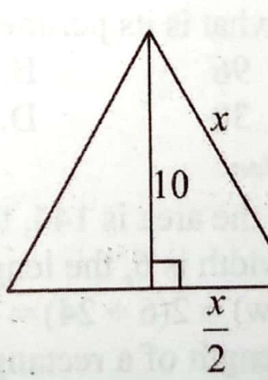
Answer: A

10. What is the area of an equilateral triangle whose altitude is 10?

A. 30 B. $\frac{100\sqrt{3}}{3}$ C. $30\sqrt{3}$ D. $\frac{10\sqrt{3}}{3}$

Solution: Let x be the side length

$$\Rightarrow \left(\frac{x}{2}\right)^2 + 10^2 = x^2$$



$$\Rightarrow x^2 - \frac{x^2}{4} = 100$$

$$\Rightarrow 3x^2 = 400$$

$$\Rightarrow x^2 = \frac{400}{3} \Leftrightarrow x = \frac{20}{\sqrt{\ }}$$

$$\therefore\ A = \frac{x^2\sqrt{3}}{4} = \frac{400\sqrt{3}}{12} = \frac{100\sqrt{3}}{3}$$

Answer: B

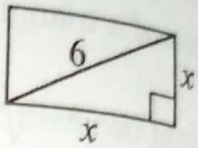
11. What is the area of square whose diagonal is 6?

A. 12 B. 24 C. 18 D. 36

Solution: Let x = side length

$$x^2 + x^2 = 6^2 \Leftrightarrow 2x^2 = 36$$

$$x^2 = \frac{36}{2} = 18$$



Answer: C

12. What is the length of each side of a square if its diagonals are 8?

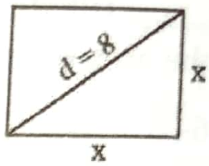
A. $4\sqrt{3}$ B. $8\sqrt{2}$ C. $4\sqrt{2}$ D. $12\sqrt{2}$

Solution:

$$x^2 + x^2 = d^2$$

$$2x^2 = 8^2 = 64$$

$$\Rightarrow x^2 = 32 \Rightarrow x = \sqrt{32} = 4\sqrt{2}$$



Answer: C

Important Fact

Here are the area formula you need to know:

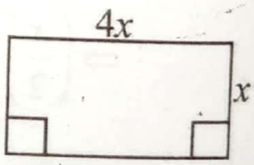
- Area of a parallelogram: $A = bh$
- Area of a rectangle: $A = \ell w$
- Area of a square: $A = s^2$ or $A = \frac{1}{2}d^2$ where d – is diagonal
- Area of a circle: $A = \pi r^2 = \frac{\pi d^2}{4}$ where d – is diameter

Illustrative Example

13. If the length of a rectangle is 4 times its width, and if its area is 144, what is its perimeter?

A. 96 B. 60

C. 30 D. 24



Solution:

Since the area is 144, then $(4x)(x) = 144 \Leftrightarrow x^2 = 36 \Rightarrow x = 6$

The width is 6, the length is 24, and the perimeter is $2(l + w) = 2(6 + 24) = 60$.

Answer: B

14. The length of a rectangle is 5 more than the side of a square, and the width of the rectangle is 5 less than the side of the square. If the area of square is 45. What is the area of the rectangle?

A. 20 B. 25 C. 45 D. 50

Solution: Let x = the side of the square

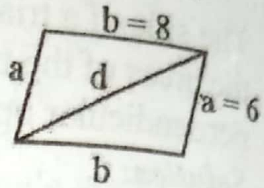
Then, width (W) of rectangle: $W = x - 5$

Solution:

Use: $A = 2\sqrt{S(S-a)(S-b)(S-d)}$ where $S = \frac{a+b+d}{2}$

$\Rightarrow A = 2\sqrt{13(13-6)(13-8)(13-12)}$

$\therefore A = 2\sqrt{455}m^2$



29. If the perimeter of a triangle is 27m, then the length of one of sides **CANNOT** be:

A. 1 B. 10 C. 15 D. 13

Solution: Let a, b and c the side length of a triangle then semi-perimeter of a triangle is greater than any side length of a triangle therefore $\frac{27}{2} = 13.5$

$\Rightarrow 13.5 > 1, \quad 13.5 > 10, \quad \text{and } 13.5 > 13$

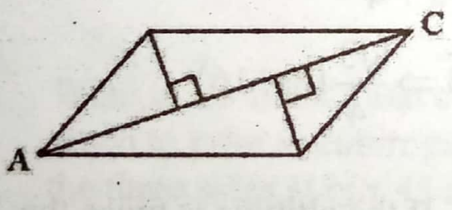
Answer: C

30. Find the area of a quadrilateral of whose diagonal is 24m long and the lengths of perpendicular from the other two vertices are 19m and 11m respectively.

A. $150m^2$ C. $240m^3$
B. $360m^2$ D. $180m^2$

Solution: Let the length of perpendicular be h_1 and h_2 respectively

$\therefore \text{Area} = \frac{1}{2}(AC)h_1 + \frac{1}{2}(AC)h_2$



$= \frac{1}{2}(AC)(h_1 + h_2)$

$= \frac{1}{2}(24m)(19m + 11m) = 360m^2$

Answer: B

8.3 SOLID GEOMETRY

Important fact

Here are the area formula you need to know:

Surface – Area and Volume

B = area of base, P = perimeter

C = Circumference, h = height

ℓ = slant height, r = radius

	Surface area	Volume
Prism	$S = 2B + ph$	$V = Bh$
Cylinder	$S = 2B + Ch$ $= 2\pi r^2 + 2\pi rh$	$V = \pi r^2 h$
Regular Pyramid	$S = B + \frac{1}{2}P\ell$	$V = \frac{1}{3}Bh$
Right cone	$S = B + \frac{1}{2}C\ell$ $= \pi r^2 + \pi r\ell$	$V = \frac{1}{3}\pi r^2 h$
Sphere	$S = 4\pi r^2$	$V = \frac{4}{3}\pi r^3$

31. The diagonals of a rhombus are 15m and 20m, find the side, area and the height of the rhombus

Solution:

To find the side we use the formula: $_2d_1^2 + d_2^2 = 4a^2$

$$\Rightarrow a^2 = \frac{d_1^2 + d_2^2}{4} = \frac{(15m)^2 + (20)^2}{4} = \frac{625}{4}$$

$$\therefore a = \sqrt{\frac{625}{4}} = \frac{25m}{2}$$

- Area $= \frac{1}{2}d_1 d_2 = \frac{1}{2}(15)(20) = 150m^2$
- To find the height: $A = bh \Rightarrow 150 = \frac{25m}{2} xh$

$$\Rightarrow h = \frac{2 \times 150}{25} = 12m.$$

32. The diagonals of a rhombus are 10m and 24m respectively. Find the perimeter of the rhombus.

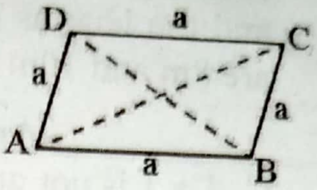
Solution: Each side: $a = \sqrt{\left(\frac{d_1}{2}\right)^2 + \left(\frac{d_2}{2}\right)^2}$

$$\therefore a = \sqrt{5^2 + 12^2} = \sqrt{169} = 13m$$

Hence the perimeter: $P = 4a = 4(13) = 52m$.

8.4 RHOMBUS

- A rhombus is a quadrilateral whose all sides are equal.
- Diagonals AC = d_1 and BD = d_2 are bisect each other at 90°, but AC ≠ BD



Important fact on Rhombus.

- Area: $A = \frac{1}{2} \times$ product of its diagonals $= \frac{1}{2} d_1 d_2$

$$A = \frac{1}{2} \times d_1 \times \sqrt{a^2 - \left(\frac{d_1}{2}\right)^2}, \text{ since } d_2^2 = a^2 - \left(\frac{d_1}{2}\right)^2.$$

$$A = \frac{1}{2} \times d_2 \times \sqrt{a^2 - \left(\frac{d_2}{2}\right)^2}, \text{ since } d_1^2 = a^2 - \left(\frac{d_2}{2}\right)^2.$$

$$d_1^2 + d_2^2 = 4a^2 \text{ and } (d_1 + d_2)^2 = 4(a^2 + A)$$

$$(d_1 - d_2)^2 = 4(a^2 - A)$$

Review Exercise

1. Which of the following sets of numbers **cannot** represent the sides of a triangle?
   A. 9, 40, 41 B. 7, 7, 3 C. 4, 5, 1 D. 6, 6, 6
2. If the lengths of two sides of a triangle are 12 and 20 and the third side is represented by x, then:
   A. $x = 32$ B. $x > 32$ C. $x < 8$ D. $8 < x < 32$
3. The diagonals of rhombus are 18 and 24. Find
   a) the area of the rhombus
   b) the length of a side of the rhombus
4. Find the length of a side of a square if a diagonal has a length of 8.
5. The circumference of the base of a cone is 8π cm. If the volume of the cone is 16πcm^3. What is the height?
6. The sides of a triangle are 7cm, 8cm and 9cm. find its area.
7. The sides of a triangle are 13cm, 14cm and 15cm. find the perpendicular distance from the vertex to the longest side.
8. The diagonals of a rhombus are 8m and 6cm respectively. Find the side and the height

UNIT ONE

ANTONYMS

(Verbal Tactics)

- Besides meaning, words have another element called **relationship**. In the family of words, words can relate closely to one another or be distinctly opposite.
- A word that means the **opposite** of another word is called **antonym.** (i.e. An **antonym** is a word that has the **opposite** meaning). Nearly all words have a synonyms. Not all words have antonym. When a word doesn't have a suitable **antonym**, the space provided for the response has been removed. In most cases, you will find a single best answer available among the options. In a few cases there may be two synonyms that would be suitable for one word.
- Some of words in the list of **synonyms** and **antonyms** may be difficult or unfamiliar. For these words, consult your dictionary or thesaurus.

Antonym Questions

- The **antonym** questions are always the **first group of questions in a** verbal section.
- **Antonym** questions provide a single word and ask you to select from a list of words the one that is **most opposite** in meaning.
  - To answer an **antonym question,** use the following strategies.

1. Look for a word that is **opposite** in meaning. Do not be thrown off by any synonyms - words that are **similar** in meaning-that are included among the choices.
2. Before looking at the choices, be sure that you know the meaning of the first word. Define it using any of the following methods:
   a. Think of another word or group of words that means the same thing.

b. Think of a sentence or sentences that use the word then try to arrive at an exact definition.
c. Try analyzing the parts of the word.

3. Once you know the meaning of the first word, look at the choices. If none of these is obviously the correct antonym, try either or both of the following strategies.
   a. Eliminate any obviously incorrect answers.
   b. Remember that many words have more than one meaning. If none of the choices seems to be opposite in meaning, think of other meanings of the first word.

◆ **A typical antonym question looks like this:**

DEPRESS: A. force C. clarify E. loosen
B. allow D. elate

✧ Based on the tactics given above, the answer to the sample question is **D.**

- Bekan knew the **joy** of a hard workout would come later from the sheer **pleasure** of victory.
- Bekan knew the **pain** of a hard workout would be rewarded by the sheer **pleasure** that comes from victory.

✧ The first sentence displays **"pleasure"** as a synonym for **"joy,"** permitting the author to get an idea across without being dull and repetitions. In the second sentence, the use of **"pain"** as an antonym helps the author to state an entirely different thought by changing the sentence only slightly.

✧ Thus, **synonyms** and **antonyms** become important elements in the word mastery process.

UNIT TWO

SYNONYMS

SYNONYMS:

When you write or speak, you want to express your meaning exactly. English is so rich in words that you can often choose among words with **similar meanings** to find just the right one.

- ✓ A word that has **the same** or **nearly the same meaning**, is therefore, called **synonym**.
- ✓ Although **synonyms** mean about the same thing, they often convey slightly different shades of meaning **(see antonyms – unit one).**

Here are some examples

felt **drowsy:** A. ill B. content C. sleepy D. livery

Clues: Although all the words make sense when used with **felt**, only one word is close in meaning to **drowsy**. Notice that you might feel **drowsry** because you are ill, but 'ill' doesn't mean the same thing. Notice that content and lively are not **synonyms** of **drowsy**. The answer is **C, sleepy.**

Words	Synonyms	Words	Synonyms
eat	consume	panic	crowds
hurt	injũre	source	beginning
big	large	accurate	correct
clash	conflict	occasion	event
fir	discharge	crude	rough
slim	slender	apparently	seemingly
enormous	huge	dynamic	energetic
fortune	lucky	frustrate	foil

responsibility	duty	utmost	greatest
artificial	fake	prominent	well known
benefit	gain	reluctant	hesitant
pattern	design	tremble	shake
notion	idea	tedious	boring
imitate	copy	refrain	resist
ritual	ceremony		

UNIVERSITY ENTRANCE EXAMINATION AND OTHER QUESTION WITH DITAILED EXAMPLES AND EXPLANATIONS (UEE/EHEECE 2001 – 2009)

1. **PARTISAN**

A. neutral B. biased C. objective D. impartial

Clues: **partisan (adj)** someone who is partisan strongly supports a particular person or cause, often without thinking carefully about the matter.

Biased (adj) if someone is biased, they refer one group of people to another, and behave **unfairly** as a result.

Answer: B

2. **NEUTRAL**

A. unkind B. precious C. mean D. indifferent

Clues: **Neutral (adj)** if a person or a country adopts **neutral** position or remains **neutral**, they don't support anyone in disagreement, war, or contest. **Answer: D**

3. **CONCEAL**

A. reveal B. hide C. ignorant D. respected

Clue: **conceal (v)** - to cover or hide something carefully.

- to keep from sight.

Answer: B

4. **EXCEED**

A. outstrip B. magnify C. delimit D. offset

Clue: **exceed (v)** if something **exceeds** a particular amount or number, it is greater or larger than that amount or number.

- to pass beyond the measure of something, to surpass

Answer: C

UNIT THREE

ANALOGIES

What are analogies?

- A **verbal analogy** expresses a relationship or comparison between sets of words.
- A complete analogy compares the two pairs of words and makes a statement about them. It asserts that the relationship between the first pair of words is the same as the relationship between the second.
- Generally, **analogies** require you to define how two sets of words are **alike** or **different**. Sometimes those words express a **similar** relationship, and sometimes the words express an **opposite** relationship. (i.e to choose the pair of words that best matches or parallels the relationship of key, or given, pair of words.)

☞ **Here are two examples**

1. **maple** is to **tree** is as
   a. acorn is to oak.
   b. hen is to rooster
   c. rose is to flower
   d. shrub is to lilac
2. **joyful** is to **gloomy** as
   a. cheerful is to happy
   b. strong is to weak
   c. quick is to famous
   d. hungry is to starving

Clues: In order to find the correct answer to exercise **1**, you must first determine the relationship between the two key words, **maple** and **tree**. In this case, that relationship might be expressed as "a **maple** is a kind (or type) of the tree." The next is to select from choices **a, b, c** and **d** the pair of words that best reflects the same relationship.

Clearly, the correct answer is **(C)**; it is the only choice that parallels the relationship of the key words:

A **rose** is a kind (or type) of flower, just as a **maple** is a kind (or type) of tree. The other choices do not express the same relationship.

In exercise **2,** the relationship between the key words can be expressed as **"joyful** means the opposite of **gloomy."** Which of the choices best represents the same relationship? The answer, of course, is **(b)**; **"strong"** means the opposite of **"weak."**

☞ **Follow also the following techniques with its patterns.**

✧ We have said that **analogies** are very helpful in understanding word **relationships**. It is also a partial similarity between things that are somewhat different.

☞ **see these words:**

dog : cat

✧ These two words can be used to start an **analogy**. the sign [:] means **"is to"**. The analogy is continued by adding another sign [::] meaning **"as"** Let's add these two signs to our two words:

dog : cat ::

(dog is **to** cat **as** cat is **to**)

✧ What you are looking at so far is expressed like this. As indicated above, **"dog is to cat as..."** but now we need a work which expresses a relationship with cat. So continuing, the pattern will appear like this:

dog : cat :: cat :

✧ Dog and cat are natural enemies. The analogy can be completed by inserting the name of an animal that is the cat's natural enemy. The completed analogy will look like this.

dog : cat :: cat : rat

✧ You can probably see how **analogies, synonyms,** and **antonyms** go hand in hand. It is easy to see that just knowing whether key words are the same or opposite can be very helpful. In this book is a typical analogy problems (questions), the kind of problem you will be dealing with as you work your way through this book. What answer would you choose to complete the following analogy?

pleasure : pain :: win : ______?

a. victory b. joy c. lose

These pair of words have the opposite relationship **(pleasure = happiness; pain = ailness).**

Answer: C

Analogy Questions

(General Tactics)

✧ As earlier mentioned, in the **analogy question,** you are given a **pair of words** which have some kind of relationship. You are asked to select from four and sometimes five pairs of words the pair which has the same relationship as the first two words. Follow the following **strategies:**

1. Before you look at the choices, try to state the relationship between the **CAPITALIZED WORDS** in a good sentences. Then use the word pairs from the answer choices in the same sentence. Frequently, only one will make sense, and you'll have the correct answer.
2. Don't be misled if the choices are from different fields or areas, or seem to deal with different items, from the given pair. Study the capitalized words until you see the connection between them; then search for the same relationship among the choices.

   Examples

   BOTANIST: MICROSCOPE :: CARPENTER : HAMMER, even though the two workers may have little else in common besides their uses of tools.
3. If more than one answer fits the relationship in your sentence, look for a narrower approach.

   Examples

   MITTEN : HAND ::

   A. bracelet : wrist
   B. belt : waist
   C. muffler : neck
   D. ring : finger
   E. sandal : foot

✧ You make up the sentence, "you wear a **mitten** on your **hand**." unfortunately, **all** the answer choices will fit that sentence. So you say to yourself "why do you wear a **mitten?** You wear a **mitten** to keep your hand warm." Now when you try to substitute, only choice **(C)** works, so you've your answer.

4. Watch out for errors steaming from grammatical reveals. Ask yourself **who** is doing what to whom.

Examples

FUGITIVE : FLEE is not the same as **LAUGHINGSTOCK: MOCK**.

- A **fugitive** is a person who **flees**. A **laughingstock** is a person who **is mocked**.

5. Be familiar with the whole range of common analogy types. Know the usual ways in which pairs of words on the **SAT** are linked.

Examples

DAUNTLESS: COURAGEOUS

- **dauntless (fearless)** and **courageous** are synonyms.

DAUNTLESS: COWARDLY

- Someone **dauntless** doesn't exhibit cowardice.

POET: SONNET

- A **poet** creates a **sonnet**.

PAINTER: BRUSH

- A **painter** uses a **brush**.

SAW: WOOD

- A **saw** cuts **wood**.

CROWBAR: PRY

- A **crowbar** is a tool used to **pry**.

NOD: ASSENT

- A **nod** is a sign of **assent** (agreement).

STAMMER: TALK

- To **stammer** is to **talk** in a halting manner.

LUKEWARM: BOILING

- **Lukewarm** is less intense than **boiling**.

WHALE: MAMMAL

- A **whale** is a member of the class know as **mammal**.

TIGER: CARNIVOROUS

- A **tiger** is by definition a **carnivorous** (meat-eating) animal.

ARCHIPELAGO: ISLAND

- An **archipelago** (chain of islands) is made up of many **islands**.

DOE: STAGE

- A **doe** is a female deer; a **stag**, a male deer.

DOVE: PEACE

- A **dove** is the symbol of peace.

Common Relationship found in analogy Questions

☞ There are many different kinds of relationships represented in the **analogy questions** you will find in this book.

The common relationships include:

Type of Analogy		Examples
actions to object	⟶	play: clarinet
causes to effect	⟶	sun: sunburn
item to category	⟶	iguana: reptile
object to its material	⟶	curtains : cloth
object to its function	⟶	pencil; writing
part to whole	⟶	page : book
time sequence	⟶	recent: current
type to characteristic	⟶	dancer : agile
word to antonym	⟶	help: hinder
word to synonym	⟶	provisions: supplies
worker and creation	⟶	artist: sketch
worker and tool	⟶	lumberjack: saw

More common Analogy Types

A. Definition

REFUGE : SHELTER

- **A refuge** (place of asylum) by definition **shelters**

NOMAD ; WANDER

- **A nomad** by definition is **wanders.**

HAGGLER : BARGAIN

- **A haggler,** a person who argues over prices, by definition bargain.

B. Defining Characteristic

TIGER : CARNIVOROUS

- **A tiger** is defined as a carnivorous or meat eating animal.

INTOMOLOGIST : INSECTS

- **An intomologist** is defined as a person who studies insects.

HIVE : BEE

- **A hive** is defined as a home for bees.

C. Class and member

RODENT : SQUUIRREL

- **A squirrel** is a kind rodent.

SOFA : FURNITURE

- **A sofa** belongs to the category known as furniture.

SONNET : POEM

- **A sonnet** is a kind of poem.

D. Group and Member

DANCER : ENSEMBLE

- **A dancer** is a member of an ensemble or troupe.

LION : PRIDE

- **A lion** is a member of a pride or company.

GAGGLE : GEESE

- A **gaggle** is group or flock of geese.

E. **Antonyms**

✧ **Antonyms** are words that are opposite in meaning. Both words belong to the same part of speech.

CONCERNED : INDIFFERENT

- indifferent means **unconcerned**.

WAX : WANE

- wax, to grow larger, and wane, to dwindle, are opposites.

ANARCHY : ORDER

- **Anarchy** is the opposite of **order**.

F. **Antonym Variants**

In an **antonym variant**, the words are not strictly antonyms; however, their meanings are opposed. Take the adjective "nervous." A strict antonym for the adjective **nervous** would be the adjective **poised**. However, where an Antonym would put the adjective poised. An Antonym variant puts the noun poise. It looks like this:

NERVIOUS : POISE

- Nervous means lacking in **poise**.

WICKED : VIRTUE

- Something **wicked** lacks **virtue.** It is the opposite of **virtuous**.

WILLFUL : OBDIENT

- Willful means lacking in obedience. It is the opposite of **obedient**.

G. **Synonyms**

Synonyms are words that have the same meaning. Both words belong to the same part of speech.

MAGNIFICENT : GRANDIOSE

- Grandiose means magnificent.

NARRATE : TELL

- To **narrate** is to **tell.**

EDIFICE : BUILDING

- An **edifice** is a building.

H. **Synonym Variants**

In a **synonym variant,** the words are not strictly synonyms; however, their meanings are similar. For example, take the adjective **"willful"** a strict synonym for the adjective willful would be the adjective **unruly.** However, where a synonym would put the noun **unruliness.** It looks like this:

WILLFUL: UNRULINESS

- **Willful** means exhibiting unruliness.

VERBOSE : WORDINESS

- Someone verbose is **wordy;** he or she exhibits wordness.

FRIENDLY : AMICABILITY

- Someone friendly is amicable; he or she shows **amicability.**

I. Degree of Intensity

LUKEWARM : BOILING

- **Lukewarm** is less extreme than boiling.

FLURRY : BLIZZARD

- A flurry or shower of snow is less extreme than a blizzard.

ANNOYED : FURIOUS

- To be **annoyed** is less intense an emotion than to be **frious.**

J. Part to whole

ISLAND : ARCHIPELAGO

- Many islands make up an archipelago.

LETTER : ALPHABET

- The English alphabet is made up of 26 letters.

FINGER : HAND

- The finger is part of the hand.

K. Part to whole

ASYLUM : REFUGE

- An **asylum** provides **refuge** or protection.

FEET : MARCH

- A function of **feet** is to march.

LULL : STORM

- A lull temporarily interrupts a storm.

L. **Manner**

MUMBLE : SPEAK

- To **mumble** is to speak indistinctly, that is, to speak in an indistinct manner.

STRUT : WALK

- To **strut** is to **walk** proudly, that is, to walk in a proud manner.

STRAINED : WIT

- **Wit** that is strained is forced in manner.

M. **Worker and Article created**

POET : SONNET

- A **poet** creates a **sonnet**.

ARCHITECT : BLUEPRINT

- An **architect** designs a **blueprint**.

MASON : WALL

- A **mason** builds a **wall**.

N. **Worker and Tool**

PAINTER : BRUSH

- A painter uses a brush.

GOLFER : CLUB

- A **golfer** uses a **club** to strike the ball.

CARPENTER : VISE

- A carpenter uses a vise to hold the object being worked on.

N. **Worker and Tool**

PAINTER : BRUSH

- A painter uses a brush.

GOLFER : CLUB

- A **golfer** uses a **club** to strike the ball.

CARPENTER : VISE

- A carpenter uses a vise to hold the object being worked on.

O. Worker and Action

ACROBAT : CARTIWHEEL

- An **acrobat** performs a cartwheel.

FINANCIER : INVEST

- A financier invests.

TENOR : ARIA

- A tenor sings an aria.

P. Worker and workplace

TEACHER : CLASSROOM

- A teacher works in a classroom.

SCULPTOR : STUDIO

- A sculptor works in a studio.

DRUGGIST : PHARMACY

- A **druggist** works in a pharmacy.

Q. Tool and object it acts upon

KNIFE : BREAD

- A knife cuts bread.

PEN : PAPER

- A pen writes on paper.

RAKE : LEAVES

- A rake gathers leaves.

R. Tool and its Action

SAW : CUT

- **Saw** is a tool used to cut wood.

CROWBAR : PRY

- A **crowbar** is a tool used to pry things apart.

SIEVE : SIFT

- A sieve is a tool used to strain or sift.

S. **Action and its Significance**

HUG : AFFECTION

- A hug is a sign of affection.

NOD : ASSENT

- A nod signifies assent or agreement.

WINCE : PAN

- A wince is a sign that one feels pain.

T. **Cause and effect**

VIRUS : INFLUENZA

- A virus causes influenza.

FIRE : ASHES

- Fire causes ashes.

U. **Time Sequence**

FIRST : LAST

- First and **last** mark the beginning and end of a sequence.

V. **Spatial sequence**

ATTIC : BASEMENT

- The attic is the highest point in the house; the basement, the lowest point.

W. **Gender**

DOE : STAG

- A doe is a female deer, a stag, a male deer.

X. **Age**

COLT : STALLION

- A **colt** is a young stallion.

Y. **Symbol and Abstraction it represents**

DOVE : PEACE

- A **dove** is a symbol of peace.

Practice

For each of the following Paris of words, name the common analogy type to which it belongs.

1. SURGEON : SCALGEL ______________
2. BARK : TREE ______________
3. FLOWER : PEONY ______________
4. DRILL : BORE ______________
5. MINUTE : HOUR ______________
6. COW : HERBIVOROUS ______________
7. TENT : SHETER ______________
8. EWE : RAM ______________
9. SHOAT : PIG ______________
10. LAUREL : VICTORY ______________
11. FAWN : DEER ______________
12. MEAL : LUNCH ______________
13. SCULPTOR : STATUE ______________
14. TELLER : BANK ______________
15. DISPERSE : ASSEMBLE ______________
16. DRENCHED : MOIST ______________
17. ADORE : LOATHE ______________
18. SCULPTOR : MALLET ______________
19. VERACIOUS : TRUTHFUL ______________
20. ANGLER : FISH ______________
21. BUTCHER : CLEAVER ______________
22. RIDDLE : CRYPTIC ______________
23. KAYAK : BOAT ______________
24. BROOM : SWEEP ______________
25. SPORIFIC : SLEEP ______________

Answer for the above analogy type

1. Worker and tool
2. Part of whole
3. Class and member
4. Tool and its action. A drill is a tool used to bore holes.
5. Part to whole. A minute is part of an hour.
6. Defining characteristic. A cow is defined as herbivorous.
7. Class and member. A tent is a kind of shelter.
8. Defining characteristics. An ewe is a female sheep: a ram, a male sheep.
9. Defining characteristic. A shoat is a young pig.
10. Symbol and what it represents. The laurel is the symbol of victory.
11. Defining characteristic. A fawn is a young deer.
12. Class and member. One example of a meal is lunch.
13. Worker and article created. A sculptor creates a statue.
14. Worker and workplace. A teller works in a bank.
15. Antonyms disperse (scatter) and assemble are opposites.
16. Degree of intensity. Drenched means extremely wet; moist, only moderately so.
17. Antonyms. Adore and loathe (hate) are opposites.
18. Worker and tool. A sculptor uses a mallet.
19. Synonyms. Veracious and truthful have the same meaning.
20. Function. An angler tries to catch fish.
21. Worker and tool. A butcher uses clever.
22. Defining characteristic. A riddle is by definition cryptic (mysterious).
23. Class and member. A kayak is a kind of boat.
24. Tool and object. A broom is a tool used to sweep.
25. Cause and effect. Something soporific induces sleep.

UNIT FOUR

SENTNCE COMPLETION

- In this type of question, you are given a sentence with **one** or **two words** omitted. You are also given **four** possible choices. You have to select the best of the **four** possible answers provided.

Basic Strategies

1. Before you look at the choices, read the sentence and think of a word that makes sense. The word you think of may not be the exact word that appears in the answer choices, but it will probably be similar in meaning to the right answer.
2. Look at all the possible answers before you make your final choice. You are looking for the word that **best** fits the meaning of the sentence as a whole. In order to be sure you have not bean hasty in making your decision, substitute all the answer choices for the missing word. That way you can satisfy yourself that you have come up with the answer that best fits.
3. In double-blank sentence, go through the answers, testing the **first** word in each choice (and eliminating those that don't fit). Read through the entire sentence. Then, insert the first word of each answer pair in the sentence's first blank. Ask yourself whether this particular word makes sense in this blank. If the initial word of an answer pair makes no sense in the sentence, you can eliminate that answer pair.
4. Use your knowledge of word parts and context clues to get at the meanings of unfamiliar words. If a word used unknown to you, look at its context in the sentence to see whether the context provides a clue to the meaning of the word. Often authors will use an unfamiliar word and then immediately define it within the same sentence. Similarly, look for familiar word parts **-prefixes, suffixes,** and **roots-** in unfamiliar words.

5. Watch out for negative words and words signaling frequency or duration. Only a small change makes these two sentences very different in meaning.

They were not lovers.

They were not often lovers.

6. Look for words which indicate that the omitted portion of the sentence continues a thought developed elsewhere in the sentence. Examples are **and, moreover, in addition,** and **furthermore** In such cases, a synonym or near-synonym for another words in the sentences should provide the correct answer.

7. Look for words or phrases which indicate a contrast between one idea and another.

Examples are **but, nevertheless, although, despite, however, eventhough, even though,** and **on the other hand**. In such cases, an antonym or near-antonym for another word in the sentence should provide the correct answer.

8. Look for words or phrases that indicate that one thing causes another –words like **because, since, therefore, thus, consequently, accordingly, hence, for**, etc.

UNIVERSITY ENTRANCE EXAMINATION AND OTHER QUESTIONS WITH DETAILED EXPLANATIONS. (2001 – 2009)

1. My ________ were ______ when I heard his explanation I was convinced that he was telling the truth.
   A. doubts… dispelled　　C. fears…distracted
   B. suspicious… confirmed　　D. misgivings…aroused

Clues: If the listener believed the speaker, any **doubts** that he might have had, must have been **dispelled** (made to disappear).

Answer: A

2. The owners of the spa advertised that their ________ were especially ________ for the arthritic.
   A. waters … healthful　　C. water…deleterious
   B. mountains…beneficial　　D. spring … toxic

Clues: A spa is a mineral spring. Its **waters** are thought to be **healthful** (conductive to health).

Answer: A

Note: The difference in meaning between **healthful** and **healthy** (having good health).

UNIT FIVE

LANGUAGE USAGE

(standard written English)

Usage Questions And Sentence Correction

In **Usage Questions,** four words or groups of words will be underlined in each sentence. You'll be asked to find the **error**, if there is one, in one of the underlined parts. You don't have to correct the sentence; you only have to correct the sentence; you only have to identify the **error**.

With **sentence correction Questions** you have to do more than just spot the error. You have to find the correction as well. These questions give you sentences in which one section is underlined. The answer choices repeat the underlined section and give you four other versions of the same section. You must decide which version is best. Many of these questions cover errors in the structure or logic of a sentence.

Usage Questions Tactics

1. In most cases, you should be able to spot the error when you read sentence. Think how it would sound if you were to read it would sound if you were to read it aloud. Pay special attention to any phrases that sound awkward.
2. If you cannot identify the error immediately. Use the following check list:
   a. If the underlined part is noun, look for errors is case, agreement with verb, parallel structure, and direction.
   b. If the underlined part is pronoun, look for errors in case, agreement with antecedent, agreement with verb, parallel structure, and diction.
   c. If the underlined part is verb, look for errors in agreement with subject, tense, mood, parallel, structure, and diction.

d. If the underlined part is an adjective, look for errors in degree of comparison and in diction. Make certain that the word modifies a noun or pronoun.
e. If the underlined part is an adverb, look for errors in degree of comparision and in diction. Make certain that the word modifies a verb, an adjective, or another adverb.
f. If the underlined part is a participle, check whether it has a word to modify in the sentence. Otherwise, it is a dangling participle.
g. If the underlined part is a phrase, look for errors, in sentence structure and in parallel structure.
h. If the underlined part is a clause, look for errors in sentence structure and in parallel structure.
i. If you find no errors, choose E as your answer.

Sentence Correction Questions Tactics

1. Use the check list supplied above to test for errors in the underlined part of the sentence.
2. Examine the four or five choices provided to find the one which corrects the error you have found.
3. Note that there may be more than one error in the underlined part. Be sure to correct all errors.
4. Don't change the meaning of the sentence when you make correction.
5. If you think the underlined part is correct, you should select choice "A" as your answer choice "A" repeats the underlined part without any changes.

UNIVERSITY ENTRANCE EXAMINATION AND OTHERS QUESTIONS WITH ANSWER AND EXPLANATIONS. (2001-2009)

✧ The following sentences contain problems in **grammar, usage, diction (choice of words)**, and **idioms**.

Some sentences are correct.

No sentence contains more than one error.

You'll find that the error, if there is one, is underlined and lettered.

Assume that elements of the sentence that are not underlined are correct

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