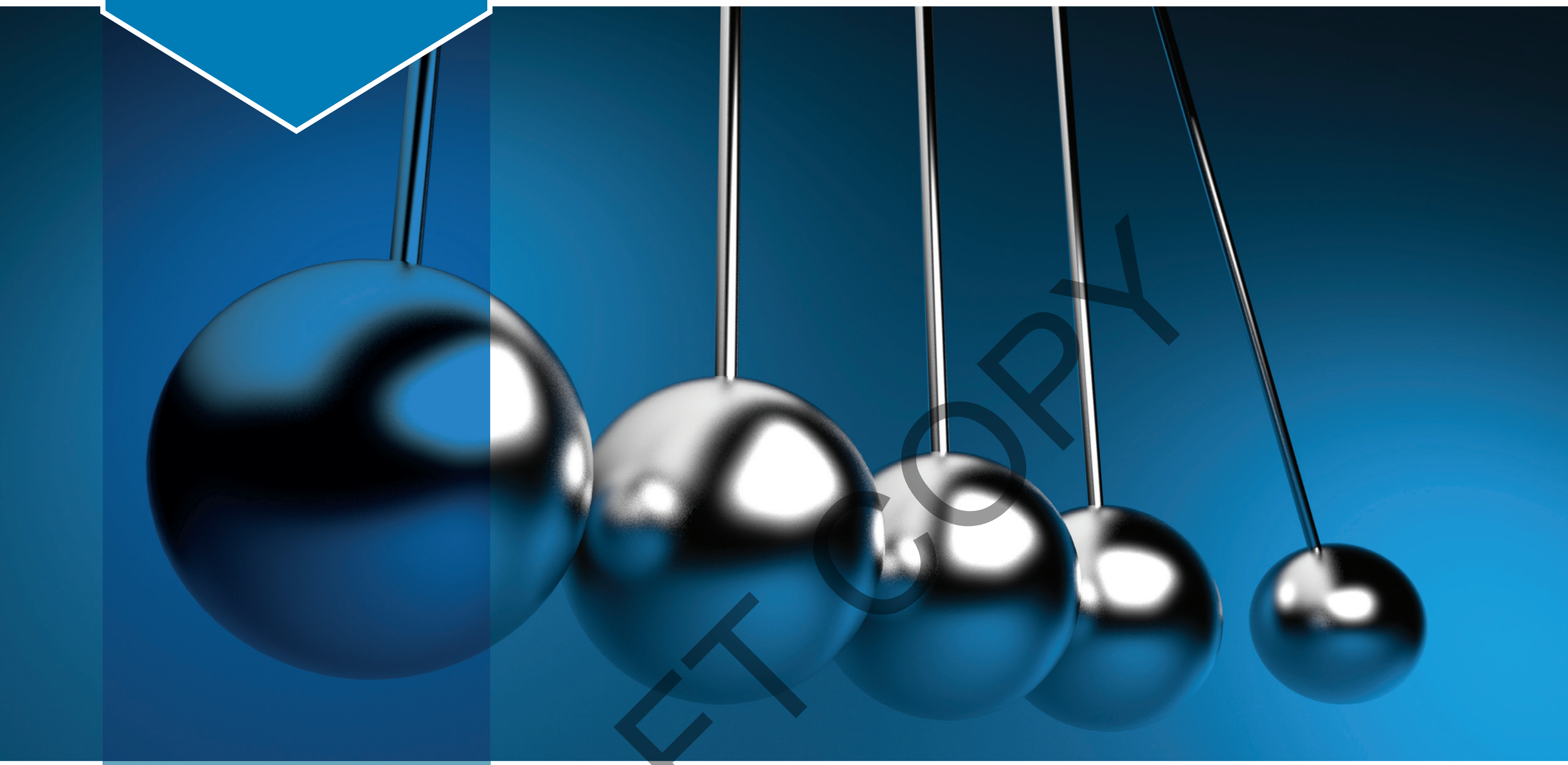
5 Work, energy and power



I like work: it fascinates me. I can sit and look at it for hours.

Jerome K. Jerome

Discussion point

This photograph shows the tidal power station at Annapolis in Nova Scotia, Canada. Where does the energy it produces come from?

Figure 5.1

1 Energy and momentum

When describing the motion of objects in everyday language the words *energy* and *momentum* are often used quite loosely and sometimes no distinction is made between them. In mechanics they must be defined precisely.

For an object of mass m moving with velocity $\mathbf{v}$:

- Kinetic energy $= \frac{1}{2}mv^2$ (this is the energy it has due to its motion)
- Momentum $= m\mathbf{v}$

v is the speed, the magnitude of the velocity $v = |\mathbf{v}|$.

Notice that kinetic energy is a scalar quantity with magnitude only, but momentum is a vector in the same direction as the velocity.

Both the kinetic energy and the momentum are liable to change when a force acts on a body and you will learn more about how the energy is changed in this chapter. You will meet momentum again in Chapter 6.

2 Work and energy

In everyday life you encounter many forms of energy such as heat, light, electricity and sound. You are familiar with the conversion of one form of energy to another: from chemical energy stored in wood to heat energy when you burn it; from electrical energy to the energy of a train's motion, and so on. The S.I. unit for energy is the joule, J.

Mechanical energy and work

In mechanics, two forms of energy are particularly important.

- *Kinetic energy* is the energy which a body possesses because of its motion.
- The kinetic energy of a moving object $= \frac{1}{2} \times \text{mass} \times (\text{speed})^2$.
- *Potential energy* is the energy which a body possesses because of its position. It may be thought of as stored energy which can be converted into kinetic energy or other forms of energy. You will meet this again later in this chapter.
- The energy of an object is usually changed when it is acted on by a force. When a force is applied to an object which moves in the direction of its line of action, the force is said to do *work*.
  The work done by a constant force = force × distance moved in the direction of the force.

The following examples illustrate how to use these ideas.

Example 5.1

A brick, initially at rest, is raised by a force averaging 40 N to a height of 5 m above the ground, where it is left stationary. How much work is done by the force?

Solution

The work done by the force in raising the brick is given by

$$\begin{aligned}\text{work done} &= \text{force} \times \text{distance}\\ &= 40 \times 5\\ &= 200\text{ J}\end{aligned}$$

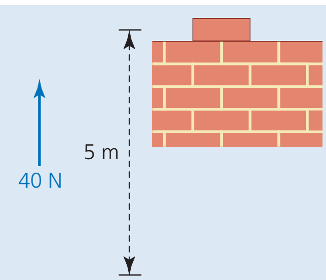


Figure 5.2

Examples 5.2 and 5.3 show how the work done by a force can be related to the change in kinetic energy of an object.

Example 5.2

A train travelling on level ground is subject to a resistive force (from the brakes and air resistance) of 250 kN for a distance of 5 km. How much kinetic energy does the train lose?

Note

Work and energy have the same units.

Solution

The forward force is −250 000 N.

The work done by it is −250 000 × 5000 = −1 250 000 000 J.

So −1 250 000 000 J of kinetic energy are gained by the train. In other words, +1 250 000 000 J of kinetic energy are lost and the train slows down. This energy is converted to other forms such as heat and perhaps a little sound.

Example 5.3

A car of mass m kg is travelling at $u\,\mathrm{m\,s^{-1}}$ when the driver applies a constant driving force F N. The ground is level and the road is straight and resistance to motion can be ignored. The speed of the car increases to $v\,\mathrm{m\,s^{-1}}$ in a period of t s over a distance of s m. Show that the change in kinetic energy of the car is equal to the work done by the driving force.

Solution

Treating the car as a particle and applying Newton's second law:

$$F = ma$$

$$a = \frac{F}{m}$$

Since F is assumed constant, the acceleration is constant also, so using the constant acceleration equation $v^2 = u^2 + 2as$

$$v^2 = u^2 + \frac{2Fs}{m}$$

$$\Rightarrow \frac{1}{2}mv^2 = \frac{1}{2}mu^2 + Fs$$

change in kinetic energy

$$Fs = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

⇒ work done by force = final kinetic energy − initial kinetic energy of the car.

Note

This is an important result given in symbols and in words.

The work–energy principle

Examples 5.4 and 5.5 illustrate the *work–energy principle* which states that:

! The total work done by the forces acting on a body is equal to the increase in the kinetic energy of the body.

Example 5.4

A sledge of total mass 30 kg, initially moving at $2\,\mathrm{m\,s^{-1}}$, is pulled 14 m across smooth horizontal ice by a horizontal rope in which there is a constant tension of 45 N. Find its final velocity.

This is the horizontal force on the sledge.

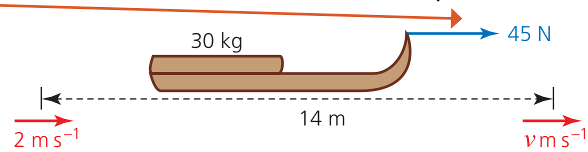


Figure 5.3

Solution

Since the ice is smooth, the work done by the force is all converted into kinetic energy and the final velocity can be found by using

work done by force = final kinetic energy − initial kinetic energy

$$45 \times 14 = \frac{1}{2} \times 30 \times v^2 - \frac{1}{2} \times 30 \times 2^2$$

So $v^2 = 46$ and the final velocity of the sledge is $6.8\,\mathrm{m\,s^{-1}}$ (to 2 s.f.).

Example 5.5

The combined mass of a cyclist and her bicycle is 65 kg. She accelerates from rest to $8\,\mathrm{m\,s^{-1}}$ in 80 m along a horizontal road.

(i) Calculate the work done by the net force in accelerating the cyclist and her bicycle.

(ii) Hence calculate the net forward force (assuming the force to be constant).

Solution

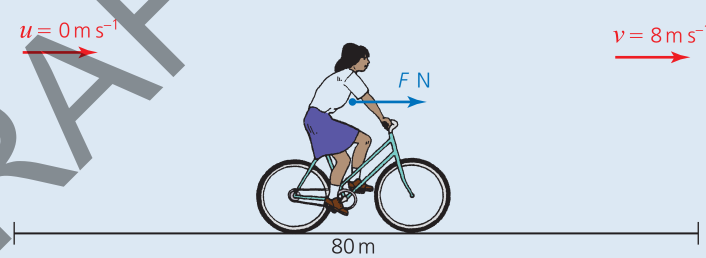


Figure 5.4

(i) The work done by the net force F is given by

$$\begin{aligned} \text{work} &= \text{final K.E.} - \text{initial K.E.} \\ &= \frac{1}{2}mv^2 - \frac{1}{2}mu^2 \\ &= \frac{1}{2} \times 65 \times 8^2 - 0 \\ &= 2080\,\mathrm{J} \end{aligned}$$

The work done is 2080 J.

(ii) $$\begin{aligned} \text{Work done} &= Fs \\ &= F \times 80 \end{aligned}$$

$$\text{So } 80F = 2080$$

$$F = 26$$

The net forward force is 26 N.

Work

It is important to realise that:

- work is done by a force
- work is only done when there is movement
- a force only does work on an object when it has a component in the direction of motion of the object.

Notice that if you stand holding a brick stationary above your head, painful though it may be, the force you are exerting on it is doing no work. Nor is the vertical force doing any work if you walk round the room holding the brick at the same height. However, once you start climbing the stairs, a component of the brick's movement is in the direction of the upward force that you are exerting on it, so the force is now doing some work.

When applying the work–energy principle, you have to be careful to include all the forces acting on the body. In the example of a brick of weight 40 N being raised 5 m vertically, starting and ending at rest, the change in kinetic energy is clearly 0.

This seems paradoxical when it is clear that the force which raised the brick has done $40 \times 5 = 200$ J of work. However, the brick was subject to another force, namely, its weight, which did $-40 \times 5 = -200$ J of work on it, giving a total of $200 + (-200) = 0$ J.

Conservation of mechanical energy

The net forward force on the cyclist in Example 5.5 is her driving force minus resistive forces such as air resistance and friction in the bearings. In the absence of such resistive forces, she would gain more kinetic energy. Also the work she does against them is lost; it is dissipated as heat and sound. Contrast this with the work a cyclist does against gravity when going uphill. This work can be recovered as kinetic energy on a downhill run. The work done against the force of gravity is conserved and gives the cyclist potential energy (see page 94).

Forces such as friction which result in the dissipation of mechanical energy are called *dissipative forces*. Forces which conserve mechanical energy are called *conservative forces*. The force of gravity is a conservative force and so is the tension in an elastic string; you can test this using an elastic band.

The mechanical energy of a system is the sum of its potential energy and its kinetic energy. If there are no dissipative forces, the mechanical energy of the system is conserved.

Example 5.6

A bullet of mass 25 g is fired directly at a wooden barrier 3 cm thick so that it hits it at a right angle. When it hits the barrier it is travelling at $200\,\mathrm{m\,s^{-1}}$. The barrier exerts a constant resistive force of 5000 N on the bullet.

(i) Does the bullet pass through the barrier and, if so, with what speed does it emerge?

(ii) Is energy conserved in this situation?

Solution

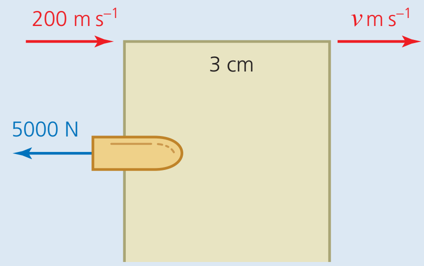


Figure 5.5

(i) The work done *by* the force is defined as the product of the force and the distance moved *in the direction of the force*. Since the bullet is moving in the direction opposite to the net resistive force, the work done by this force is negative.

$$\text{Work done} = -5000 \times 0.03$$
$$= -150\,\text{J}$$

3 cm = 0.03 m

The initial kinetic energy of the bullet is

$$\text{Initial K. E.} = \frac{1}{2}mu^2$$
$$= \frac{1}{2} \times 0.025 \times 200^2$$
$$= 500\,\text{J}$$

25 g = 0.025 kg

A loss in energy of 150 J will not reduce kinetic energy to zero, so the bullet will still be moving on exit.

Since the work done is equal to the change in kinetic energy,

$$-150 = \frac{1}{2}mv^2 - 500$$

Solving for v

$$\frac{1}{2}mv^2 = 500 - 150$$
$$v^2 = \frac{2 \times (500 - 150)}{0.025}$$
$$v = 167 \text{ (to nearest whole number)}$$

So the bullet emerges from the barrier with a speed of 167 m s^{-1}.

(ii) Total energy is conserved but there is a loss of mechanical energy of $\frac{1}{2}mu^2 - \frac{1}{2}mv^2 = 150$ J. This energy is converted into non-mechanical forms such as heat and sound.

Example 5.7

An aircraft of mass m kg is flying at a constant velocity v m s^{-1} horizontally. Its engines are providing a horizontal driving force F N.

(i) (a) Draw a diagram showing the driving force, the lift force L N, the air resistance (drag force) R N and the weight of the aircraft.

(b) State which of these forces are equal in magnitude.

(c) State which of the forces are doing no work.

(ii) In the case when $m = 100\,000$, $v = 270$ and $F = 350\,000$ find the work done in a 10-second period by those forces which are doing work, and show that the work–energy principle holds in this case.

At a later time the pilot increases the horizontal driving force of the aircraft's engines to 400 000 N. When the aircraft has travelled a distance of 30 km, its speed has increased to $300\,\mathrm{m\,s^{-1}}$.

(iii) Find the work done against air resistance during this period, and the average resistance force.

Solution

(i) (a)

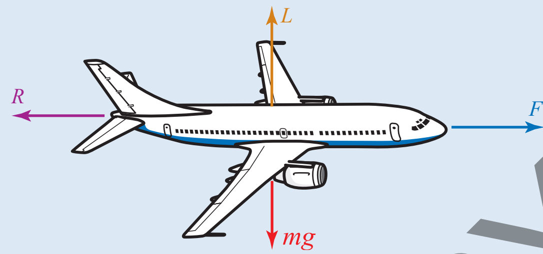


Figure 5.6

(b) Since the aircraft is travelling at constant velocity it is in equilibrium.

Horizontal forces: $F = R$

Vertical forces: $L = mg$

(c) Since the aircraft's velocity has no vertical component, the vertical forces, L and mg, are doing no work.

(ii) In 10 s at $270\,\mathrm{m\,s^{-1}}$ the aircraft travels 2700 m.

Work done by driving force $F = 350\,000 \times 2700 = 945\,000\,000\,\mathrm{J}$

Work done by resistance force $R = 350\,000 \times -2700 = -945\,000\,000\,\mathrm{J}$

The work–energy principle states that in this situation

work done by F + work done by R = change in kinetic energy.

Now work done by F + work done by R = (945 000 000 − 945 000 000) = 0 J, and change in kinetic energy = 0 (since velocity is constant), so the work–energy principle does indeed hold in this case.

(iii) Final K.E. − initial K.E. $= \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

$= \frac{1}{2} \times 100\,000 \times 300^2 - \frac{1}{2} \times 100\,000 \times 270^2$

$= 855 \times 10^6\,\mathrm{J}$

Work done by driving force $= 400\,000 \times 30\,000$

$= 12\,000 \times 10^6\,\mathrm{J}$

Total work done = K.E. gained

Work done by resistance force + $12\,000 \times 10^6 = 855 \times 10^6$

Work done by resistance force $= -11\,145 \times 10^6\,\mathrm{J}$

Average force × distance = work done by force

Average force $\times 30\,000 = 11\,145 \times 10^6$

⇒ The average resistance force is 371 500 N (in the negative direction).

Note

When an aircraft is in flight, most of the work done by the resistance force results in air currents and the generation of heat. A typical large jet cruising at 35 000 feet has a body temperature about 30°C above the surrounding air temperature. For supersonic flight the temperature difference is much greater. Concorde used to fly with a skin temperature more than 200°C above that of the surrounding air.

Exercise 5.1

1. Find the kinetic energy of the following objects.
   (i) An ice skater of mass 50 kg travelling with speed $10\,\mathrm{m\,s^{-1}}$.
   (ii) An elephant of mass 5 tonnes moving with speed $4\,\mathrm{m\,s^{-1}}$.
   (iii) A train of mass 7000 tonnes travelling with speed $40\,\mathrm{m\,s^{-1}}$.
   (iv) The Moon of mass 7.4×10^{22} kg, travelling at $1000\,\mathrm{m\,s^{-1}}$ in its orbit around the Earth.
   (v) A bacterium of mass 2×10^{-16} g which has a speed of $1\,\mathrm{mm\,s^{-1}}$.
2. Find the work done by a man in the following situations.
   (i) He pushes a packing case of mass 35 kg a distance of 5 m across a rough floor against a resistance of 200 N. The case starts and finishes at rest.
   (ii) He pushes a packing case of mass 35 kg a distance of 5 m across a rough floor against a resistance of 200 N. The case starts at rest and finishes with a speed of $2\,\mathrm{m\,s^{-1}}$.
   (iii) He pushes a packing case of mass 35 kg a distance of 5 m across a rough floor against a resistance of 200 N. Initially the case has speed $2\,\mathrm{m\,s^{-1}}$ but it ends at rest.
   (iv) He is handed a packing case of mass 35 kg. He holds it stationary, at the same height, for 20 s and then someone else takes it away from him.
3. A sprinter of mass 60 kg is at rest at the beginning of a race and accelerates to $12\,\mathrm{m\,s^{-1}}$ in a distance of 30 m. Assume air resistance is negligible.
   (i) Calculate the kinetic energy of the sprinter at the end of the 30 m.
   (ii) Write down the work done by the sprinter over this distance.
   (iii) Calculate the forward force exerted by the sprinter, assuming it to be constant, using work = force × distance.
   (iv) Using force = mass × acceleration and the constant acceleration formulae, show that this force is consistent with the sprinter having a speed of $12\,\mathrm{m\,s^{-1}}$ after 30 m.
4. A sports car of mass 1.2 tonnes accelerates from rest to $30\,\mathrm{m\,s^{-1}}$ in a distance of 150 m. Assume air resistance to be negligible.
   (i) Calculate the work done in accelerating the car. Does your answer depend on an assumption that the driving force is constant?
   (ii) If the driving force is, in fact, constant, what is its magnitude?
   (iii) Is it reasonable to assume that air resistance is negligible?
5. A car of mass 1600 kg is travelling at a speed of $25\,\mathrm{m\,s^{-1}}$ when the brakes are applied so that it stops after moving a further 75 m. Assuming that other resistive forces can be neglected, find
   (i) the work done by the brakes.
   (ii) the retarding force from the brakes, assuming that it is constant.
6. The forces acting on a hot air balloon of mass 500 kg are its weight and the total uplift force.
   (i) Find the total work done when the vertical speed of the balloon changes from
      (a) $2\,\mathrm{m\,s^{-1}}$ to $5\,\mathrm{m\,s^{-1}}$ (b) $8\,\mathrm{m\,s^{-1}}$ to $3\,\mathrm{m\,s^{-1}}$.
   (ii) If the balloon rises 100 m vertically while its speed changes, calculate, in each case, the work done by the uplift force.

⑦ A bullet of mass 20 g, found at the scene of a police investigation, had penetrated 16 cm into a wooden post. The speed for that type of bullet is known to be $80\,\mathrm{m\,s^{-1}}$.

(i) Find the kinetic energy of the bullet before it entered the post.

(ii) What happened to this energy when the bullet entered the wooden post?

(iii) Write down the work done in stopping the bullet,

(iv) Calculate the resistive force on the bullet, assuming it to be constant.

Another bullet of the same mass and shape had clearly been fired from a different and unknown type of gun. This bullet had penetrated 20 cm into the post.

(v) Estimate the speed of this bullet before it hit the post.

⑧ The highway code gives the braking distance for a car travelling at $22\,\mathrm{m\,s^{-1}}$ (50 mph) to be 38 m (125 ft). A car of mass 1300 kg is brought to rest in just this distance. It may be assumed that the only resistance forces come from the car's brakes.

(i) Find the work done by the brakes.

(ii) Find the average force exerted by the brakes.

(iii) What happened to the kinetic energy of the car?

(iv) What happens when you drive a car with the handbrake on ?

⑨ A car of mass 1200 kg experiences a constant resistance force of 600 N. The driving force from the engine depends upon the gear, as shown in the table.

Gear	1	2	3	4
Force (N)	2800	2100	1400	1000

Starting from rest, the car is driven 20 m in first gear, 40 m in second, 80 m in third and 100 m in fourth. How fast is the car travelling at the end?

⑩ In this question take g to be $10\,\mathrm{m\,s^{-2}}$. A crate of mass 60 kg is resting on a rough horizontal floor. The coefficient of friction between the floor and the crate is 0.4. A woman pushes the crate in such a way that its speed–time graph is as shown below.

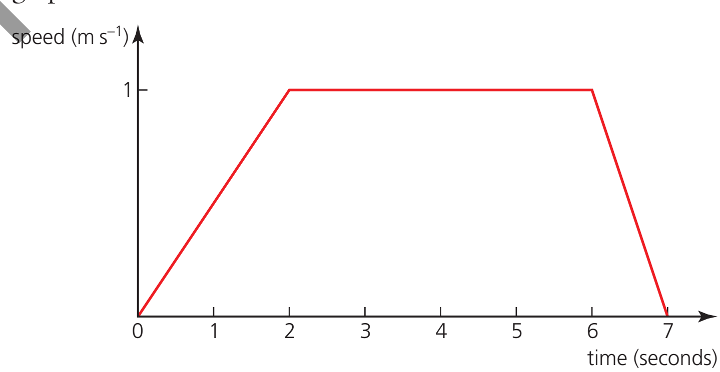


Figure 5.7

(i) Find the force of frictional resistance acting on the crate when it moves.

(ii) Use the speed–time graph to find the total distance travelled by the crate.

(iii) Find the total work done by the woman.

(iv) Find the acceleration of the crate in the first 2 seconds of its motion and hence the force exerted by the woman during this time, and the work done.

(v) In the same way find the work done by the woman during the time intervals 2 to 6 seconds, and 6 to 7 seconds.

(vi) Show that your answers to parts (iv) and (v) are consistent with your answer to part (iii).

3 Gravitational potential energy

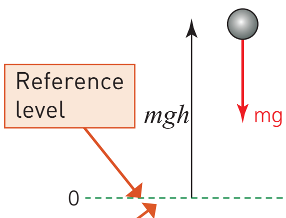


Figure 5.8

You will often have to choose the reference level for a particular situation. It could be ground level, the top of a building or the height of an aircraft. There is no right answer but choosing a suitable level can make your calculations easier.

As you have seen, kinetic energy (K.E.) is the energy that an object has because of its motion. Potential energy (P.E.) is the energy an object has because of its position. The units of potential energy are the same as those of kinetic energy or any other form of energy, namely, joules.

One form of potential energy is *gravitational potential energy*. The gravitational potential energy of the object in Figure 5.8 of mass m kg at height h m above a particular reference level, 0, is mgh J. If it falls to the reference level, the force of gravity does mgh J of work and the body loses mgh J of potential energy.

A loss in gravitational potential energy is an alternative way of accounting for the work done by the force of gravity.

If a mass m kg is *raised* through a distance h m, the gravitational potential energy *increases* by mgh J. If a mass m kg is *lowered* through a distance h m the gravitational potential energy *decreases* by mgh J.

Example 5.8

Calculate the gravitational potential energy, relative to the ground, of a ball of mass 0.15 kg at a height of 2 m above the ground.

Solution

Mass $m = 0.15$, height $h = 2$.

$$\text{Gravitational potential energy} = mgh$$
$$= 0.15 \times 9.8 \times 2$$
$$= 2.94\text{ J}$$

Note

Assuming no other forces, the gain in K.E. is 2.94 J so that the maximum theoretical speed of the ball when it hits the ground is $\sqrt{\frac{2\times 2.94}{0.15}} = 6.26\text{ m s}^{-1}$.

In reality, there would be air resistance, so the ball's speed would be less than this.

Note

If the ball falls:

loss in P.E. = work done by gravity

= gain in K.E.

There is no change in the total energy (P.E. + K.E.) of the ball assuming there are no other forces acting on the ball.

Using conservation of mechanical energy

When gravity is the only force which does work on a body, mechanical energy is conserved. When this is the case, many problems are easily solved using energy. This is possible even when the acceleration is not constant.

Example 5.9

A skier starts from rest and slides down a ski slope 400 m long which is at an angle of 30° to the horizontal. The slope is smooth and air resistance can be neglected.

(i) Find the speed of the skier when he reaches the bottom of the slope.

At the foot of the slope the ground becomes horizontal and is made rough in order to help skiers stop. The coefficient of friction between the skis and the ground is $\frac{1}{4}$.

(ii) Find how far the skier travels before coming to rest.

(iii) In what way is your model unrealistic?

Solution

(i) The skier is modelled as a particle.

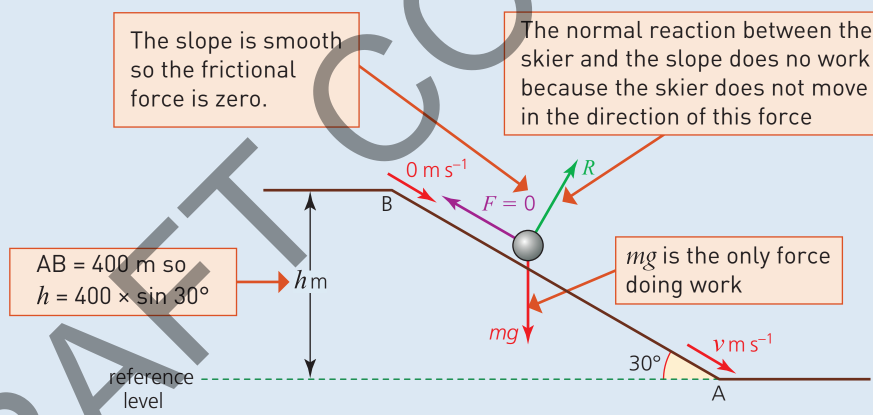


Figure 5.9

Mechanical energy is conserved.

$$\text{Total mechanical energy at B} = mgh + \frac{1}{2}mu^2$$
$$= m \times 9.8 \times 400 \sin 30° + 0$$
$$= 1960\,m \text{ J}$$

$$\text{Total mechanical energy at A} = \left(0 + \frac{1}{2}mv^2\right) \text{J}$$

Since mechanical energy is conserved.

$$\frac{1}{2}mv^2 = 1960m$$
$$v^2 = 3920$$
$$v = 62.6\ \ldots..$$

Notice that the mass of the skier cancels out. In this model all skiers should arrive at the bottom of the slope with the same speed.

The slope could be curved and the model would give the same speed as long as the total height lost is the same.

The skier's speed at the bottom of the slope is 62.6 m s⁻¹ (to 3 s.f.).

(ii) For the horizontal part there is some friction. Suppose the skier travels a further distance s m before stopping.

Note

An alternative approach to this problem is to use the constant acceleration formulae. Make sure you are comfortable with both approaches.

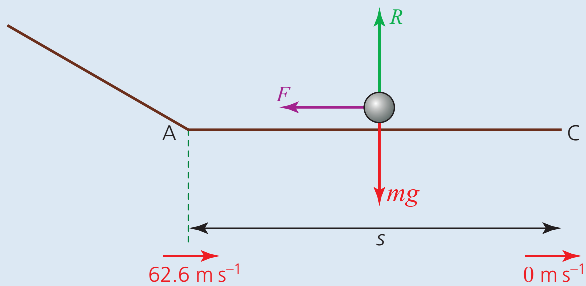


Figure 5.10

The frictional force is $F = \mu R = \frac{1}{4}R$.

There is no vertical acceleration, so $R = mg$

and so $F = \frac{1}{4}mg$.

Negative because the force is in the opposite direction to the motion.

Work done by the friction force $= -F \times s = -\frac{1}{4}mgs$

The increase in kinetic energy between A and C $= \left(0 - \frac{1}{2}mv^2\right)$ J.

$$-\frac{1}{4}mgs = -\frac{1}{2}mv^2 = -1960m \text{ from } ①$$

The increase in kinetic energy $0 - \frac{1}{2}mv^2$ = work done

Solving for s gives $s = 800$.

So the distance the skier travels before stopping is 800 m.

(iii) The assumptions made in solving this problem are that friction on the slope and air resistance are negligible, and that the slope ends in a smooth curve at A. Clearly the speed of 62.6 m s^{-1} is very high, so the assumption that air resistance is negligible must be suspect.

Example 5.10

Ama, whose mass is 40 kg, is taking part in an assault course. The obstacle shown in Figure 5.11 is a river at the bottom of a ravine that is 8 m wide. She has to cross the ravine by swinging on an inextensible rope 5 m long secured to a point on the branch of a tree, immediately above the centre of the ravine.

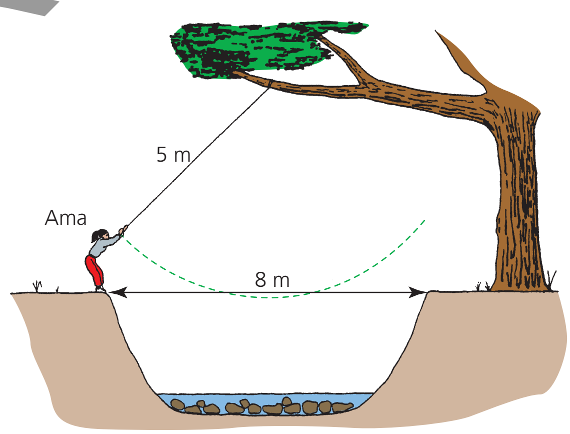


Figure 5.11

(i) Find how fast Ama is travelling at the lowest point of her crossing

(a) if she starts from rest

(b) if she launches herself off at a speed of 1 m s^{-1}.

(ii) Will her speed be 1 m s^{-1} faster throughout her crossing?

Solution

(i) (a) The vertical height Ama loses is HB in the diagram.

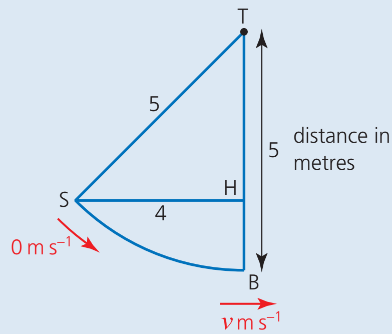


Figure 5.12

Using Pythagoras' theorem in triangle TSH.

$TH = \sqrt{5^2 - 4^2} = 3$

$HB = 5 - 3 = 2$

P.E. lost $= mgh$

$= 40 \times 9.8 \times 2 = 784$

K.E. gained $= \frac{1}{2}mv^2 - 0$

$= \frac{1}{2} \times 40 \times v^2 = 20v^2$

Conservation of mechanical energy, K.E. gained = P.E. lost

$20v^2 = 784$

$v = \sqrt{39.2} = 6.26$

Ama is travelling at $6.26\,\mathrm{m\,s^{-1}}$.

(b) If she has initial speed $1\,\mathrm{m\,s^{-1}}$ at S and speed $v\,\mathrm{m\,s^{-1}}$ at B, her initial K.E. is $\frac{1}{2} \times 40 \times 1^2$ and her K.E. at B is $\frac{1}{2} \times 40 \times v^2$.

Using conservation of mechanical energy,

$\frac{1}{2} \times 40 \times v^2 - \frac{1}{2} \times 40 \times 1^2 = 40 \times 9.8 \times 2$

$20v^2 - 20 = 784$

$v = 6.34$

Ama is travelling at $6.34\,\mathrm{m\,s^{-1}}$.

(ii) Ama's speed at the lowest point is only $0.08\,\mathrm{m\,s^{-1}}$ faster in part (i) (b) compared with that in part (i) (a), so she clearly will not travel $1\,\mathrm{m\,s^{-1}}$ faster throughout the crossing in part (i) (b).

Historical note

James Joule was born in Salford in Lancashire on Christmas Eve 1818. He studied at Manchester University at the same time as the famous chemist, John Dalton.

Joule spent much of his life conducting experiments to measure the equivalence of heat and mechanical forms of energy to ever increasing degrees of accuracy. Working with Thompson, he also made the discovery that when a gas is allowed to expand without doing work against external forces it cools. It was this discovery that paved the way for the development of refrigerators. Joule died in 1889; his contribution to science is remembered with the S.I. unit for energy named after him.

4 Work and kinetic energy for two-dimensional motion

Discussion point

Imagine that you are cycling along a level winding road in a strong wind. Suppose that the strength and direction of the wind are constant, but, because the road is winding, sometimes the wind is directly against you but at other times it is from your side or even behind you.

How does the work you do in travelling a certain distance, say 1 m, change with your direction?

Work done by a force at an angle to the direction of motion

You have probably decided that as a cyclist you would do work against the component of the wind force that is directly against you. The sideways component does not resist your forward progress.

Suppose you are cycling along the straight bit of the horizontal road OP shown (from above) in the diagram. The force of the wind on you is F N. In a certain time you travel a distance s m along the road. The component of this distance in the direction of F is d m, as shown in the diagram.

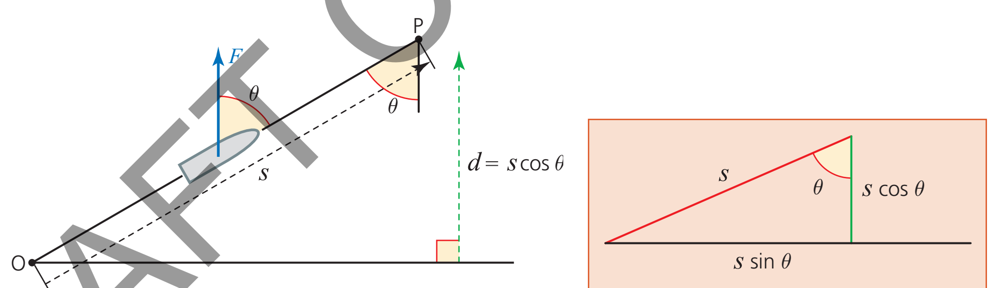


Figure 5.13

Work done by $F = Fd$

The work done by the force F is $Fs \cos\theta$. This can also be written as the product of the component of F along OP, $F \cos\theta$, and the distance moved along OP, s.

$$F \times s\cos\theta = F\cos\theta \times s$$

Example 5.11

A car of mass m kg drives up a slope inclined at an angle α to the horizontal, along a line of greatest slope. It experiences a constant resistive force F N and a driving force D N.

(i) Draw a diagram showing the forces acting on the car.

(ii) State what can be said about the work done by each of the forces as the car moves a distance d up the slope.

(iii) What can be deduced about the work done by the force D when

(a) the car moves at constant speed?

(b) the car slows down?

(c) the car gains speed?

The initial and final speeds of the car are denoted by $u\,\mathrm{ms}^{-1}$ and $v\,\mathrm{ms}^{-1}$, respectively.

(iv) Write v^2 in terms of the other variables.

Solution

(i)

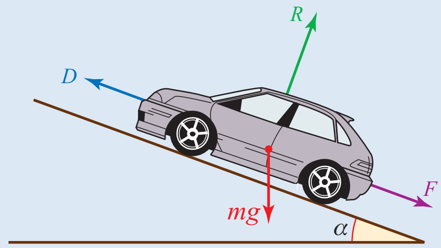


Figure 5.14

(ii)

Force	Work done
Resistance F	$-Fd$
Normal reaction	0
Force of gravity mg	$-mgd\sin\alpha$
Driving force D	Dd
Total work done	$Dd - Fd - mgd\sin\alpha$

(iii) (a) If the car moves at a constant speed, there is no change in kinetic energy so the total work done is zero, giving

work done by D is

$$Dd = Fd + mgd\sin\alpha$$

(b) If the car slows down, the total work done by the forces is negative, and hence work done by D is

$$Dd < Fd + mgd\sin\alpha$$

(c) If the car gains speed, the total work done by the forces is positive so work done by D is

$$Dd > Fd + mgd\sin\alpha.$$

(iv) Total work done = Final K.E. – initial K.E.

$$\Rightarrow \quad Dd - Fd - mgd\sin\alpha = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

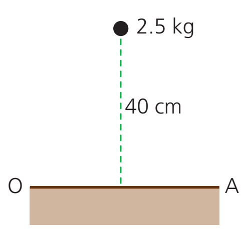
Multiplying by $\frac{2}{m}$

$$\Rightarrow \quad v^2 = u^2 + \frac{2d}{m}(D - F) - 2gd\sin\alpha$$

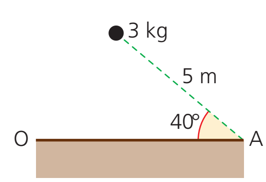
Exercise 5.2

① Calculate the gravitational potential energy, relative to the reference level OA, for each of the objects shown.

(i)



(ii)



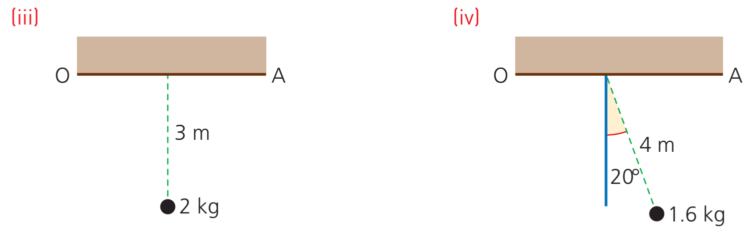


Figure 5.15

② Calculate the change in gravitational potential energy when each object moves from A to B in the situation shown below. State whether the change is an increase or a decrease.

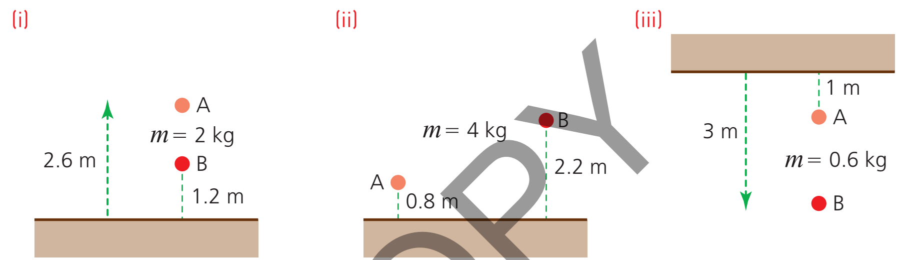


Figure 5.16

③ A vase of mass 1.2 kg is lifted from ground level and placed on a shelf at a height of 1.5 m. Find the work done against the force of gravity.

④ Find the increase in gravitational potential energy of a woman of mass 60 kg who climbs to the twelfth floor of a block of flats. The distance between the floors is 3.3 m.

⑤ A sledge of mass 10 kg is being pulled across level ground by a rope which makes an angle of 20° with the horizontal. The tension in the rope is 80 N and there is a resistance force of 14 N.

(i) Find the work done while the sledge moves a distance of 20 m by

(a) the tension in the rope

(b) the resistance force.

(ii) Find the speed of the sledge after it has moved 20 m

(a) if it starts at rest

(b) if it starts at $4\,\mathrm{m\,s^{-1}}$.

⑥ A bricklayer carries a hod of bricks of mass 25 kg up a ladder of length 10 m inclined at an angle of 60° to the horizontal.

(i) Calculate the increase in the gravitational potential energy of the bricks.

(ii) If instead he had raised the bricks vertically to the same height, using a rope and pulleys, would the increase in potential energy be less, the same, or more than in part (i)?

⑦ A girl of mass 45 kg slides down a smooth water chute of length 6 m inclined at an angle of 40° to the horizontal.

(i) Find

(a) the decrease in her potential energy

(b) her speed at the bottom.

(ii) How are answers to part (i) affected if the slide is not smooth?

8. A gymnast of mass 50 kg swings on a rope of length 10 m. Initially the rope makes an angle of 50° with the vertical.
   (i) Find the decrease in her potential energy when the rope has reached the vertical.
   (ii) Find her kinetic energy and hence her speed when the rope is vertical, assuming that air resistance may be neglected.
   (iii) The gymnast continues to swing. What angle will the rope make with the vertical when she is next temporarily at rest?
   (iv) Explain why the tension in the rope does no work.
9. A car of mass 0.9 tonnes is driven 200 m up a slope inclined at 5° to the horizontal. There is a resistive force of 100 N.
   (i) Find the work done by the car against gravity.
   (ii) Find the work done against the resistance.
   (iii) When asked to work out the total work done by the car, a student replied '$(900g + 100) \times 200$ J'. Explain the error in this answer.
   (iv) If the car slows down from $12\,\mathrm{m\,s^{-1}}$ to $8\,\mathrm{m\,s^{-1}}$, what is the total work done by the engine?
10. A stone of mass 0.2 kg is dropped from the top of a building 80 m high. After t s it has fallen a distance x m and has speed $v\,\mathrm{m\,s^{-1}}$. Air resistance may be neglected.
   (i) What is the gravitational potential energy of the stone relative to ground level when it is at the top of the building?
   (ii) What is the potential energy of the stone t s later?
   (iii) Show that, for certain values of t, $v^2 = 19.6x$ and state the range of values of t for which it is true.
   (iv) Find the speed of the stone when its kinetic energy is 10 J.
   (v) Find the kinetic energy of the stone when its gravitational potential energy relative to the ground is 66.8 J.
11. Wesley, whose mass is 70 kg, inadvertently steps off a bridge 50 m above water. When he hits the water, Wesley is travelling at $25\,\mathrm{m\,s^{-1}}$.
   (i) Calculate the potential energy Wesley has lost and the kinetic energy he has gained.
   (ii) Find the size of the resistive force acting on Wesley while he is in the air, assuming it to be constant.

   Wesley descends to a depth of 5 m below the water surface, then returns to the surface.
   (iii) Find the total upthrust (assumed constant) acting on him while he is moving downwards in the water.
12. A hockey ball of mass 0.15 kg is hit from the centre of the pitch. Its position vector (in m), t s later is modelled by

$$\mathbf{r} = 10t\,\mathbf{i} + \left(10t - 4.9t^2\right)\mathbf{j}$$

   where the unit vectors $\mathbf{i}$ and $\mathbf{j}$ are in the directions along the line of the pitch and vertically upwards.
   (i) What value of g is used in this model?
   (ii) Find an expression for the gravitational potential energy of the ball at time t. For what values of t is your answer valid?

(iii) What is the maximum height of the ball? What is its velocity at that instant?

(iv) Find the initial velocity, speed and kinetic energy of the ball.

(v) Show that, according to this model, mechanical energy is conserved and state what modelling assumption is implied by this. Is it reasonable in this context?

13 A ski-run starts at altitude 2471 m and ends at 1863 m.

(i) If all resistive forces could be ignored, what would the speed of a skier be at the end of the run?

A particular skier of mass 70 kg actually attains a speed of $42\,\mathrm{m\,s^{-1}}$. The length of the run is 3.1 km.

(ii) Find the average force of resistance acting on a skier.

Two skiers are equally skilful.

(iii) Which would you expect to be travelling faster by the end of the run, the heavier or the lighter?

14 A tennis ball of mass 0.06 kg is hit vertically upwards with speed $20\,\mathrm{m\,s^{-1}}$ from a point 1.1 m above the ground. It reaches a height of 16 m.

(i) Find the initial kinetic energy of the ball, and its potential energy when it is at its highest point.

(ii) Calculate the loss of mechanical energy due to air resistance.

(iii) Find the magnitude of the air resistance on the ball, assuming it to be constant while the ball is moving.

(iv) With what speed does the ball land?

15 Akosua draws water from a well 12 m below the ground. Her bucket holds 5 kg of water and by the time she has pulled it to the top of the well it is travelling at $1.2\,\mathrm{m\,s^{-1}}$.

(i) How much work does Akosua do in drawing the bucket of water?

On an average day, 150 people in the village each draw 6 such buckets of water. One day, a new electric pump is installed that takes water from the well and fills an overhead tank 5 m above ground level every morning. The flow rate through the pump is such that the water has speed $2\,\mathrm{m\,s^{-1}}$ on arriving in the tank.

(ii) Assuming that the villagers' demand for water remains unaltered, how much work does the pump do in one day?

16 A block of mass 20 kg is pulled up a slope passing through points A and B with speeds $10\,\mathrm{m\,s^{-1}}$ and $2\,\mathrm{m\,s^{-1}}$ respectively. The distance between A and B is 100 m and B is 12 m higher than A. For the motion of the block from A to B, find

(i) the loss in kinetic energy of the block

(ii) the gain in potential energy of the block.

The resistance to motion of the block has magnitude 10 N.

(iii) Find the work done by the pulling force acting on the block.

The pulling force acting on the block has constant magnitude 25 N and acts at an angle α upwards from the slope.

(iv) Find the value of α.

5 Power

Figure 5.17

It is claimed that a motorcycle engine can develop a maximum *power* of 26.5 kW at a top *speed* of 165 km h^{-1}. This suggests that power is related to speed and this is indeed the case.

Power is the rate at which work is being done. A powerful car does work at a greater rate than a less powerful one.

Think of a force, F, acting for a very short time t over a small distance s. Assume F to be constant over this short time.

Power is the rate of working so

$$\text{power} = \frac{\text{work}}{\text{time}}$$

$$= \frac{Fs}{t}$$

$$= Fv$$

Note

This gives you the power at an *instant* of time. The result is true whether or not F is constant.

The power of a vehicle moving at speed v under a driving force F is given by Fv.

For a motor vehicle the power is produced by the engine, whereas for a bicycle it is produced by the cyclist. They both make the wheels turn, and the friction between the rotating wheels and the ground produces a forward force on the machine.

The unit of power is the watt (W), named after James Watt. The power produced by a force of 1 N acting on an object that is moving at 1 m s^{-1} is 1 W. Because the watt is such a small unit, you will probably use kilowatts more often (1 kW = 1000 W).

Example 5.12

A car of mass 1000 kg can produce a maximum power of 45 kW. Its driver wishes to overtake another vehicle. Ignoring air resistance, find the maximum acceleration of the car when it is travelling at

(i) $12\,\mathrm{m\,s^{-1}}$

(ii) $28\,\mathrm{m\,s^{-1}}$.

Solution

(i) Power = force × velocity

The driving force at $12\,\mathrm{m\,s^{-1}}$ is F_1 N, where

$$45\,000 = F_1 \times 12$$

$$\Rightarrow \quad F_1 = 3750$$

By Newton's second law, $F = ma$

$$\Rightarrow \quad \text{acceleration} = \frac{3750}{1000} = 3.75\,\mathrm{m\,s^{-2}}$$

(ii) Now the driving force F_2 is given by

$$45\,000 = F_2 \times 28$$

$$\Rightarrow \quad F_2 = 1607$$

$$\Rightarrow \quad \text{acceleration} = \frac{1607}{1000} = 1.61\,\mathrm{m\,s^{-2}}.$$

This example shows why it is easier to overtake a slow moving vehicle.

Example 5.13

A car of mass 900 kg produces a power of 45 kW when moving at a constant speed. It experiences a resistance of 1700 N.

(i) What is its speed?

(ii) The car comes to a downhill stretch inclined at 2° to the horizontal. What is its maximum speed downhill if the power and resistance remain unchanged?

Solution

(i) As the car is travelling at a constant speed, there is no resultant force on the car. In this case, the forward force of the engine must have the same magnitude as the resistive forces, i.e. 1700 N.

Denoting the speed of the car by $v\,\mathrm{m\,s^{-1}}$, $P = Fv$ gives

$$v = \frac{P}{F} = \frac{45\,000}{1700} = 26.5$$

The speed of the car is $26.5\,\mathrm{m\,s^{-1}}$ (which is approximately 60 mph).

(ii) The diagram shows the forces acting.

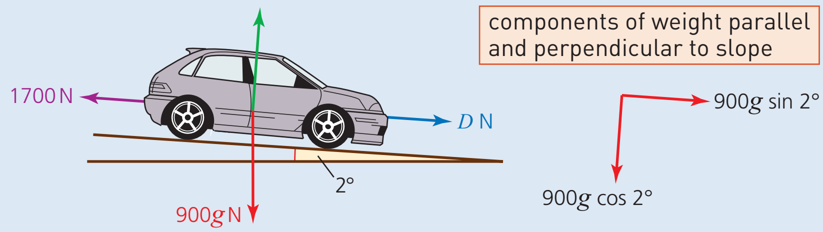


Figure 5.18

At maximum speed there is no acceleration so the resultant force down the slope is zero.

When the driving force is D N

$$D + 900g\sin 2° - 1700 = 0$$

$$\Rightarrow \qquad D = 1392$$

But power is Dv so $45\,000 = 1392v$

$$\Rightarrow \qquad v = \frac{45\,000}{1392}$$

The maximum speed is $32.3\,\text{m s}^{-1}$ (about 73 mph).

Historical note

James Watt was born in 1736 in Greenock in Scotland, the son of a house- and ship-builder. As a boy, James was frail and he was taught by his mother rather than going to school. This allowed him to spend time in his father's workshop where he developed practical and inventive skills.

As a young man, he manufactured mathematical instruments: quadrants, scales, compasses and so on. One day, he was repairing a model steam engine for a friend and noticed that its design was very wasteful of steam. He proposed an alternative arrangement, which was to become standard on later steam engines.

This was the first of many engineering inventions which made possible the subsequent industrial revolution. James Watt died in 1819, a well-known and highly respected man. His name lives on as the S.I. unit for power.

Example 5.14

A car of mass 1000 kg travels along a horizontal straight road. The power provided by the car's engine is constant and equal to 20 kW. The resistance to the car's motion is constant and equal to 800 N. The car passes two points A and B with speeds 15 m s^{-1} and 25 m s^{-1}, respectively and takes 40 seconds to travel from A to B. Find the distance AB.

Solution

The work done by the engine in travelling from A to B is

$$20\,000 \times 40 = 800\,000 \text{ J}$$

The work done against the resistance is

$$800 \times AB\ J$$

The increase in the kinetic energy of the car is

$$\frac{1}{2}\times 1000\times 25^2 - \frac{1}{2}\times 1000\times 15^2 = 200\,000\ J$$

Using the work–energy principle:

$$800\,000 - 800AB = 200\,000$$

$$AB = \frac{600\,000}{800} = 750\ m$$

The distance AB is 750 m.

Example 5.15

A car of mass 1200 kg is travelling at a constant speed of 15 m s^{-1}, while climbing for 30 s along a uniform slope of elevation arcsin (0.05) against a constant resistance of 900 N.

Calculate the power generated by the driving force.

Solution

The distance travelled by the car along the slope is $15 \times 30 = 450$ m

The work done against the resistance force is thus $900 \times 450 = 405\ 000$ J

The vertical height gained by the car is $450 \times 0.05 = 22.5$ m

The gravitational potential energy gained by the car is therefore

$$1200 \times 9.8 \times 22.5 = 264\,600\ J$$

The total work done by the driving force is

$$405\,000 + 264\,600 = 669\,600\ J$$

This work is produced in 30 s, and thus the rate of doing work is equal to

$$669\,600 \div 30 = 22\,320\ W$$

The power generated by the driving force is 22 320 W.

Exercise 5.3

① A builder hoists bricks up to the top of the house he is building. Each brick weighs 3.5 kg and the house is 9 m high. In the course of one hour, the builder raises 120 bricks from ground level to the top of the house, where they are unloaded by his assistant.

(i) Find the increase in gravitational potential energy of one brick when it is raised in this way.

(ii) Find the total work done by the builder in one hour of raising bricks.

(iii) Find the average power with which he is working.

② A weightlifter takes 2 seconds to lift 120 kg from the floor to a position 2 m above it, where the weight has to be held stationary.

(i) Calculate the work done by the weightlifter.

(ii) Calculate the average power developed by the weightlifter.

The weightlifter is using the 'clean and press' technique. This means that in the first stage of the lift he raises the weight 0.8 m from the floor in 0.5 s. He then holds it stationary for 1 s before lifting it up to the final position in another 0.5 s.

(iii) Find the average power developed by the weightlifter during each of the stages of the lift.

③ A winch is used to pull a crate of mass 180 kg up a rough slope of angle 30° to the horizontal against a frictional force of 450 N. The crate moves at a steady speed of $1.2\,\mathrm{m\,s^{-1}}$.

(i) Calculate the gravitational potential energy given to the crate during 30 s.

(ii) Calculate the work done against friction during this time.

(iii) Calculate the total work done by the winch in 30 seconds.

The cable from the winch to the crate runs parallel to the slope.

(iv) Calculate the tension, T, in the cable.

(v) What information is given by the product of T and the speed of the crate?

④ The power output from the engine of a car of mass 50 kg which is travelling along level ground at a constant speed of $33\,\mathrm{m\,s^{-1}}$ is 23 200 W.

(i) Find the total resistance on the car under these conditions.

(ii) You were given one piece of unnecessary information. Which is it?

⑤ A motorcycle has a maximum power output of 26.5 kW and a top speed of 103 mph (about $46\,\mathrm{m\,s^{-1}}$). Find the force exerted by the motorcycle engine when the motorcycle is travelling at top speed.

⑥ A crane is raising a load of 500 tonnes at a steady rate of $5\,\mathrm{cm\,s^{-1}}$. What power is the engine of the crane producing? (Assume that there are no forces from friction or air resistance.)

⑦ A cyclist, travelling at a constant speed of $8\,\mathrm{m\,s^{-1}}$ along a level road, experiences a total resistance of 70 N.

(i) Find the power which the cyclist is producing.

(ii) Find the work done by the cyclist in 5 minutes under these conditions.

⑧ A mouse of mass 15 g is stationary 2 m below its hole when it sees a cat. It runs to its hole, arriving 1.5 seconds later with a speed of $3\,\mathrm{m\,s^{-1}}$.

(i) Show that the acceleration of the mouse is not constant.

(ii) Calculate the average power of the mouse.

⑨ A train consists of a diesel shunter of mass 100 tonnes pulling a truck of mass 25 tonnes along a level track. The engine is working at a rate of 125 kW. The resistance to motion of the truck and shunter is 50 N per tonne.

(i) Calculate the constant speed of the train.

While travelling at this constant speed, the truck becomes uncoupled. The shunter engine continues to produce the same power.

(ii) Find the acceleration of the shunter immediately after this happens.

(iii) Find the greatest speed the shunter can now reach.

10. A supertanker of mass 4×10^8 kg is steaming at a constant speed of $8\,\mathrm{m\,s^{-1}}$. The resistance force is 2×10^6 N.

(i) What power is being produced by the ship's engines?

One of the ship's two engines suddenly fails but the other continues to work at the same rate.

(ii) Find the deceleration of the ship immediately after the failure.

The resistance force is directly proportional to the speed of the ship.

(iii) Find the eventual steady speed of the ship under one engine only, assuming that the single engine maintains constant power output.

11. A car of mass 850 kg has a maximum speed of $50\,\mathrm{m\,s^{-1}}$ and a maximum power output of 40 kW. The resistance force, R N, at speed $v\,\mathrm{m\,s^{-1}}$ is modelled by $R = kv$

(i) Find the value of k.

(ii) Find the resistance force when the car's speed is $20\,\mathrm{m\,s^{-1}}$.

(iii) Find the power needed to travel at a constant speed of $20\,\mathrm{m\,s^{-1}}$ along a level road.

(iv) Find the maximum acceleration of the car when it is travelling at $20\,\mathrm{m\,s^{-1}}$

(a) along a level road

(b) up a hill at 5° to the horizontal.

12. A truck of mass 1800 kg is towing a trailer of mass 800 kg up a straight road which is inclined to the horizontal at an angle α, where $\sin\alpha = \frac{1}{20}$.
The truck is connected to the trailer by a light inextensible rope which is parallel to the direction of motion of the truck. The resistances to motion of the truck and the trailer from non-gravitational forces are modelled as constant forces of magnitudes 300 N and 200 N, respectively. The truck is moving at constant speed $v\,\mathrm{m\,s^{-1}}$ and the engine of the truck is working at a rate of 40 kW.

(i) Find the value of v.

As the truck is moving up the road, the rope breaks.

(ii) Find the acceleration of the truck immediately after the rope breaks.

[June 2014, 6678/01, Question 4]

13. A lorry of mass 1800 kg travels along a straight horizontal road. The lorry's engine is working at a constant rate of 30 kW. When the lorry's speed is $20\,\mathrm{m\,s^{-1}}$, its acceleration is $0.4\,\mathrm{m\,s^{-2}}$. The magnitude of the resistance to the motion of the lorry is R newtons.

(i) Find the value of R.

The lorry now travels up a straight road which is inclined at an angle α to the horizontal, where $\sin\alpha = \frac{1}{12}$. The magnitude of the non-gravitational resistance to motion is R newtons. The lorry travels at a constant speed of $20\,\mathrm{m\,s^{-1}}$.

(ii) Find the new rate of working of the lorry's engine.

[January 2013, 6678/01, Question 2]

14. A tractor and its plough have a combined mass of 6000 kg. When developing a power of 5 kW, the tractor is travelling at a steady speed of $2.5\,\mathrm{m\,s^{-1}}$ along a horizontal field.

    (i) Calculate the resistance to the motion.

    The tractor comes to a patch of wet ground where the resistance to motion is different. The power developed by the tractor during the next 10 seconds has an average value of 8 kW over this time. During this time, the tractor accelerates uniformly from $2.5\,\mathrm{m\,s^{-1}}$ to $3\,\mathrm{m\,s^{-1}}$.

    (ii) Show that the work done against the resistance to motion during the 10 seconds is 71 750 J. Assuming that the resistance to the motion is constant, calculate its value.

    The tractor now comes to a slope at $\arcsin\left(\frac{1}{20}\right)$ to the horizontal. The non-gravitational resistance to motion on this slope is 2000 N. The tractor accelerates uniformly from $3\,\mathrm{m\,s^{-1}}$ to $3.25\,\mathrm{m\,s^{-1}}$ over a distance of 100 m while climbing the slope.

    (iii) Calculate the time taken to travel this distance of 100 m and the average power required over this time period.

15. A car of mass 1250 kg has a maximum power of 50 kW. Resistive forces have a constant magnitude of 1500 N.

    (i) Find the maximum speed of the car on level ground.

    The car is now ascending a hill with inclination θ and $\sin\theta = 0.1$.

    (ii) Calculate the maximum steady speed of the car when ascending the hill.

    (iii) Calculate the acceleration of the car when it is descending the hill at $20\,\mathrm{m\,s^{-1}}$ working at half the maximum power.

16. A car of mass 1000 kg travels along a horizontal straight road. The power provided by the car's engine is a constant 15 kW. The resistance to the car's motion is a constant 400 N. The car passes through two points A and B with speeds $10\,\mathrm{m\,s^{-1}}$ and $20\,\mathrm{m\,s^{-1}}$ respectively. The car takes 30 seconds to travel from A to B.

    (i) Find the acceleration at A.

    (ii) Find the distance AB.

KEY POINTS

1 The work done by a constant force F is given by Fs where s is the distance moved in the direction of the force.

2 The kinetic energy (K.E.) of a body of mass m moving with speed v is given by $\frac{1}{2}mv^2$. Kinetic energy is the energy a body possesses on account of its motion.

3 The work–energy principle states that the total work done by all the forces acting on a body is equal to the increase in the kinetic energy of the body.

4 The gravitational potential energy of a body of mass m at a height h above a given reference level is given by mgh. It is the work done against the force of gravity in raising the body.

5 Mechanical energy (kinetic energy + gravitational potential energy) is conserved when no forces other than gravity do work.

6 Power is the rate of doing work, and is given by Fv.

7 Average power = total work done ÷ total time taken

8 The S.I. unit for energy is the joule and for power is the watt.

LEARNING OUTCOMES

When you have completed this chapter, you should be able to

- understand the language relating to work, energy and power
- calculate the work done by a force which moves either along its line of action or at an angle to it
- calculate kinetic energy and gravitational potential energy
- understand and use the principle of conservation of energy
- understand and use the work–energy principle
- understand and use the concept of power.