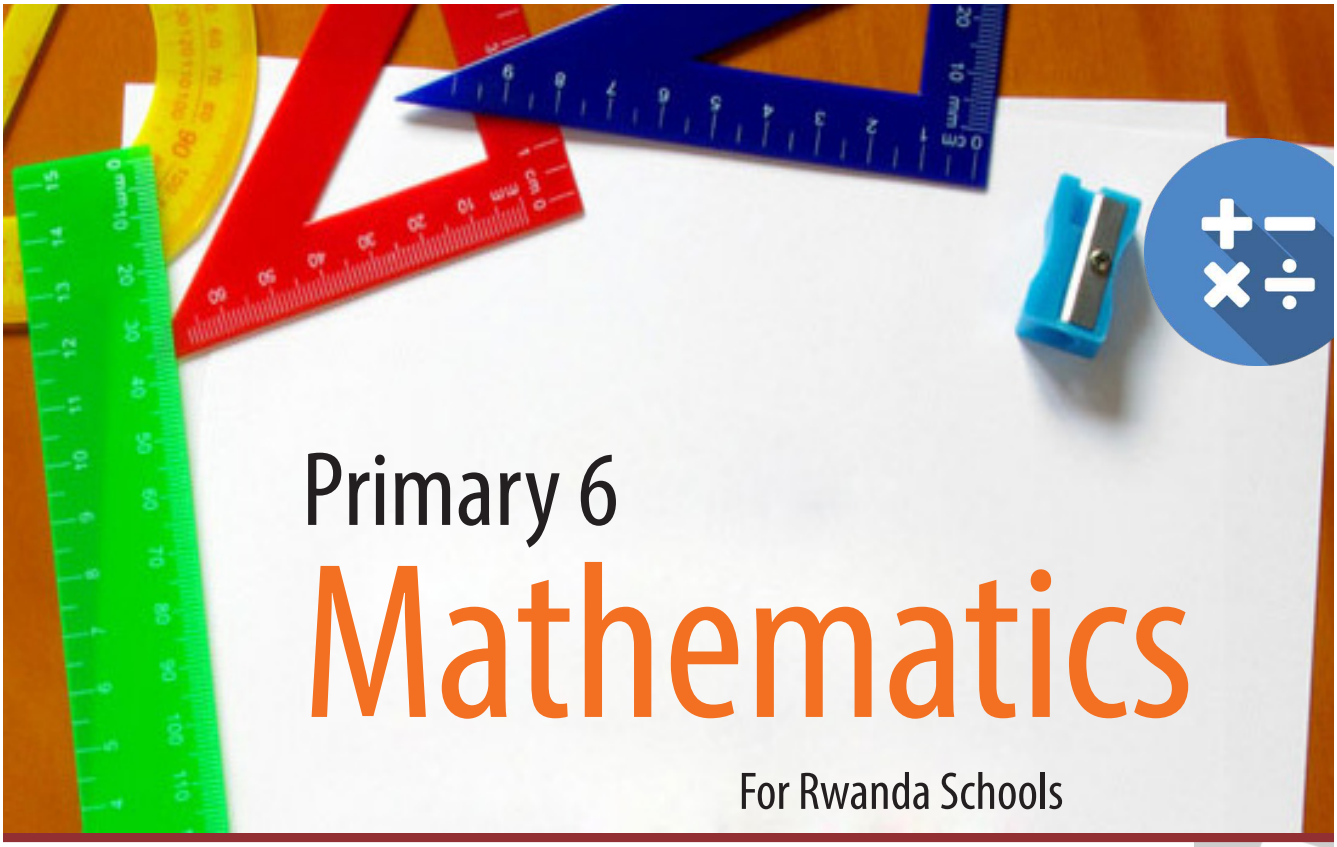


Primary 6
Mathematics

For Rwanda Schools

Learner's Book

Kigali, January 2019

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Preface

Primary 6 Mathematics Book is based on the competence based curriculum called "Curriculum for sustained development". This Curriculum is developed by Rwanda Education Board (REB) in 2015.

Each unit is introduced with an activity to enable learners work and discover concepts on their own.

Learners are then taken through examples, study tips and thereafter questions are given to test whether they have learnt the concept. These equip the learner with knowledge, skills and attitude necessary to succeed in this era of technological growth and socio-economic growth.

The knowledge, skills and attitudes developed will enable learners to count, estimate, measure, calculate, handle and manage money, interpret statistics and also represent data on Mathematical tools such as pie-charts, bar graphs and so on.

Generally, this book is organized in a way that enables the learners to investigate, practice, summarize, integrate and finally assess him / herself.

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Unit 1 Reading, writing and comparing whole numbers beyond 1,000,000

Key unit competence: To be able to read, write and compare whole numbers beyond 1,000,000.

Introduction

In daily situations, numbers are used to count, record or compare things or objects. Big numbers beyond a million are also used in daily situation such as counting a big sum of money in millions.

(a) In your experience give other real life situations where numbers beyond millions are used.

(b) From your experience, are numbers beyond millions useful in daily life? Explain your answer.

1.1 Reading and writing numbers beyond 1,000,000 in words

Activity

1. A chart shows 999,999.
   (a) If you add 1 to 999,999, what will be your next number?
   (b) How many digits will the next number have?
   (c) Draw an abacus with 7 spikes. Starting from ones (right), fill in the number you get in (a) above to the last place value on the left.
   (d) What is the place value of 1 in the number you get in (a) above?
   (e) What name would you give to the number you get in (a) above?
2. Now match the number figures to their corresponding number words.

2,999,999	Fourteen million, one hundred forty thousand, two hundred nineteen.
1,259,000	Two million, nine hundred ninety-nine thousand nine hundred ninety-nine.
14,140,219	One million, two hundred fifty-nine thousand.

(a) Explain the steps you took when matching the numbers.

(b) Use the same steps in (a) above to write the population of Rwanda 12,279,742 in words.

(c) Why is it necessary to know how to write in number words?

Example 1

A country has a population of 5,600,002. Write the population in words.

Solution

Group the population in digits of threes. (5),(600),(002)

Draw a place value table and fill in the digits.

Write in words the values in the three-digit groups below.

Millions			Thousands			Units		
			H	T	O	H	T	O
		5	6	0	0	0	0	2
Five			Six hundred			two		

The population is five million, six hundred thousand two.

Example 2

A water tank holds 82,999,555 litres of water. Write the litres in words.

Solution

Millions			Thousands			Units		
H	T	O	H	T	O	H	T	O
	8	2	9	9	9	5	5	5
Eighty-two			Nine hundred ninety-nine			Five hundred fifty-five		

Eighty-two million nine hundred ninety-nine thousand, five hundred fifty-five litres.

Study tip

When you add 1 to 999,999, you get 1,000,000.

To write 1,000,000 in words:

- First group the given digits in threes starting from the right to the left side of the whole number.
- Each group of three digits is known as a period.
- Draw a place value table and then write each digit under its place value.

- Write the value of each period in words. Then write the names of the periods to separate one period from another.
- Use a comma to separate millions from thousands and thousands from units.
- The periods that are millions, thousands and units are written without 's'. So, not in plural form.



Application 1.1

1. Write the following whole numbers in words:
   (a) 12,456,678 (b) 9,700,956 (c) 59,648,200 (d) 721,569,216
2. Kalisa bought a cow at 456,700 Frw. Write the amount in words and explain your working out.
3. Abeli collected 5,417,257 litres of milk from his farm in five months. Write the number of litres he collected in words.



4. Agatha deposited 4,565,090 Frw in the bank. Write the amount she deposited in words.
5. A non-government organisation spent 12,468,250 Frw on educating young people about the dangers of alcohol. Write the amount of money they spent in words.
6. On the inauguration ceremony of a district chairperson, 4,412,567 people were invited. Write the number of people invited in words.
7. A district planted 9,998,888 saplings. Write the number in words.

1.2 Reading and writing numbers beyond 1,000,000 in figures

Activity

Match the following cards accordingly:

Four million, five hundred sixty-five thousand, two hundred seventy.	1,900,999
One million, nine hundred thousand, nine hundred ninety-nine.	1,300,860
One million, three hundred thousand, eight hundred sixty.	4,565,270

1. Explain the steps you took when matching the cards.
2. Why is it necessary to know how to write numbers in figures?

Example 1

Write "one million, two hundred seventy thousand, one hundred thirty-six" in figures.

Solution

One million ----------	1,000,000
Two hundred seventy thousand ----------	270,000
One hundred thirty-six ----------	136
	1,270,136

Example 2

Mahama Refugee Camp received three hundred forty-two million, six hundred two thousand, six hundred thirty-one Rwandan francs from the government for buying food for the refugees in 2016. Write the amount in figures.

Solution

Three hundred forty-two million	342,000,000
Six hundred two thousand -------	602,000
Six hundred thirty-one ----------	631
	342,602,631

Study tip

When writing whole number in figures:

- First, group the number words in threes starting from the right of the whole numbers.
- Write the digits of each period in numbers.
- Read and write each period separately.
- Use a comma to separate millions from thousands and units.
- Add the values to get the right figure.



Application 1.2

Write the following numbers in figures:

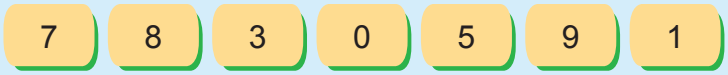
1. Fifteen million, three hundred fifty-six thousand, four hundred thirteen.
2. Eighty-three million, sixty- six thousand, two hundred thirty.
3. Eight hundred million, eighteen thousand, seven hundred seventeen.
4. A school collected four hundred fifty-six million, five hundred forty-five thousand, two hundred Rwandan francs as school fees from its learners. Write the amount in figures.

5. Five hundred twelve million, five hundred forty-nine thousand Rwanda francs was spent by a school in a year. Write the amount in figures.
6. Nineteen million Rwanda francs was spent by a company to print books. Write the amount in figures.

1.3 Finding place value and values of numbers up to 7 digits

Activity

1. Look at the cards below and form a number.



2. Draw a place value table in your exercise book.
3. Fill in the digits of the number 7,830,591 in the correct place values in the place value table.
4. Write down the place value of each digit from the table.
5. Use the place value table to write 7,830,591 in words.
6. In which areas of your life can you apply place values?

Example 1

What is the place value of each digit in 8,356,421?

Solution

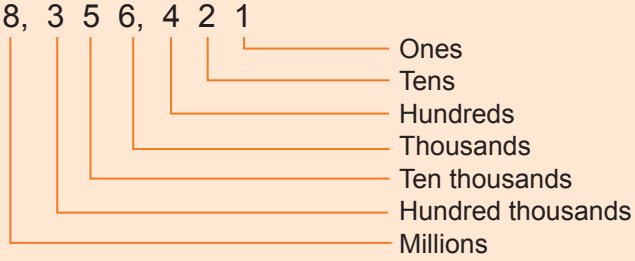
Method 1

Count the digits in the number. There are 7 digits, now draw a place value table as shown below.

Period	Millions			Thousands			Units		
Place value	H	T	O	H	T	O	H	T	O
Number			8	3	5	6	4	2	1
			8 x 1,000,000	3 x 100,000	5 x 10,000	6 x 1,000	4 x 100	2 x 10	1 x 1
Value			8,000,000	300,000	50,000	6,000	400	20	1

- The place value of 8 is millions.
- The place value of 3 is hundred thousands.
- The place value of 5 is ten thousands.
- The place value of 6 is thousands.
- The place value of 4 is hundreds.
- The place value of 2 is tens.
- The place value of 1 is ones.

Method 2



Example 2

What is the value of 8 in 4,835,634?

Solutions

Period	Millions			Thousands			Units		
Place value	H	T	O	H	T	O	H	T	O
Number			8	3	5	6	4	2	1
			4 x 1,000,000	8 x 100,000	3 x 10,000	5 x 1,000	6 x 100	3 x 10	4 x 1
Value			4,000,000	800,000	30,000	5,000	600	30	4

Therefore the value of 8 is 800,000.

Study tip

- The place values of numbers are the positions of Ones, Tens, Hundreds, Thousands, Ten thousands, Hundred thousands, Millions, Ten millions and Hundred millions.
- To find the place values of digits, draw the place value table and place the digits in it from the right.
- To find the value of digits, find the product of the digits and their place values.



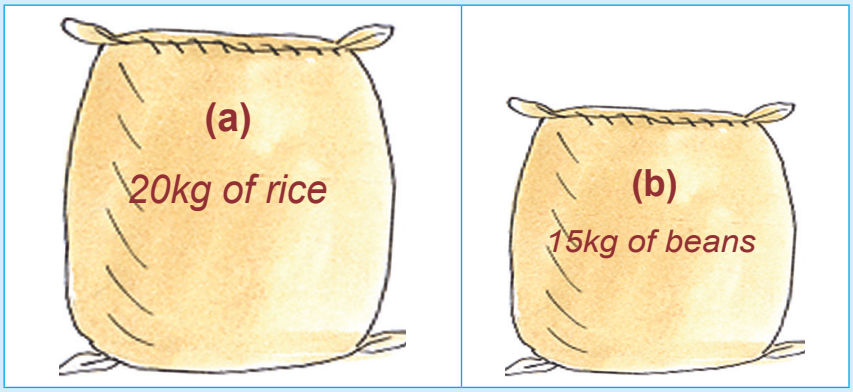
Application 1.3

1. Write the place value of each digit in the following:
   (a) 2,312,983 (b) 7,676,405 (c) 10,101,899 (d) 853,925,732
2. Write the place value of the underlined digits in the following:
   (a) 3,459,874 (b) 356,295,712 (c) 7,879,631 (d) 54,382, 345
3. Write the value of each digit in the following:
   (a) 24,567,400 (b) 894,500,678 (c) 208,567,120 (d) 120,394,456
4. Write the value of the underlined digits in the following:
   (a) 34,475,576 (b) 687,034,239 (c) 126,687,893 (d) 653,592,721
5. Describe how you can identify the value of 6 in 4,567,890.

1.4 Comparing numbers using <, > or =

Activity

1. Study the figure below and answer the questions that follow.



   (a) Which one is bigger than the other?

   (b) Explain your answer.

   (c) Complete the sentence using either <, > or =. That is A ..... B.

2. Count the total number of learners in your class.

   (a) Use <, > or = to compare the number of boys to girls in your class.

   (b) How can you apply comparison in your daily life situations?

Example 1

Compare 6,312,542 and 6,312,452 using <, > or =.

Solution

Draw a place value table and fill in the numbers.

Millions			Thousands			Units		
H	T	O	H	T	O	H	T	O
		6	3	1	2	**5**	4	2
		6	3	1	2	**4**	5	2

Compare the digits in each place value from left to right.

6 = 6, 3 = 3, 1 = 1, 2 = 2, **5 > 4** in the hundreds of units place value.
Therefore, 6,312,542 > 6,312,452

Example 2

Compare 42,635,989 and 42,543,129 using <, > or =.

Solution

Draw a place value table and fill in the numbers.

Millions			Thousands			Units		
H	T	O	H	T	O	H	T	O
	4	2	**6**	3	5	9	8	9
	4	2	**5**	4	3	1	2	9

Compare the digits in each place value from left to right.

4 = 4, 2 = 2, **6 > 5** in the hundred thousands place value.

Therefore, 42,635,989 > 42,543,129.

Example 3

Imanirere sold clothes worth 2,560,320 Frw in 2015. She sold clothes worth 4,576,670 Frw in 2016. Compare the sales over the two years using <, > or =.

Solution

Draw a place value table and fill in the numbers.

Millions			Thousands			Units		
H	T	O	H	T	O	H	T	O
		2	5	6	0	3	2	0
		4	5	7	6	6	7	0

Compare the digits in each place value from left to right.

2 < 4 in the millions place value.

Therefore 2,560,320 < 4,576,670

Study tip

- The symbol > means greater than, < means less than and = means equal to.
- Compare numbers using place values. Check the biggest number against the smallest number.
- You can compare numbers by counting the number of digits of the whole numbers given.

- For numbers with the same number of digits, compare the digits with the same place value from the left. The number with the bigger digit is greater than the other. The number with the smaller digit is less than the other.
- For numbers with a different number of digits, the one with the highest number of digits is greater than the one with the lowest number of digits.
- If two numbers have different place values, compare them looking for the biggest or the smallest.
- If two numbers have the same place values and the same number of digits, compare them starting from the left until you get two different digits to tell the biggest or the smallest number.

Application 1.4

1. Use <, > or = to compare the following:
   (a) 260,340……60,430,730.
   (b) 8,855,631……..8,855,136
   (c) 302,831,547 …… 30,283,154.
   (d) 9,991,999 …… 9,991,999
2. Camille harvested 5,562 tonnes of beans and Kajile harvested 5,256 tonnes of beans. Who harvested more beans?
3. Mukagasana deposited 2,506,590 Frw in the bank. His son deposited 259,000 Frw. Who deposited more money? Explain your answer.



Sacks of beans

4. A school received a donation of 67,957,800 Frw while another school received 67,854,590 Frw. Find the difference. Which school received less money?
5. Hospital A admitted 45,679 patients in 2016 while hospital B admitted 67,890 patients in the same year. Which hospital admitted more patients? Explain your answer.
6. A district collected 4,853,825 Frw in taxes while another district collected 4,197,900 Frw. Which district collected more money?



Patients in the hospital.

1.5 Arranging numbers in ascending and descending order

Activity

Use cards below.

942,704 942,407 1,496,066 1,496,606

- Order pairs of cards in ascending order.
- Write the order on slips of paper.
- Change to ordering cards in descending order.
- Write down the order.

What did you consider when ordering the numbers in the cards? Explain your procedure to the class.

Example

Arrange the following numbers in ascending and descending order.

1,707,055 1,770,550 3,025,446 3,205,446

Solution

Use a place value table to compare the digits.

Millions			Thousands			Units		
H	T	O	H	T	O	H	T	O
		1	7	0	7	0	5	5
		1	7	7	0	5	5	0
		3	0	2	5	4	4	6
		3	2	0	5	4	4	6

- Start comparing from the highest place value to the lowest place value.
- In ones of millions, 1 = 1, 3 = 3, but 1 < 3 and 3 > 1.
- In hundred thousands, 7 = 7, 0 < 2, so, 3,205,446 > 3,025,446; 3,025,446 > 1,770,550; 1,770,550 > 1,707,055.
- Also, 1,707,055 < 1,770,550; 1,770,550 < 3,025,446 and 3,025,446 < 3,205,444.
- Ascending order is the arrangement from the smallest to the biggest. So, the ascending order is 1,707,055; 1,770,550; 3,025,446; 3,205,446.
- Descending order is the arrangement from the biggest to the smallest. So, the descending order is 3,205,446; 3,025,446; 1,770,550; 1,707,055.

Study tip

- A number is greater than another if its corresponding digit of another number in the same place value is smaller if they have the same number of digits.
- A number is smaller than another if its corresponding digit of another number in the same place value is bigger.
- A number with more digits is bigger than the other.
- A number with fewer digits is smaller than the other.
- Use a place value table to compare and arrange numbers in ascending and descending order.



Application 1.5

1. Order the following numbers in ascending order.
   (a) 1,673,421; 1,065,345; 1,671,241; 1,065,234
   (b) 2,303,874; 2,033,874; 2,330,874; 2,874,303
   (c) 9,827,623; 6,827,623; 8,279,625; 9,623,829
   (d) 11,046,305; 11,460,305; 4,116,305; 4,611,530
2. Order the following numbers in descending order.
   (a) 4,963,427; 4,427,963; 4,369,427; 4,724,963
   (b) 8,306,396; 8,693,306; 8,369,306; 8,063,963
   (c) 12,042,994; 12,420,994; 12,994,609; 12,499,906
   (d) 3,625,113; 625,113; 6,253,311; 652,311

1.6 Adding numbers beyond 1,000,000

Activity

- Write two 7-digit numbers on flash cards.
- Arrange them in vertical order by the place value of their digits.
- Work out their sum.
- What answers do you get?
- Compare the answers in your groups.

Example 1

Add: 6,325,904 and 2,834,978.

Solution

Arrange the digits according to place values.

Then add the two whole numbers starting from the right (ones) to the left.

Millions			Thousands			Units		
H	T	O	H	T	O	H	T	O
		$^{1}6$	3	$^{1}2$	$^{1}5$	9	$^{1}0$	4
	+	2	8	3	4	9	7	8
		9	**1**	**6**	**0**	**8**	**8**	**2**

Units

- Add ones: 4 + 8 = 12. Write 2 under ones and carry **1** to tens.
- Add tens: 0 + 7 + **1** = 8. Write 8 under tens.
- Add hundreds:9 + 9 = 18. Write 8 under hundreds and carry **1** to thousands.

Thousands

- Add thousands: 5 + 4 + **1** = 10. Write 0 under thousands and carry **1** to ten thousands.
- Add ten thousands: 2 + 3 + **1** = 6. Write 6 under ten thousands.
- Add hundred thousands: 3 + 8 = 11. Write 1 under hundreds thousands and carry **1** to millions.

Millions

- Add Millions: 1 + 6 + 2 = 9 write 9 under millions.

Therefore, 6,325,904 + 2,834,978 = 9,160,882.

Example 2

Find the sum of 4,629,208; 2,823,004 and 5,987,253.

Solution

Arrange the digits according to place values.

Then add the two whole numbers starting from the right (ones) to the left.

Millions			Thousands			Units		
H	T	O	H	T	O	H	T	O
		$^{2}4$	$^{1}6$	$^{1}2$	9	2	$^{1}0$	8
		2	8	2	3	0	0	4
	+	5	9	8	7	2	5	3
	1	**3**	**4**	**3**	**9**	**4**	**6**	**5**

Units:

- Add ones: 8 + 4 + 3 = 15. Write 5 under ones and carry **1** to tens.
- Add tens: 0 + 0 + 5 + **1** = 6. Write 6 under tens.
- Add hundreds: 2 + 0 + 2 = 4. Write 4 under hundreds.

Thousands:

- Add thousands: 9 + 3 + 7 = 19. Write 9 under thousands and carry **1** to ten thousands.
- Add ten thousands: 2 + 2 + 8 + **1** = 13. Write 3 under ten thousands and carry **1** to hundred thousands.
- Add hundred thousands: 6 + 8 + 9 = 24. Write 4 under hundred thousands and carry **2** to millions.

Millions: Add millions: 4 + 2 + 5 + **2** = 13. Write 3 under millions and carry 1 to ten millions. Therefore, 4,629,208 + 2,823,004 + 5,987,253 = 13,439,465

Study tip

- When adding whole numbers, start from right to left. That is, from ones, to tens, thousands, ten thousands, hundred thousands and millions.
- Words used for addition include; total, sum, altogether, combined.



Application 1.6

Add the following:

(a) 4,985,670 + 2,322,502 =

(b) 2,069,012 + 2,625,044 =

(c) 6,232,343 + 2,432,234 + 1,067,103 =

(d) 3,807,233 + 2,067,943 + 6,723,623 =

(e) 9,088,033 + 9,000,046 =

(f) 1,602,444 + 2,622,433 + 5,789,987 =

(g) 8,421,982 + 3,723,848 + 3,921,982 =

1.7 Solving problems involving addition of numbers beyond 1,000,000

Activity

In a country, there are 12,000,000 males and 3,000,000 females.

1. What should you do to know the total number of people in the country?
2. Compare the number of males to that of females.
3. Create a list of instances where addition is applied in your daily life.
4. How is addition relevant to you?

Example

Builders used 5,762,426 bricks to build the foundation of a house and 3,028,987 bricks to put up walls of the house. Find the total number of bricks that were used to complete the house.

Solution

Arrange the digits according to place values.

Then add the two whole numbers starting from the right (ones) to the left.

Millions			Thousands			Units		
H	T	O	H	T	O	H	T	O
		5	7	6	2	4	2	6
	+	3	0	2	8	9	8	7
		8	**7**	**9**	**1**	**4**	**1**	**3**

Units

- Add ones: 6 + 7 = 13. Write 3 under ones and carry **1** to tens.
- Add tens: 2 + 8 + **1** = 11. Write 1 under tens and carry **1** to hundreds.
- Add hundreds: 4 + 9 + **1** = 14. Write 4 under hundreds and carry **1** to thousands.

Thousands

- Add thousands: 2 + 8 + 1 = 11. Write 1 under thousands and carry **1** to ten thousands.
- Add ten thousands: 6 +2 + **1** = 9. Write 9 under ten thousands.
- Add hundred thousands: 7 + 0 = 7. Write 7 under hundreds thousands.

Millions

- Add Millions: 5 + 3 = 8 write 8 under millions.

The total bricks that were used to complete the house are 8,791,413.

Study tip

- First read and interpret the question correctly.
- When adding whole numbers, arrange the digits in the table according to place values.
- If numbers do not have the same number of digits, use zeros to act as place holders to ensure proper alignment of each digit according to place values.
- Start adding from right to the left, that is from ones to tens, thousands, ten thousands, hundred thousands and millions.
- Other words used for addition include total, sum, altogether and combined.



Application 1.7

1. A dairy cooperative sold 1,123,456 and 8,467,619 litres of milk on Monday and Tuesday respectively. How much milk was sold in the two days?
2. Publishing companies A, B and C supplied the following number of textbooks to primary schools in the same district last month. A supplied 1,345,346 copies, B supplied 1,206,460 copies and C supplied 1,600,400 copies. What is the total number of books supplied by all three publishing companies?
3. 3,460,782 people participated in a Run for Water marathon organised by a telecom company. About 5,525,448 people participated in a cancer marathon organised by the same telecom company. How many people participated in both marathons?
4. Kayitesi sold 1,625,255 kg of maize flour in one year and 3,268,450 kg of maize flour in another year. What was her total sales in the two years?

1.8 Subtracting numbers beyond 1,000,000

Activity

- Write two 7-digit numbers on slips of paper.
- Calculate the difference between the bigger and the smaller number.
- What is the most suitable method you can use to work out the difference? Explain your procedure to the class.
- Now try out this: Mukangarambe went to the shop with 20,000 Frw, she spent 17,240 Frw. How much was her balance?

Example 1

Subtract: 6,345,625 - 2,124,304

Solution

Arrange the digits according to place values.

Put the larger number at the top of the table followed by the smaller number.

Subtract the two whole numbers starting from the right to the left.

Remember to borrow then re-group where necessary as you subtract.

Millions			Thousands			Units		
H	T	O	H	T	O	H	T	O
		6	3	4	5	6	2	5
	-	2	1	2	4	3	0	4
		4	**2**	**2**	**1**	**3**	**2**	**1**

Units

- Subtract ones: 5 - 4 = 1, write 1 under ones.
- Subtract tens: 2 - 0 = 2, write 2 under tens.
- Subtract hundreds: 6 - 3 = 3, write 3 under hundreds.

Thousands

- Subtract thousands: 5 - 4 = 1, write 1 under thousands.
- Subtract ten thousands: 4 - 2 = 2, write 2 under ten thousands.
- Subtract hundred thousands: 3 - 1 = 2, write 2 under hundred thousands.

Millions

- Subtract Millions: 6 - 2 = 4, write 4 under millions.

Therefore, 6,345,625 - 2,124,304 = 4,221,321.

Example 2

Subtract 1,899,550 litres from 2,985,620 litres.

Solution

Arrange the digits in the table according to place values.

Put the larger number at the top of the smaller number.

Subtract the two whole numbers starting from the right to the left.

Remember to borrow then re-group where necessary.

Millions			Thousands			Units		
H	T	O	H	T	O	H	T	O
		2	~~9~~ 8	~~8~~ 7	15	~~6~~ 5	12	0
	-	1	8	9	9	5	5	0
		1	**0**	**8**	**6**	**0**	**7**	**0**

Units

- Subtract ones: 0 - 0 = 0, write 0 under ones.
- Subtract tens: 2 - 5 = (not possible), borrow 1 from hundreds, then regroup with tens to get 12 - 5 = 7. Now write 7 under tens.
- Subtract hundreds: 5 - 5 = 0, write 0 under hundreds.

Thousands

- Subtract thousands: 5 - 9 (not possible), borrow 1 from ten thousand, then regroup with thousands to get 15 - 9 = 6. Now write 6 under thousands.
- Subtract ten thousands: 7 - 9 (not possible), borrow 1 from hundred thousand, then regroup with ten thousands to get 17 - 9 = 8. Write 8 under ten thousands.
- Subtract hundred thousands: 8 - 8 = 0, write 0 under hundred thousands.

Millions

- Subtract Millions: 2 - 1 = 1, write 1 under millions.

Therefore, 2,985,620 litres - 1,899,550 litres = 1,086,070 litres.

Study tip

- When subtracting large numbers, arrange the numbers in vertical order, placing each digit in its correct place value.
- When subtracting a bigger digit from a smaller one in the same place value, borrow 1 ten (10) from the digit in the next place value to the left. Add it to the smaller digit on your right being subtracted from. This is called re-grouping. Then subtract.



Application 1.8

Subtract the following:

1. 6,000,101 – 4,999,011 =
2. 6,291,569 – 4,687,263 =
3. 3,562,560 – 1,670,340 =
4. 9,003,087 – 6,334,050 =
5. 3,642,110 kg – 1,039,042 kg =
6. 6,334,050 trees from 9,003,087 trees =
7. 9,008,200 Frw – 8,000,200 Frw =
8. 6,326,428 books from 8,040,249 books =
9. 9,462,490 – 5,233,982 =

1.9 Solving problems involving subtraction of numbers beyond 1,000,000

Activity

Gabiro borrowed 5,345,600 Frw from the bank. He has so far cleared 3,000,560 Frw.

1. Determine the amount of money Gabiro still owes the bank.
2. Give instances where subtraction is applied in your daily life.
3. How is subtraction relevant to you?

Example

A juice company produced 7,003,453 litres last week. It sold only 5,654,000 litres in the week. How many litres of juice remained unsold?

Solution

Arrange the digits in the table according to place values.

Put the large number at the top of the small number.

Subtract the two whole numbers starting from the right to the left.

Remember to borrow then re-group where necessary.

Millions			Thousands			Units		
H	T	O	H	T	O	H	T	O
		6 ~~7~~	9 ~~0~~	9 ~~0~~	1 3	4	5	3
	-	5	6	5	4	0	0	0
		1	**3**	**4**	**9**	**4**	**5**	**3**

Units

- Subtract ones: 3 - 0 = 3, write 3 under ones.
- Subtract tens: 5 - 0 = 5, write 5 under tens.
- Subtract hundreds: 4 - 0 = 4, write 4 under hundreds.

Thousands

- Subtract thousands: 3 - 4 (not possible), borrow 1 from millions, then first regroup with hundred thousands, then later regroup with ten thousands to get 13 - 4 = 9. Write 9 under thousands.
- Subtract ten thousands: 9 - 5 = 4, write 4 under ten thousands.
- Subtract hundred thousands: 9 - 6 = 3, write 3 under hundred thousands.

Millions

- Subtract Millions: 6 - 5 = 1, write 1 under millions.

  1,349,453 litres of juice remained.

Study tip

- First read and interpret the question correctly.
- When subtracting whole numbers, arrange the digits in the table according to place values. Make sure the large number is on top of the small number.
- If numbers do not have the same number of digits, use zeros to act as place holders to ensure proper alignment of each digit according to place values.
- Start subtracting from the right (ones) as you go to the left side. Make sure you borrow and re-group where you find that the top digit is smaller than the bottom digit.
- Words to mean subtraction are; take away, minus, reduce and difference.
- To subtract, means to reduce a given number by a certain number. The result of subtraction is called difference.
- When faced with a problem involving both addition and subtraction, always carry out addition first and subtract last.



Application 1.9

1. What is the difference between 2,798,576 pens and 2,745,568 pens?
2. 3,567,342 babies were born in a country in 2016. Of these, 1,593,599 babies were girls. Find the number of boys.
3. A farmer harvested 12,000,500 kg of maize in the first season. By the end of the first month, he had sold 6, 400,400 kg of maize. How many kilograms are still in his store?
4. A truck carrying 2,560,000 litres of milk was in an accident. 1,756,950 litres were split. How much milk remained?
5. There are nine million three hundred twelve thousand six hundred eight animals in a park. Of these, three million six hundred nine thousand, three hundred twenty-three are zebras. How many animals are not zebras?
6. Gitego harvested twenty-five million, five thousand two hundred fifty kilograms of Irish potatoes. He took away sixteen million, four hundred twenty-eight thousand, five hundred kilograms to distribute to schools. How many kilograms remained?

1.10 Multiplying numbers beyond 1,000,000

Activity

- Write 1,235,265 on a sheet of paper.
- Multiply the number by 4. Note the answer.
- Multiply the number by 20. Write the answer.
- Now multiply 1,235,263 by 100. What do you get?
- Add the three answers. What is the result?

Why did you multiply by 4, then by 20, then by 100? Explain to the class.

Example

Multiply 1,603,421 by 132.

Solution

Arrange in vertical order according to the place values of each digit.

	1 6 0 3 4 2 1	
	× 1 3 2	
	3 2 0 6 8 4 2	1,603,421 x 2 (ones)
	4 8 1 0 2 6 3 0	1,603,421 x 30 (tens)
+	1 6 0 3 4 2 1 0 0	1,603,421 x 100 (hundreds)
	2 1 1, 6 5 1, 5 7 2	

Therefore, 1,603,421 multiplied by 132 = 211,651,572.

Study tip

- When multiplying, arrange numbers in vertical order placing each digit in its correct place value.
- Multiplication is done by multiplying ones, tens, and hundreds.
- Align the multiplied digits in their correct place values, then add the products.



Application 1.10

Work out the following:

(a) 986,342 x 76

(b) 1,112,025 x 111

(c) Multiply 1,076,033 by 104.

(d) 896,234 x 121

(e) Multiply 2,316,310 by 99.

(f) What is the product of 1,404,055 and 121?

1.11 Solving problems using calculation strategies on multiplication

Activity

In January, a school admitted 500 learners. Each learner paid 30,000 Frw in school fees to the bursar.

1. How would you find the total school fees paid by all learners?
2. Describe the steps you take to get the answer.
3. What mathematics operation are you likely to use to carry out the calculation easily?
4. In what other ways can you use the operation in your daily life?

Example 1

A petrol station operated 36 trucks each carrying 456,798 litres of petrol between its branches in Rwanda. How much petrol was distributed?

Solution

36 trucks each carrying 456,798 litres

	456,798	
	× 36	
	2,740,788	(456,798 x 6)
+	13,703,940	(456,798 x 30)
	16,444,728	litres

16,444,728 litres were distributed.

Example 2

Mr. Kamanutsi sells 1,200,350 litres of milk in a month. How many litres does he sell in a year?

Solution

1,200,350 litres in a month

A year has 12 months, therefore,

	1,200,350	
	× 12	
	2,400,700	(1,200,350 x 2)
+	12,003,500	(1,200,350 x 10)
	14,404,200	litres

Kamanutsi sells 14,404,200 litres a year.

Study tip

- First read and interpret the question correctly. This will help you to apply the right operation.
- Arrange the whole number according to its place value.
- Then multiply the lower value by all of the upper values starting from ones to the left. Write your answer.
- Next multiply the second lowest number by all of the upper values from ones (right side) all through to the left side. Write your answer by skipping one place value from the right.
- Lastly add the two answers from multiplication to get a product.



Application 1.11

1. A non-government organisation was supposed to deposit five hundred thousand five hundred Rwanda francs as tuition fees for each of the students it sponsors at university. If it sponsors 25 students, how much money should it deposit?
2. Habimana wanted to save tuition fees for her daughter to study at University. At university the tuition fees are 400,000 Frw per year. How much money must she save in order for her daughter to complete three years?
3. In a store, there are 382,324 bags of mangoes. Each bag contains 250 mangoes. How many mangoes are in the store?
4. 44 drums of the same capacity contain 2,200,000 litres of oil each. How many litres are there altogether?
5. If light travels 300,000,000 metres in one second, how far does light travel in one minute?
6. A private school has 617 learners. If one learner pays 10,000 Frw in school fees per term, how much money do they pay altogether?
7. A shoe factory makes 600,000 pairs of shoes per day. How many pairs of shoes can the same factory produce in 15 days if they work at the same rate?

1.12 Dividing numbers beyond 1,000,000

Activity

- Get 1,240,000 paper notes in pretend of 500 Frw notes.
- Shared it equally among 40 learners. How much money does each get?
- Explain to the class the procedure.
- Is sharing equally the things we use with friends good? Discuss.

Example 1

Divide: 2,448,768 by 32

Solution

```
      76524
32 ) 2448768
   - 224
     208
   - 192
      167
   - 160
        76
     - 64
        128
     - 128
          0
```

← 7 x 32
← 6 x 32
← 5 x 32
← 2 x 32
← 4 x 32

Therefore 2,448,786 ÷ 32 = 76,524.

Example 2

Share 1,175,576 *kg* of beans among 184 parishes.

Solution

```
      6389
184 ) 1175576
   - 1104
      715
   - 552
      1637
   - 1472
      1656
   - 1656
         0
```

← 6 x 184
← 3 x 184
← 8 x 184
← 9 x 184

Therefore, each parish gets 6,389 *kg*.

Study tip

- When dividing, start with the digits in the highest place value.
- Estimate the nearest number of times a number can be divided.
- Carry the remainder to the next place value if it does not divide exactly.
- Align the digits in order to subtract correctly.



Application 1.12

Work out the following.

(a) 2,026,648 ÷ 26 =

(b) 8,123,518 ÷ 34 =

(c) 7,562,296 ÷ 56 =

(d) 9,561,978 ÷ 73 =

(e) Share equally 8,164,904 saplings among 124 villages.

(f) Distribute equally 7,827,831 *kg* of maize among 333 parishes. How many kilograms does each parish get?

1.13 Solving problems using calculation strategies on division

Activity

Get 300 sticks and share them among 10 learners.

(a) How many sticks does each learner get?

(b) What operation have you carried out?

(c) Write the operation statement and work it out.

(d) Present your working out to the class.

Example 1

The electricity board distributes equally 2,026,800 units of electricity to 24 districts. How many units of electricity does each district receive?

Solution

```
          8 4, 4 5 0
24 ) 2, 0 2 6, 8 0 0
   – 1 9 2                ←  8 x 24
       1 0 6
     –   9 6              ←  4 x 24
       1 0 8
     –   9 6              ←  4 x 24
         1 2 0
     –   1 2 0            ←  5 x 24
             0 0
           – 0            ←  0 x 24
             –
```

Therefore each district received 84,450 units.

Example 2

Traders in a certain country equally contributed 1,305,425 Frw for their association. If there are 235 traders, how much did each contribute?

Solution

```
            5,555
235 ) 1, 3 0 5, 4 2 5
    – 1   1 7 5             ←  5 x 235
        1 3 0 4
      – 1 1 7 5             ←  5 x 235
          1 2 9 2
      – 1 1 7 5             ←  5 x 235
          1 1 7 5
        – 1 1 7 5           ←  5 x 235
              0
```

Therefore, each trader contributed 5,555 Frw.

Study tip

- First read and interpret the question correctly. This will help you to apply the right operation.
- When dividing, start with the digits in the highest place value.
- Estimate the nearest number of times a number can be divided. If it does not divide exactly, carry the remainder to the next place value.
- Then multiply and subtract.



Application 1.13

Divide the following:

1. Share equally 2,026,800 Frw among 24 employees.How much does each get!
2. 500 members of the congregation contributed equally 5,501,000 Frw. How much did each contribute?
3. A soda bottling company packed 8,462,376 bottles of soda in crates each containing 24 bottles. Find the number of crates that were packed.
4. A school paid its employees 28,559,925 Frw salary for the month just ended. If there are 135 employees who get the same salary, how much money does each employee receive?
5. A sugar factory manufactured 12,960,648 *kgs* of sugar in a year. How many *kgs* of sugar were produced every month if the factory produces equal amounts of sugar monthly?

1.14 Rounding off whole numbers to the nearest tens

Activity

Study the number cards shown and answer the questions that follow.

7.9 | 8.2 | 3.5 | 4.3 | 9.7 | 1.4 | 5.0

(a) Write all the number cards in whole numbers.

(b) Describe the steps you took to write the above number cards as whole numbers.

(c) What do you refer to during the process of converting a decimal number into a whole number?

(d) Round off the following numbers to the nearest tens.

(i) 23<u>4</u>5 (ii) 87<u>0</u>3

(e) Of what importance is rounding off in daily life?

Example 1

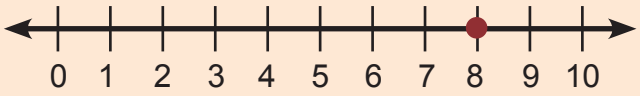
Round off 2,458,548 to the nearest tens.

Solution

2, 4 5 8, 5 4 8

Number to the right

Required place value



8 is in the upper limit so it is nearer to 10 than to 0. So 8 is rounded to 1 ten (10).

2,458,540
\+ 10
2,458,550

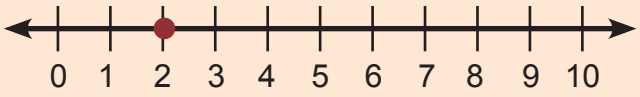
Therefore, 2,458,548 rounded to the nearest tens is 2,458,550.

Example 2

Round off 1,602,162 to the nearest tens.

Solution

1, 6 0 2, 1 6 2
Number to the right
Required place value



2 is in the lower limit so it is nearer to 0 than to 10. So 2 is rounded to 0 ten (00).

1,602,160
\+ 00
1,602,160

Therefore, 1,602,162 rounded to the nearest tens is 1,602,160

Study tip

- To round off whole numbers, use the place values and the value of digits.
- To round off to the nearest tens, first look at the digit in the place value of tens in the whole number.
- If the digit on the right of the required place value is greater or equal to 5, (that is, 5, 6, 7, 8, 9), you round up. Add 1 to the digit in the required place value.
- If the digit on the right of the required place value is less than 5, (that is, 0, 1, 2, 3, 4), you round down. The digit in the required place value doesn't change but all digits to the left change to 0.

Application 1.14

1. Round off to the underlined place value.
   (a) 4,856,7<u>9</u>6 (b) 6,789,7<u>3</u>5 (c) 2,234,5<u>8</u>7
   (d) 3,654,8<u>6</u>7 (e) 62,453,7<u>9</u>2 (f) 5,459,5<u>9</u>9
2. The average number of goats on a farm is 6,753,927, round off this number to the nearest tens.
3. Find the product of 23,000 and 30. Round off your answer to the nearest tens.
4. A school used 100,000 Frw to buy computers and 598,999 Frw to buy books for its library. How much money did it spend altogether? Round off the answer to the nearest tens.
5. A restaurant sells food at 3,000 Frw per plate. On the first day it sold 413 plates, on the second day it sold 123 plates. Round off the plates sold in the two days to nearest tens.

1.15 Rounding off whole numbers to the nearest hundred and thousands

Activity

- In pairs, pick four number cards from the pack.
- Look keenly at the digits in each particular place value.
- Write on the sheets of paper the approximate numbers to hundred.
- Now round off 39,289 to the nearest hundred. Explain your working.

Example 1

Round off 6,773,543 to the nearest hundreds.

Solution

6, 7 7 3, 5 4 3
Number to the right
Number to the right
Required place value



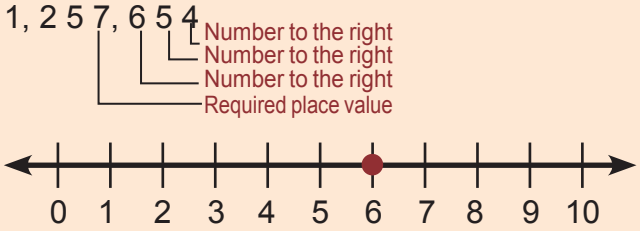
6,773,500
\+ 000
6,773,500

4 is in the lower limit. It is nearer to 0. It is added to 5 to give 5.
∴ 6,773,543 rounded to the nearest hundreds is 6,773,500.

Example 2

Round off 1,257,654 to nearest thousands.

Solution



	1,257,000
+	1,000
	1,258,000

6 is in the upper limit. It is nearer to 1 thousand, 1 is added to 7 to give 8.
Therefore, 1,257,654 rounded to the nearest thousands is 1,258,000.
The number on the right hand side of the digit in the desired place value becomes zero

Study tip

- To round off whole numbers to the nearest hundreds, consider the number in the tens place value.
- To round off whole numbers to the nearest thousands, consider the number in the hundreds place value.
- If the digit is greater or equal to 5, it is converted to one hundred or thousand then added to the place value of hundreds or thousands according to the required place value.
- If the digit is less than 5, it is converted to zero hundred or zero thousand and then added to the required place value.



Application 1.15

1. Round off the following numbers to the nearest hundreds.
   (a) 3,654,597 (b) 22,987,635 (c) 564,323,990
   (d) 3,890,909 (e) 4,361,367 (f) 12,642,298
2. Round off the following numbers to the nearest thousands.
   (a) 6,068,602 (b) 8,523,174 (c) 64,565,438
   (d) 70,309,985 (e) 4,236,201 (f) 17,099,924

1.16 Rounding off whole numbers to the nearest ten thousands, hundred thousands and millions

Activity

Study the number cards and answer the questions that follow:

2,345,789 3,604,800 5,687,231 1,342,798

(a) Identify the digits to the right of the ten thousands place value. Round it up or down.

(b) Add the rounded digit to the ten thousands place value.

(c) Replace all the digits to the right of the ten thousands place value with zeros. What do you notice?

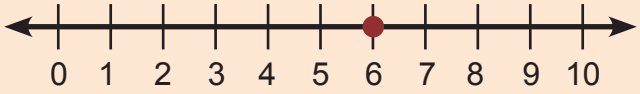
Example 1

Round off 1,576,798 to the nearest ten thousands.

Solution

1, 5 7 6, 7 9 8

Number to the right
Number to the right
Required place value



1,570,000
\+ 10,000
1,580,000

6 is in the upper limit. It is nearer to 1 ten thousands.

So, 6 is rounded to 1 ten thousands (10,000).Therefore, 1,576,798 rounded to the nearest ten thousands is 1,580,000.

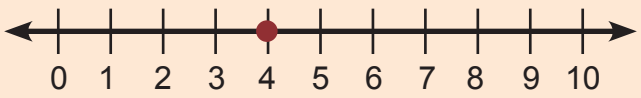
Example 2

Round off 3,540,750 to the nearest hundred thousands.

Solution

3, 5 4 0, 7 5 0

Number to the right
Number to the right
Required place value



4 is in the lower limit. It is nearer to 000,000.
So 4 is rounded to 000,000.

$$\begin{array}{r} 3{,}540{,}000 \\ +\ \ 000{,}000 \\ \hline \mathbf{3{,}500{,}000} \end{array}$$

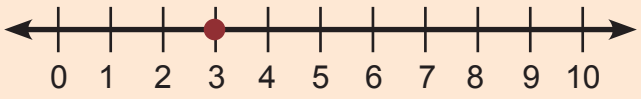
Therefore, 3,540,750 rounded to the nearest hundred thousands is 3,500,000.

Example 3

Round 7,398,500 to the nearest millions.

Solution

7, 3 9 8, 5 0 0
Number to the right
Number to the right
Required place value



$$\begin{array}{r} 7{,}000{,}000 \\ +\ 0{,}000{,}000 \\ \hline \mathbf{7{,}000{,}000} \end{array}$$

3 is in the lower limit. It is nearer to 0 millions. So, 3 is rounded to 0 million (0,000,000).

Therefore, 7,398,500 rounded to the nearest millions is 7,000,000.

Study tip

- To round off to the nearest ten thousands, consider the digit in the thousands place value.
- To round off to the nearest hundred thousands, consider the digit in the ten thousands place value.
- To round off to the nearest million, consider the digit in the hundred thousands place value.
- If the digit is greater or equal to 5, it is converted to one ten

thousands, one hundred thousands, one million respectively, then added to the required place value.

- If the digit is less than 5, it is converted to zero ten thousands, zero hundred thousands, zero million respectively, then added to the required place value.



Application 1.16

1. Round off the following numbers to the nearest ten thousands:
   (a) 4,546,401 (b) 2,560,456 (c) 1,564,670
2. A farmer sold ten of his cows and earned 4,687,300 Frw. Round off his earned money to the nearest ten thousand.
3. Round off the following numbers to the nearest hundred thousands.
   (a) 8,576,700 (b) 61,223,789 (c) 7,890,650
4. Mutesi paid 1,520,500 Frw in tuition fees for her first semester at university. Round off her tuition fees to the nearest hundred thousands.
5. Round off the following numbers to the nearest millions:
   (a) 3,120,600 (b) 8,670,798 (c) 7,456,982
6. Agatesi bought her car for 9,561,000 Frw. Round off the money she paid to the nearest millions.

End of unit 1 assessment

1. Write the place value of the underlined digits?
   (a) 7̲6,76̲7,709 (b) 5,9̲9̲9,9̲9̲9 (c) 1̲,80̲8,06̲4
2. Compare the following using >, < or =.
   (a) 1,121,277.....1,121,207
   (b) 9,876,534..... (3,232,456 + 1,087,653)
   (c) 92,268 ÷ 2..... 7,689 x 12
3. Round off to the underlined digits:
   (a) 8,765,42̲3 (b) 6,545,6̲77 (c) 98,77̲6,113
   (d) 45,36̲7,789 (e) 9̲,999,958 (f) 32,6̲94,689

4. Round off 1,140,038 to the nearest millions.
5. A company printed 19,884,345 books last year and 26,326,150 books this year. How many books were printed altogether over the two years?
6. A farmer's hens gave him 16,764 eggs per day. How many eggs will he get in 30 days?
7. An NGO released five million four hundred and seventy-two thousand six hundred fifty francs to an organisation that rehabilitate street children. Write the amount in figures.
8. A total of nine million, seven hundred and ninety-six thousand, eight hundred and seventeen text books were bought by a library in the 4 years. Write the total number of books in figures.
9. Kamali collected 5,678,950 Frw from milk sales in this month. Write the amount in words.
10. Muhire has 8,434,579 tea shrubs on his tea estate. Write the number in words.
11. Complete the table below.

Quantity one	Sign	Quantity two	Total
1,456,776 kg	+	2,456,767 kg	
2,555,550 Frw	–		1,365,890 Frw
	×	450	10,688,400 Frw
49,560,000 Frw	÷	7,080 Frw	

12. A company produced 10,964,329 bottles of soda in January, 12,726,455 bottles in February, 18,612,900 bottles in March and 5,046,500 bottles in April.
    (a) How many bottles of soda did the company produce over the four months?
    (b) If the company sold 20,892,600 bottles during those four months, how many bottles of soda remained?

Unit 2 Multiplication and division of integers

Key unit competence: To be able to multiply and divide integers.

Introduction

In Mathematics, operations on integers are performed in a range of situations. Suppose you are moving and counting from point A to point B with strides forward for a distance of 20 strides.

(a) What happens if you reach the point B and go back jumping 2 strides four times without changing the direction you are facing? Is the movement positive or negative?

(b) How many jumps are needed to cover the distance from point B to A? Give the mathematical operation used to get the answer.

(c) Consider the starting point A as zero. What will happen if you continue jumping and pass the starting point A four strides?

2.1 Multiplying integers using a number line

Activity

- Draw a number line on the ground.
- Jump from 0 to 4, then from 4 to 8 and from 8 to 12.
- How many jumps have you made?
- Record your findings and discuss.
- What do you notice?

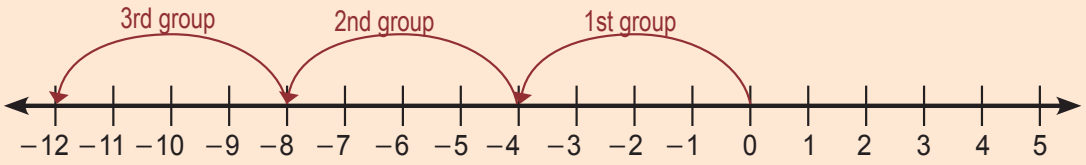
Example 1

Multiply (3) × (−4) using a number line:

Solution

3 × −4 = **−4** + **−4** + **−4**

3 × −4 = 3 groups of −4 (multiplication is repeated addition)



Therefore, 3 × −4 = −12

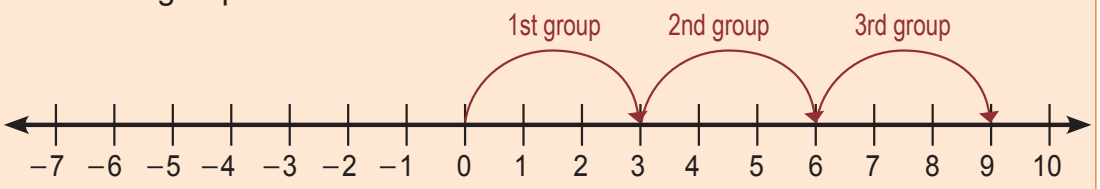
Example 2

Multiply (3) × (+3) using a number line:

Solution

3 × +3 = **+3** + **+3** + **+3** (add +3, three times).

3 × +3 = 3 groups of **+3**



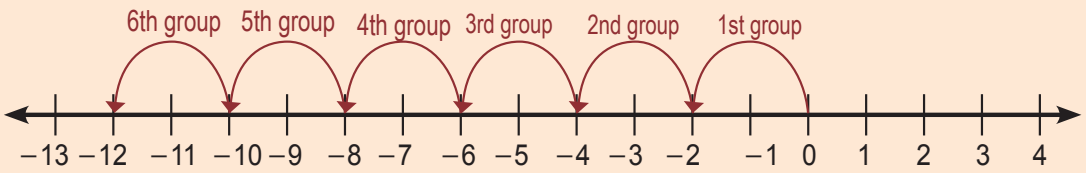
Therefore, 3 × +3 = +9

Example 3

Multiply (6) × (−2) using a number line:

+6 × −2 = **−2** + **−2** + **−2** + **−2** + **−2** + **−2** (add −2, six times).

+6 × −2 = 6 groups of **−2**



Therefore, +6 × −2 = −12

Study tip

- When multiplying integers using a number line, move intervals which are equal to the multiplied number, times the number of the multiplier.
- Multiplication is like repeated addition.
- To multiply a positive integer by a positive, move to the positive side.
- To multiply a negative integer by a positive odd or even number, move to the negative side.
- To multiply a negative integer by a negative numbers, move to the positive side.



Application 2.1

1. Use a number line to multiply the following:
   (a) (+8) × (+5) = (b) (+11) × (-2) = (c) (+6) × (−4) =
   (d) (+3) × (−9) = (e) (+12) × (+3) = (f) (+5) × (−4) =
2. Kankera makes a stride of 3 gaps on a number line to its right. Find the total integers he makes in 6 strides.
3. Gashumba made 9 jumps. Each jump is represented by 2 gaps on a number line to its left. Write an expression that describes the statement. What is the answer?

2.2 Multiplying integers without using a number line

Activity

Pick number cards that complete the following statements:

(a) −3 x +6 = (b) −4 x −9 = (c) +8 x +6 = (d) +2 x −6 =

36 −12 48 −18

Explain the steps taken to arrive at the answer.
Make a presentation to the class.

Example

Multiply the following without using a number line:

(a) +7 × +9 = (b) −3 × +12 = (c) +5 × −8 = (d) −7 × −8 =

Solution

(+ × + = +) (− × + = −) (+ × − = −) (− × − = +)

(a) +7 × +9 = +63 (b) −3 × +12 = −36 (c) +5 × −8 = −40 (d) −7 × −8 = +56

Study tip

- Multiplying integers with the same sign gives a positive. That is, positive × positive gives a positive, and negative × negative gives a positive.
- Multiplying integers with different signs gives a negative. That is, positive × negative gives a negative and negative × positive gives a negative.

Application 2.2

Work out the following:

(a) $+4 \times +4 =$ (b) $-5 \times -8 =$ (c) $+10 \times -7 =$ (d) $-9 \times +6 =$

(e) $12 \times -8 =$ (f) $-4(+3) =$ (g) $-11 \times -9 =$ (h) $+7 \times +6 =$

(i) $-24 \times -25 =$ (j) $120 \times -5 =$ (k) $-125 \times 12 =$ (l) $-123 \times -101 =$

2.3 Dividing integers using a number line

Activity

- Draw a number line on the ground.
- Jump from +28 to +21, then from +21 to +14, then from +14 to +7 and finally from +7 to 0.
- How many jumps have you made?
- Record your findings and discuss.
- What do you notice?

Example

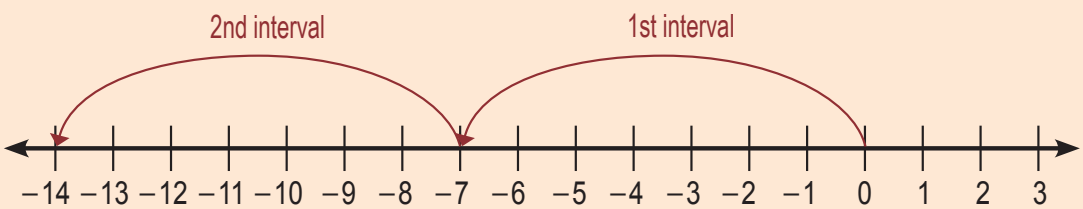
Work out the following using a number line.

(a) $(-14) \div (+7)$ (b) $(+18) \div (+3)$

Solution

(a) $(-14) \div (+7)$

Start moving from 0 making intervals of 7 steps up to -14.

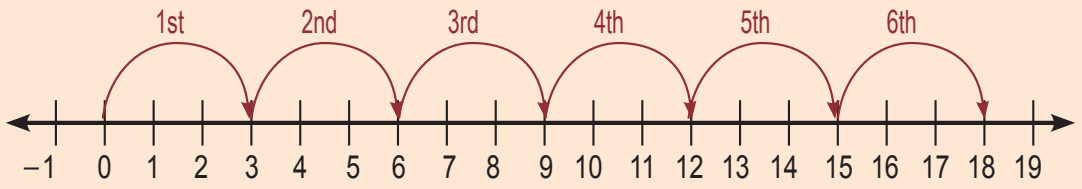


The number of intervals is 2 in the negative direction.

Therefore, $(-14) \div (+7) = -2$

(b) (+18) ÷ (+3)

Start moving from 0 making intervals of 3 steps up to +18.



The number of intervals is 6 in the positive direction.

Therefore, (+18) ÷ (+3) = +6

Study tip

- Division is like repeated subtraction.
- Start from zero and make intervals equivalent to the divisor in length until you reach the dividend.
- The number of intervals becomes the answer.
- If the dividend is in the positive direction (quotient), the integer is positive.
- If the dividend is in the negative direction, the integer is negative.

Application 2.3

Use number lines to divide the following:

(a) +20 ÷ +5 = (b) −16 ÷ +2 = (c) +18 ÷ −3 = (d) −28 ÷ −4 =

(e) +12 ÷ −3 = (f) +27 ÷ +3 = (g) −9 ÷ −3 = (h) −24 ÷ +8 =

2.4 Dividing integers without using a number line

Activity

Pick number cards that complete the following statements:

(a) (+18) ÷ (−3) = (b) (+77) ÷ (+7) =

(c) (−50) ÷ (10) = (d) (+12) ÷ (−3) =

−5 −4 −6 +11

Explain the steps taken to get the answer.

Make a presentation to the class.

Example

Divide the following without using a number line.

(a) $+12 \div +3 =$ (b) $-21 \div +7 =$ (c) $-48 \div -8 =$

Solution

$(+ \div + = +)$ $(- \div + = -)$ $(- \div - = +)$

(a) $+12 \div +3 = +4$ (b) $-21 \div +7 = -3$ (c) $-48 \div -8 = +6$

Study tip

- A positive integer divided by a positive integer the answer is a positive integer.
- A positive integer divided by a negative integer the answer is a negative integer.
- A negative integer divided by a negative integer the answer is a positive integer.



Application 2.4

Divide the following without using a number line.

(a) $(+39) \div (-13) =$ (b) $(-21) \div (-7) =$ (c) $(-50) \div (+10) =$

(d) $(+44) \div (+11) =$ (e) $(-18) \div (-6) =$ (f) $+24 \div +8 =$

(g) $-32 \div -4 =$ (h) $-96 \div -8 =$ (i) $-125 \div -25 =$

(j) $+625 \div +125 =$ (k) $-3{,}500 \div +700 =$ (l) $+299{,}160 \div +12465 =$

2.5 Solving problems involving multiplication and division of integers

Activity

- Think of a number.
- Divide it by $+4$.
- Multiply the answer by -2.
- If the final answer is 6 what is the number?
- Defend your answer.

Example 1

Work out the following:

Temperatures of an area increases by $+4^{o}C$ per hour during the day. What is the total increase in 6 hours?

Solution

1 hr = $+4^{o}C$

6 hrs = $+4^{o}C \times +6$

= $+24^{o}C$

Example 2

James measured the temperature through the day. In the morning it was 16°C, at noon it was 24°C and in the evening it was 18°C. What was the average temperature of the day?

Solution

We have been given: +16°C, +24°C and +18°C.

$$\text{Average} = \frac{\text{Sum of items}}{\text{Number of items}}$$

$$\text{Average} = \frac{+16^{\circ}C + +24^{\circ}C + +18^{\circ}C}{3} = \frac{58^{\circ}C}{3} = 19.3^{\circ}C$$

The average temperature of the day was +19.3°C

Study tip

- Integers may be applied in real life situation, such as loss and profit, temperature rise and fall, ascending and descending altitude.
- You can use multiplication to check your division.



Application 2.5

1. In a village, there is a famous trader called Mugisha. He bought a 100 kg sack of rice at 230,000 Frw. How much did he pay for each kilogramme of rice?
2. Annet, Juma and Shasa shared 18 mangoes among themselves. How many mangoes did each get?
3. A school spent 23,500 Frw on 47 calculators. What was the cost of each calculator?
4. Joshua paid 15,000 Frw for milk from Bizimana's farm during the month of April. How much money did he pay daily?
5. Temperature in Summer increased by +3°C per hour. What was the increase in 9 hours?
6. Each receipt book contains 60 debtors. Each debtor owes 24,000 Frw to the company. Calculate the total debt.
7. The temperature change climbing a mountain was 3°C every after 1 decametre. A climber covered 5 decametre. What was the temperature when he stopped?
8. Arsenal lost 3 points in each of the last 5 games played. What was the loss altogether?

End of unit 2 assessment

1. Solve without using a number line.

   (a) $(+24) \div (+8) =$ (b) $(-10) \times (+3) =$ (c) $(-66) \div (-11) =$

   (d) $(-12) \times (-12) =$ (e) $(+3) \times (-9) =$ (f) $(-20) \div (+4)$

2. Work out the following using a number line.

   (a) $3 \times -4 =$ (b) $+24 \div -3 =$ (c) $+30 \div +6 =$

   (d) $+12 \times -4 =$ (e) $+9 \times +6 =$ (f) $+36 \div -4 =$

3. Ngabire bought 4 kg of sugar each costing 1,200 FRW. How much did she pay?

4. Mukahirwa owed 4 friends of hers 12,000 FRW each. She earned 40,000 FRW and paid it off to them equally. How much is she still owed by each one?

5. Share 4,800 books among 8 learners.

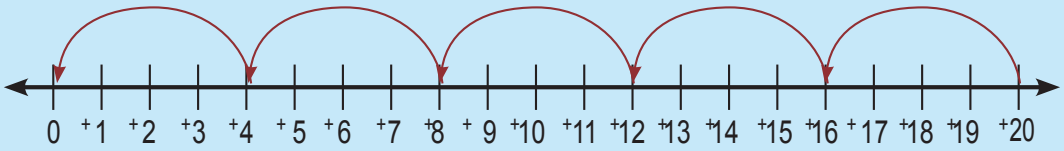
6. The sum of two integers is +20. Dividing the largest integer by the smallest integer gives a quotient of -3. What are the two integers?

7. A debt of 1,600 FRW was cleared by 4 people. How much did each clear?

8. Uwera took a loan of 6,000 FRW. She paid it in 4 equal installments. How much was each installment?

9. A debt of 1,200 FRW was cleared equally by 6 people. How much did each person clear?

10. Write the division statement shown on the number line.



Unit 3 Powers and indices, LCM and GCF

Key unit competence: To be able to use powers and indices, and apply the Lowest Common Multiple (LCM) and the Greatest Common Factors (GCF) when solving problems.

Introduction

To write big numbers in figures, is sometimes challenging for some people.

For example, the distance from the earth to the sun is of many millions of kilometers 149,600,000 km. To facilitate people in writing such kind of numbers, Mathematicians found a way of writing them in short form using powers or indices.

The distance from the earth to the sun can be easily written in short form as follows: 149.6×10^5.

(a) Have you ever come across numbers written in this form?

(b) How is writing big numbers in short form helpful? Explain.

3.1 Indices

Activity

Write 64 on a sheet of paper and prime factorise it.
(a) What common prime factor did you use?
(b) How many times did you multiply it?
Write the common prime factor once and write the number of times to its right, above it. Explain your working.

Example 1

Write $3 \times 3 \times 3 \times 3$ using indices.

Solution

$3 \times 3 \times 3 \times 3 = 3^4$

(3 has been multiplied by itself 4 times. Write 4 to the right, above 3)

Example 2

Write $8 \times 8 \times 8 \times 8 \times 8$ using indices.

Solution

$8 \times 8 \times 8 \times 8 \times 8 = 8^5$

(8 has been multiplied by itself 5 times. Write 5 to the right, above 8)

Study tip

- When writing in indices, write the number that has been multiplied repeatedly once.
- Count and write the number of times the number has been multiplied to the right, above that number.



Application 3.1

1. Write the following numbers using indices:
   (a) $7 \times 7 \times 7 \times 7 \times 7 =$ (b) $6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 =$
   (c) $10 \times 10 \times 10 \times 10 \times 10 =$ (d) $8 \times 8 \times 8 =$
   (e) $9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9 =$ (f) $12 \times 12 \times 12 \times 12 \times 12 \times 12 =$
2. Complete the following statements:
   (a) $4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4^{?}$ (b) $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^{?}$
   (c) $11 \times 11 \times 11 \times 11 \times 11 \times 11 \times 11 = ?^{9}$ (d) $20 \times 20 \times 20 \times 20 = ?^{4}$
3. Write the following powers in expanded form:
   (a) $3^{7} =$ (b) $5^{9} =$ (c) $2^{12} =$ (d) $100^{6} =$

3.2 Defining base and exponent

Activity

Study the number 7^{4} and answer the questions that follow.

(a) Which number has been multiplied repeatedly? What is its name?

(b) Which number shows the number of times the number has been multiplied repeatedly? What is its name?

Explain to the class.

Example 1

Given 6^{3}, which number shows the base and exponent?

Solution

6 is the number that has been multiplied repeatedly. It is called the **base**.

3 shows the number of times 6 must be multiplied by itself. It is called the **exponent** or **index**.

Example 2

Expand 8^{6}.

Solution

8 is the number that has been multiplied repeatedly. It is the **base**.

6 shows the number of times 8 must be multiplied by itself. It is the **exponent**.

Therefore, $8^{6} = 8 \times 8 \times 8 \times 8 \times 8 \times 8$

Study tip

- The number that is multiplied repeatedly is the base.
- The number that shows the number of times a number has been multiplied repeatedly is the exponent.

- Another name for exponent is the index. a^n (exponent, base)
- a^n is "a" to the "n^{th}" power.
- The expression of a number with a base and exponent is called the power. a^n is a power notation.



Application 3.2

1. State the number that represents the base in the following:
   (a) 5^2 (b) 6^4 (c) 10^7 (d) 8^5 (e) 2^9
2. State the exponent in the following:
   (a) 7^5 (b) 15^9 (c) 11^4 (d) 4^7 (e) 18^6
3. Given that 9 is the base and 6 is the exponent, write the number in power notation.
4. Expand 11^7.
5. $6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6$. Write the number in power notation. State the exponent and the base.

3.3 Multiplying and the law of multiplication of indices

Activity

- Given $6^2 \times 6^5$, expand and simplify.
- How many times has 6 been multiplied?
- Now add the indices in the first expression.
- Leave your answer in power notation.
- Compare your results. What do you notice?

Example 1

Simplify $3^3 \times 3^2 \times 3^3$.

Solution

Method 1

Use expanded notation.

$3^3 \times 3^2 \times 3^3 = (3 \times 3 \times 3) \times (3 \times 3) \times (3 \times 3 \times 3)$

$= 3^8$

Method 2

Add the indices.

$3^3 \times 3^2 \times 3^3 = 3^{(3+2+3)}$

$= 3^8$

Example 2

Simplify $7^2 \times 7^2$.

Solution

Method 1

Use expanded notation.

$7^2 \times 7^2 = (7 \times 7) \times (7 \times 7)$

$= 7^4$

Method 2

Add the indices.

$7^2 \times 7^2 = 7^{(2+2)}$

$= 7^4$

Study tip

The law of multiplying indices: When multiplying numbers with the same base, maintain the base and add the exponents or indices.



Application 3.3

1. Simplify the following: Leave your answers in power notation.
   (a) $6^2 \times 6^3 =$ (b) $9^2 \times 9^2 =$ (c) $10^5 \times 10^6 =$
   (d) $12^4 \times 12^5 =$ (e) $4^3 \times 4^3 \times 4^2 =$ (f) $5^3 \times 5 =$
   (g) $7^6 \times 7^4 =$ (h) $20 \times 20 \times 20^5 =$ (i) $13 \times 13 =$
2. Evaluate the following:
   (a) $2^3 \times 2^4 =$ (b) $10^2 \times 10^2 \times 10^1 =$ (c) $4^2 \times 4^1 \times 4^3 =$
   (d) $5^1 \times 5^3 \times 5^1 =$ (e) $11^2 \times 11^1 \times 11^2 =$ (f) $3 \times 3^3 =$
   (g) $2 \times 2^2 \times 2^3 =$ (h) $12 \times 12 \times 12 =$ (i) $1 \times 1^{10} \times 1^4 =$

3.4 Dividing and the law of division of indices

Activity

- Study the statement: $27 \div 9$ and prime factorise the numbers.
- Write the numbers in power notation.
- Simplify, leaving the answer in power notation.
- Now write the first power notation expression again.
- Maintain the base and subtract the indices. Compare your answers.
- Now expand $6^5 \div 6^3$. Show your working.
- Then work out $6^{(5-3)}$. Compare your results. What do you notice?

Example 1

Simplify: $4^3 \div 4^1$.

Solution

Method 1

Expand and divide.

$$4^3 \div 4^1 = \frac{4 \times 4 \times \overset{1}{\cancel{4}}}{\cancel{4}_1} = 4^2$$

Method 2

Subtract the indices.

$4^3 \div 4^1 = 4^{(3-1)} = 4^2$

Example 2

Work out: $8^5 \div 8^3$

Solution

Method 1

Expand and divide.

$$8^5 \div 8^3 = \frac{8 \times 8 \times \overset{1}{\cancel{8}} \times \overset{1}{\cancel{8}} \times \overset{1}{\cancel{8}}}{{}_1\cancel{8} \times {}_1\cancel{8} \times \cancel{8}_1} = 8^2 = 8 \times 8 = 64$$

Method 2

Subtract the indices.

$8^5 \div 8^3 = 8^{(5-3)} = 8^2 = 8 \times 8 = 64$

Study tip

- The law of dividing indices states that, when dividing numbers with the same base, you should maintain the base and subtract the exponents or indices.
- Simplify means the answer is given in power notation.
- Work out or evaluate means getting the value out of the expression.



Application 3.4

1. Use method one to simplify the following:
   (a) $4^2 \div 4^1$ (b) $6^5 \div 6^3$ (c) $9^7 \div 9^4$ (d) $11^6 \div 11^4$
2. Use method two to simplify the following:
   (a) $5^3 \div 5^1$ (b) $8^6 \div 8^3$ (c) $10^8 \div 10^6$ (d) $12^6 \div 12^4$
3. Use both methods to simplify the following:
   (a) $9^5 \div 9^3$ (b) $11^{10} \div 11^4$ (c) $12^6 \div 12^4$ (d) $4^{10} \div 4^4$
4. Use both methods to evaluate the following:
   (a) $4^5 \div 4^4$ (b) $10^5 \div 10^2$ (c) $12^8 \div 12^6$ (d) $8^5 \div 8^2$

3.5 Multiplying and dividing indices

Activity

- On a slip of paper, write the numbers 32, 8 and 16 in power form.
- Multiply the powers of 32 by the powers of 8, then divide by the powers of 16.
- What answer do you get in power form?

Example

Simplify: $2^6 \times 2^8 \div 2^4$.

Method 1

$2^2 \times 2^3 \div 2^4 = \dfrac{2^2 \times 2^3}{2^4}$

Expand and divide by canceling.

$$= \frac{(\cancel{2}^{1} \times 1\cancel{2}) \times (\cancel{2}^{1} \times \cancel{2}^{1} \times 2)}{1\cancel{2} \times 1\cancel{2} \times 1\cancel{2} \times 1\cancel{2}}$$

$$= 2^1 = 2$$

Method 2

Use the law of multiplying indices and the law of dividing indices.

$$= 2^2 \times 2^3 \div 2^4$$

$$= 2^{2+3} \div 2^4$$

$$= \frac{2^5}{2^4} = 2^{(5-4)}$$

$$= \frac{2^5}{2^4} = 2^{(5-4)} = 2^1 = 2$$

Study tip

When multiplying and dividing indices, use both the law of multiplying and that of dividing.



Application 3.5

1. Use method 1 to simplify the following:
   (a) $a^2 \times a^3 \div a^3$ (b) $c^5 \times c^4 \div c^6$ (c) $10^5 \times 10^6 \div 10^2$
2. Use method 2 to simplify the following:
   (a) $q^4 \times q^5 \div q^6$ (b) $m^4 \times m^{-2} \div m$ (c) $8^{-5} \times 8^4 \div 8^2$
3. Evaluate
   (a) $5^3 \times 5^2 \div 5^4$ (b) $7^6 \times 7^{-3} \div 7^{-2}$ (c) $11^{10} \times 11^{-2} \div 11^6$

3.6 Finding unknown and the law of multiplying indices

Activity

- Write two multiples of 5.
- Express them in power form.
- Use the law of multiplying indices to find the answer in power notation.
- If the index of one of the numbers is not given, and the second is given, use the answer to find the unknown index.
- Present your answer to the class.

Example 1

Simplify: $4^y \times 4^3 = 4^9$

Solution

Apply the law of multiplying indices.

$4^y \times 4^3 = 4^{(y+3)} = 4^9$

Form an equation of the indices.

$y + 3 = 9$

Solve by subtracting 3 on both sides.

$y + 3 - 3 = 9 - 3$

$y = 6$

Therefore, the index y is 6.

Example 2

Work out: $7^7 \times 7^m = 7^{11}$

Solution

$7^7 \times 7^m = 7^{11}$

$7^{(7+m)} = 7^{11}$

Form an equation, then solve.

$7 + m = 11$

$7 - 7 + m = 11 - 7$

$m = 4$

Therefore, the index m is 4.

Study tip

To find the unknown index, form an equation using the law of multiplying indices. Then solve it.



Application 3.6

Simplify the following:

(a) $3^2 \times 3^y = 3^3$ (b) $5^6 \times 5^x = 5^{10}$ (c) $9^n \times 9^5 = 9^9$

(d) $2^{14} \times 2^k = 2^{17}$ (e) $11^7 \times 11^9 = 11^n$ (f) $13^p \times 13^6 = 13^7$

(g) $15^6 \times 15^6 = 15^y$ (h) $12^3 \times 12^5 = 12^y$ (i) $16^r \times 16^3 = 16^3$

(j) $4^5 \times 4^x = 4^2$ (k) $6^6 \times 6^3 = 6^k$ (l) $x^3 \times x^y = x^4$

3.7 Finding the unknown and the law of dividing indices

Activity

- Write two numbers in power form.
- Form a statement of dividing and the law of dividing indices for the above numbers.
- Explain your working to the class.

Example 1

Simplify: $5^6 \div 5^y = 5^3$

Solution

Apply the law of dividing indices.

$5^6 \div 5^y = 5^{6-y} = 5^3$

Form an equation of the indices.

$6 - y = 3$

Solve to find y by subtracting y both sides.

$6 - 6 - y = 3 - 6$

$-y = -3$

Divide by -1 both sides to make the terms positive.

$-y \div -1 = -3 \div -1$

Negative ÷ negative = positve.

$y = 3$

Therefore, index y is 3.

Example 2

Work out: $x^n \div x^4 = x^5$

Solution

$x^n \div x^4 = x^5$

Apply the law of dividing indices.

$x^n \div x^4 = x^{n-4} = x^5$

Form an equation of the indices.

$n - 4 = 5$

Solve to find n by adding 4 both sides.

$n - 4 + 4 = 5 + 4$

$n = 9$

Therefore, index n is 9.

Study tip

- To find the unknown index in dividing and the law of dividing indices, form an equation of the indices, then solve it.



Application 3.7

Simplify the following:

(a)	$2^3 \div 2^x = 2^2$	(b)	$5^y \div 5^2 = 5^1$	(c)	$7^m \div 7^4 = 7^2$
(d)	$9^n \div 9^2 = 9$	(e)	$12^k \div 12^3 = 12^3$	(f)	$10^4 \div 10^x = 10^2$
(g)	$k^3 \div k^y = k^1$	(h)	$8^n \div 8^2 = 8^2$	(i)	$6^y \div 6^1 = 6^3$
(j)	$x^a \div x = x^{11}$	(k)	$7^q \div 7^5 = 7^3$	(l)	$y^x \div y = y$

3.8 Finding the lowest common multiple (lcm) of numbers

Activity

Write 8 and 12 on slips of paper.

- Write down the first ten multiples of each number.
- Identify their common multiples.
- What is their lowest common multiple?

Example

Find the LCM of 6 and 9.

Method 1 (prime factorise)

	6	9
2	3	9
3	1	3
3	1	1

LCM = The product of all the prime factors.

LCM = 2 × 3 × 3 = **18**

Method 2 (list multiples)

Multiples of 6 are: 6, 12, 18, 24, 30, 36, 42, ...

Multiples of 9 are: 9, 18, 27, 36, 45, 54, ...

The common multiples are 18, 36, ...

The LCM is **18**.

Study tip

- To find the LCM, either list the multiples of the given numbers and choose the lowest common multiple, or prime factorise the given numbers then find the product of all the prime factors.
- The Lowest Common Multiple (LCM) is also known as the Lowest Common Denominator (LCD) in case of denominators of fractions.



Application 3.8

Find the LCM of the following:

(a) 8 and 6 (b) 18 and 16 (c) 7 and 11

(d) 48 and 32 (e) 10, 12, and 15 (f) 20, 25 and 30

(g) 25, 30 and 45 (h) 45, 60, and 70

3.9 Solving problems involving LCM

Activity

- Two alarm clocks ring in intervals of 10 and 15 minutes.
- Make a list of time intervals for each of the two clocks.
- What is the first common time interval of the two clocks?
- What does the first common time interval mean?
- Present your findings to the class.

Example

Mugisha was diagnosed with malaria and the doctor prescribed the treatment as follows: 4 tablets of Coartem to be swallowed every 12 hours and 2 tablets of Cotrimazole to be swallowed every 8 hours. How long will Mugisha swallow both drugs together again?

Solution

Multiples of 12 are 12, **24**, 36, **48**, 60, 72, ...

Multiples of 8 are 8, 16, **24**, 32, 40, **48**, 56, 64, ...

Common multiples are 24 and 48.

LCM is 24. Therefore, Mugisha will swallow both medicines together again after 24 hours.

Study tip

- To solve problems involving LCM, list the multiples of the given numbers then choose the lowest common multiple.
- Read and understand the question thoroughly.

Application 3.9

Work out the following:

1. Three taxis leave the park at intervals of 15, 20 and 25 minutes. After how long will the taxis leave the park at the same time?
2. Ganza packed books in boxes that carry 15 books each. Ines could not carry the boxes because they were too heavy for her. She repacked the books in boxes carrying 9 books each. How many books were there if she did not leave any books out?
3. Two nurses administer medication to two patients in intervals of 30 and 45 minutes respectively. How long will the nurses administer medication to the patients at the same time if they started at the same time?
4. Three buses arrive at a bus park at intervals of 30, 40 and 45 minutes. How long will the buses take to arrive at the park at the same time if their first arrival time was the same?
5. Kamana plants maize and groundnuts in spaces 12 cm and 15 cm apart respectively. At what distance will both plants be planted together?



Buses arriving at the bus park

3.10 Finding the greatest common factor (GCF) of numbers

Activity

- List 3 numbers of your choice.
- Find the factors of each number.
- List the factors that are common.
- What is the greatest of all the common factors?
- Present your working out to the whole class.

Example

What is the greatest common factor (GCF) of 20 and 24?

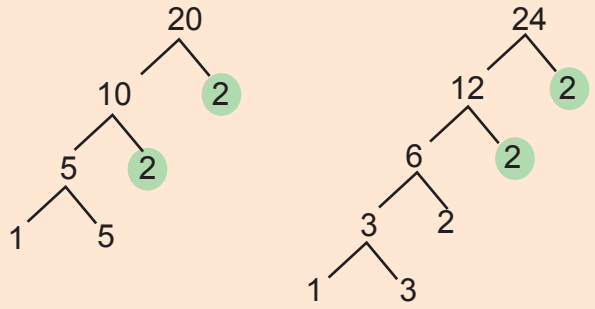
Solution

Method 1

List the factors of 20 and 24.
Factors of 20 are 1, 2, 4, 5, 10, and 20.
Factors of 24 are 1, 2, 3, 4, 6, 8, 12 and 24.
The common factors are 1, 2 and 4.
The GCF is 4.

Method 2

Prime factorise to get the common prime factors.



The common factors are (2, 2)

GCF is 2 x 2 = 4. Therefore, the GCF of 20 and 24 is 4.

Study tip

- To find the greatest common factor, list all the factors of the given numbers. Pick out the common factors then the greatest common factor.
- You can also use a factor tree to find the common factors, then work out their product.



Application 3.10

Find the greatest common factor (GCF) of the following:

1. Use listing of factors.
   (a) 30 and 40 (b) 120 and 180 (c) 180 and 240
   (d) 60, 120 and 180 (e) 120 and 60 (f) 45, 60, and 75
2. Use prime factorisation.
   (a) 40 and 64 (b) 64 and 128 (c) 42, 56, and 84
   (d) 120, 220, and 360 (e) 48, 72 and 96 (f) 50, 75, and 90

3.11 Solving problems involving GCF

Activity

- Two wires 448 cm and 616 cm in length are to be cut into pieces of the same length without any remainder.
- What do we do to find the greatest possible length of the pieces?
- Try to work out the problem.
- Present your findings to the class.

Example

Joel fetched 60 litres of water in the morning and 72 litres in the evening. Find the capacity of the biggest container Joel used in both instances.

Solution

The factors of 60 are: **1**, **2**, **3**, **4**, 5, **6**, 10, **12**, 15, 20, 30, 60.
The factors of 72 are: **1**, **2**, **3**, **4**, **6**, 8, **9**, **12**, 18, 24, 36, 72.
The common factors are: **1**, **2**, **3**, **4**, **6**, **12**.
The greatest common factor is 12.
The biggest container Joel can use in both instances has a capacity of 12 litres.

Study tip

- GCF is useful in packaging products in factories.
- GCF is useful in simplifying fractions to the lowest term.
- GCF is used in dividing two or more numbers without leaving a remainder.

Application 3.11

1. What is the greatest number that can divide 36 and 54 without leaving a remainder?
2. Mutesi collects 30 litres of milk from her farm in the morning. She collects 35 litres of milk in the afternoon and 45 litres in the evening. What is the capacity of the biggest container that can be used in all instances with no milk remaining?
3. Musa collected 48 kg of okra seeds from one garden and 84 kg from another. Find the mass of the pack that can be used in both instances without leaving any okra seeds in the garden.



Mutesi collecting milk.

4. The workers at a maize mill have 3 sacks of maize flour weighing 90 kg, 120 kg and 150 kg respectively. What is the mass of the biggest pack that can be used so that no flour remains in any of the sacks?
5. What number when divided by 6, 8 and 12 gives 3 no remainder?

3.12 Finding the unknown number using lcm and GCF

Activity

- Write two numbers of your choice on slips of paper.
- Work out their LCM.
- Now find the GCF of the two numbers you chose.
- Calculate the product of the two numbers, then divide by the LCM. What do you observe?
- Now calculate by dividing the previous product by the GCF.
- What is the relationship between the product, GCF and LCM?

Example 1

The LCM of two numbers is 30. One of the numbers is 10. If the GCF is 5, find the second number.

Solution

LCM = 30, GCF = 5, 1st number = 10, 2nd number = ?

The product of the two numbers = the product of LCM and GCF.

Let the 2nd number be y.

10 x y = 30 x 5

10 y = 150 (divide by 10 both sides) = $\frac{\overset{1}{\cancel{10}}y}{\cancel{10}_1} = \frac{\cancel{150}}{\cancel{10}} = 15$

Therefore, y = 15.

Example 2

The GCF of two numbers is 3. The two numbers are 6 and 9. Find their LCM.

Solution

GCF = 3, LCM = ?, 1st number = 6, 2nd number = 9

The product of the GCF and LCM = the product of the two numbers.

Let LCM be x.

3 x x = 6 x 9

$3x$ = 54 (divide both sides by 3) = $\frac{\overset{1}{\cancel{3}}x}{\cancel{3}_1} = \frac{\overset{18}{\cancel{54}}}{\cancel{3}_1} = 18$

x = 18 therefore, the LCM is 18.

Study tip

- The product of two numbers is equal to the product of the GCF and LCM.
- LCM is equal to the product of two numbers divided by the GCF.
- GCF is equal to the product of two numbers divided by the LCM.
- The unknown number is equal to the product of the LCM and GCF divided by the given number.



Application 3.12

1. The LCM of two numbers is 72. Find the second number if one of the numbers is 8 and GCF is 12
2. The GCF of two numbers is 14. The numbers are 42 and x. Find x. If LCM is 84
3. The two numbers are 9 and 15. What is their GCF and LCM?
4. The LCM of two numbers is 180. GCF is 25, one of the numbers is 50. What is the second number?
5. Calculate the GCF of 45 and 60.
6. The LCM of two numbers is 144. The GCF is 2. If one of the numbers is 18, what is the other number?
7. The GCF of two numbers is 3. The LCM is 60. If one of the numbers of 12, find the second number.
8. One of the numbers in a pair is h. The other number is 8. If the LCM is 24 and the GCF is 4, what is the value of h?

End of unit 3 assessment

1. Work out the following:
   (a) $8^4 \div 8^3 =$ (b) $3^3 \times ..... = 3^5$ (c) $..... \div 7^2 = 7^3$
   (d) $5^5 \div ... = 5^2$ (e) $12^3 \times 12^{-2} = ?$ (f) $13^4 \times ... \div 13^2 = 13^5$
2. Find the LCM and GCF of the following:
   (a) 60 and 120 (b) 36, 48 and 92 (c) 9, 15 and 36
3. John and Jesca were each given sugarcane of equal length. John cut his sugarcane into equal lengths of 20 cm, while Jesca cut her sugar cane into equal lengths of 50 cm. If they don't have any remainder, find the shortest possible length of sugarcane should each equally have
4. 3 bells at a factory ring at intervals of 15 minutes, 20 minutes and 30 minutes to mark shifts of different departments. If they first ring together at 8:00 am. When do they ring together again?
5. Uwera prepared 96 litres of orange juice, 72 litres pineapple juice and 84 litres of watermelon juice she kept it small containers. What is the capacity of the biggest container she used if no juice remained?
6. At exactly 6:00 a.m, two buses leave the bus terminal. Thereafter, every 40 and 70 minutes, a bus leaves the terminal. Find the time the buses will leave the terminal together.

Unit 4 Operations on fractions

Key unit competence: To be able to apply fractions in daily life situations and solve related problems.

Introduction

In real life, people share many things. They share food, drinks, money, land and so many other things. When people are sharing, each person receives a portion of a whole and this portion is known as fraction in Mathematics.

Suppose that a pineapple is shared equally among 3 people.

(a) What portion/fraction does each person get?

(b) If two of them decide to combine their portions, what mathematical operations do they carry out?

(b) Find out some examples of equal sharing involving Mathematical operations in real life, and present it using fractions.

4.1 Multiplying a whole number by a fraction

Activity

- Collect 9 halves of beans.
- Write one part as a fraction of a whole bean.
- Form a multiplication statement of the number of halves of the beans multiplied by a half of a bean.
- Explain your working out to the class.

Example 1

Multiply: $15 \times \frac{2}{3}$

Solution

Multiply the whole by the numerator.

$\frac{15 \times 2}{3} = \frac{30}{3}$

Divide the numerator by the denominator.

$30 \div 3 = 10$

Example 2

Multiply: 169 by $\times \frac{4}{13}$

Solution

Multiply the whole by the numerator.

$\frac{169 \times 4}{13} = \frac{676}{13}$

Divide the numerator by the denominator.

$673 \div 13 = 52$

Example 1

What is the product of 62 and $3\frac{1}{4}$.

Solution

$62 \times 3\frac{1}{4} = 62 \times \frac{13}{4}$

Multiply the whole by the numerator.

$\frac{62 \times 13}{4} = \frac{806}{4}$

Divide the numerator by the denominator.

$806 \div 4 = 201\frac{1}{2}$

Example 2

Find the product of 13 and $2\frac{1}{3}$.

Solution

$13 \times 2\frac{1}{3} = 13 \times \frac{7}{3}$

Multiply the whole by the numerator.

$\frac{13 \times 7}{3} = \frac{91}{3}$

Divide the numerator by the denominator.

$91 \div 3 = 30\frac{1}{3}$

Study tip

To multiply a whole number by a fraction, multiply the whole number by the numerator, then divide the product by the denominator. Simplify where necessary.



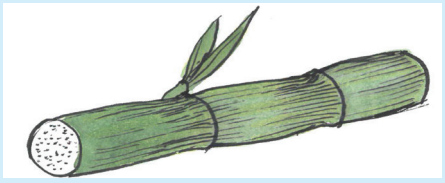
Application 4.1

1. Multiply the following fractions by whole numbers.
   (a) $7 \times \frac{1}{3}$ (b) $12 \times \frac{3}{4}$ (c) $144 \times \frac{1}{12}$ (d) $21 \times 1\frac{1}{4}$
   (e) $121 \times \frac{9}{11}$ (f) $169 \times \frac{3}{13}$ (g) $108 \times \frac{11}{12}$ (h) $100 \times 2\frac{1}{3}$
2. What is the product of 42 and $\frac{5}{7}$?
3. Find the product of 80 and $3\frac{1}{3}$.
4. Multiply 148 by $2\frac{3}{4}$.
5. Calculate the product of 102 and $4\frac{1}{5}$.

4.2 Multiplying a fraction by a whole number

Activity

Look at the sugarcane below:



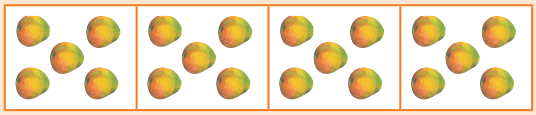
1. How many parts does the sugarcane have?
2. Write the fraction representing one part.
3. Multiply the fraction above with the number of parts.
4. What do you get?

Example 1

What is $\frac{1}{4}$ of 20 mangoes?

Solution

$\frac{1}{4}$ of 20 mangoes $= \frac{1}{4} \times \frac{20}{1}$



$\frac{1}{\cancel{4}_{1}} \times \frac{\cancel{20}^{5}}{1} = 5$ mangoes

Example 2

Multiply: $\frac{5}{12} \times 24$

Solution

$= \frac{5}{\cancel{12}_{1}} \times \frac{\cancel{24}^{2}}{}$

$= 5 \times 2$

$= 10$

Study tip

To multiply a fraction by a whole number, multiply the numerator by the whole number. Divide the product by the denominator and simplify where possible.

Application 4.2

1. Multiply the following fractions by whole numbers.

   (a) $\frac{3}{7} \times 7$ (b) $\frac{3}{4} \times 8$ (c) $1\frac{2}{5} \times 15$ (d) $2\frac{2}{3} \times 18$

   (e) $4\frac{1}{7} \times 35$ (f) $\frac{1}{5} \times 6$ (g) $\frac{3}{8} \times 24$ (h) $\frac{9}{42} \times 7$

2. What is $2\frac{3}{4}$ of 80 minutes?
3. What is $9\frac{2}{3}$ of 27 chairs?
4. What is $7\frac{2}{5}$ of 75 books?
5. What is $13\frac{1}{7}$ of 91 oranges?
6. $\frac{1}{7}$ x 14
7. $\frac{3}{5}$ x 25
8. $\frac{3}{4}$ x 264
9. $1\frac{1}{6}$ x 648

4.3 Multiplying a fraction by a fraction

Activity

- Draw a shape of a whole.
- Divide it into 10 equal parts.
- Shade 5 parts.
- Now divide each part into halves.
- Shade $\frac{1}{2}$ of the shaded part.
- Count and form a fraction of the double shaded part out of all the parts of the whole. What is your answer?
- Now multiply $\frac{1}{2}$ by $\frac{5}{10}$. What do you get? Present your findings to the class.

Example 1

Simplify $\frac{5}{6} \times \frac{3}{4}$

Solution

$\frac{5}{6} \times \frac{3}{4} = \frac{3 \times 3}{6 \times 4} = \frac{15}{24} = \frac{15 \div 3}{24 \div 3}$

Divide by 3 on both numerator and denominator.

$= \frac{5}{8}$

Example 2

What is $2\frac{1}{4}$ multiplied by $1\frac{1}{2}$?

Solution

$2\frac{1}{4} \times 1\frac{1}{2} = \frac{9}{4} \times \frac{3}{2} = \frac{9 \times 3}{4 \times 2} = \frac{27}{8}$

Change to a mixed fraction.

$= 3\frac{3}{8}$

Study tip

When multiplying a fraction by a fraction, multiply both numerators separately and then the denominators separately. Simplify where possible.



Application 4.3

Multiply the following fractions.

(a) $\frac{1}{4} \times \frac{2}{3} \times \frac{1}{9}$ (b) $\frac{3}{7} \times \frac{3}{5}$ (c) $\frac{4}{5} \times \frac{1}{2} \times \frac{1}{3}$ (d) $\frac{1}{11} \times \frac{1}{6}$

(e) $\frac{1}{2} \times \frac{1}{6}$ (f) $1\frac{2}{5} \times \frac{3}{4}$ (g) $\frac{7}{5} \times 2\frac{5}{8}$ (h) $\frac{1}{15} \times \frac{5}{2}$

(i) $3\frac{3}{8} \times 9\frac{4}{12}$ (j) $7\frac{5}{6} \times 1\frac{1}{2}$ (k) $\frac{1}{10} \times \frac{9}{4} \times 3\frac{6}{17}$ (l) $4\frac{5}{6} \times 5\frac{1}{2}$

4.4 Solving problems involving multiplying fractions

Activity

Mbabazi and Rugari stood for the post of Chairperson for their saving group. Mbabazi got $\frac{1}{3}$ of the votes and Rugari got the rest.

If 3,600 people voted:

(a) How many people voted for Mbabazi?

(b) What fraction voted for Rugar?

(c) How many people voted Rugari?

Example 1

Mary is a businesswoman. Her income in a year is 12,000,000 Frw. She calculated her business expenditure and found out that $\frac{1}{5}$ of her income is spent on advertisement of her products.

She spends $\frac{3}{20}$ of her income on paying her staff.

The goods that she bought cost $\frac{1}{3}$ of her income. She saved the rest.

1. How much money does Mary pay for advertising?
2. How much money does Mary pay her staff?
3. What is the cost of the goods that Mary buys?
4. How much does she save?

Solution

Income = 12,000,000 Frw

1. $\frac{1}{5}$ of income = $\frac{1}{\cancel{5}_{1}} \times \overset{2,400,000}{\cancel{12,000,000}} = 1 \times 2,400,000 = 2,400,000$ Frw.
   Mary pays 2,400,000 Frw for advertisement.
2. $\frac{3}{20}$ of income = $\frac{3}{{}_{1}\cancel{20}} \times \overset{600,000}{\cancel{12,000,000}} = 3 \times 600,000 = 1,800,000$ Frw.
   Mary pays her staff 1,800,000 Frw.
3. $\frac{1}{3}$ of income = $\frac{1}{\cancel{3}_{1}} \times \overset{4,000,000}{\cancel{12,000,000}} = 1 \times 4,000,000 = 4,000,000$ Frw.
   The cost of the goods that Mary buys is 4,000,000 Frw.
4. Expenditure = $\frac{1}{3} + \frac{3}{20} + \frac{1}{3} = \frac{12 + 9 + 20}{60} = \frac{41}{60}$

   Fraction saved = $1 - \frac{41}{60} = \frac{60 - 41}{60} = \frac{19}{60}$

   Amount saved = $\frac{19}{{}_{1}\cancel{60}} \times \overset{200,000}{\cancel{12,000,000}} = 19 \times 200,000 = 380,000$ Frw.

Example 2

Uwera had land. She gave $\frac{2}{5}$ of it to her son, Kwizera. She also gave $\frac{3}{8}$ of the land to her daughter, Cissy.

(a) What fraction of the land did she remain with?

(b) If Uwera remained with 18 hectares, calculate the hectares she had originally.

Solution

(a) Land is represented by 1.

Kwizera got $\frac{2}{5}$ of 1 whole.

Cissy got $\frac{3}{8}$ of 1 whole.

Uwera remained with ? of the whole.

Land got by both Kwizera and Uwera was:

$\frac{2}{5} + \frac{3}{8} = \frac{16 + 15}{40} = \frac{31}{40}$ of the whole land.

Fraction of the land Uwera remained with:

$1 - \frac{31}{40} = \frac{40}{40} - \frac{31}{40} = \frac{9}{40}$ of the whole land.

Solution

(b) Let the whole land be x hectares.

$\frac{9}{40}$ of $x = 18$

$\frac{9}{40} \times x = 18$

$\frac{\cancel{40}}{1} \times \frac{9x}{\cancel{40}_{1}} = 18 \times 40$

$\frac{{}^{1}\cancel{9}x}{{}_{1}\cancel{9}} = \frac{{}^{2}\cancel{18} \times 40}{\cancel{9}_{1}}$

Therefore, $x = 2 \times 40 = 80$

Uwera had 80 hectares originally.



Application 4.4

1. Jasmine had 300,000 Frw in her purse. She gave $\frac{3}{10}$ of it to Jane, $\frac{1}{5}$ to Julian and saved the rest.
   (a) What fraction of the money did she save?
   (b) How much money did Jasmine save?
2. Three men shared 800 kg of beans. Ali got $\frac{1}{4}$ of the beans, Moses got $\frac{3}{8}$ and Katto got the remaining beans.
   (a) Calculate the fraction Katto got.
   (b) Work out the amount of beans in kilogram each got.
3. A painter painted $\frac{1}{5}$ of the room with white colour and $\frac{4}{10}$ of it with blue colour. He then painted the remaining part with 5 litres of green colour.
   (a) What part was painted green?
   (b) Work out the amount of paint the painter used to paint the entire classroom.
4. Moses spent $\frac{1}{4}$ of his money on food. He also spent $\frac{1}{3}$ of the remaining money on transport. He was left with 12,000 Frw. How much money did he have originally?
5. Kanyange planted $\frac{1}{8}$ of her land with sorghum, $\frac{1}{4}$ of it with maize and $\frac{1}{2}$ of the remaining land with beans. She planted other crops on the remaining sacres. How much land did she have in acres?

4.5 Finding reciprocals

Activity

- Write a number and multiply it by any unknown.
- Equate it to one (1) to form an equation.
- Calculate the value of the unknown.
- What do you observe about the number you wrote at first compared to the result?
- Present your findings to the class.

Example 1

What is the reciprocal of 5?

Solution

Let the reciprocal be r.

$5 \times r = 1$

$5r = 1$ (divide both sides by 5)

$\frac{5r}{5} = \frac{1}{5}$ therefore, $r = \frac{1}{5}$

The reciprocal of 5 is $\frac{1}{5}$.

Example 2

Find the reciprocal of $\frac{1}{3}$.

Solution

Let the reciprocal be y

$\frac{1}{3} \times y = 1$

$\frac{1}{3}y = 1$

$\cancel{3}^{1} \times \frac{1}{\cancel{3}_{1}} y = 1 \times 3$ therefore, $y = 3$

The reciprocal of $\frac{1}{3}$ is 3.

Study tip

- The reciprocal of a given number is the other number that is multiplied by it to get a product (1).
- The reciprocal of a whole number is a fraction.
- The reciprocal of a fraction is a whole number.



Application 4.5

Find the reciprocal of the following:

(a) 4	(b) 6	(c) 9	(d) 13	(e) 20
(f) 42	(g) $\frac{1}{2}$	(h) $\frac{1}{8}$	(i) $\frac{1}{11}$	(j) $\frac{2}{3}$
(k) $\frac{3}{7}$	(l) $\frac{8}{15}$	(m) $1\frac{1}{3}$	(n) $1\frac{9}{10}$	(o) $2\frac{2}{5}$
(p) $3\frac{1}{4}$	(q) $4\frac{1}{7}$	(r) $11\frac{11}{20}$	(s) 10	(t) 18

4.6 Dividing a whole number by a fraction

Activity

- Collect 2 sheets of paper.
- Cut each in four equal parts.
- What fraction of a whole sheet is each portion?
- Count all the portions altogether.
- Formulate a Mathematical statement for the activity.

Example 1

Divide 32 by $\frac{3}{5}$.

Solution

Multiply the whole by the reciprocal of $\frac{3}{5}$.

$32 \div \frac{3}{5} = 32 \times \frac{5}{3}$

$= \frac{32 \times 5}{3} = \frac{160}{3}$

$160 \div 3 = 53\frac{1}{3}$

Example 2

Divide 38 by $1\frac{8}{11}$.

Solution

Express the mixed fraction as an improper fraction.

$38 \div 1\frac{8}{11} = 38 \div \frac{19}{11}$

The reciprocal of $\frac{19}{11}$ is $\frac{11}{19}$.

$\cancel{38}^{2} \times \frac{11}{\cancel{19}_{1}} = 2 \times 11$

$= 22$

Study tip

- To divide a whole by a fraction, multiply the whole by the reciprocal of the fraction.
- Simplify where necessary.



Application 4.6

Work out the following:

(a) $4 \div \frac{1}{2}$ (b) $10 \div \frac{1}{5}$ (c) $15 \div \frac{2}{5}$ (d) $25 \div \frac{5}{11}$

(e) $60 \div 1\frac{1}{3}$ (f) $108 \div 2\frac{1}{4}$ (g) $144 \div \frac{12}{13}$ (h) $49 \div 5\frac{1}{6}$

(i) $54 \div 1\frac{1}{5}$ (j) $60 \div 2\frac{1}{2}$ (k) $96 \div 2\frac{2}{7}$ (l) $100 \div 3\frac{1}{3}$

4.7 Dividing a fraction by a whole number

Activity

- Draw an orange and divide it into two equal parts.
- Divide each portion into 3 portions.
- Now count the portions you have.
- What is the fraction of one portion out of all the portions?
- Present your findings to the class.

Example 1

Divide $\frac{1}{2}$ by 3.

Solution

The reciprocal of 3 is $\frac{1}{3}$.

Multiply by the reciprocal of 3.

$\frac{1}{2} \div 3 = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

Example 2

Work out: $3\frac{1}{4} \div 13$.

Solution

The reciprocal of 13 is $\frac{1}{13}$.

Multiply by the reciprocal of 13.

$3\frac{1}{4} \div 13 = \frac{\cancel{13}^{1}}{4} \times \frac{1}{\cancel{13}_{1}} = \frac{1}{4}$

Study tip

To divide a fraction by a whole number, first write the fraction, then multiply the fraction by the reciprocal of the whole number.



Application 4.7

Work out the following:

(a) $\frac{5}{6} \div 7$ (b) $\frac{3}{4} \div 9$ (c) $\frac{1}{3} \div 2$ (d) $\frac{9}{5} \div 3$

(e) $\frac{7}{2} \div 5$ (f) $4\frac{2}{13} \div 8$ (g) $2\frac{4}{5} \div 4$ (h) $2\frac{1}{2} \div 5$

(i) $\frac{5}{6} \div 2$ (j) $\frac{7}{10} \div 7$ (k) $1\frac{1}{4} \div 5$ (l) $3\frac{1}{9} \div 14$

4.8 Dividing a fraction by a fraction

Activity

- Draw a circle and divide it into 8 equal portions.
- Shade 3 portions and write down the fraction of the shaded part.
- Divide the shaded portion into 2 equal parts.
- Shade one portion. Write it as a fraction of the first shaded portion.
- Divide the circle into more equal small portions.
- How many are there?
- What is the relationship between the first and final shaded parts?
- What is the fraction of the final shaded portion out of all the small portions of the circle? Write it down.
- Share the procedure with the whole class.

Example 1

Work out $\frac{3}{4} \div \frac{1}{3}$

Solution

$$\frac{3}{4} \div \frac{1}{3} = \frac{3 \times 3}{4 \times 1}$$

$$= \frac{9}{4}$$

$$= 2\frac{1}{4}$$

Example 2

Divide $4\frac{2}{3} \div 1\frac{3}{4}$

Solution

$$4\frac{2}{3} \div 1\frac{3}{4} = \frac{14}{3} \div \frac{7}{4}$$

Change both into improper fractions.

$$= \frac{{}^{2}\cancel{14}}{3} \times \frac{4}{\cancel{7}_{1}}$$

$$= \frac{8}{3} = 2\frac{2}{3}$$

Study tip

When dividing a fraction by a fraction, multiply the first fraction by the reciprocal of the second fraction. Simplify where possible.



Application 4.8

Work out the following:

(a) $\frac{3}{4} \div \frac{2}{9}$ (b) $\frac{5}{6} \div \frac{1}{3}$ (c) $\frac{5}{3} \div \frac{3}{4}$

(d) $1\frac{1}{3} \div \frac{3}{7}$ (e) $3\frac{2}{5} \div 1\frac{2}{9}$ (f) $4\frac{2}{11} \div 1\frac{1}{2}$

(g) $5\frac{1}{3} \div 2\frac{1}{4}$ (h) $\frac{1}{2} \div \frac{109}{89}$ (i) $\frac{56}{57} \div \frac{19}{44}$

(j) $\frac{4}{7} \div \frac{2}{5}$ (k) $\frac{1}{2} \div \frac{3}{4}$ (l) $\frac{9}{5} \div \frac{3}{7}$

4.9 Solving problems involving dividing fractions

Activity

Juma collected 15 litres of water which is three quarters of a jerrycan.
What is the capacity of the container Juma used?
Present your working to the class.

Example

A shopkeeper sold 50 kg of maize flour in a week. This was half of a full sack of maize flour. How many kilograms does a full sack carry?

Solution

$\frac{1}{2}$ of a sack = 50 kg

1 sack = $50 \div \frac{1}{2}$ (The reciprocal of $\frac{1}{2}$ is 2)

= $50 \times 2 = 100$ kg

Study tip

- In order to work out problems involving dividing fractions, read and interpret in order to understand.
- Identify the operations to be used. The given portion is divided by the given amount to find the total quantity.



Application 4.9

Work out the following and explain your working:

1. The product of two number is $\frac{7}{3}$. The first number is $\frac{7}{9}$. Find the second number.
2. A car covered 240 km of a journey. This is only $\frac{6}{7}$ of the whole journey. What is the distance of the whole journey?
3. Kalisa planted 25 hectares of maize. This is $\frac{5}{8}$ of his land. How big is his land?
4. A learner attempted 30 questions in an examination. This was $\frac{3}{5}$ of whole examination. How many questions were in whole examination paper?
5. A teacher used 75 pieces of chalk to write on the blackboard for a month. This is $\frac{3}{4}$ of the full box of chalk. How many pieces of chalk does the box contain?

4.10 Multiplying and dividing fractions

Activity

- Write three fractions of your choice.
- Multiply the first two.
- Divide the product of the first two by the third fraction.
- Which steps did you carry out to get the answer?
- Present your working to the class.

Example 1

Work out: $\frac{1}{2} \times \frac{2}{3} \div \frac{5}{6}$

Method 1

Work out division first.

$\frac{2}{3} \div \frac{5}{6}$ (Multiply by the reciprocal of $\frac{5}{6}$).

$= \frac{2}{3} \times \frac{6}{5} = \frac{2 \times 6}{3 \times 5} = \frac{12}{15}$

Multiply by $\frac{1}{2}$

$= \frac{1}{\cancel{2}_{1}} \times \frac{\cancel{12}^{6}}{15} = \frac{1 \times \cancel{6}^{2}}{1 \times \cancel{15}_{5}}$

$= \frac{2}{5}$

Method 2

Combine the working out.

$\frac{1}{2} \times \frac{2}{3} \div \frac{5}{6}$

Multiply by the reciprocal of $\frac{5}{6}$.

$= \frac{1}{2} \times \frac{2}{3} \times \frac{6}{5}$

Simplify where possible.

$= \frac{1}{\cancel{2}_{1}} \times \frac{\cancel{2}^{1}}{\cancel{3}_{1}} \times \frac{\cancel{6}^{2}}{5}$

$= \frac{1 \times 1 \times 2}{1 \times 1 \times 5} = \frac{2}{5}$

Example 2

Work out: $1\frac{1}{3} \div \frac{3}{4} \times 2\frac{1}{2}$

Method 1

Work out division first.

$1\frac{1}{3} \div \frac{3}{4} = \frac{4}{3} \div \frac{3}{4}$

Multiply by the reciprocal of $\frac{3}{4}$.

$= \frac{4}{3} \times \frac{4}{3} = \frac{16}{9}$

Multiply $\frac{16}{9}$ by $2\frac{1}{2}$.

$= \frac{\cancel{16}^{8}}{9} \times \frac{5}{\cancel{2}_{1}} = \frac{40}{9}$

$= 4\frac{4}{9}$

Method 2

Combine the working out.

$1\frac{1}{3} \div \frac{3}{4} \times 2\frac{1}{2}$

Change to improper fraction.

$= \frac{4}{3} \div \frac{3}{4} \times \frac{5}{2}$

Multiply by the reciprocal of $\frac{3}{4}$.

$\frac{4}{3} \times \frac{4}{3} \times \frac{5}{2}$

Simplify where possible.

$= \frac{4}{3} \times \frac{\cancel{4}^{2}}{3} \times \frac{5}{\cancel{2}_{1}} = \frac{4 \times 2 \times 5}{3 \times 3 \times 1}$

$= \frac{40}{9} = 4\frac{4}{9}$

Study tip

- When calculating combined multiplying and dividing fractions, work out division first, then multiply and simplify where possible. Use BODMAS.
- Multiplication and division of fractions can be worked by changing the

divided fraction to its reciprocal. Then the fractions are multiplied together.



Application 4.10

(a) $\frac{3}{4} \times \frac{1}{2} \div \frac{1}{3}$	(b) $\frac{2}{3} \div \frac{1}{4} \times \frac{3}{5}$	(c) $\frac{5}{7} \times \frac{4}{5} \div \frac{2}{3}$
(d) $\frac{9}{10} \div \frac{4}{5} \times \frac{1}{2}$	(e) $1\frac{1}{3} \times \frac{3}{4} \div \frac{2}{3}$	(f) $2\frac{1}{3} \div 1\frac{1}{2} \times 1\frac{1}{4}$
(g) $3\frac{1}{2} \times \frac{4}{5} \div 1\frac{1}{3}$	(h) $6\frac{1}{3} \div 3\frac{2}{3} \times 2\frac{1}{4}$	(i) $\frac{1}{3} \times 3\frac{2}{3} \div \frac{1}{4}$

End of unit 4 assessment

1. Work out the following

   (a) $\frac{2}{3} \times \frac{2}{5}$ (b) $1\frac{3}{5} \div \frac{1}{3}$ (c) $\frac{1}{2} \times 3$ (d) $5\frac{5}{6} \times 4$

   (e) $12\frac{1}{5} \times 11\frac{1}{4}$ (f) $\frac{1}{9} \div \frac{2}{3}$ (g) $6\frac{1}{4} \div 2\frac{1}{8}$ (h) $10\frac{1}{2} \div 5\frac{1}{4}$

2. Mutesi's land is $2\frac{1}{2}$ times of Uwacu's land. Uwacu has 5 hectares. How big is Mutesi's land?

3. Akida's salary is 640,000 Frw. He spends $\frac{3}{5}$ on food, $\frac{1}{4}$ of it on school fees and $\frac{3}{20}$ on other requirements. How much does he spend on each item?

4. Mukamusoni planted saplings on her land. $\frac{1}{3}$ of them were eucalyptus and $\frac{2}{5}$ of them were pine saplings. The rest were cypress saplings.

   (a) What was the fraction for cypress saplings?

   (b) If she planted 60 cypress saplings, how many saplings did she plant altogether?

5. Reverend Kalisa was traveling on a journey. After covering $\frac{4}{9}$ of it, his car broke down. He continued the remaining 20 km by bus. Find the distance of the whole journey.

6. Sylvia made a mug of tea. $\frac{3}{5}$ of it was milk. The remaining 200 ml were water. How many milliliters filled the cup?

Unit 5 Rounding and converting decimals, fractions/numbers

Key unit competence: To be able to round off decimals, convert fractions to decimals and vice versa.

Introduction

In counting and mathematical operations, some numbers may be found difficult to be memorized or used. These can be simplified to their nearest numbers which can be easily memorized or used. For example money used by different schools per year can be rounded to simple figures to summarize.

Let's consider an example of school party where Primary 6 learners contributed to buy all items needed. After planning all needed items, they found that the quantity of rice needed for the party is 40.97 kg which is near 50 kg. Then they decided to buy 50 kg of rice instead of 49.97 kg.

(a) Why do you think Primary 6 learners prefer to buy 50 kg instead of 49.97 kg?

(b) Do you think both quantities: 50 kg and 49.97 kg are easily memorized?

(c) How is rounding off useful in daily life?

5.1 Rounding off decimal vumbers to the nearest tenths

Activity

- Pick a flash card from A and match it with its corresponding card in B containing a value nearest to it.

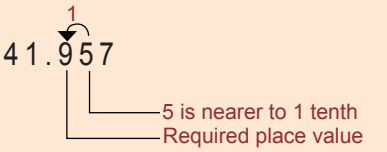
A	B
0.243	10.1
66.98	27.2
199.605	0.2
27.166	67.0
10.09	0.6
0.5623	199.6

- What have you noticed?
- Present your findings to the class.

Example 1

Round off 41.957 to the nearest tenths.

Solution



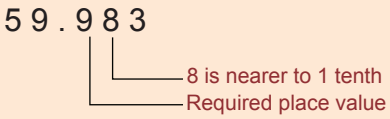
41.957 = 41.95 + 1 = 42.0

Therefore, 41.957 is rounded off to 42.0.

Example 2

Round off 59.983 to the nearest tenths.

Solution



59.98 = 59.89 + 1 = 60.0 *(Because 8 is nearer to 1.)*

Therefore, 59.983 is rounded off to 60.0.

Study tip

- To round off to the nearest tenths, first identify the place value of tenths. Then find the digit to its right..
- If it is 0, 1, 2, 3 or 4, add 0 to the digit in the tenths place value. This is because the digits are nearest to 0.
- If it is 5, 6, 7, 8 or 9, add 1 to the digit in the tenths place value. This is because the digits are nearest to 10.
- The rounded off answer must be to one decimal place.



Application 5.1

Work out the following.

1. Round off to the nearest tenths:

   (a) 0.86 (b) 8.51 (c) 12.97

   (d) 2.15 (e) 15.03 (f) 18.26

2. Round off the following to the underlined place value:

   (a) 56,748.<u>9</u>3 (b) 875.<u>3</u>9 (c) 264,537.<u>2</u>992

   (d) 12,354.<u>6</u>78 (e) 0.<u>8</u>7 (f) 455.<u>9</u>99

5.2 Rounding off decimal numbers to the nearest hundredths

Activity

Read, think and respond quickly, then explain.

1. 123.1234 rounded to the nearest hundredths is equal to….
2. 239.5589 rounded to the nearest hundredths is equal to….
3. Explain the difference between those two numbers.
4. Present your findings to the class.

Example 1

Round off 0.107 to the nearest hundredths.

Solution

0 . 1 0 7
7 is nearer to 1 hundredth
Required place value

$$0.107 = \begin{array}{r} 0.107 \\ +\ 1 \\ \hline 0.11 \end{array}$$

Therefore, 0.107 is rounded off to 0.11.

Example 2

Round off 123.123 to the nearest hundredths.

Solution

1 2 3 . 1 2 3
3 is nearer to 0 hundredth
Required place value

$$123.123 = \begin{array}{r} 123.123 \\ +\ 0 \\ \hline 123.12 \end{array}$$

Therefore, 123.123 is rounded off to 123.12.

Study tip

- To round off to the nearest hundredths, first identify the place value of hundredths and find the digit to its right.
- If it is 0, 1, 2, 3 or 4, add 0 to the digit in the hundredths place value. If it is 5, 6, 7, 8 or 9, add 1 to the digit in the hundredths place value.
- The rounded off answer must be to two decimal places.



Application 5.2

Round off the following decimals to the nearest hundredths:

(a) 16.597	(b) 4.221	(c) 0.571	(d) 8.009
(e) 367.812	(f) 6.0027	(g) 5.2743	(h) 0.6451
(i) 7.0095	(j) 4.3214	(k) 6.7890	(l) 9.248

5.3 Rounding off decimal numbers to the nearest thousandths

Activity

- Pick a flash card from A and match it with its corresponding card in B containing a value nearest to it.

A	B
10.2435	10.244
66.9804	27.167
199.6057	0.2
27.1665	66.980
10.0928	0.562
0.5623	199.606

- What have you noticed?
- Present your findings to the class.

Example 1

Round off 5.6877 to the nearest thousandth.

Solution

5 . 6 8 7 7

7 is nearer to 1 thousandth

Required place value

$$\begin{array}{rr} 5.6877 = & \begin{array}{r} 5\,.\,6\,8\,7\,\cancel{7}^{\,1} \\ +\qquad 1 \\ \hline 5\,.\,6\,8\,8 \end{array} \end{array}$$

5.6877 is rounded off to 5.688.

Example 2

Round off 142.13534 to the nearest thousandth.

Solution

1 4 2 . 1 3 5 3 4

3 is nearer to 0 thousandth

Required place value

$$\begin{array}{rr} 142.13534 = & \begin{array}{r} 1\,4\,2\,.\,1\,3\,5\,\cancel{3}^{\,0}\,4 \\ +\qquad\quad 0 \\ \hline 1\,4\,2\,.\,1\,3\,5 \end{array} \end{array}$$

142.13534 is rounded off to 142.135.

Study tip

- To round off to the nearest thousandth, first identify the thousandths place value.
- If the digit to the right of the thousandths place value is 0, 1, 2, 3, or 4, add 0 to the digit in the digit in the thousandths place value.
- If the digit to the right of the thousandths place value is 5, 6, 7, 8, or 9, add 1 to the thousandths place value.
- The rounded off answer must possess three decimal places.



Application 5.3

Round off the following to the nearest thousandth:

(a) 2.7985	(b) 12. 3421	(c) 5.6874	(d) 125.8213
(e) 1.4685	(f) 25.0098	(g) 7.23567	(h) 34.69745
(i) 0.69736	(j) 67.7983	(k) 89.8357	(l) 295.2110

5.4 Rounding off decimal numbers to the nearest ten thousandths

Activity

- Study the number cards below:

  7.04683 | 0.62437 | 1,625.00016 | 0.99685

- Write the approximate values of the numbers estimated to ten thousandths.
- Explain the procedure to the class.

Example 1

Round off 3.34507 to the nearest ten thousandths.

Solution

3 . 3 4 5 0 7

7 is nearer to 1 ten thousand

Required place value

$$3.34507 = \begin{array}{r} 3.34507^{1} \\ + \quad 1 \\ \hline 3.3451 \end{array}$$

3.34507 is rounded off to 3.3451.

Example 2

What is 0.07124 rounded off to the nearest ten thousandths?

Solution

0 . 0 7 1 2 4

4 is nearer to 0 ten thousandth

Required place value

$$0.07124 = \begin{array}{r} 0.0712^{4^{0}} \\ + \quad 0 \\ \hline 0.0712 \end{array}$$

0.07124 is rounded off to 0.0712.

Study tip

- To round off to the nearest ten thousandths, first identify the ten thousandths place value.
- If the digit to the right of the ten thousandths place value is 0, 1, 2, 3, or 4, add zero (0) to the digit in the ten thousandths place value.
- If the digit to the right of the ten thousandths place value is 5, 6, 7, 8 or 9, add 1 to the digit in the ten thousandths place value.
- The rounded off answer must be to four decimal places.



Application 5.4

Round off the following to the nearest ten thousandths:

(a) 0.06951	(b) 12.106798	(c) 482.00311	(d) 0.002456
(e) 4.99999	(f) 19.00283	(g) 0.97568	(h) 73.01010
(i) 1,206.07989	(j) 63,006.7099	(k) 723.909	(l) 0.917801

5.5 Rounding off decimal numbers to the nearest hundred thousandth

Activity

- Study the decimal numbers in A and B.

A	B
0.063768	0.99365
4.739256	18.88235
13.004021	4.73926
0.993652	0.06377
18.882346	13.00402

- Match the corresponding decimal numbers in A to those in B.
- Defend your answers.

Example 1

Round off 0.076423 to the nearest hundred thousandths.

Solution

0 . 0 7 6 4 2 3

3 is nearer to 0 hundred thousandth

Required place value

$$0.076423 = \begin{array}{r} 0.0764230 \\ + \quad\quad 0 \\ \hline 0.07642 \end{array}$$

0.076423 is rounded off to 0.07642.

Example 2

What is 76.2569293 rounded off to the nearest hundred thousandths?

Solution

7 6 . 2 5 6 9 2 9 3

9 is nearer to 1 hundred thousandth

Required place value

$$76.2569293 = \begin{array}{r} 76.2569293 \\ + \quad\quad 1 \\ \hline 76.25693 \end{array}$$

76.2569293 is rounded off to 76.25 693.

Study tip

- To round off to the nearest hundred thousandths, first identify the place value of hundred thousandth, then find the digit to its right.
- If it is 0, 1, 2, 3, or 4, add 0 to the digit in the hundred thousandths place value.
- If it is 5, 6, 7, 8 or 9, add 1 to the digit in the hundred thousandths place value.
- The rounded off answer must be to five decimal places.



Application 5.5

Round off the following to the nearest hundred thousandths:

(a) 1.1010110	(b) 0.063285	(c) 0.006192
(d) 15.73912	(e) 92.093018	(f) 0.185306
(g) 100.683048	(h) 9.999999	(i) 45.789093

5.6 Rounding off decimal numbers to the nearest millionths

Activity

- Pick five cards with 7 and 8 decimal places from the pack.
- Study each card critically.
- On slips of paper, write the approximate of each number to six decimal places.
- What did you consider when approximating?

Example 1

Round off 0.1550536 to the nearest millionths.

Solution

0 . 1 5 5 0 5 3 6

6 is nearer to 1 millionth

Required place value

0 . 1 5 5 0 5 3 6

0.01550536 = + 1

0 . 1 5 5 0 5 4

0.1550536 is rounded off to 0.155054.

Example 2

Round off 31.0938173 to the nearest millionths.

Solution

3 1 . 0 9 3 8 1 7 3

3 is nearer to 0 millionth

Required place value

3 1 . 0 9 3 8 1 7 3

31.0938173 = + 0

3 1 . 0 9 3 8 1 7

31.0938173 is rounded off to 31.093817.

Study tip

- To round off to the nearest millionths, first identify the place value of millionth and find the digit to its right.
- If the digit is 0, 1, 2, 3 or 4, add 0 to the digit in the millionth place value.
- Add 1 to the digit in the millionth place value if the digit is 5, 6, 7, 8 or 9
- The rounded off answer must be with six decimal places.



Application 5.6

Round off the following decimals to the nearest millionths.

(a) 0.1251076	(b) 4.1407324	(c) 0.0506985
(d) 13.1303643	(e) 1.1109427	(f) 46.9328342
(g) 0.8342071	(h) 0.0321969	(i) 85.0732804

5.7 Solving problems involving rounding off decimal numbers

Activity

- Measure the length, then the width, of the football pitch.
- Note the measurements on slips of paper.
  (a) What is the area to the nearest whole? (ones)
  (b) Round off the width to the nearest tenth.
- Discuss how rounding off can be applied in daily life.

Example

The area of a district is 11,033.1416 square kilometres.
(a) Round off the area to the nearest tenths.
(b) What is the area rounded off to the nearest thousandths?

(a) 11,033.1416

1 1 0 3 3 . 1 4 1 6

4 is nearer to 0 tenths
Required place value

$$\begin{array}{r} 11{,}033.1\overset{0}{4}16 \\ +\quad 0 \\ \hline 11{,}033.1 \end{array}$$

Therefore, the area rounded to the nearest tenth is 11,033.1 square kilometres.

(b) 11,033.1416

11033.1416

6 is nearer to 1 thousandth
Required place value

$$\begin{array}{r} 1\,1\,0\,3\,3\,.\,1\,4\,1\overset{1}{6} \\ +\qquad\qquad 1 \\ \hline 1\,1\,0\,3\,3\,.\,1\,4\,2 \end{array}$$

Therefore, the area rounded to the nearest thousandth is 11,033.142 square kilometres.

Study tip

- Read and understand the statements.
- Consider rounding off when estimating quantities.
- If the digit in the place value to the right of the required place value is 0, 1, 2, 3, or 4, round down.
- If the digit in the place value to the right is 5, 6, 7, 8, or 9, round up.



Application 5.7

1. A trader bought beans weighing 1,672.3 kg. Approximately how many kilograms were there when rounded to the nearest ones?
2. Richard's height is 177.3 cm. Write this height to the nearest whole number.
3. In a school store, there are 362.51 kg of beans and 506.78 kg of maize flour. Round off the total kilograms to the nearest tenths.
4. Michael Jordan ran 100 m in 9.83 seconds. Round off the time to the nearest seconds.
5. A forest has an area of 6.234 hectares. Round off this area to the nearest hundredths.
6. At birth, Winfred weighed 3.27 kg. Round off the weight to the nearest kilograms.
7. The area of a certain district is 1,467.49 hectares. Round off the area of that district to the nearest tenths.
8. The distance between two towns is 236.17 km. Round off the distance to the nearest kilometres.

5.8 Converting fractions into decimals

Activity

Look at the fractions in part A and decimals in part B. Match them accordingly. Explain your matching process to the whole class.

Part A	Part B
$\frac{6}{5}$	2.56
$\frac{64}{25}$	0.88
$5\frac{4}{10}$	1.2
$\frac{88}{100}$	5.4

Example 1

Convert $\frac{5}{8}$ into decimals.

Solution

$$
\begin{array}{r}
0.625 \\
8\overline{)\,5000} \\
(5\times 0)= \quad 0 \\
\hline
50 \\
(6\times 8)= \quad 48 \\
\hline
20 \\
(2\times 8)= \quad 16 \\
\hline
40 \\
(5\times 8)= \quad 40 \\
\hline
00
\end{array}
$$

Therefore, $\frac{5}{8} = 0.625$.

Example 2

Convert $\frac{5}{4}$ into decimals.

Solution

$$
\begin{array}{r}
1.25 \\
4\overline{)\,500} \\
(1\times 4)= \quad 4 \\
\hline
10 \\
(2\times 4)= \quad 8 \\
\hline
20 \\
(5\times 4)= \quad 20 \\
\hline
00
\end{array}
$$

Therefore, $\frac{5}{4} = 1.25$

Example 3

Convert $9\frac{2}{5}$ into decimals.

Solution

$$9\frac{2}{5} = \frac{(9\times 5)+2}{5} = \frac{47}{5}$$

$$
\begin{array}{r}
9.4 \\
5\overline{)\,470} \\
(5\times 9)= \quad 45 \\
\hline
20 \\
(5\times 4)= \quad 20 \\
\hline
00
\end{array}
$$

Therefore, $9\frac{2}{5} = 9.4$

Example 4

Convert $\frac{1}{3}$ into decimals.

Solution

$$
\begin{array}{r}
0.33\ldots \\
3\overline{)\,100} \\
(3\times 0)= \quad 0 \\
\hline
10 \\
(3\times 3)= \quad 9 \\
\hline
10 \\
(3\times 3)= \quad 9 \\
\hline
1
\end{array}
$$

Therefore, $\frac{1}{3} = 0.333\ldots$

Study tip

- When converting fractions into decimals, we divide the numerator by the denominator.
- To change a fraction to a decimal, take a number which you can multiply by the denominator to get multiples of 10, 100, 1000, etc. Multiply the number by both the numerator and the denominator. Then convert to decimals considering the place value of the denominator.
- For mixed fractions like in example 3, first change the mixed fractions to an improper fraction, then divide the numerator by the denominator.
- When a fraction gives a decimal number with repeating digits, the decimal number is called a recurring decimal. It is written with an ellipsis to the right.



Application 5.8

Convert the following fractions into decimals.

(a) $\frac{13}{20}$	(b) $\frac{5}{16}$	(c) $\frac{59}{10}$	(d) $2\frac{1}{10}$	(e) $\frac{2}{5}$
(f) $\frac{11}{8}$	(g) $1\frac{3}{8}$	(h) $5\frac{3}{4}$	(i) $\frac{15}{11}$	(j) $\frac{12}{9}$
(k) $\frac{1}{9}$	(l) $\frac{2}{3}$	(m) $\frac{4}{9}$	(n) $\frac{6}{11}$	(o) $1\frac{1}{3}$
(p) $\frac{4}{8}$	(q) $\frac{2}{6}$	(r) $\frac{1}{6}$	(s) $\frac{12}{24}$	(t) $1\frac{2}{5}$

5.9 Converting decimals into fractions

Activity

Write 3 decimal numbers:

- The first with 1 decimal place.
- The second with 2 decimal places.
- The third with 3 decimal places.

Ignore the decimal point and write the denominator under each decimal, the zeros matching the number of decimal places.

What is your observation?

Example 1

Convert 0.236 into a fraction and show your working.

Solution

There are 3 decimal places, therefore divide by 1,000 and simplify where possible.

$$0.236 = \frac{\cancel{236}^{113}}{\cancel{1000}_{500}}$$

$$= \frac{113}{500}$$

Example 2

Convert 4.09 into a fraction and show your working.

Solution

There are 2 decimal places, therefore, divide by 100 and simplify where possible.

$$4.09 = \frac{409}{100}$$

$$= 4\frac{9}{100}$$

Example 3

Convert 0.3... into a fraction.

Solution

Assume the decimal to an unknown.

(i) Let $x = 0.3...$

Recurring starts with tenth, 3 is the repeating order, we multiply by 10.

$10 \times x = 10 \times 0.3...$

(ii) $10x = 3.3...$

Subtract expression (i) from (ii).

$$\begin{array}{rcl} 10x & = & 3.3... \\ -\ x & = & 0.3... \\ \hline 9x & = & 3.0 \end{array}$$

$$x = \frac{3}{9} = \frac{1}{3}$$

Therefore, $0.3... = \frac{1}{3}$

Example 4

Convert 0.23... into a fraction.

Solution

Assume the decimal to an unknown.

(i) Let $y = 0.23...$

Recurring starts with hundredth, 23 is the repeating order, we multiply by 100.

$100 \times y = 100 \times 0.23...$

(ii) $100y = 23.23...$

Subtract expression (i) from (ii).

$$\begin{array}{rcl} 100y & = & 23.23... \\ -\ y & = & 0.23... \\ \hline 99y & = & 23.0 \end{array}$$

$$y = \frac{23}{99}$$

Therefore, $0.23... = \frac{23}{99}$

Study tip

When converting decimals into fractions, simply ignore the decimal point and write all digits of decimal number as the numerator, then the denominator depends on the number of decimal place. To know the denominator, you count the number of digits to your right after the decimal places.

- If the decimal place is 1, the denominator is 10.
- If the decimal places are 2, the denominator is 100.
- If the decimal places are 3, the denominator is 1,000.
- If the decimal places are 4, the denominator is 10,000.
- If the decimal places are 5, the denominator is 100,000.
- If the decimal places are 6, the denominator is 1,000,000.

If a decimal fraction recurs in the tenth place value, equate it to unknown to form an algebraic expression. Multiply both sides by 10 to form a second expression, subtract the first expression from the second expression. If a decimal recurs in the hundredth, thousands multiply by 100 and 1000 respectively then simplify.



Application 5.9

Convert the following decimals into fractions and simplify completely.

(a) 0.1234	(b) 19.67	(c) 195.008	(d) 10.08
(e) 17.3	(f) 13.5	(g) 54.029	(h) 0.375

(i) 0.256	(j) 56.35	(k) 6.18	(l) 25.25
(m) 25.3	(n) 18.75	(o) 0.875	(p) 0.3...
(q) 0.5...	(r) 0.45...	(s) 0.125...	(t) 0.09...

5.10 Solving problems involving converting decimals into fractions and fractions into decimals

Activity

- Think of a decimal number with three decimal places.
- Write it on a piece of paper.
- Give it to your neighbour.
- Ask him/her to round it to the tenths place value and convert it into a fraction.
- Discuss the whole process.
- Is the final result the same as the first? Explain.

Example

Mugiraneza bought 2.35 m, 1.265 m and 0.75 m of cloth. What total length of cloth did Mugiraneza buy? Convert the answer to a fraction.

Solution

The total length of the cloth = 2.35 m + 1.265 m + 0.750 m = 4.365 m

There are 3 decimal places, therefore, divide by 1000 and simplify where possible.

$$4.365 = \frac{4365}{1000}$$

$$= 4\frac{\cancel{365}^{73}}{\cancel{1000}_{200}} \text{ (Divide by 5)}$$

$$= 4\frac{73}{200}$$

Study tip

When solving problems, read and try to understand the problem then choose how to solve it.



Application 5.10

Work out the following:

1. Bwiza scored the following marks in different Mathematics tests: 89.5%, 73.25% and 69%. How many marks did she score altogether? What is her average mark as a fraction?
2. If 58 out of 100 text books in the school are mathematics, write a decimal for the portion of the school text books which are Mathematics.

3. A tailor wants to cut 24 small pieces of 0.4 m from a roll of cloth. What is the length of the roll of cloth as a fraction?
4. Kanyana cooked 0.25 kg of rice for lunch. What is the amount of the rice she cooked as a common fraction?
5. A taxi covered a distance of 72.03 km. What is the distance covered by the taxi as a common fraction?
6. The distance from Kalisa's farm to his home is 12 km 200 m. Express the distance as a decimal.
7. Musa, Sarah and Akida weigh 36.7 kg, 41.3 kg and 57.6 respectively. Express their average weight as a fraction.
8. When the circumference of a circle is divided by its diameter, the result is 3.14. Express this as a fraction.



Kanyana cooking

End of unit 5 assessment

1. Round off the following to the nearest hundredths:
   (a) 12.4637 (b) 876.9975 (c) 324.6675
2. Round off the following to the nearest tenth:
   (a) 123.22 + 0.678 = (b) 0.987 + 456.66 = (c) 65.54
3. Round off the following to the nearest thousandths:
   (a) 23.4567 (b) 0.57684 (c) 567.6834
4. Round off 2.73 to the nearest tenths.
5. Irankunda wanted to save $\frac{7}{8}$ of the money she earned. Represent that fraction as a decimal number. Do you think it is necessary to save?
6. In a 100 m race, three runners achieved the following times: 9.85 s, 9.6 s and 9.625 s.
   (a) What is the winning time?
   (b) Express the average time as a fraction.
   (c) Calculate the average time to 1 decimal place.
7. A cylinder has a 3.437 cm diameter. If it is remade to be 0.095 cm larger, what is the new size of that cylinder? Round off the answer to the nearest hundredths.
8. A motorbike was filled with 5 litres of fuel. If it uses 0.6 litres for each kilometre, how many kilometres will it cover using all 5 litres?
9. Change $\frac{5}{11}$ to a recurring decimal
10. Convert 0.65.. to a fraction.
11. Convert $\frac{47}{11}$ to a decimal number.
12. Mukabazungu ate 0.6 of a cake. Express the part she ate as a fraction.

Unit 6 Ratios, proportions, percentages and mixtures

Key unit competence: To be able to work out ratios, proportions, percentages, and mixtures.

Introduction

In daily life people compare quantities, share proportionally and mix things or objects for different reasons. Do you ever wonder why knowing the following mathematical concepts (ratio, percentage, proportion, mixture) is necessary?

For example look at your classmates:

(a) Find out the number of boys, then the number of girls. Can you express the number of boys or girls in terms percentage?

(b) Can you equally share a certain number of Mathematics textbooks to different groups in your classroom and then figure out the ratio of Mathematics textbooks per learner?

(c) Give examples where the concepts of ratios, percentages and mixtures are used in real life.

6.1 Converting percentages into decimals

Activity

- You have been scoring marks in test and examinations.
- Write the percentages on slips of paper.
- Express the % mark as a denominator of 100.
- Write each percentage value out of 100.
- Considering the number of zeros in 100, write decimals with places of the same number.
- What are your answers?
- Make a class presentation.

Example 3

Convert $12\frac{1}{2}\%$ into decimals.

Solution

First convert the percentage into a fraction.

$$12\frac{1}{2}\% = \frac{25}{2} \div 100$$

$$= \frac{\overset{1}{\cancel{25}}}{2} \times \frac{1}{\cancel{100}_{4}} = \frac{1}{8}$$

Change the fraction into a decimal.

$$\begin{array}{r} 0.125 \\ 8\overline{)\,1} \\ -0 \quad \leftarrow 0 \times 8 \\ \hline 10 \\ -8 \quad \leftarrow 1 \times 8 \\ \hline 20 \\ -16 \quad \leftarrow 2 \times 8 \\ \hline 40 \\ -40 \quad \leftarrow 5 \times 8 \\ \hline 00 \end{array}$$

Therefore, $12\frac{1}{2}\%$ is equal to 0.125.

Example 4

Convert 37.5% into decimals.

Solution

First convert the percentage into a fraction.

$$37.5\% = \frac{375}{10} \div 100$$

$$= \frac{375}{10} \times \frac{1}{100}$$

$$= \frac{375}{1000}$$

Three zeros denominators give 3 decimal places.

Therefore, 37.5% is equal to 0.375.

Study tip

- Whenever you see a % symbol, it means every each hundred. The zeros in the denominator give the number of decimal places in the decimal numbers.
- To convert a percentage into decimals, first determine the number of zeros in the denominator.
- Then count the place value of the number of zeros from the right side (ones) of the number.
- Decimals are always expressed with a point in one of the digits after or before the digits of the whole number. For example, 80% can be expressed as 0.80.
- % means out of 100. Since 100 has 2 zeros, the decimal will have 2 decimal places.
- Fractional percentages differ in the number of zeros, for example, $37\frac{1}{2}\%$ $\frac{375}{1000}$ = 0.375 (3 decimal places).



Application 6.1

Convert the following percentages into decimals:

(a) 48% (b) 97% (c) 1% (d) 13% (e) 22%

(f) 15.7% (g) 1.2% (h) 125% (i) 142% (j) 33%

(k) $45\frac{1}{2}\%$ (l) $20\frac{3}{4}\%$ (m) $62\frac{1}{5}\%$ (n) 56.6% (o) 87.02%

6.2 Converting decimals into percentages

Activity

- Formulate five decimal numbers of 1, 2, and 3 decimal places.
- Express them as fractions, but do not simplify.
- Multiply each fraction by 100.
- What are your answers?

Example 1

Express 0.35 as a percentage.

Solution

Method 1

Move the decimal point two places to the right:

$0.35 \rightarrow 3.5 \rightarrow 35.$

Add a % sign: 35%

Therefore, 0.35 = 35%

Method 2

Multiply by 100%

$0.35 \times 100\% = \frac{35}{100} \times 100\%$

$= 35\%$

Therefore, 0.35 = 35%

Example 2

Convert 0.6... into a percentage.

Solution

Assume the percentage to an unknown.

(i) Let x = 0.6...

Recurring starts with tenth, 6 is the repeating order, so, we multiply by 10.

10 x x = 10 x 0.6...

(ii) $10x$ = 6.6...

Subtract expression (i) from (ii).

$$\begin{array}{rcl} 10x & = & 6.6... \\ -\ \ x & = & 0.6... \\ \hline 9x & = & 6.0 \end{array}$$

$$x = \frac{6}{9} = \frac{2}{3}$$

Change to percentage.

$$\frac{2}{3} \times 100\% = \frac{200}{3}\% = 66\frac{2}{3}\%$$

Therefore, $0.6... = 66\frac{2}{3}\%$

Study tip

- To convert decimals into percentages, change the decimal into a fraction, then multiply the fraction by 100%.
- Remember to put the percentage (%) mark to the right side.
- To covert decimals into percentages, you can also move the decimal point two places to the right. This is because 100 has 2 zeros, hence 2 decimal places.
- Then add a % symbol.



Application 6.2

Convert the following decimal numbers into percentages:

(a) 0.86	(b) 0. 2	(c) 0.12	(d) 0.05	(e) 0.56
(f) 0.125	(g) 0.075	(h) 1.46	(i) 1.2	(j) 0.195
(k) 0.3...	(l) 0.54...	(m) 0.5...	(n) 0.63...	(o) 0.4...

6.3 Converting percentages into fractions

Activity

- Write four percentages on a slip of paper.
- Express them as fractions of the denominator 100.
- Simplify each fraction.
- What do you get?

Example 1

Change 40% into a fraction.

Solution

First convert the percentage into a fraction.

$$40\% = \frac{40}{100}$$

$$= \frac{\cancel{4}2}{\cancel{10}5}$$

$$= \frac{2}{5}$$

Example 2

Change $33\frac{1}{3}\%$ into a fraction.

Solution

Change into improper fraction.

$$33\frac{1}{3} = \frac{100}{3}$$

Divide by the percentage (100).

$$\frac{100}{3} \div 100$$

Multiply by the reciprocal.

$$= \frac{100}{3} \times \frac{1}{100} = \frac{1}{3}$$

Study tip

- To change percentages into fractions, make the given figure the numerator and 100 the denominator, then reduce where possible.
- To change fractional percentages into common fractions, divide the fractional part by 100. Then multiply by the reciprocal of 100.



Application 6.3

Convert the following percentages into fractions:

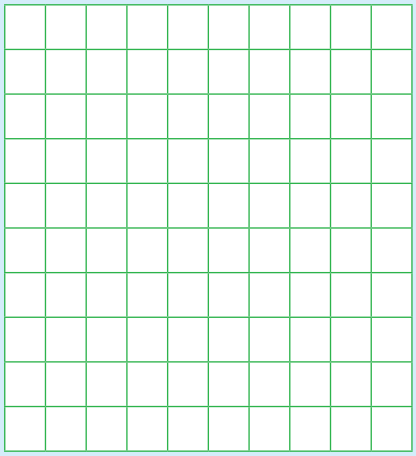
(a) 40%	(b) 50%	(c) 12.5%	(d) 75%
(e) 2%	(f) 11%	(g) $66\frac{2}{3}\%$	(h) 8%
(i) 68%	(j) 88%	(k) 90%	(l) 45%
(m) $54\frac{6}{11}\%$	(n) 51%	(o) 10%	(p) 18%
(q) 69%	(r) 81%	(s) 86%	(t) 9.8%

6.4 Converting fractions into percentages

Activity

- Get four sheets of paper of 10 by 10 square grid like the one shown.
- Study the fractions in the flash cards.

$\frac{1}{2}$ $\frac{9}{10}$ $\frac{79}{100}$ $\frac{7}{20}$



- Present the fractions in the grids as follows:

  $\frac{1}{2}$ shade 1 out of every 2 squares.

  $\frac{9}{10}$ shade 9 out of every 10 squares.

  $\frac{79}{100}$ shade 79 out of every 100 squares.

  $\frac{7}{20}$ shade 7 out of every 20 squares.

- Write the shaded squares out of the total squares in the grid.
- What are your observations?

Example 1

Convert $\frac{3}{4}$ into a percentage.

Solution

Multiply the fraction by 100%, then simplify.

$$\frac{3}{\cancel{4}_{1}} \times \cancel{100}^{25}\% = 3 \times 25\%$$
$$= 75\%$$

Example 2

Convert $\frac{1}{8}$ into a percentage

Solution

Multiply the fraction by 100%, then simplify.

$$\frac{1}{8} \times 100\% = \frac{1 \times \cancel{100}^{25}}{\cancel{8}_{2}}$$
$$= \frac{1 \times 25}{2} = 12\frac{1}{2}\%.$$

Study tip

- When converting a fraction into a percentage, multiply the fraction by 100%.
- Then simplify to the simplest terms.
- The percentage obtained is equivalent to the fraction.
- Percentages are fractions with the denominator 100.



Application 6.4

Change the following fractions into percentages:

(a) $\frac{2}{5}$ (b) $3\frac{1}{7}$ (c) $\frac{11}{9}$ (d) $5\frac{1}{2}$ (e) $\frac{1}{9}$

(f) $\frac{7}{8}$ (g) $\frac{3}{10}$ (h) $\frac{3}{8}$ (i) $\frac{5}{6}$ (j) $\frac{2}{5}$

(k) $\frac{1}{3}$ (l) $\frac{2}{3}$ (m) $\frac{3}{5}$ (n) $\frac{6}{11}$ (o) $\frac{4}{9}$

6.5 Comparing quantities as percentages

Activity

- Convert the following into percentages:
  - (a) 6 pencils out of 1 dozen pencils.
  - (b) 30 minutes out of 1 hour.
  - (c) 200 grams out of 1 kilogram
  - (d) 250 metres out of 2 kilometres.
- Present your working to the class.

Example 1

Express 750 g as a percentage of 1 kg of sugar.

Solution

Express the given grams as a fraction a kilogram.

1 kg = 1,000 grams

$750\text{ g} = \frac{750}{1{,}000}\text{kg}$

Multiply the fraction by 100%.

$\frac{75\cancel{0}}{1{,}\cancel{000}} \times 1\cancel{00} = 75\%$

Therefore, 750 g of 1 kg = 75%

Example 2

What percentage of 1 hour is 36 minutes?

Solution

Express the given minutes as a fraction an hour.

1 hr = 60 minutes

$36\text{ minutes} = \frac{36}{60}\text{ hour}$

Multiply the fraction by 100%.

$\frac{\cancel{36}^{6}}{\cancel{60}_{1}} \times 10\cancel{0} = 60\%$

36 minutes of 1 hour = 60%.

Example 3

45 out of 75 learners in P.6 got above the pass mark in a Mathematics examination. Which percentage of the class;

i) Passed?

ii) Failed?

Solution

Percentage of learners who passed = $\frac{45}{75} \times 100\% = 60\%$

Number of learners that failed = 75 – 45 = 30

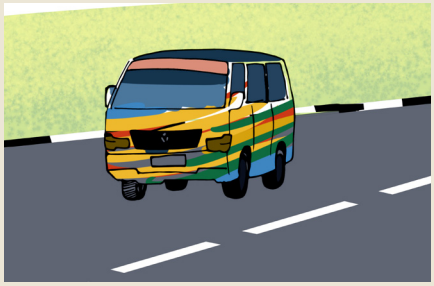
Percentage of learners who failed = $\frac{30}{75} \times 100\% = 40\%$

Study tip

- When comparing quantities as percentages, first express the quantity as a fraction then multiply the fraction formed by 100%.
- The total is always 100%. To find the original number, multiply the value of the unit percentage by 100%.
- When comparing quantities as percentages, first convert the units to be the same. Express as a fraction, then multiply the fraction by 100%.

Application 6.5

1. What percentage of 1 hour is:
   (a) 45 minutes? (b) 20 minutes?
2. A taxi covered 65 km of 130 km of the journey. What percentage of the journey was:
   (a) Covered? (b) Left?



3. There are 80 girls in a school. If there are 400 learners in the whole school, what is the percentage of girls?
4. Express 500 g as a percentage of 2 kg of beans?
5. Paul bought 6 blue pens. There were 50 pens in the whole packet. What is the percentage of blue pens in the whole packet?

6.6 Comparing percentages as quantities

Activity

- Write 10% on slips of paper.
- Let 10% represent 20 books.
- Form groups of 10% equating them to group of 20 books.
  10% is equivalent to 20 books.
  20% is equivalent to 40 books.
  30% is equivalent to 60 books.
  40% is equivalent to 80 books and so on up to 100%
  .....
  .....
  100% is equivalent to ? books.
  100% is taken to be equivalent to total books.
- How many books are there?
- Present your working to the class.

Example

5% of the learners in P.6 are boys.
If there are 20 boys in the class, how many learners are in the class?

Solution

Method 1

5% of the total learners = 20 boys

To get 1% of the total learners divide 20 by 5.

$= \frac{\cancel{20}^{4}}{\cancel{5}_{1}} = 4$ learners

1% = 4 learners

Multiply by 100 to get the total number of learners.

4 x 100% = 400 learners

Solution

Method 2

Assume the total to be an unknown.

Let the total number of learners be r.

5% of r = 20 boys

$\frac{5}{100} \times r = 20$

Multiply by 100 on both sides.

$\cancel{100}^{1} \times \frac{5}{\cancel{100}_{1}} r = 20 \times 100$

Divide both side by 5

$\frac{\cancel{5}r^{1}}{\cancel{5}_{1}} = \frac{\cancel{20}^{4} \times 100}{1\cancel{5}}$; r = 100

Therefore, there are 400 learners in P.6.

Study tip

- Divide the given quantity by the given percentage to get the equivalent of 1%.
- Multiply that fraction by 100% to get the total quantity/amount.



Application 6.6

1. 20% of a number is 60. What is the number?
2. 60 is equivalent to 10% of a number. Find the number.
3. $12\frac{1}{2}$% of a number is 1600. Calculate the number.
4. 45% of the fish in a pond are catfish. There are 900 catfish. How many fish are in the pond altogether?
5. 11% of the cars imported by RENEX Auto Mart every month are pickups. If there are 55, find the total number of cars imported.
6. $37\frac{1}{3}$% of the chicken mash prepared is made from silverfish. If this is 224 kg, what is the total mass of the mash prepared?

6.7 Increasing a Number by a Percentage

Activity

- Dusabe had 1,000 Frw.
- Her father increased it by 20%. How much does she have now?
- 20% increase means:

  Each 100 Frw, the increase is 20 Frw.
  200 Frw, the increase is 40 Frw.
  300 Frw, the increase is 60 Frw.
  ....., the increase is ? Frw.
  .....
  1,000 Frw, the increase is ? Frw.
- Add the increase to 1,000 Frw.
- What is your answer?

Study tip

- To increase a number by a given percentage, add the given percentage to 100%, then multiply by the given number.
- To get the increased new amount, work out the increased amount, then add it to the old amount.
- Increasing a given number by a given percentage means, adding the quantity on each 100 of the original quantity.

Example 1

Ancila's salary is 8,000 Frw per month.

How much money will her salary be if it is increased by 20%?

Solution

Method 1

Assume 100% to be equivalent to the old amount.

(100% + given %) of the old number.

= (100% + 20%) of 8,000 Frw.

= 120% of 8,000 Frw.

$= \frac{120 \times 8{,}000}{100}$ Frw

= 120 x 80 = 9,600 Frw

Method 2

Calculate the increase first.

(Given % increase x old amount) + old amount

= (20%) of 8,000 Frw.

$= \frac{20 \times 8{,}000}{100} = 1{,}600$ Frw

Add the increase to the old amount.

= 1,600 Frw + 8,000 Frw

= 9,600 Frw

Example 2

The price of peas was 2,400 Frw. It was increased by 20% in the first harvest season. In the second season it was increased by 10%. What is the new price?

Solution

Method 1

Assume 100% to be equivalent to the old amount.

(100% + given %) of the old price.

1st increase: (100% + 20%) = 120%

2nd increase: (100% + 10%) = 110%

New price

$= \frac{120}{100} \times \frac{110}{100} \times 2{,}400$ Frw

$= 12 \times 11 \times 24$

$= 3{,}168$

Therefore, the new price is 3,168 Frw.

Method 2

Calculate the increase first.

(Given % increase x old price) + old price

1st increase: (20% of 2,400)

$= \frac{20 \times 2{,}400}{100} = 4{,}80$ Frw

Add the increase to the old price.

$= 480 + 2{,}400 = 2{,}880$ Frw

2nd increase: (10% of 2,880)

$= \frac{10 \times 2{,}880}{100} = 288$ Frw

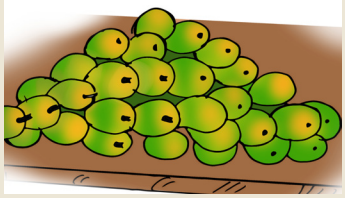
Add the increase to the old price.

$= 288 + 2{,}880 = 3{,}168$ Frw

Therefore, the new price is 3,168 Frw.

Application 6.7

1. Kamana had 50 mangoes. She purchased 30% more mangoes. How many did she have?
2. The population of a village was 600 people. If 40% migrated to the village area, how many people are in the village now?
3. A shirt was priced at 9,000 Frw last week. This week its price increased by 12%. What is the new price of the shirt?
4. The price of a pair of shoes is 40,000 Frw. This is increased by 15%. What is the new price of the shoes?
5. A cow increased its milk production of 5 litres by 10%, then by 25%. How many litres of milk does it give now?
6. Increase 14,000 by 8%, then by 12%.
7. 28,000 Frw was increased by 10%, then by 10%. What did it become?



Kamana's mangoes

6.8 Decreasing a number by a percentage

Activity

- If an item costs 500 Frw.
- Its price was decreased by 5%. What is new price now?

  Each 100 Frw, the decrease is 5 Frw.

  200 Frw the decrease is 10 Frw.

  ..... Frw the decrease is .... Frw.

  ..... Frw the decrease is .... Frw.

  ..... Frw the decrease is .... Frw.
- Subtract the total decrease from the original price.
- What is the new price?

Study tip

- To decrease a number by a given percentage, subtract the given percentage from 100%, then multiply by the given number.
- To get the new amount after decreasing, work out the decrease first, then subtract it from the given amount.

Example 1

The 1,200 litres of fuel supplied to the Ministry of Education monthly was reduced by 25%. How much fuel is supplied to the ministry now?

Solution

Method 1

Assume the old amount to be 100%.

(100% – 25%) of old number.

= 75% of 1,200 litres of fuel

= $\frac{75}{100}$ x 1,200 litres

= 75 x 12

= 900 litres of fuel

Method 2

Calculate the decreased amount first

Old amount – (given % decrease x old amount)

= 25% of 1,200 litres

= $\frac{25}{100}$ x 1,200 = 300 litres

(Old amount – decrease)

= 1200 – 300

= 900 litres of fuel

Example 2

The price of sugar decreased from 900 Frw by 10% then by 10%.
What is the new price?

Solution

Method 1

Assume 100% to be equivalent to the old amount.

(100% - given %) of the old number.

1st decrease: (100% - 10%) = 90%

2nd increase: (100% - 10%) = 90%

New price

$= \frac{90}{100} \times \frac{90}{100} \times 900$ Frw

$= 9 \times 9 \times 9$

$= 81 \times 9$

$= 729$ Frw

Therefore, the new price is 729 Frw.

Method 2

Calculate the decrease first.

(Given % decrease x old amount) + old amount

1st decrease: (10% of 900)

$= \frac{10 \times 900}{100} = 90$ Frw

Subtract the decrease from the old amount.

$= 900 - 90 = 810$ Frw

2nd decrease: (10% of 810)

$= \frac{10 \times 810}{100} = 81$ Frw

Subtract the decrease from the old amount.

$= 810 - 81 = 729$ Frw

Therefore, the new price is 729 Frw.



Application 6.8

1. Decrease 800 by 12%.
2. Decrease 960 by 30%.
3. Decrease 1,500 by 24%.
4. 400 kg is decreased by 16%, what is the new amount?
5. The marked price of a new car was 4,000,000 Frw. It was reduced by 6%. At what price was it to be sold?
6. A trader bought 650 kg of beans at 600 Frw per kg. The price decreased by 8% after the harvest season. How much did she get after the sale?
7. Decrease 101,000 kg by 30% then by 10%.
8. The marked price of a motorcycle was 1,000,000 Frw. It was decreased by 12% then by 9%. What is the new price?

6.9 More about increasing and decreasing quantities by percentage

Activity

- Think of an amount of money. Write it on slips of paper.
- If the amount is increased by 40%, it becomes 28,000 Frw.
- What was the old amount?
- Explain your working to the class.

Example 1

After increasing a number by 15%, it became 34,500. What is the number?

Solution

Method 1

Assume the number to an unknown.

Let the number be y.

Old number = (100% + 15%) of y

$= 115\%$ of y

$34{,}500 = \frac{115}{100}y$

Multiply by the reciprocal.

$\frac{100}{\cancel{115}_{1}} \times \overset{300}{\cancel{34{,}500}} = \frac{\cancel{100}}{\cancel{115}_{1}} \times \frac{\cancel{115}_{1}}{\cancel{100}}\ y$

$30{,}000 = y$

Therefore, the old number is 30,000.

Method 2

Old number = 100% + 15%

= 115%

115% of the number = 34,500

1% of the number = $\frac{\overset{300}{\cancel{34{,}500}}}{\cancel{115}_{1}} = 300$

100% of the number = $\frac{\overset{300}{\cancel{34{,}500}}}{\cancel{115}_{1}} \times 100$

$= 300 \times 100$

$= 30{,}000$

Therefore, the old number is 30,000.

Example 2

What amount when decreased by 12% becomes 528,000 Frw?

Solution

Method 1

Let the number be y.

Old number = (100% - 12%) of x

$528{,}000 = 88\%$ of x

$528{,}000 = \frac{88}{100}y$

Multiply by the reciprocal.

$\frac{100}{\cancel{88}_{1}} \times \overset{6}{\cancel{528}{,}000} = \frac{\cancel{100}}{\cancel{88}_{1}} \times \frac{\cancel{88}_{1}}{\cancel{100}}\ x$

$600{,}000 = y$

Therefore, the old amount is 600,000 Frw.

Method 2

Old number = 100% - 12%

= 88%

88% of the amount = 528,000 Frw

1% of the amount = $\frac{\overset{6}{\cancel{528}{,}000}}{\cancel{88}_{1}}$

= 6,000 Frw

100% of the amount = 6,000 x 100

= 600,000 Frw

Therefore, the old amount is 600,000 Frw.

Study tip

- To find the unknown number, find 1% of the increased amount then multiply by 100.
- To find the unknown number, find 1% of the decreased amount. Then multiply by 100.



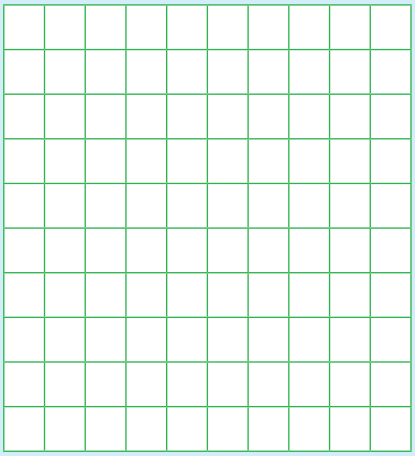
Application 6.9

1. What number when increased by 16% becomes 580?
2. After decreasing a certain amount of money by 9% it becomes 36,400 Frw. What was the amount?
3. Irebe harvested a certain amount of maize last season. This season the harvest increased by 6% to 848 kg. How many kilograms did he harvest last season?
4. What number when decreased by 20% becomes 1,600?
5. Akim's salary was increased to 540,000 Frw by 8%. Find his old salary.
6. What number when decreased by 12.5% becomes 1,050?
7. Jastine's monthly salary was increased by $7\frac{1}{3}$% to 86,000 Frw. Calculate her previous salary.

6.10 Finding percentage increase and decrease

Activity

- Study the grid.
- Shade 40 squares.
- Then shade 60 squares including the first 40 squares with different colour.
- How many more squares have you shaded?
- Express the more shaded squares as a fraction of the first 40 squares.
- Convert the fraction into a percentage.
- What do you observe?
- Share your working out with the class.



Example 1

After increasing 600 by a certain percentage it becomes 840. Find the percentage increase.

Solution

Old number = 600

New number = 840

The increase = (840 - 600) = 240

$$\% \text{ increase} = \frac{\text{Increase}}{\text{Old number}} \times 100\%$$

$$= \frac{\overset{40}{\cancel{240}}}{\underset{1}{\cancel{600}}} \times 100$$

= 40%

Example 2

When 48,000 Frw is decreased by x% it becomes 36,000 Frw. Calculate the value of x.

Solution

Old amount = 48,000 Frw

New number = 36,000 Frw

The decrease = (48,000 - 36,000) Frw

= 12,000 Frw

$$\% \text{ decrease} = \frac{\text{Decrease}}{\text{Old number}} \times 100\%$$

$$= \frac{\overset{1}{\cancel{12,000}}}{\underset{4}{\cancel{48,000}}} \times \overset{25}{\cancel{100}}$$

= 25%

Study tip

- Percentage increase = $\frac{\text{Increase}}{\text{Old number}}$ x 100%.
- Percentage decrease = $\frac{\text{Decrease}}{\text{Old number}}$ x 100%.



Application 6.10

1. By what percentage will 900 be increased to become 1200?
2. When 2,000 is decreased by x% it becomes 1,600 Frw. Find the value of x.
3. The price of a shirt was increased by a percentage. The price increased from 6,000 to 8,000 Frw. Calculate the percentage increase.
4. The marked price of a car was 4,200,000 Frw. It was decreased by x% to become 4,000,000 Frw. What is the value of x?
5. The population of a country increased by a certain percentage. The population in the previous census was 12,000,000 people. The recent population rose to 14,400,000 people. By what percentage did the population increase?
6. The price of a pair of shoes decreased from 20,000 Frw to 17,000 Frw/ Find the percentage decrease?

6.11 Finding percentage profit and percentage loss

Activity

- Role-play buying and selling of items in a class shop.
- In pairs, demonstrate when you gain more money and when you get less money than the amount you purchased the item.
- Express the difference as a fraction of the buying price, then multiply by 100. How do you describe the result?
- Discuss your answers with the class.

Example 1

A trader bought a bag at 10,000 Frw. He later sold it to a customer at 12,500 Frw. What was her percentage profit?

Solution

Buying price = 10,000 Frw

Selling price = 12,500 Frw

Profit = Selling price - buying price

= 12,500 - 10,000 Frw

= 2,500 Frw

$$\% \text{ profit} = \frac{\text{Profit}}{\text{Buying price}} \times 100\%$$

$$= \frac{2{,}500}{10{,}000} \times 100$$

= 25%

Therefore, the percentage profit is 25%.

Example 2

Muhakanizi bought a TV at 450,000 Frw. He made a loss after selling it at 300,000 Frw. Calculate the percentage loss.

Cost price = 450,000 Frw

Selling price = 300,000 Frw

Loss = Cost price - selling price

= 450,000 - 300,000 Frw

= 150,000 Frw

$$\% \text{ loss} = \frac{\text{Loss}}{\text{Cost price}} \times 100\%$$

$$= \frac{150{,}000}{450{,}000} \times 100$$

$$= \frac{100}{3}\% = 33\frac{1}{3}\%$$

Therefore, the percentage loss is $33\frac{1}{3}\%$.

Study tip

- Profit = selling price - cost price.
- Other terms used for profit are: gain, raise, increase and more.
- Percentage profit = $\frac{\text{Profit}}{\text{Buying price}} \times 100\%$.
- Loss = Cost price - Selling price.
- Other terms used for loss are reduction, less and decrease.
- Percentage loss = $\frac{\text{Loss}}{\text{Cost price}} \times 100\%$.



Application 6.11

1. Tereza sold a radio at 50,000 Frw. She had bought it at 40,000 Frw. Calculate the percentage profit.
2. Sales man made a loss after selling a dining table at 75,000 Frw which its original cost was 90,000 Frw. What was the percentage loss?
3. A grocer sold a pineapple at 500 Frw. She had purchased it at 400 Frw. Find the percentage profit.
4. Kwizera bought a school bag at 15,000 Frw. He realised that his books could not fit in it. He sold it at 12,000 Frw. What was the percentage loss?
5. A shopkeeper bought a sack of sugar at 750 Frw each kilogram. He sold every kilogram at 800 Frw. Calculate the percentage profit.
6. Mukandahiro bought a dozen of counter books at 1,500 Frw each. She later sold every book at 1,000 Frw. What was her percentage loss?

6.12 Solving problems involving percentages

Activity

- A school enrolment was 600 learners in 2016.
- The number increased by 20% in 2017.
- What is the enrollment in 2017?
- What is the meaning of 20% increase?
- Write a list of progressive increase of 20 to every 100 learners.
- What is the answer?

Example 1

150 babies were born in a hospital in the month of February. In the month of March, the number increased by 12%. How many babies were born in the month of March?

Solution

The old number was 100%

New number = 100% + 12% = 112%

Example 2

Kalisa took 5,000 Frw to school as pocket money in first term. He performed poorly in class and his father reduced it by 30% in his second term. How much did he take as pocket money in second term?

Solution

The old value is 100%

New value 100% - 30% = 70%

100% = 150 babies

1% = $\frac{150}{100}$

112% = $\frac{\cancel{150}^{3}}{\cancel{100}_{\cancel{2}\ 1}} \times \cancel{112}^{56}$

= 3 × 56

= 168 babies

100% = 5,000 Frw

1% = $\frac{5{,}000}{100}$ Frw

70% = $\frac{5{,}0\cancel{0}\cancel{0}}{1\cancel{0}\cancel{0}} \times 70\%$

= 50 × 70

= 3,500 Frw

Example 3

Uwacu's salary was increased from 50,000 Frw to 60,000 Frw. Calculate the percentage of increase.

Old salary = 50,000 Frw

New salary = 60,000 Frw

The increase = New salary - Old salary

= 60,000 Frw - 50,000 Frw = 10,000 Frw

% increase = $\frac{\text{Increase}}{\text{Old salary}}$ x 100%

= $\frac{\cancel{10{,}000}}{1\cancel{50{,}000}} \times \cancel{100}^{20}$ = 20%

Therefore, the percentage increase is 20%.

Study tip

- The original number is always a percentage (100%).
- Increase amount = (100% + % increase) of the original amount.
- Decrease amount = (100% - % decrease) of the original amount.
- A loss is a decrease and a profit is an increase.



Application 6.12

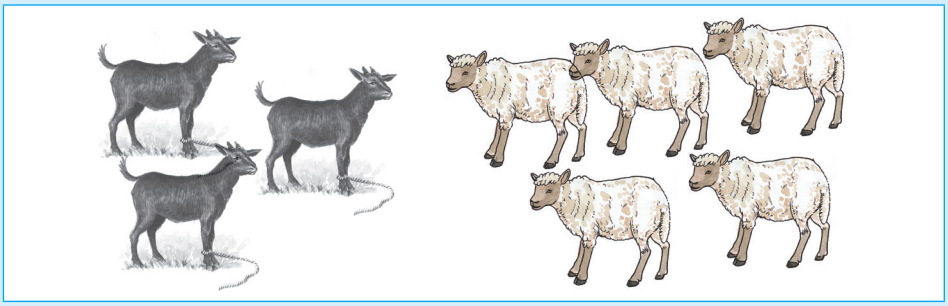
1. A cow increased its milk production from 15 litres by 10%. How much milk does it give now?
2. 450 people had accounts at a bank. 20% of the accounts were closed last month. How many people have accounts with the bank now?
3. The government tarmacked 150 roads last year. This year the number of tarmac roads has increased to 168. What is the percentage increase in the number of tarmac roads?

4. Catherine bought a dress at 5,000 Frw and sold it at 7,000 Frw. Calculate the percentage of profit.
5. A shopkeeper reduced the price of a shirt from 2,000 Frw to 1,500 Frw. Calculate the percentage of decrease in the price of the shirt.
6. 300 kg of beans bought by the school was raised to 450 kg. Calculate the percentage of increase.
7. An article was bought at 50,000 Frw and sold at 46,000 Frw. What was the percentage of loss?
8. Mukandoli sold a sofa set at 400,000 Frw. She had bought it at 350,000 Frw. Find the percentage of profit.

6.13 Finding ratios

Activity

In a farm, there are 3 goats to 5 sheep.



(a) Do you know how you can present the ratio of the goats to sheep? Write it down.

(c) How are ratios useful?

(d) Explain your working out.

Example 1

There are 25 learners in the class. The ratio of boys to girls is 2:3. Find the number of boys and girls.

Solution

Total of ratios is 2 + 3 = 5

Number of boys is $\frac{2 \times \cancel{25}^{5}}{\cancel{5}_{1}} = 10$ boys

Number of girls is $\frac{3 \times \cancel{25}^{5}}{\cancel{5}_{1}} = 15$ girls

Example 2

Write $\frac{1}{2}$ to $\frac{3}{4}$ as a ratio.

Solution

Use the LCM of 2 and 4. The LCM is 4

Write a ratio $\frac{1}{2} : \frac{3}{4}$

Multiply by 4

$\cancel{4}^{2} \times \frac{1}{\cancel{2}_{1}} : \frac{3}{\cancel{4}_{1}} \times \cancel{4}^{1} = 2:3$

Study tip

- A ratio is a comparison between quantities. No units are used.
- A ratio is an ordered pair of numbers written in the order a:b where b cannot be zero(0).
- Ratios may be used while calculating things which are compared proportionally.



Application 6.13

1. There are 5 children in a room. 2 are boys and 3 are girls. What is the ratio of boys to girls?
2. What is the ratio of 20 cm to 2 m?
3. Write 12 kg to 240 g as a ratio.
4. Express $\frac{2}{3}$ to $\frac{1}{5}$ as a ratio.
5. Ashimwe's salary was increased in the ratio of 3:5. What is his new salary if the old salary was 60,000 Frw?
6. Sorghum flour is mixed with maize flour in a ratio of 5:7. If there are 20 kg of sorghum flour, find the quantity of the mixed flour.
7. In Primary Six, learners have 44 blue pens and 22 black pens. Find the ratio of these pens.
8. A recipe for cakes uses 3 cups of wheat and 2 cups of milk. What is the ratio of that recipe? If the recipe calls for 55 cups, how many cups of wheat and how many cups of milk are there?
9. 250 bottles of soda were served to 125 people at a party. If there are 360 people, how many bottles of soda would be needed?
10. In our school, the ratio of teachers to learners is 1:8. If there are 10 teachers, how many learners are there?

6.14 Sharing quantities in ratios

Activity

Get 32 bottle tops.

- In pairs, pick from the heap at intervals.
- When the 1st learner picks 3 bottles tops, the 2nd picks 5 until all the bottle tops are completely shared.
- How many bottle tops does each get?
- Share the procedure with other classmates.

Example 1

Peter and Jane shared 25 sweets in ratio of 2:3. How many sweets did each get?

Solution

Add to find the total of the shares.

2 + 3 = 5 shares

Peter got 2 of 5 shares.

Express as a fraction

$= \frac{2}{5}$

Multiply by 25 sweets

$\frac{2}{\cancel{5}_{1}} \times \cancel{25}^{5} = 2 \times 5 = 10$ sweets

Jane got 3 out of 5 shares.

$\frac{3}{\cancel{5}_{1}} \times \cancel{25}^{5} = 3 \times 5 = 15$ sweets

Example 2

Abdul and Sharifa contributed money to start a business in the ratio of 6:5 respectively. Sharifa contributed 450,000 Frw.

(i) How much did Abdul contribute?

(ii) How much money was contributed?

Solution

(i) Abdul : Sharifa

6 : 5

5 shares = 450,000 Frw

1 share = $\frac{450{,}000}{5} \times 1$

= 90,000 Frw

6 shares = 90,000 x 6 = 540,000

Abdul contributed 540,000 Frw

(ii) Total shares = 6 + 5 = 11 shares

1 share = 90,000 Frw

11 shares = 90,000 x 11

= 990,000 Frw

Total amount contributed was 990,000 Frw.

Study tip

- Sharing in ratio; express each share as a fraction, then multiply by the quantity.
- Sharing in ratio; if the quantity of equivalent to some shares is given, get the unit quantity value to each share. Then multiply by the rest of the shares by the unit value.
- The sum of the given ratio is equal to the total quantity.



Application 6.14

1. Share 500 in the ratio of 2:3.
2. Share 420 kg in the ratio of 1:5.
3. Kaibanda and Mukamusoni shared 12,000 Frw in the ratio of 3:5. Find how much each got.
4. Two farmers shared 125 kg of beans to sow. How many kilograms did each get if they shared in a ratio of 3:2?

5. Anita and Alpha contributed money to buy land in the ratio of 7:8. Alpha contributed 640,000 Frw.
   (a) How much more did Anita contribute?
   (b) Calculate the total amount of money both contributed.
6. The ratio of girls to boys in a primary school is 9:8. There are 450 girls in the school.
   (a) Work out the number of boys.
   (b) Find the total enrolment in the school.

6.15 Increasing and decreasing quantities in ratios

Activity

- Get counters like beads or small stones.
- Pick 30 of them and form groups of 5.
- How many group have your formed?
- Now form a 6th group of the same number of counters.
- How many counters are in all the 6 groups altogether?
- How many more counters have you added to the 30 counters?
- Discuss and make a class presentation.

Example 1

Increase 900 litres in the ratio of 3:2.

Solution

Method 1

The new part is 3

The old part is 2

Match the parts with their quantity.

New part : Old part

3 : 2

? : 900 litres

2 parts = 900 litres

1 part = $\frac{\cancel{900}^{450}}{\cancel{2}_{1}} = 450$

3 parts = 450 x 3 = 1,350 litres

The new litres are 1,350 litres

Method 2

The new part is 3

The old part is 2

New amount = $\frac{\text{New part}}{\text{Old part}}$

3:2 = $\frac{3}{2}$

Multiply by the old quantity

New amount = $\frac{3}{\cancel{2}_{1}} \times \cancel{900}^{450}$

= 3 x 450

= 1,350 litres

The new litres are 1,350 litres

Example 2

Decrease 1,400 Frw in the ratio of 4:7.

Solution

Method 1

The new part is 4

The old part is 7

Match the parts with their quantity.

New amount : Old amount

4 : 7

? : 1,400

7 parts = 1,400

1 part = $\frac{1{,}400^{200}}{\cancel{7}_{1}}$ = 200

4 parts = 200 x 4 = 800 Frw

The new amount is 800 Frw.

Method 2

The new part is 4

The old part is 7

New amount = $\frac{\text{New part}}{\text{Old part}}$

4:7 = $\frac{4}{7}$

Multiply by the old amount

New amount = $\frac{4}{\cancel{7}_{1}} \times \cancel{1{,}400}^{200}$

= 4 x 200

= 800 Frw

The new amount is 800 Frw.

Study tip

- The increased amount by a ratio = $\frac{\text{New part}}{\text{Old part}}$ x old quantity.
- In a ratio, the first part is the new part and the second is the old part.



Application 6.15

1. Increase 150 kg in the ratio of 7:3.
2. Decrease 490 in the ratio of 3:7.
3. Increase 720 in the ratio of 5:4.
4. Decrease 1,400 in the ratio of 7:10.
5. What amount do you get after increasing 21,000 Frw in a ratio of 9:7.
6. 800 books were decreased in the ratio of 3:4. How many books are there?
7. Increase 1,200 litres in the ratio of 8:5.
8. Decrease 4,000 Frw in the ratio of 5:8.
9. Find the amount you get after increasing 14,400 Frw in a ratio of 17:12.

6.16 Finding the ratio of increase and decrease

Activity

- Write 400 on slips of paper. Increase it to 600.
- Express the new number as a fraction of the old number.
- Reduce it to the simplest terms, then write as a ratio.
- What do you notice?

Example 1

A school bought 2,000 kg of beans in term one. In term 2, it bought 2,500 kg. Find the ratio of increase.

Solution

New amount = 2,500 kg

Old amount = 2,000 kg

$$\text{Increase in ratio} = \frac{\text{New amount}}{\text{Old amount}}$$

$$= \frac{\cancel{2{,}500}^{5}}{\cancel{2{,}000}^{4}}$$

$$= \frac{5}{4}$$

Therefore, the ratio of increase is 5:4.

Example 2

Lucumu's salary was 45,000 Frw. It was decreased to 36,000 Frw. In what ratio was the salary decreased?

Solution

New salary = 36,000 Frw

Old salary = 45,000 Frw

$$\text{Decrease in ratio} = \frac{\text{New salary}}{\text{Old salary}}$$

$$= \frac{\cancel{36{,}000}^{4}}{\cancel{45{,}000}^{5}}$$

$$= \frac{4}{5}$$

Therefore, the ratio of decrease is 4:5.

Study tip

- Ratio of increase = New amount to old amount. The new amount is bigger than the old amount.
- Ratio of decrease = New amount to old amount. The new amount is smaller than the old amount.



Application 6.16

1. 500 was increased to 600. Find the ratio of increase.
2. 4,000 was reduced to 2,500. What is the ratio of decrease?
3. A trader used to purchase 144 cartons of markers. Now he purchases 168 cartons every week. Work out the ratio of increase.
4. The enrollment in a school dropped from 700 learners to 560 learners. What was the ratio of decrease?
5. What is the ratio increase from 108,000 to 156,000?

6. The price of a radio was 70,000 Frw. It was decreased to 63,000 Frw. Find the ratio of decrease.
7. The school fees in a school was 75,000 Frw. Now it is 90,000 Frw. What is the ratio of increase?
8. Mrs. Kwizera's farm used to produce 900 litres of milk. It now produces 810 litres. Find the ratio of decrease.

6.17 Solving problems involving ratios

Activity

- Get 40 bottle tops.
- Group them in twos and threes at the same time.
- Write a statement about the above.
- How many bottle tops are in each group?
- Discuss, then make a presentation to class.

Example 1

Linda and Harry got sweets in the ratio of 3:4 respectively. If Harry got 12 sweets,
(i) How many sweets did Linda get?
(ii) How many sweets did both share?

Solution

(i) The ratio of Linda to Harry is 3:4.

Each got ? : 12

Each part of the ratio $= \frac{\cancel{12}^{3}}{\cancel{4}_{1}}$

$= \frac{3}{1} = 3$

Linda got: 3 x 3 = 9 sweets

(ii) Both shared (12 sweets + 9 sweets)

Linda got 9 sweets

Harry got 12 sweets

Both got (12 + 9) = 21 sweets

Example 2

A class had 49 learners. The number decreased to 42 learners. In what ratio did the number decrease.

Solution

New number = 42

Old number = 49

Decrease in ratio $= \frac{\text{New number}}{\text{Old number}}$

$= \frac{\cancel{46}^{6}}{\cancel{49}_{7}}$

$= \frac{6}{7}$

= 6:7

Ratio of decrease = 6:7

Study tip

- Read and understand the question for clear interpretation.
- Writing in ratio is a simple way of comparing number.



Application 6.17

1. The cost of a pen and a pencil is in the ratio of 5:2. What is the cost of the pen if the cost of a pencil is 100 Frw.
2. There are 120 sheep on a farm. The ratio of ewes to rams is 5:7. How many rams are in the farm?
3. The amount of money collected by P.6 learners to go for a picnic last year was 200,000 Frw. This year it increased in the ratio of 7:4. How much was collected this year?
4. The price of paraffin was 700 Frw a litre last month. It was increased to 900 Frw a litre. In what ratio was the price increased?
5. The marked price of a television is 124,000 Frw. A customer bought the television at a reduced price in a ratio of 3:4. How much did she buy the television?
6. The population in Munyaneza's district increased from 3,200 people to 4,800 people. In what ratio did the population increase?
7. Alpha and Anne got 360 saplings from the forestry official. They shared them in the ratio of 4:5 respectively.
   (a) How many saplings did Alpha get?
   (b) How many saplings did Anne get?
8. The ratio of boys to girls in a school is 9:11. If there are 330 girls;
   (a) How many boys are in the school?
   (b) Find the number of learners in the school.

6.18 Finding indirect proportions

Activity

Work done by 12 people can be completed in 5 hours.

If the time to accomplish the work is reduced, the number of people to accomplish the work should be increased.

(a) Discuss the situation.

(b) Why do you think the workers were increased as time reduced?

(c) Relate indirect proportions to your daily life.

Example 1

2 people can dig the school garden in 8 days. How many people working at the same rate are needed to dig the same garden in 4 days?

Solution

8 days need 2 people to do the work
More people are needed, so multiply.
1 day needs (2 × 8) people
Then divide to share the work.

4 days needs $\left(\frac{2 \times 8^{2}}{4_{1}}\right)$ = 4 people

Example 2

8 builders take 10 days to complete a house. How many builders are needed to complete the house in only 5 days?

Solution

10 days needs 8 builders to do the work
Multiply to increase the number of builders to share the work.
1 day needs (10 × 8) builders
Then divide to share the work.

5 days needs $\left(\frac{10^{2} \times 8}{5_{1}}\right)$ = 16 builders

Study tip

When solving indirect proportion problems, you must note that:

- Time to do any piece of work increases as the number of people decreases.
- Time to do any piece of work decreases as the number of people increases.
- A task to be completed in a short period of time needs many participants.
- Work done by many people is shared, so it takes a shorter period of time to accomplish than work done by few people.



Application 6.18

1. 2 painters can paint a building in 5 days. How long will 20 painters take to do the same piece of work, working at the same rate? Show your working out.
2. 3 men can do a piece of work in 6 days. How long will 9 men working at the same rate take to do the piece of work? Show your working out.
3. 8 girls can paint 24 square metres of a building. How many square metres can 6 girls paint if they work at the same rate? Show your working out.
4. In a village, 16 men can dig a pit in 5 days. How many men working at the same rate can dig the same pit in 20 days? Show your working out.
5. 28 learners take 30 minutes to clean a school compound. How many minutes fewer will 40 learners need to clean the compound?



Men painting a room

6.19 Finding the average price of a mixture

Activity

Take 3 kg of millet with 2 kg of maize. Ask how much 1 kg of millet and 1 kg of maize costs in the shop. Buy them and mix in a basket.

(a) Measure how many kilograms you have got altogether

(b) Calculate the money you have spent on your mixture.

(c) Work out the average price.

(d) What do you notice?

(e) Why do you think it is important to study mixtures?

(f) Explain your working out to the class.

Example

10 kg of curry powder cost 400 Frw per kilogram mixed with 15 kilograms of another type of curry powder which cost 250 Frw per kilogram was mixed by Janine. What is the average price per kilogram of the mixture? Explain the steps taken in solving this problem.

Solution

1st carry powder

1 kg costs 400 Frw

10 kg cost (10 × 400 Frw) = 4,000 Frw

2nd carry powder

1 kg costs 250 Frw

15kg (15 × 250 Frw) = 3,750 Frw

The mixture

(Add to find the total mass)

10 kg + 15 kg = 25 kg

(Add to get the total cost)

4,000 Frw + 3,750 Frw = 7,750 Frw

25 kg of the mixture will cost 7,750 Frw

(Divide the total cost by the total mass to get the unit cost)

1 kg of the mixture will cost 7,750 ÷ 25 = 310 Frw.

So the mixture cost 310 Frw per kg

Study tip

To find the average price of the mixture:

- First find the total cost of both types of mixture.
- Then divide by the total kilograms of the mixture.

Application 6.19

1. Kambanda has 12 kg of brown sugar which cost 1,100 Frw per kilogram and 8 kg of white sugar which cost 1,200 Frw per kilogram. What will be the average price of the mixture? Show your working out.
2. Kalisa mixed 6 litres of honey which cost 4,500 Frw per litre with 9 litres of another quality of honey which cost 4,200 Frw per litre. Calculate the average price per litre of the mixture.
3. Charles mixed 30 litres of orange juice which cost 1,000 Frw per litre with 20 litres of passion fruit juice which cost 1,200 Frw per litre. Calculate the average price of each litre of juice. Show your working out.
4. Agnes mixed 20 kg of sorghum flour which cost, 2,000 Frw per kg with millet flour which cost 1,500 Frw per kg and formed a 40 kg mixture.
   (a) Calculate the average price of the mixture. Show your working out.
   (b) What was the mass of the millet flour?

6.20 Finding quantity of one type of the mixture

Activity

- Get 2 kg of soya flour which costs 2,000 Frw per kilogram.
- Mix it with maize flour which costs 700 Frw per kilogram.
- Selling the mixture at 1,400 Frw, on which type of flour is a profit or loss achieved? And, by how much?
- What do you get?
- Selling maize flour alone, do you gain or make a loss? Explain.
- Try find the mass of the maize flour before forming a mixture.
- Make a class discussion.

Example

A mixture of yellow maize flour and white maize flour cost 800 Frw per kg. 4 kg of yellow maize flour cost 900 Frw per kg and the white maize flour costs 700 Frw per kg. Find the kilograms for the white maize flour.

Show your working out.

Solution

1 kg of the mixture cost = 800 Frw

Selling the yellow maize flour alone causes a loss of:

(900 – 800) = 100 Frw per kilogram

Selling 4 kg results in a loss of:

(100 × 4) = 400 Frw

Selling the white maize flour alone results in a gain of

(800 – 700) = 100 Frw

The number of kilogram for the second type is ... 400 ÷ 100 = 4 kg

Study tip

- To get a profit, the price of an item is lower than the price of the mixture.
- To get a loss, the price of an item is higher than the price of the mixture.
- Then balance it up by multiplying the ratio of profit and loss by the kilograms given.
- To find the weight of the second item in a mixture, divide the total gain in the first item by the gain on each kilogram of the second item.



Application 6.20

1. Kamanzi mixed 5 kg of yellow beans which cost 600 Frw per kg with white beans which cost 700 Frw per kg. He sold the mixture at 680 Frw per kg. Find the number of kg for the white beans. Show your working.
2. Nkubiri mixed 10 kg of peas which cost 800 Frw per kg with another type which cost 500 Frw per kg and sold the mixture at 600 Frw. Find the number of kg for the second type of mixture. Show your working out.
3. Mutesi mixed 10 kg of white sugar that cost 1,300 Frw per kg with brown sugar that cost 1,100 Frw per kg. She sold the mixture at 1,200 Frw per kg. How many kg of brown sugar were there?
4. Kayinamura mixed 6 kg of yellow beans which cost 600 Frw per kg with coloured beans which cost 400 Frw per kg. If he sold the mixture at 500 Frw, how many kg of coloured beans did he mix?
5. Kayibanda mixed 5 kg of one type of rice which costs 2,000 per kg with another type which costs 1,600 Frw per kg. If he sold the mixture at 1,800 Frw, how many kg of another type did he mix?

6.21 Finding the price of one type of ingredient in the mixture

Activity

Kalisa mixed 20 kg of brown peas which cost 150 Frw per kg with 30 kg of green peas. Find the unit price of the green peas if the mixture cost 90 Frw per kg.
Present your findings to the class.

Example

Mugabo had 150 kg of a mixture of red beans and yellow beans that cost 600 Frw per kg. 90 kg of red beans cost 500 Frw per kg. Find the price per kg of the yellow beans. Show your working out.

Solution

Mixture = 150 × 600 = 90,000 Frw

Red beans = 90 × 500 = 45,000 Frw

Find the difference between the kilograms and the cost

Yellow beans (150 – 90) kg and (90,000 – 45,000) Frw

Yellow beans = 60 kg = 45,000 Frw

$$1 \text{ kg} = \frac{\cancel{45{,}000}^{750}}{\cancel{60}^{1}} = 750 \text{ Frw}$$

Study tip

- After finding the total mass and the cost of the mixture, subtract the cost of the first type from the cost of the mixture. Lastly, divide the result by the weight of the second type.



Application 6.21

1. Jeanne mixed 40 kg of beans which cost 700 Frw per kg with 60 kg of another type. What is the price per kg of the second type if the price of the mixture is 600 Frw per kg? Show your working out.
2. Gasana has 120 kg of mixed sugar and sells each at 1,200 Frw. If there are 80 kg of white sugar which cost 1,300 Frw per kilogram, find the price per kilogram of the second type of sugar. Show your working out.
3. Geoffrey sold 150 kg of a mixture of Super rice and Pakistani rice at 1,600 Frw per kilogram. 65 kg was Super Rice at 2,000 Frw per kilogram. Calculate the price of Pakistani rice.

4. A farmer sells 50 kg of a mixture of groundnuts and simsim at 3,000 Frw per kilogram. She sold 20 kg of simsim at 4,000 Frw per kilogram. Find the price of the groundnuts.
5. Grace had 95 kg of mixed rice that she sold for 152,000 Frw altogether. She had bought 40 kg of one type of rice at 1,200 Frw per kilogram. Find the price of the second type of rice.

6.22 Finding both quantities of a mixture

Activity

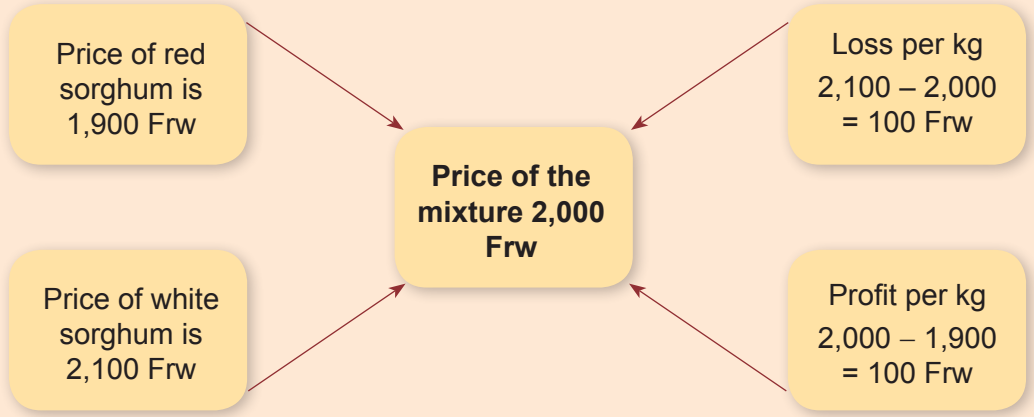
Joan had 150 kg of mixed beans. The cost of the mixture was 650 Frw per kg. One type cost 700 Frw per kg and the second type cost 600 Frw per kg. What was the weight of each type of bean in the mixture?

Show your working out.

Example

Juliet has a mixture of red sorghum and white sorghum weighing 40 kg. If one type costs 1,900 Frw per kg and another cost 2,100 Frw per kg, find the number of kg of each type if the mixture is sold at 2,000 Frw per kg.

Solution



Ratio of mixture = 100:100

For every 100 kg sold at 2,000 Frw, add 100 kg sold at 2,100 Frw

Red sorghum sold at 1,900 Frw $= \frac{\cancel{100}^{1}}{\cancel{200}_{1}} \times \cancel{40}^{20} = 20$ kg

White sorghum sold at 2,100 Frw $= \frac{\cancel{100}^{1}}{\cancel{200}_{1}} \times \cancel{40}^{20} = 20$ kg

Study tip

- To find the quantity of the two types in the mixture, use the ratio of each mixture over the total ratio and multiply by the cost of each mixture.



Application 6.22

1. In the shop, there are 60 kg of mixed beans. A kilogram costs 650 Frw. If the first type is sold at 700 Frw and another at 600 Frw, what is the weight of each type?
2. Mugabe mixed 120 kg of rice which he sells at 1,600 Frw per kilogram. The first type is sold at 2,000 Frw while the second is sold at 1,200 Frw. Find the quantity of each type.
3. Mutabazi mixed two types of sugar and formed 50 kg which he sold at 1,200 Frw per kg. If he sold the first type at 1,300 Frw per kilogram and the second type at 1,100 Frw per kilogram, find the kilograms of each type.
4. Umuhoza mixed two types of beans weighing 20 kg altogether and sold each kg at 900 Frw. The first type is sold at 1,200 Frw per kg and the second type at 700 Frw per kg. Find the kilograms of each type of beans.
5. Irish potatoes that cost 280 Frw per kg were mixed with another type of Irish potato that costs 240 Frw per kg to form a mixture. The price of 1 kg of mixed potatoes is 260 Frw per kg. Find the weight of each type of Irish potato if there were 60 kg of mixed Irish potato. Show your working out.

6.23 Solving problems involving ratios, percentages, mixtures and inverse proportions

Activity

- Do you remember what you learned about ratios, percentages, mixtures and inverse proportions?
- Why do you think it is important to study them?
- If it is helpful to you, give an example for each and explain how they can be applied in real life situations.
- Present your working out to the class.

Example

Everyday learners in school increase or decrease in enrolment. During the last academic year, there were 600 learners in the school. At the beginning of this year, the number increased by 15%. What is the new number?

Solution

Increased number is $\frac{15 \times 600}{100} = 15 \times 6 = 90$ learners.

The increased number in the school is 600 learners + 90 learners = 690 learners

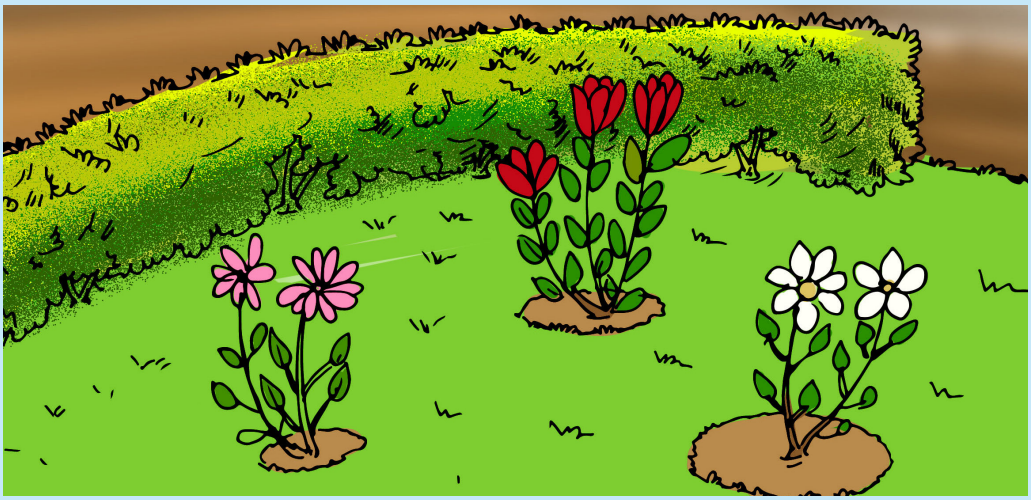


Application 6.23

1. There were 60 learner in a class. The number increased by 15%. What was the new number?
2. A trader mixed 80 kg of one type of maize flour that cost 700 Frw per kilogram with a second type of maize flour that cost 500 Frw per kilogram. He sold 120 kg of the mixture at 600 Frw per kg. Find how much of the second type of maize the trader had in kg.
3. In a school library, there are 800 textbooks. 10% of them are novels, 30% are Social Studies books and 40% are Mathematics textbooks. The rest are English books. How many textbooks of each type are there? Explain your working out.
4. 8 boys can dig a hole in 3 days. How many boys can dig the same hole in 2 days? Show your working out.
5. Express the ratio of $\frac{36}{3}:\frac{45}{5}$ in its simplest form.
6. A rectangular field has an area of 600 square metres and a length of 40 metres. What is the ratio of the length to the width of this field?
7. Katto's salary was decreased by 10% and it became 270,000 Frw. What was his salary before?
8. Convert 0.6... into a percentage.
9. Change 12.5% into a fraction.
10. Share 600 in a ratio of 3:4.

End of unit 6 assessment

1. The perimeter of a rectangle is equal to 280 metres. The ratio of its length to its width is 5:2. Find the area of a rectangle and show your working out.
2. There are (A) red flowers, (B) roses and (C) white flowers in a school garden. Write the ratio of the number of red flowers to the total number of flowers in terms of A, B and C.



3. If 35 men can reap a field in 8 days, in how many days can 20 men reap the same field?
4. Kambanda mixed 50 kg of sorghum flour which cost 2,000 Frw per kg with 20 kg of another type of sorghum. What is the price of a kg of the second type if the price of the mixture is 1,600 Frw per kg?
5. 6 boys working 5 hours a day can take 16 days to cultivate a garden. How many days will 4 boys take to do the same job, each working 5 hours?
6. There were 60 learners in a class. The number increased by 15%. What was the new number?
7. An examination paper has 25 questions. If Kamali got 70% of the answers correct, how many questions did Kamali fail? Show your working out.
8. How many people can dig 96 hectares in 8 days if 6 people can dig 64 hectares in 8 days?
9. In a boarding school of 600 learners, students have enough food for 90 days out of the whole term. How long will that food last if 50 more learners join the school?
10. How many kilograms of rice will 42 people need for 90 days if 10 people need 540 kilograms of rice for 72 days?

Unit 7 Relationship between volume, capacity and mass

Key unit competence: To be able to convert between units of volume, capacity and mass.

Introduction

In daily life, people use different containers (bottles, jerry cans, buckets, etc to carry water or other liquids. Each container has volume, capacity and mass and when it is full of water it has a certain mass.

(a) Did you ever try to think about the relationship between the capacity of a bottle, the quantity of water to fill the bottle and the mass of the bottle full of water?

(b) Do you think that the volume of a container, its capacity and its mass may have a relationship? Explain.

7.1 Revision on mass measurement

Activity

- Take a stone outside of the classroom and measure its mass.
- Measure mass of different stones with different sizes and record the results in your note book.
- Explain the meaning of mass and give an example.
- What do you notice?
- Present your findings to the class.

Example 1

Convert: 30 *dag* = ? *kg*

Solution

1 *dag* = 10 *g*

30 *dag* = (30 × 10) *g* = 300 *g*

Example 2

Convert: 2,500 g = ? *kg*

Solution

1,000 *g* = 1 *kg*

$2{,}500\ g = \frac{2{,}500}{1{,}000} = 2.5\ kg$

Study tip

- The mass of an object is the amount of material in its weight.
- The standard unit of mass is kg (kilogram) and g (grams).
- For small quantities, g (gram) is used.
- For large quantities, use kilograms.



Application 7.1

1. Work out and convert the given units:
   (a) 50 *kg* + 23 *hg* =… *dg*
   (b) 66 *hg* 56 *g* + 55 *dg* = … *mg*
2. Muziranenge bought 50 tonnes of beans. She sold 30 sacks each containing 80 kg. How many kg did she remain with?
3. Lydia bought the following items:1 *kg* of sugar: 5 *kg* of tomatoes, 250 *g* of salt, 400 *g* of tea bags. How many *kg* of items did she buy?
4. The following are the results of 5 learner's body mass tests: 57.4 *kg*, 30 *kg*, 49.7 *kg*, 50.02 *kg*, and 44.3 *kg*. What is their total mass?
5. A shopkeeper sold the following items to a customer: $3\frac{1}{2}$ kg of salt, 250 g of tea bags, 500 g of curry powder, 1 kg of wheat flour and $2\frac{1}{4}$ kg of sugar. Calculate the total mass in grams.
6. In your own words, explain why learning of mass measurement is helpful.

7.2 Revision on Capacity Measurement

Remember that:

1 *kl* = 10 *hl*
1 *kl* = 10,000 *dl*
1 *hl* = 1000 *dl*
1 *hl* = 10,000 *cl*
1 *dal* = 100 *dl*
1 *l* = 10 *dl*
1 *dl* = 10 *cl*

1 *kl* = 100 *dal*
1 *kl* = 100,000 *cl*
1 *hl* = 100 *l*
1 *hl* = 100,000 *ml*
1 *dal* = 1,000 *cl*
1 *l* = 100 *cl*
1*dl* = 100 *ml*

1 *kl* = 1,000 *l*
1 *kl* = 1,000,000 *ml*
1 *hl* = 1,000 *dl*
1 *dal* = 10 *l*
1 *dal* = 10,000 *ml*
1 *l* = 1,000 *ml*
1 *dl* = 10 *ml*

Activity

- Get an empty jerrycan.
- Fill it with water.
- Pour water in one litre bottles.
- How many litre bottles are filled from one jerrycan?
- Compare the capacity of a jerrycan with 1 litre bottle.

Example 1

Convert: 3,450 *l* to *kl*.

Solution

1 *kl* = 1,000 *l*

3,450 *l* = $\frac{3{,}450}{1{,}000}$

= 3.45 *kl*

Example 2

Work out: 15 *kl* 657 *dl* + 34 kl 23 *dl* = ...*dl*

Solution

15 *kl* 657 *dl*

+ 34 *kl* 023 *dl*

49 kl 680 dl

Study tip

- Capacity is the amount of liquid that fills a container.
- The standard units of capacity are litres and milliliters: 1,000 ml = 1 *l*



Application 7.2

1. A tank of water contains 5 *kl* of water. How many jerrycans of 20 *l* can be fully filled from that tank?
2. A container of 10 *l* of juice was shared equally among 15 children, how many *litres* did each one get?
3. On a farm, 100 *kl* of milk is collected in a month. How many *litres* of milk is collected in 5 months if milk is produced at the same rate?
4. A jerrycan of 20 *litres* is poured into 3 *litres* small jerrycans.
   (a) How many 3 *litres* small jerrycans are fully filled?
   (b) How much water remains?
5. Convert 16.24 *hl* to *dl*.
6. How can you apply capacity measurement in your daily life?

7.3 Measurement of volume

Activity

- Find a box container, for example chalk box.
- Measure the length, width and height of the box with a ruler.
- Write down the results.
- What is the volume of the box if:
  (a) You measure in cm?
  (b) You measure in mm?
- List the units of volume.
- How is the measurement of volume useful in daily life? Explain your findings to the class.

Example 1

Convert: 12 cm^3 to mm^3.

Solution

Place 12 in the first 2 places of cm^3.

dm^3			cm^3			mm^3		
				1	2	0	0	0

Fill all the places to mm^3 with zeros (0).

Therefore 12 cm^3 = 12,000 mm^3.

Example 2

Convert: 3,400,000 cm^3 to m^3.

Solution

Place 3 in the first place of m^3.

Place 4 in the last place of dm^3.

Find all the places to cm^3 with zeros.

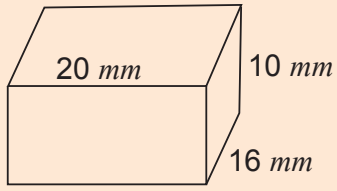
m^3			dm^3			cm^3		
		3.	4	0	0	0	0	0

3,400,000 cm^3 = 3.4 m^3.

Example 3

A cuboid has a length, width and height of 20 mm, 16 mm and 10 mm respectively. Calculate its volume.

Solution



L= 20 *mm*, W = 16 *mm* H = 10 *mm*

Volume = Length x width x height

= 20 *mm* x 16 *mm* x 10 *mm*

= 3,200 mm^3

Example 4

A rectangular tank has length, width and height of 2m, 1.5m and 3m respectively. Calculate its volume in cubic centimetres.

Solution

Length (l) = 2 m = (2 x 100) = 200 cm

Width (w) = 1.5 m = (1.5 x 100) = 150 cm

Height (h) = 3 m = (3 x 100) = 300 cm

Volume = Length x width x height

= 200 cm x 150 cm x 300 cm

= 9,000,000 cm^3

Study tip

- Volume is measured in mm^3, cm^3, dm^3, m^3, dam^3.
- Volume is the space occupied by an object.
- Volume determines how big or how small an object is.
- Conversion of units of volume is done better using a conversion table.



Application 7.3

1. With the help of the conversion table, work out the following:
   (a) 3 m^3 = ____ cm^3
   (b) 15 dm^3 = ____ cm^3
   (c) 0.6 m^3 = ____ m^3
   (d) 32 m^3 ____ dm^3
   (e) 4,000 dm^3 = ____ m^3
   (f) 9,000,000 cm^3 = ____ m^3
   (g) 700,000 mm^3 = ____ cm^3
   (h) 400,000 mm^3 = ____ cm^3
2. Calculate the volume of shapes with the following dimensions.
   (a) 12 mm by 15 mm by 9 mm
   (b) 15 cm by 8 cm by 10 cm
   (c) 14 m by 11 m by 7 m.
3. A rectangular water tank has a length of 4 m, width of 6 m and height of 2 m. Calculate its volume in cm^3.
4. Find the volume of your mathematics textbook.
5. Calculate the volume of the chalk box in your classroom.

7.4 Finding the relationship between units of volume, capacity and mass

Activity

- Find a rectangular cup or tin of one litre and fill it with water.
- Measure the mass of water in the cup.
- Measure and calculate the volume of the cup.
- What is the relationship between these measurements?
- Explain your working to the class.

Example

What is the relationship between the following measurements?

Litre(*l*), kilogram(*kg*) and decimetre cubed (dm^3).

Solution

With the help of this conversion table, the relationship is:

m^3					dm^3	cm^3			mm^3		
		kl	*hl*	*dal*	*l*	*dl*	*cl*	*ml*			
		t	*q*	-	*kg*	*hg*	*Dag*	*g*	*dg*	*cg*	*mg*
					1						

1 *l* = 1 *kg* = 1 dm^3

Study tip

- Volume, capacity and mass measurements are related only when measuring water.
- 1 *l* = 1 *kg* = 1 dm^3.
- This concept is helpful if you want to know the measurement of water when you already know its measurement in one of the above measurements.



Application 7.4

1. Match the following units of measurements:

Mass measurements	Volume measurements	Capacity measurements
1 *kg*	1 cm^3	1 *ml*
1 *g*	1 m^3	1 *dl*
1 t	1 dm^3	1 *kl*
1 *hg*	100 cm^3	1 *l*

2. Find out how many litres are in a 20 kg buckets.
3. Does 1 litre of cooking oil equal 1 kg? Explain.
4. Why is the relationship between these measurements applicable when measuring water?

7.5 Converting between units of volume, capacity and mass

Activity

- Convert the following units:
  10 *kg* = ... *kl* = ... cm^3
  1 dm^3 = ... *ml* = ... *dg*
- What do you notice?

Example

Convert: (a) 13 dm^3 to hg (b) 7,800 l to m^3

Solution

m^3			dm^3			cm^3			mm^3		
		kl	*hl*	*dal*	*l*	*dl*	*cl*	*ml*			
		t	*q*	-	*kg*	*hg*	*Dag*	*g*	*dg*	*cg*	*mg*
		7	8	1 0	3 0	0					

Therefore, 13 dm^3 = 130 *hg;* 7,800 *l* = 7.8 m^3

Study tip

When converting capacity, volume and mass, use a conversion table including all the units.



Application 7.5

1. Convert the following:
   (a) 6,767 *kg* = ...*cl*
   (b) 4 m^3 =..... *g*
   (c) 9,865 *l* = ... *q*
   (d) 469 *dl* to *ml*
   (e) 1.7m^3 to cm^3
   (f) 3.25 *mg* to *g*
2. Jacques fetches water with a jerrycan of twenty litres twice a day. How many litres does he fetch through the whole day? How many kilograms of water does he fetch?
3. In order to manage water for building a school, 2 water tanks of 5,000 litres were bought. Convert the capacity of both tanks into cubic decimetres.
4. Volume of water in a drum is 200 dm^3. There are $3\frac{1}{2}$ drums in a store. Calculate the capacity in litres.
5. A rectangular water tank has a capacity of 10,000 litres.
   (a) Calculate its volume in cm^3. (b) What is the mass of water in q?

End of unit 7 assessment

1. (i) Fill in the gaps:
   (a) 777 *dl* = _____ *kg*
   (b) 589,000 *g* = _____ m^3
   (c) 678.654 m^3 =_____ *dl*

   (ii) Convert in given units
   (d) 469 dl to ml
   (e) 1.7 m^3 to cm^3
   (f) 3.25 mg to g
2. 800 jerrycans fill a tank. One jerrycan weighs 20 *kg*. How many litres does the tank hold?
3. A bucket contains 45.6 litres of water. The empty bucket weighs 3 *kg*.
   (i) What is the mass of the full bucket?
   (ii) What is the volume of the water that fills the bucket in cm^3?
4. There are 500 learners in a school. Lunch time each learner takes one cup of tea. If the cup holds 50 *cl*, what is the volume of tea taken throughout the whole school in cm^3?
5. How many cubic centimetres make up a litre?
6. 5 jerrycans full of water weigh 109 kg. Each of the empty jerrycans weighs 1,800 g.
   (a) Find the capacity of the water in the jerrycans.
   (b) If Kalisa uses $\frac{1}{2}$ of that water, what is the mass of the water that will remain in kilograms?
7. The mass of water a school uses daily is 1.4 tonnes:
   (a) Find the daily capacity of water the school uses in litres.
   (b) Calculate the capacity of water it uses in a week excluding the weekend in decalitres.

Unit 8 Speed, distance and time

Key unit competence: To be able to calculate speed, distance and time, solve problems related to different time zones and convert speed from km/hr to m/sec and vice versa.

Introduction

People move everyday, whether to school, home, place of work, visiting friends and in any other journeys. How fast people move depends on the distance and time. Imagine you are a business person who has to deliver goods to a customer, who also needs to distribute them to his or her customers.

(a) What should you do? Travel faster or slower? By which means?

(b) What might be the result for late delivery of goods to the customer?

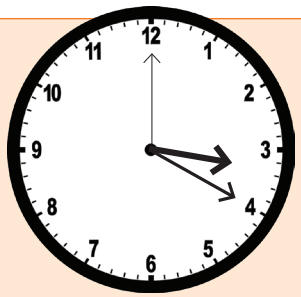
(c) Do you think that there is a relationship between the time, the speed and the distance traveled? Establish that relationship.

8.1 Comparing the 12-hour format to the 24-hour format

Activity

- You have learned about telling time.
- Choose any time in the morning and in the afternoon.
- Tell time using the 12-hour and 24-hour formats.
- Show time charts for 12-hour and 24-hour format.
- Compare those formats and explain them.

Example



The clock shows that we are in the afternoon.
What time is it?

Solution

It is twenty minutes past three.

Therefore, we write **3:20 p.m** in the 12-hour format.

To change 3:20 p.m to the 24-hour clock, add 12:00 because 3 hours 20 minutes have passed the noon time of 12:00 noon.

Therefore, 3:20 p.m will become;

$$\begin{array}{r} 03:20 \\ +\ 12:00 \\ \hline \mathbf{15:20} \end{array}$$

Therefore, 3:20 p.m will become **15:20** hrs in the 24-hour clock.

Study tip

- 12-hour format starts immediately after midnight until 12.00 noon, then from 12:00 noon to 12:00 midnight.
- 24-hour clock starts immediately after 24:00 hours, then back to 24:00 hrs.
- To change from 12-hour clock to 24-hour clock, add 12.00 hours to the given time if it is post meridiem or past mid day (p.m).
- To change from 12-hour clock to 24-hour clock, add 00.00 hrs to the given time if it is ante meridiem or before mid day (a.m).



Application 8.1

1. Compare the 12-hour clock in the 1st column with the 24-hour clock in the 2nd column. Then match each time in column to its respective 24-hour clock in the 2nd column.

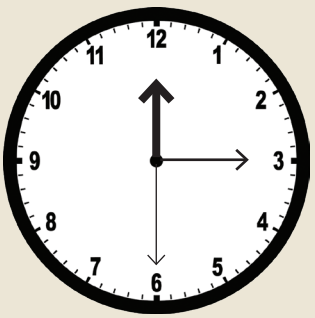
1st column	2nd column
3 :23 p.m.	06:45
6:45 a.m.	20:44
7:15 a.m.	15:23
12:12 p.m.	07:15
8:44 p.m.	12:12

2. Indicate the time in form of 12-hours and then in form of 24-hours.

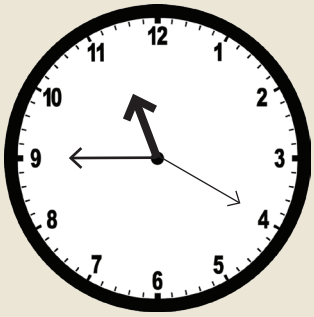
a) It is in the afternoon



b) It is in the morning



c) It is in the morning



d) It is in the evening



3. Change the time from 12-hour format to 24-hour format.

(a) 4:21 p.m.	(b) 5:56 p.m.	(c) 9:12 a.m.
(d) 8:45 a.m.	(e) 12:46 a.m.	(f) 10:43 a.m.
(g) 1:59 p.m.	(h) 7:18 a.m.	(i) 3:14 a.m.
(j) 2:49 p.m.	(k) 5:00 p.m.	(l) 9:56 a.m.

8.2 Converting 12-hr format to 24-hr format

Activity

Do it personally, with your partner or in a group.

- Match the time in 24-hour format with that in 12-hour format.

24-hour	12-hour
12:45	11:30 p.m
07:30	12:45 p.m
23.30	9:15 p.m
21:15	4:20 a.m
04:20	7:30 a.m

- What do you notice?

Example 1

Convert 6:24 a.m to the 24-hr format.

Solution

a.m time changes directly to 24-hour format, so add 00:00 hours.

$$\begin{array}{rr} & 06:24 \\ + & 00:00 \\ \hline & \mathbf{06:24} \end{array}$$

Therefore, 6:24 a.m. = 06:24 hrs.

Example 2

Convert 11:28 p.m to the 24-hr format.

Solution

p.m. time changes to 24-hour format by adding 12:00 hours.

$$\begin{array}{rr} & 11:28 \\ + & 12:00 \\ \hline & \mathbf{23:28} \end{array}$$

Therefore, 11:28 p.m. = 23:28 hrs.

Example 3

Convert 4:02 a.m to the 24-hr format.

Solution

a.m. time changes to 24-hour format by adding 00:00 hours.

$$\begin{array}{rr} & 04:02 \\ + & 00:00 \\ \hline & \mathbf{4:02} \end{array}$$

Therefore, 4:02 a.m. = 04:02 hrs.

Example 4

Convert 3:30 p.m to the 24-hr format.

Solution

p.m. time changes to 24-hour format by adding 12:00 hours.

$$\begin{array}{rr} & 03:30 \\ + & 12:00 \\ \hline & \mathbf{15:30} \end{array}$$

Therefore, 3:30 p.m. = 15:30 hrs.

Study tip

- To change a.m. time to the 24-hr format, add 00:00 hours.
- To change p.m. time to the 24-hr format, add 12:00 hours.



Application 8.2

Change the time from the 12-hour format to the 24-hour format.

(a) 4:21 p.m.	(b) 5:56 p.m.	(c) 9:12 a.m.
(d) 8:45 a.m.	(e) 12:46 a.m.	(f) 10: 43 a.m.
(g) 1:59 p.m.	(h) 7:18 a.m.	(i) 3:14 a.m.
(j) 2:49 p.m.	(k) 5:00 p.m.	(l) 9:56 a.m.

8.3 Converting 24-hr format to 12-hr format

Activity

- A watch shows 22:22.
- It is in 24-hour format.
- What would it display in the 12-hours format?
- Explain to your classmates.



24 hr clock

Example 1

Change 06:46 hr to the12-hour format.

Solution

The time begins with 0 so it is a.m.

Time below 12:00 hrs subtract 00:00 hrs.

06:46 – 00:00 = 6:46 a.m.

$$\begin{array}{rr} & 06{:}46 \\ - & 00{:}00 \\ \hline & \mathbf{06{:}46} \end{array}$$

Time from 12:00 midnight to 12:59 a.m. continues unchanged.

Therefore, 06:46 hrs = 6:46 a.m

Example 2

Change 00:25 hr to the 12-hour format.

Solution

The time begins with 0 so it is a.m time.

Time after midnight, add 12:00 hrs.

00:25 + 12:00 = 12:25 a.m

$$\begin{array}{rr} & 00{:}25 \\ - & 12{:}00 \\ \hline & \mathbf{12{:}25} \end{array}$$

Time from 12:00 midnight to 12:59 a.m. continues unchanged.

Therefore, 00:25 hrs = 12:25 a.m

Example 3

Change 13:56 hr to the 12-hour format.

Solution

The time is above 12:00 hrs, so it is p.m.

Time above 12:00 subtract 12:00.

13:56 – 12:00 = 1:56 p.m

$$\begin{array}{rr} & 13{:}56 \\ - & 12{:}00 \\ \hline & \mathbf{01{:}56} \end{array}$$

Therefore, 13:56 hrs = 1:56 p.m.

Example 4

Change 21:40 hr to the 12-hr format.

Solution

The time is above 12:00 hrs, so it is p.m.

Time above 12:00 subtract 12:00.

21:40 – 12:00 = 9:40 p.m

$$\begin{array}{rr} & 21{:}40 \\ - & 12{:}00 \\ \hline & \mathbf{09{:}40} \end{array}$$

Therefore, 21:40 hrs = 9:40 p.m.

Study tip

- To convert 24-hour format to the 12-hour format, time below 12:00 hours always add 00:00 hours.
- To convert 24-hour format to 12-hour format, time above 12:00 hours always subtract 12:00 hours.
- 12:00 a.m. to 12:59 noon do not change. They remain as they are.
- 24-hours = 00:00 a.m. or 12:00 midnight, 00:40 hrs = 12:40 a.m, 12:30 p.m. = 12:30 hours.
- Time above 12:00 hours, that is 12:00 hours to 24:00 hours, is always p.m.
- Time below 12:00 hours, that is 00:01 hours to 11:59 is always a.m.
- 12-hours = 12:00 midday or 12:00 noon.



Application 8.3

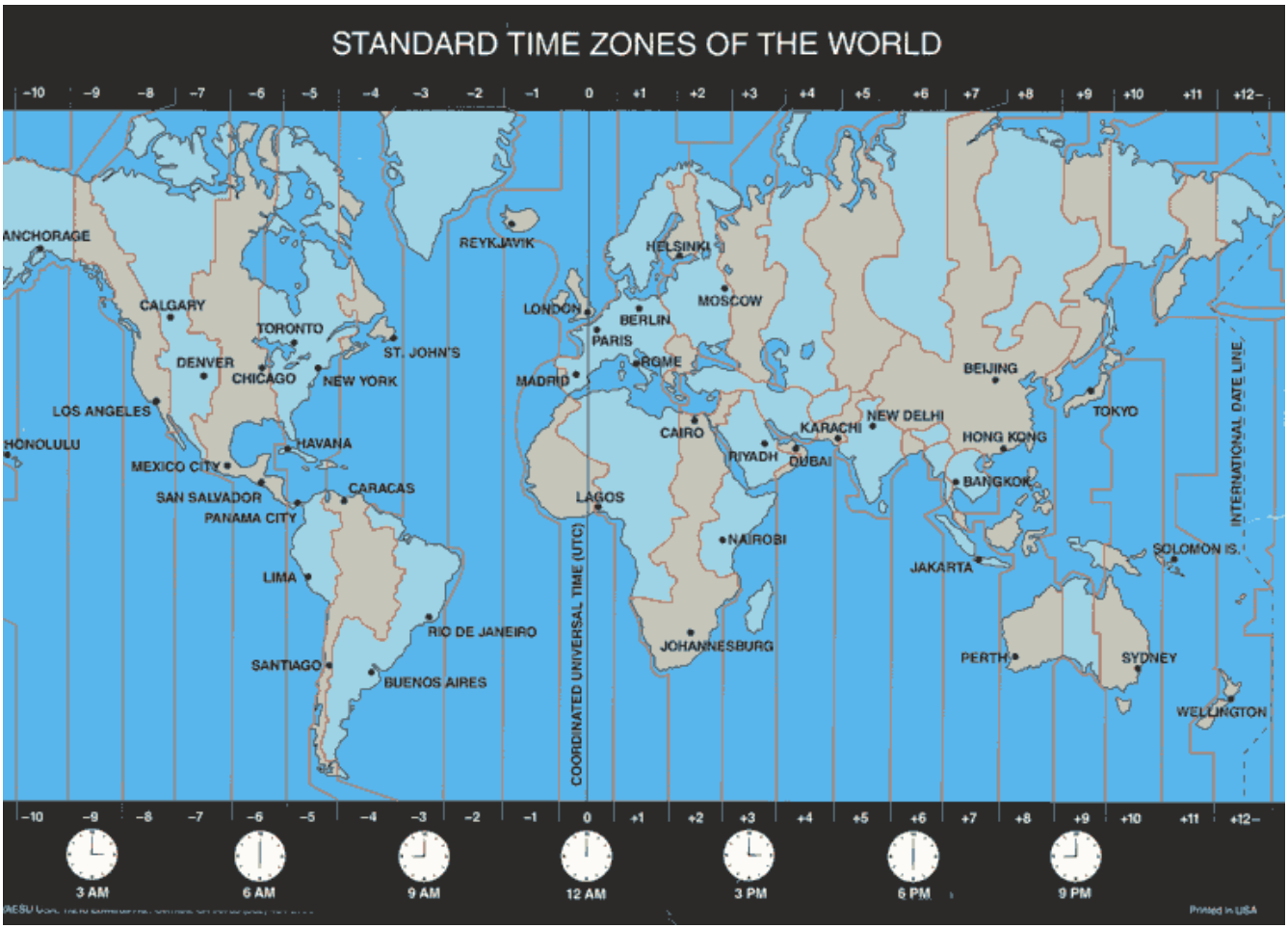
Change the following time from 24-hour format to 12-hour format.

(a) 04:12 hr	(b) 15:54 hr	(c) 06:32 hr
(d) 22:10 hr	(e) 01:23 hr	(f) 23:45 hr
(g) 05:15 hr	(h) 17: 38 hr	(i) 13:38 hr
(j) 03:45 hr	(k) 19:40 hr	(l) 09: 30 hr

8.4 The Concept of time zones

Activity

- Have you ever felt you needed more time?
- Have you ever wished you could turn the clock back?
- Places that are West of your country's time zone are earlier in time.
- Places that are East of your country's time zone are further in time.
- Look at the world map showing standard time zones and answer the following:
  - (a) In how many time zones is the world divided?
  - (b) How many hours does each zone represent?
  - (c) Locate Rwanda on the world map.
  - (d) What do you observe above time of places left of Rwanda?
  - (e) What about time of countries to Rwanda's right?



Example 1

Look at the world map showing the standard time zones. What is the time in the 9th time zone towards the West?

Solution

9th time zone to the West is represented by -9. The time reading is 3:00 a.m.

Example 2

Look at the world map showing the standard time zones. Find the time in the 11th time zone towards the East

Solution

11th time zone to the East is represented by +11. The time reading is 23:00 hrs or 11:00 p.m.

Study tip

- A time zone is a region of the globe that observes a uniform standard time for legal, commercial and social purposes.
- The world is divided into 24 time zones.
- Each time zone represents 1 hour difference in time.

- Moving to the West we reduce time and to the East, we add time. One hour per time zone.
- Remember that between two longitudes (time zones) there is 15^{O} which is equal to 1 hour.



Application 8.4

Use the world map showing standard time zones to answer the following:

Find the time reading in the time zones below:

(a) 2^{nd} time zone to the East.
(b) 7^{th} time zone to the West.
(c) 10^{th} time zone to the East.
(d) 12^{th} time zone to the West.
(e) 9^{th} time zone to the East
(f) 5^{th} time zone to the West.
(g) Time at 0 time zone.
(h) 12^{th} time zone to the East.

8.5 Solving mathematical problems relating to time zones

Activity

- Study the map of the world showing the standard time zones.
- Name the meridian that is representing 0.
- How important is the meridian that you named?
- What important line is represented by -12 or +12?
- Give the importance of that line.

Example 1

It is 11:30 p.m. in Rwanda. What time is it in New York which is located in the 7^{th} time zone to the West of Rwanda?

Solution

Places to the West are earlier in time.

The places in the first time zone to the left are 1 hour earlier.

New York is in the 7th time zone from Rwanda.

Example 2

The time in a certain time zone is 3:45 p.m. What is the time in the 6^{th} time zone to the East?

Solution

Places to the East are further in time.

The places in the first time zone to the right are 1 hour less.

Read off the time in the 6th zone to the East.

1st time zone = 1 hour 7th time zone = 7 x 1 hour = 7 hours Subtract 7 hours from the time in Rwanda. 11:30 - 7:00 **4:30** Therefore, the time in New York is 4:30 pm.	1st time zone = 1 hour 6th time zone = 6 x 1 hour = 6 hours Add 6 hours to the time given. 3:45 + 6:00 **9:45** Therefore, the time is 9:45 pm.

Study tip

- To find time of another time zone to the West, multiply its position from the time in the given time zone by 1 hour. Then subtract from the given time.
- To find time of another time zone to the East, multiply its position from the time in the given time zone by 1 hour. Then add to the given time.
- In all calculations, use the 24-hour format to make calculations easier.
- There is a difference of one hour between two time zones.



Application 8.5

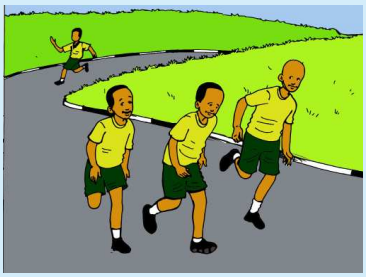
Work out the following:

1. It is 2:00 p.m. in Cairo. What is the time in Delhi which is in the 3rd time zone to the East of Delhi?
2. The time in Accra is midnight. What time is it in Buenos Aires, in the 4th time zone to the West?
3. It is 12:00 noon at the Greenwich meridian. What time is it at the International Date Line?
4. The time in Sydney is 10:00 a.m. Baghdad is in the 7th time zone to the West of Sydney. What is the time in Baghdad?
5. The time zone is 3:50 a.m. What is the time in the 11th time zone to the East?
6. What is the difference in time from the 3rd time zone to the East and 5th time zone to the West?

8.6 Calculating the speed

Activity

- Take a stop clock.
- Start it and start running around the field once.
- Stop it when you return to the starting point.
- Count the time you have taken.
- Divide the distance with the time you took.
- What do you notice?



Example 1

A motor cyclist travelled for 3 hours and covered a distance of 210 kilometres. What speed was he moving?

Solution

Distance = 210 km, Time = 3 hours

To calculate speed, divide the distance covered by the time taken to cover the distance.

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$= \frac{70\,\cancel{210}\ \text{km}}{1\,\cancel{3}\ \text{hours}}$$

$$= 70\ \text{km/hr.}$$

The speed of the motorcyclist was 70 km/hr.

Example 2

An athlete ran 600 *m* in 1 minute. Calculate his speed in *m/sec*.

Solution

Distance = 600 m, Time = 60 sec

To calculate speed, divide the distance covered by the time taken to cover the distance.

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$= \frac{10\,\cancel{600}\ \text{km}}{1\ 60\,\cancel{\text{sec}}}$$

$$= 10\ \text{m/sec.}$$

The athlete ran at a speed of 10 m/sec.

Study tip

- Speed is the distance travelled by an object in a unit of time. The standard international unit is metre per second (m/sec) and the everyday unit is kilometre per hour (km/hr).
- To calculate speed, the distance covered is divided by the time taken.
- Speed = $\frac{\text{Distance covered}}{\text{Time taken}}$.
- Speed determines how fast or how slow an object is moving.

Application 8.6

Work out the following:

1. A bus travelled for 3 hours to cover a distance of 180 km. At what speed was the bus moving?
2. At what speed was the car travelling if it covered 540 km in 6 hours?
3. What is the speed, when the distance and time are:
   (a) 160 km and 4 hours?
   (b) 150 km and 3 hours?
   (c) 200 km and 5 hours?
4. A trailer left city A for city B covering 720 km. If it took 12 hours to arrive, what was its speed?
5. A truck covered a distance of 3,600 metres in 3 minutes. Calculate its speed in metres per second.

8.7 Converting the speed from km/hr to m/sec

Activity

- Muziranenge ran 6 km in one hour.



- If she was running at the same rate, can you find the distance in metres she covered in one second?
- Discuss and defend your answer by showing your working steps?

Example 1

Express 72 km/hr as m/sec.

Solution

Change *km* to *m* and *hr* to *sec*.

1 km = 1,000 m
1 hr = 60 minutes
1 min = 60 sec
1 hr = 1 x 60 x 60 sec
1 hr = 3600 sec

$$72\ km/hr = \frac{2\ \cancel{72} \times 1{,}0\cancel{00}\ m}{1\ 1 \times 3{,}6\cancel{00}\ sec} = \frac{2 \times 10\ m}{1 \times 1\ sec} = 20\ m/sec$$

Example 2

Convert 108 km/hr into m/sec.

Solution

Change *km* to *m* and *hr* to *sec*.

1 km = 1,000 m
1 hr = 60 minutes
1 min = 60 sec
1 hr = 1 x 60 x 60 sec
1 hr = 3600 sec

$$108\ km/hr = \frac{3\ \cancel{108} \times 1{,}0\cancel{00}\ m}{1\ 1 \times 3{,}6\cancel{00}\ sec} = \frac{3 \times 10\ m}{1 \times 1\ sec} = 30\ m/sec$$

Study tip

- 1 hour = 60 minutes.
- 1 minute = 60 seconds.
- 1 hour = 60 x 60 seconds = 3,600 seconds.
- 1 km = 1,000 m.
- Speed = $\frac{\text{Distance covered}}{\text{Time taken}}$.
- Changing from km/hr to m/sec = $\frac{\text{Speed x 1,000 m}}{\text{3,600 sec}}$.



Application 8.7

Work out the following:

1. Change the following km/hr into m/sec:
   (a) 90 km/hr (b) 60 km/hr (c) 180 km/hr
   (d) 54 km/hr (e) 252 km/hr
2. The distance from village A to village B is 720 km. A car takes 6 hours to cover the journey. Calculate its speed in m/sec.
3. Jane covered a distance of 50 km in 2 hours.
   (a) Calculate the speed in km/hr.
   (d) What was the speed in m/sec?

4. The distance from one town to another town is 216 km. Kalisa covered the distance in 4 hours. Calculate the speed in m/sec.
5. A truck driver covered a distance at a speed of 70 km/hr. Express the speed in m/sec.

8.8 Converting the speed from m/sec to km/hr

Activity

During breaktime, learners of P.6 had a 100 m race.

- The winner covered 5 metres in one second.
- Suppose he runs at the same rate, what distance can he cover in one hour?
- Explain your working out.



Example 1

Express 200 m/sec as km/hr.

Solution

Change m into km and sec to hr.

1 km = 1,000 m

So 200 m = $\frac{200}{1000}$ km

1 hr = 60 minutes

1 min = 60 sec

1 hr = 1 x 60 x 60 sec

1 hr = 3600 sec

So 1 sec = $\frac{1}{3600}$ hr

200 *m/sec* = $\frac{200}{1,000} \div \frac{1}{3,600}$

= $\frac{\cancel{200}}{1,\cancel{000}} \times \frac{3,6\cancel{00}}{\cancel{1}}$

= $\frac{20 \times 36}{1 \times 1}$

= 720 *km/hr*

Example 2

Express 45 m/sec as km/hr.

Solution

Change m into km and sec to hr.

1 km = 1,000 m

So 45 m = $\frac{45}{1000}$ km

1 hr = 60 minutes

1 min = 60 sec

1 hr = 1 x 60 x 60 sec

1 hr = 3600 sec

So 1 sec = $\frac{1}{3600}$ hr

45 *m/sec* = $\frac{45}{1,000} \div \frac{1}{3,600}$

= $\frac{45}{1,\cancel{000}1} \times \frac{3,\cancel{600}3.6}{1}$

= $\frac{45 \times 3.6}{1}$

= 162 *km/hr*

Study tip

- Changing from m/sec to km/hr = $\frac{\text{Speed x 3,600 sec}}{\text{1,000 m}}$.
- 1,000 metres = 1 kilometre.
- 3,600 seconds = 1 hour.
- To change m/sec to km/hr, change m to km and sec to hrs, then divide.

Application 8.8

Work out the following

1. Convert the following into km/hr:
   (a) 15 m/sec (b) 45 m/sec (c) 25 m/sec (d) 100 m/sec
2. An aeroplane covered 1,000 metres in 5 seconds. Express the speed in km/hr.
3. During a bicycle Rwanda Tour Racing, a cyclist covered 100 metres in 10 seconds. Calculate the speed in kilometres per hour.



Bicylce race

4. In a school competition, one athlete ran 100 metres in 15 seconds. What was the speed in kilometres per hour?
5. A cyclist was riding at a speed of 10 mertes every second. Change his speed into kilometres per hour.
6. In the East African Safari Rally, Mansoor drove at a speed of 40m/sec. What speed was he driving in kilometres per hour?

8.9 Calculating the distance

Activity

- Mugenzi and Mutoni had a journey from home to visit a friend called James.
- They took 2 hours moving at a speed of 13 kilometres per hour to reach there.
- What is the distance from their home to James' home?
- Explain your working out to the class.

Example 1

A bus left Kigali at 10:00 a.m travelling at 60 km/hr. It arrived at its destination at 1:00 p.m. Find the distance it covered.

Solution

Distance = Speed × Time

Time taken = 1:00 p.m to 10:00 a.m

The duration crossed from morning to afternoon

Time = (12.00 - 10.00) + 1.00

= 2 hrs + 1 hr

= 3 hours

Speed = 60 km/hr

Distance = 60 × 3

= 180 km

The distance covered is 180 km.

Example 2

A car took 4 hours to cover a distance. It moved at a speed of 90 km/hr. Calculate the distance it covered.

Solution

Speed = 90 km/hr

Time = 4 hours

Distance = speed × time

Distance = 90 × 4

= 360 km

Study tip

- Distance = speed × time.
- If departure and arrival time is given, calculate the duration to get the time.
- Distance is measured in kilometres (km), hectometres (hm), decimetres (dam), metres (m), centimetres (cm) and millimetres (mm).
- The standard unit of measuring distance is metre (m).



Application 8.9

Work out the following:

1. It takes Mugeni 2 hours walking at a speed of 5 km/hr to arrive at the market. What is the distance between her home and the market?
2. Teacher Mugenzi rode his bicycle for 2 hours at a speed of 12 km/h to arrive at school. What is the distance from his residence to the school?
3. Find the distance that Muhire covers in 5 hours moving at a speed of 80 km/hr.
4. What is the distance covered by Therese in 15 seconds, if she moves at a speed of 72 km/hr in her car?
5. Abbas drove his car at a speed of 80 km/hr for $1\frac{1}{2}$ hours. What distance did he cover?

8.10 Calculating the time

Activity

- Joseph walks to school every day.
- The distance from home to school is 3 km.
- One day, he walked at a speed of 2km/hr. How long did he take to reach school?
- Present your working out to the class.

Example

(a) Juliet walked a distance of 45 km at a speed of 5 km/h. What time did she take?

Solution

Distance = 45 *km,* Speed = 5 *km/h.*

$$\text{Time} = \frac{\text{Distance covered}}{\text{Speed}}$$

$$= \frac{45 \text{ km}}{5 \text{ km/hr}}$$

$$= \frac{\overset{9}{\cancel{45}\ \cancel{\text{km}}}}{\underset{1}{\cancel{5}\ \cancel{\text{km}}}} \times \text{hrs} = 9 \text{ hrs}$$

Express the time in the units given in the question. Therefore, she took 9 hours.

(b) Mutesi drove a racing car for a distance of 8,000 metre at a speed of 250 m/sec. What time did she take?

Solution

Distance = 8,000 m, speed = 250 m/sec.

$$\text{Time} = \frac{\text{Distance covered}}{\text{Speed}}$$

$$= \frac{8{,}000 \text{ m}}{250 \text{ m/sec}}$$

$$= \frac{\overset{32}{\cancel{8{,}000}\ \cancel{\text{m}}}}{\underset{1}{\cancel{250}\ \cancel{\text{m}}}} \times \text{sec} = 32 \text{ sec}$$

Express the time in the units given in the question. Therefore, she took 32 sec.

Study tip

- Time = $\frac{\text{Distance covered}}{\text{Speed}}$.
- Time taken is worked out by dividing the distance covered by the speed.
- The units of time are hours, minutes and seconds.



Application 8.10

Work out the following:

1. Christine drives her car at a speed of 72 km/hr. What time does she take for the whole journey if she drives to cover a distance of 216 km?
2. A bus travelled from Kigali to Kampala. It was moving at a speed of 60 km/h. It stopped after moving 180 km. What time did it take to cover the journey?
3. Murengezi walks at 2 km/hr when he is going to school. If he moves a distance of 3 km, how long does it take him to arrive at school?
4. How long did a car travelling at 60 km/h take to cover 240 km?
5. How long will it take a cyclist to cover a distance of 105 km at a speed of 35 km/hr?
6. A bus covered a distance of 474 km at a speed of 79 km/hr. Calculate the time it took to arrive at the destination.

8.11 Moving bodies towards each other

Activity

- You studied how to calculate speed, distance and time.
- Try to solve the following:

  Mugenzi drove a car from city (A) towards city B at 7:00 a.m. He was travelling at a speed of 60 km/hr and Mukamukunzi left city (B) at the same time driving towards city A at the speed of 40 km/hr on the same road. If its is 100 km from city (A) to city (B):

  (a) At what time did they meet?

  (b) What is the distance each had covered by the time they met?
- Explain your working out.
- Present your work to the class.

Example

Bosco and Nadine moved towards each other. They started moving at 8:00 a.m and met at 8:40 a.m. Their speeds were 60 km/hr and 51 km/hr respectively. What distance had each covered by the time they met?

Solution

Calculate the duration first.

Duration = Ending time (ET) - Starting time (ST)

8:40 a.m – 80:00 a.m = 40 minutes

Express the minutes to hours

$$\frac{\cancel{40}^{2}}{\cancel{60}_{3}} = \frac{2}{3} \text{ hrs}$$

Get the distance covered by each before meeting;

Distance = speed × time

Bosco = $\cancel{60}^{20}$ km/hr × $\frac{2}{\cancel{3}_{1}}$ *hr*

20 km x 2 = 40 *km*

Nadine = $\cancel{51}^{17}$ km/hr × $\frac{2}{\cancel{3}_{1}}$ *hr*

17 km x 2 = 34 *km*

Study tip

- Distance is worked out by finding the product of speed and time.
- Duration is the time between two events.
- Duration is worked out by subtracting the starting time (ST) from the ending time (ET).
- To get distance covered by each, when they meet, multiply each one's speed by the duration.



Application 8.11

Work out the following:

1. Two motorcyclists started the journey at 9:00 a.m. and met at 11:00 a.m. One of them moved from town (A) at a speed of 45 km/hr and another one moved from town (B) at a speed of 35 km/hr. What distance had each covered by the time they met?

2. Clementine and Justin were at a distance of 60 km apart. They left their homes at 7:00 a.m and met at 9:00 p.m. If Justin moved at a speed of 12 km/h, what was the speed of Clementine?
3. Tom and Silas started the journey towards each other; moving at 30 km/hr and 24 km/hr respectively. If between their cities there are 135 km. At what time did they meet?
4. Mugenzi and Murengezi left their residences and moved towards each other at 9:00 a.m. They met at 11:40 a.m. Their speeds were 40 km/hr and 35 km/hr respectively. What was the distance between them?
5. Town A and Town B are 240 km apart. Anna started travelling from town A at the same time, Chantal moved from town B. Chantal was moving at a speed of 50 km/hr. They met after 3 hours. Find Anna's speed.

8.12 Moving bodies following each other

Activity

A car left Musanze at 8:00 a.m moving at a speed of 40 km/hr and the second car also left Musanze at 9:00 a.m moving at a speed of 60 km/hr.

(a) When do you think the second car will catch up with the first car?

(b) What distance will they have covered?

(c) Show your working out to the class.

Example

A bus travelling at 40 km/hr left Kigali at 8:30 a.m. Another bus travelling at 60 km/hr followed it after 1 hour. When did the second bus over take the first bus? Show your working.

Solution

The first bus was covering 40 km in 1 hour.

The first bus had covered 40 km/hr before the second started.

Difference in speed = (Second speed - first speed)

= (60 km/hr - 40 km/hr)

= 20 km/hr

Then calculate the time the second bus will catch up with the first.

Distance apart between the two buses after 1 hr

= (60 km – 40 km) = 20 km

Time taken by the 2nd bus to catch up with the 1st bus = $\frac{2\ \cancel{40\ km}/hr}{1\ \cancel{20\ km}}$ = 2 hours

Starting time of the second bus

Add 1 hour to the starting time of the 1st bus (8:30 a.m) because the 2nd bus started the journey 1 hour after the 1st bus.

8:20 + 1 hr = 9:30

The second bus will overtake the first bus at:

9:30 + 2 hrs = 11:30 a.m.

Study tip

- Distance is a product of speed and time.
- To work out the distance covered by one object following another, find the difference between the speeds.
- To find the duration the second moving object overtakes the first, divide the speed of the first object by the difference in distance moved by the two objects after 1 hour.
- To find the time the second moving object overtakes the first moving object, add the duration for overtaking to 1 hour.



Application 8.12

Work out the following:

1. A car moving at a speed of 60 km/hr followed a bus which had departed 2 hours earlier. If the bus was traveling at 45 km/hr, when did the car overtake the bus if the bus left at 9:00 a.m?
2. Car A moves at a speed of 40 km/hr and car B moves at a speed of 60 km/hr. What is the distance between them after 3 hours if they are moving in the same direction?
3. Mukakalisa left Kayonza at 8:00 a.m moving at 60 km/hr and Muhire followed her and met her at 11:00 a.m. What distance had Muhire covered?
4. A cyclist moving at 30 km/hr started from city (A). After 2 hours a lorry followed, moving at 40 km/hr.
   (a) How long did it take the lorry to catch up with the cyclist?
   (b) What distance had both covered?

8.13: Calculating average speed

Activity

- The distance from town A to town B is 120 km.
- A bus covers the distance in 2 hours.
- It continues to town C a distance of 180 km from town B.
- The bus takes $2\frac{1}{2}$ hours to cover the distance.
  - (a) Find the total distance.
  - (b) What was the total time taken?
  - (c) Divide the total distance covered by the total time taken. What do you get?
- Make a class presentation.

Example 1

A taxi moved from town P to town Q, a distance of 160 km. It took $2\frac{1}{2}$ hours. From town Q to town R, it covered 200 km in $3\frac{1}{2}$ hours. Find the average speed for the whole journey.

Solution

Find the total distance covered (TDC)

P to Q to R = 160 + 200 = 360 km

Find the total time taken (TTT)

P to Q to R = $2\frac{1}{2} + 3\frac{1}{2}$ = 6 hours

$$\text{Average speed} = \frac{\text{Total distance covered}}{\text{Total time taken}}$$

$$= \frac{360 \text{ km}}{6 \text{ hrs}} = 60 \text{ km/hr}$$

The average speed is 60 km/hr.

Example 2

A car takes 6 hours to cover a journey at 70 km/hr. It takes only 4 hours to return through the same distance. Calculate the average speed for the whole journey.

(i) Going	(ii) Returning	(iii) Average speed
Distance = Speed x Time Speed = 70 km/hr Time = 6 hours Distance = 70 x 6 = 420 km	Distance = 420 km Time = 4 hours	$= \frac{TDC}{TTT}$ $= \frac{(420 + 420)\text{ km}}{(6 + 4)\text{ hrs}}$ $= \frac{840\text{ km}}{10\text{ hrs}}$ = 84 km/hr

Study tip

- Average speed is distance covered per unit time by an object moving, assuming it moved at a constant speed.
- Average speed = $\frac{\text{Total distance covered}}{\text{Total time taken}}$.
- Average speed can also be calculated for two or more distances covered within different time taken.



Application 8.13

1. Town A is 40 km from Town B. A bus took 1 hour to cover the distance. The bus continued from Town A to another town 60 km away taking $1\frac{1}{2}$ hrs. What was its average speed?
2. A lorry takes 4 hours to travel from town (A) to town (B) covering a distance of 180 km. It returns in 2 hours. Calculate the average speed.
3. A bus takes 6 hours to cover a distance of 480 km. It returns in only 4 hours. What was its average speed?
4. A motorcycle travelled from Kigali to Kibuye covering a distance of 63 km. It took 1 hour 40 minutes. It returned through the same route covering the distance in 1 hour 20 minutes. What was the biker's average speed?
5. A town is 180 km from another town. It took a truck $3\frac{1}{2}$ hours to cover the distance. On return, it took $1\frac{1}{2}$ hours to cover the same distance. Calculate the truck's average speed.
6. The distance from City A to City B is 67 km. A truck covered it in 1 hrs 25 minutes. It returned via the same route taking 1 hour 35 minutes. What was its average speed?

7. A car takes 4 hours to cover a journey at 60 km/hr. On return, it takes only 2 hours. Calculate its average speed
8. A trailer took 4 hours to travel between two town at 45 km/hr. But it returned through the same route taking 6 hours. Work out its average speed.

End of unit 8 assessment

1. Angel and Karemera started moving towards each other at 7:00 a.m. Their speeds were 20 km/hr and 30 km/hr respectively. The distance between them was 150 km.
   (a) Find the time they met.
   (b) Find the distance each had covered.
2. A car moving at a speed of 70 km/hr and a lorry moving at a speed of 30 km/hr started moving towards each other at 8:00 a.m. What was the distance between them if they met at 10:30 a.m?
3. The distance from town A to town B is 720 km. A car takes 8 hours to cover the journey. Calculate its speed in m/sec.
4. Johnson is in Bangladesh which is at 6 time zones East. He wants to be in Greenland which is 2 time zones West at 16:30 hr. What time should he fly?
5. Two motorists started moving at the same time towards each other. One motorist was moving at 60 km/hr and another at 80 km/hr. They met after 4 hours.
   (a) What distance did both cover after 1 hour?
   (b) What was the distance between them before they started moving?
6. A lorry and a bus were travelling in the same direction. They set off at the same time from the park. The lorry was travelling at 50 km/hr. The bus

was travelling at 70 km/hr.

(a) How far apart will both be after 3 hours?

(b) What will be their a part distance after 5 hours?

7. A bus takes 4 hours to cover a journey of 240 km and takes only 2 hours to return. Calculate the average speed.

8. A truck took 3 hours to cover a journey at $45\frac{1}{2}$ km/hr. On return journey, it took 4 hours. Find its average speed.

9. It is 8:20 pm in a city. What time is it in the 5^{th} time zone to the East?

10. Lucimango arrived at 9:15 p.m. to his destination. He started his journey from the 7th time zone East of his destination. What time did he start the journey?

Unit 9 Simple interest and problems involving saving

Key unit competence: To be able to work out simple interest and solve problems involving saving.

Introduction

In business, people aim at getting more money in order to become rich. Many people become rich by investing their money in businesses. When people bank, borrow or lend money, they expect to get more added money to their originally used money and this is called interest.

Consider the following situation and answer the related questions:

A person put money in the bank for a year amounting to 200,000 Frw. The bank gave him/her 10 out of each 100 Frw for using his/her money.

(a) How much more money was he/she given?

(b) Why do you think we should save our money in the bank?

(c) Can you give different possibilities of using money and getting interest without saving it in the bank?

(d) How does interest benefit us in daily life?

9.1 Calculating the simple interest

Activity

- Visit a bank or SACCO if available.
- Ask how it operates.
- Write down the information on a sheet of paper.
- Back to class, make a class discussion.

Example 1

Given; principal (P) = 100,000 Frw, interest rate (R) = 10% p.a and Time (T) = 2 years. Find the simple interest (S.I).

Solution

P = 100,000 Frw, R = 10%, T = 2 years, S.I = ? Frw

$$\text{Simple Interest} = P \times \frac{R}{100} \times T$$

$$\text{Simple Interest} = 100{,}000 \times \frac{10}{100} \times 2$$

$$= 10{,}000 \times 1 \times 2$$

$$= 10{,}000 \times 2$$

$$= 20{,}000$$

Therefore, the simple interest is 20,000 Frw.

Example 2

Given; principal (P) = 1,600,000 Frw, Interest Rate (R) = 20% p.a and Time (T) = 4 years. Calculate the interest.

Solution

P = 1,600,000 Frw, R = 20%, T = 4 years, S.I = ? Frw

$$\text{Simple Interest} = P \times \frac{R}{100} \times T$$

$$\text{Simple Interest} = 1{,}600{,}000 \times \frac{20}{100} \times 4$$

$$= 160{,}000 \times 2 \times 4$$

$$= 160{,}000 \times 8$$

$$= 1{,}280{,}000$$

Therefore, the simple interest is 1,280,000 Frw.

Study tip

- The money borrowed, saved or lent is principal (P).
- The percentage used to calculate interest is Rate (R).
- Rate is the amount on every one hundred of the principal that is earned or paid back.
- The period in years that the principal is invested is Time (T).
- The additional amount offered or paid is Simple Interest (S.I).
- To find simple interest (S.I), use $S.I = P \times \frac{R}{100} \times T$.



Application 9.1

Calculate simple interest (S.I) given;

1. Principal = 400,000 Frw, Rate = 5% p.a, Time = 1 year.
2. Principal = 650,000 Frw, Rate = 12% p.a, Time = 3 years.
3. Principal = 800,000 Frw, Rate = 15% p.a, Time = 4 years.
4. Principal = 1,000,000 Frw, Rate = 18% p.a, Time = 2 years.
5. Principal = 1,200,000 Frw, Rate = 20% p.a, Time = 5 years.
6. Principal = 1,350,000 Frw, Rate = 24% p.a, Time = 3 years.

9.2 More about calculating simple interest

Activity

- Express the following as years:
  (a) 4 months (b) 6 months (c) 8months (d) 3months
- Change the following into fractions:
  (a) $2\frac{1}{2}\%$ (b) $12\frac{1}{2}\%$ (c) $6\frac{1}{4}\%$ (d) $37\frac{1}{2}\%$
- What are your answers?
- Share your working out with the class.

Example

Given; principal (P) = 1,680,000 Frw, Interest rate (R) = $12\frac{1}{2}\%$ p.a and Time (T) = 8 months. Find the simple interest.

Solution

P = 1,680,000 Frw, R = $12\frac{1}{2}\%$, T = 8 months, S.I = ? Frw

Change 8 months to years = $\frac{8}{12}$ of a year.

Change $12\frac{1}{2}\%$ into a fraction

$= \frac{12 \times 2 + 1}{100} \times \frac{1}{100} = \frac{25}{2} \times \frac{1}{100} = \frac{25}{200}$

$S.I = P \times \frac{R}{100} \times T$

$S.I = \cancel{1,680,000}^{1,400} \times \frac{\cancel{8}^{4}}{1\ \cancel{12}} \times \frac{25}{1\ \cancel{200}}$

$= 1,400 \times 4 \times 25$

$= 1,400 \times 100$

$= 140,000$ Frw

Therefore, the simple interest is 140,000 Frw

Study tip

- If rate is a fractional percentage, change it into a common fraction first.
- If time is in months, change it into years first.
- Remember 12 months make one year. Express the months given out of 12.



Application 9.2

Find the simple interest given;

1. Principal = 1,200,000 Frw, interest rate = $2\frac{1}{2}$% p.a and Time = 6 months.
2. Principal = 10,050,000 Frw, interest rate = $6\frac{1}{5}$% p.a and Time = $2\frac{1}{2}$ years.
3. Principal = 1,800,000 Frw, interest rate = $16\frac{1}{2}$% p.a and Time = 3 months.
4. Principal = 960,000 Frw, interest rate = $37\frac{1}{2}$% p.a and Time = 8 months.
5. Principal = 2,000,000 Frw, interest rate = $12\frac{1}{2}$% p.a and Time = 9 months.
6. Principal = 1,041,000 Frw, interest rate = $5\frac{1}{2}$% p.a and Time = 4 months.
7. Principal = 1,460,000 Frw, interest rate = $17\frac{1}{2}$% p.a and Time = $1\frac{1}{4}$ years.
8. Principal = 1,840,000 Frw, interest rate = $7\frac{1}{2}$% p.a and Time = 6 months.

9.3 Solving problems involving simple interest

Activity

- Suppose you deposit 6,500,000 Frw in a bank.
- The bank promises to give you interest rate of 10% per year in 3 years.
- What will be your simple interest after the 3 years?
- Explain how you get it and then present your working out to the class.

Study tip

- Read and interpret the words in the problem correctly.
- Identify the principal, rate and time.
- The money banked borrowed or lent is Principal **(P)**.
- The percentage used to calculate interest is Rate **(R)**.
- The period that the principal is invested is the Time **(T)**.

- The additional amount offered or paid back is the simple interest **(S.I).**
- Simple interest = $P \times \frac{R}{100} \times T$.

Example 1

A man deposited 45,000 Frw in a bank that offers an interest rate of $3\frac{1}{2}$% per year (per annum). How much interest will he get in 2 years?

Solution

Principal (P) = 45,000 Frw, Rate (R) = $3\frac{1}{2}$ %, Time (T) = 2 years, Simple interest = ?

$$S.I = P \times \frac{R}{100} \times T$$
$$= 45{,}000 \times \frac{7}{200} \times 2$$
$$= 450 \times 7 \times 1$$
$$= 3{,}150 \text{ Frw}$$

Example 2

Ndahiro wanted to buy a car at 1,000,000 Frw. He borrowed from the bank at a rate of 8% per annum. What interest did he pay after 6 months?

Solution

P = 1,000,000 Frw, R = 8%, T = 6 months = $\frac{6}{12}$, S.I = ?

$$S.I = P \times \frac{R}{100} \times T$$
$$= 1{,}000{,}000 \times \frac{8}{100} \times \frac{6}{12}$$
$$= 10{,}000 \times 4$$
$$= 40{,}000 \text{ Frw}$$



Application 9.3

Work out the following:

1. Karoli deposited 60,000 Frw in the bank. The interest rate was 10% per year. Calculate the simple interest after 5 months.
2. Mutuyimana borrowed 120,000 Frw from a bank at an interest rate of 20% per year. How much interest did she pay after 3 year?
3. Uwase banked 350,000 Frw. How much interest did she earn in 6 months if the rate was 10% per year?
4. Mugenzi borrowed 150,000 Frw at an interest rate of 30% per year. Calculate the interest he earned after 4 years.
5. Calculate the simple interest on 390,000 Frw at the rate of $7\frac{1}{2}$ % per year for 8 months.
6. Zaninka lent Uwera 400,000 Frw. She was to pay back the money after 4 years at an interest rate of 15% per year. How much did she pay?

9.4 Calculating interest rate

Activity

- The formula for simple interest is: SI = P x $\frac{R}{100}$ x T.
- Multiply the formula by 100 both side.
- Divide both sides by P and simplify.
- Divide both sides by T. Simplify, then write the achieved formula.
- What do you notice. Explain your working out steps to the class.

Example

Calculate the interest rate given Principal = 100,000 Frw, Time = 2 years, interest = 20,000 Frw.

Solution

Method 1

P = 100,000 Frw, T = 2 years, S.I = 20,000 Frw, R = ?

Rate = $\frac{\text{Simple interest x 100}}{\text{Principal x Time}}$

= $\frac{^{10}\cancel{20,000} \times 100}{100,000 \times \cancel{2}_{1}}$

= $\frac{10 \times 1}{1 \times 1}$

= 10 (rate is always a percentage)

Therefore, rate is 10%.

Method 2

P = 100,000 Frw, T = 2 years, S.I = 20,000 Frw, R = ?

S.I = P x $\frac{R}{100}$ x T

20,000 = $100,000 \times \frac{R}{100} \times 2$

20,000 = 2,000 x R

Divide by 20,000 both sides.

$\frac{^{10}\cancel{20,000}}{_{1}\cancel{2,000}} = \frac{_{1}\cancel{2,000}}{_{1}\cancel{2,000}}$

10 = R (rate is a percentage)

Therefore, the rate is 10%

Study tip

- Read and interpret the question correctly.
- Identify the principal, simple interest and time.
- To find rate, substitute for principal, simple interest and time.
- To find rate, solve for R in R = $\frac{\text{Simple interest x 100}}{\text{Principal x Time}}$.
- Rate is a percentage.



Application 9.4

Find the interest rate given:

1. Principal = 1,200,000 Frw, Time = 4 years and interest = 400,000 Frw.
2. Principal = 960,000 Frw, Time = 2 years and interest = 60,000 Frw.
3. Principal = 100,000 Frw, Time = 1 year and interest = 25,000 Frw.
4. Principal = 1,440,000 Frw, Time = $1\frac{1}{2}$ years and interest = 72,000 Frw.
5. Principal = 1,050,000 Frw, Time = 3 years and interest = 63,000 Frw.
6. Principal = 512,000 Frw, Time = $2\frac{1}{2}$ years and interest = 64,000 Frw.
7. Principal = 2,000,000 Frw, Time = 2 years and interest = 200,000 Frw.
8. Principal = 720,000 Frw, Time = 5 years and interest = 120,000 Frw.

9.5 Solving problems involving interest rate

Activity

- Your father has a loan of 800,000 Frw from a bank.
- In one year, he pays interest of 96,000 Frw.
- Help him to calculate the interest rate.
- Present your working out as you explain to him how to get it.

Example 1

Uwacu a P.6 leaner's mother deposited 8,500 Frw on her account. After 2 years, she earned an interest of 2,450 Frw. Find the interest rate she was offered.

Solution

P = 8,500 Frw, S.I = 2,450 Frw

T = 2 years R = ?

$$\text{Rate} = \frac{\text{Simple interest} \times 100}{\text{Principal} \times \text{Time}}$$

$$= \frac{\overset{245}{\cancel{2,450}} \times \cancel{100}}{\underset{17}{\cancel{8,500}} \times 2_{1}}$$

$$= \frac{245}{17} = 14\frac{7}{17}, \text{ or } 14.4\%$$

Therefore, rate is 14.4%.

Example 2

Zaninka deposited 60,000 Frw on her savings account. After 8 months, the interest gained was 4,000 Frw. Calculate the rate of interest.

Solution

P = 60,000 Frw, S.I = 4,000 Frw

T = 8 months R = ?

$$\text{Rate} = \frac{\text{Simple interest} \times 100}{\text{Principal} \times \text{Time}}$$

$$= \frac{\overset{5}{\cancel{4,000}} \times \overset{2}{\cancel{12}} \times \cancel{100}}{{}_{1}\cancel{60,000} \times \cancel{8}_{1}}$$

$$= 5 \times 2$$

Therefore, rate is 10%.

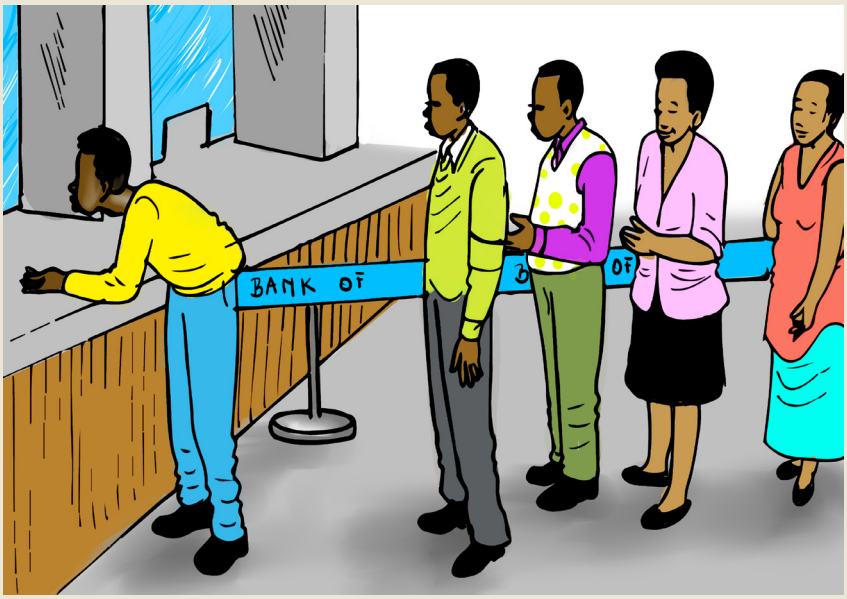
Study tip

- Rate = $\frac{\text{Simple interest x 100}}{\text{Principal x Time}}$.
- The percentage used to calculate interest is the rate.
- Rate is worked out by substituting simple interest, principal and time.

Application 9.5

Work out the following:

1. Akaliza banked 60,000 dollars and earned an interest of 15,000 dollars in 4 years. What was the rate of interest?



Attaining bank services.

2. Mugenzi borrowed 320,000 Frw from KCB for one year. He payed an interest of 32,000 Frw. Find the interest rate.
3. Uwacu banked 50,000 Frw in Equity Bank and earned an interest of 15,000 Frw after 3 years. What was the rate of interest?
4. Gitego borrowed 125,000 Frw for 2 years. He paid back 43,750 as interest. What was the interest rate?
5. A business woman requested for a loan of 3,600,000 Frw from Coge Bank that requires her to pay 12,000 Frw as interest after 2 months. What is the interest rate?

9.6 Calculating principal

Activity

On a slip of paper, write S.I formula

- Multiply both sides by 100.
- Divide both sides by R.
- Then divide both sides by T.
- Write the expression as an equation equated to P.

Now attempt this:

- Write 20,000 Frw on a slip of paper. Multiply it by 100.
- Divide the product by 10, then by 4.
- What is your result?
- Present your working out to the class.

Example

Find the principal given simple interest = 10,000 Frw, interest rate = 5% and time = 4 years.

Solution

Method 1

S.I = 10,000 Frw, R = 5% p.a,
T = 4 years, P = ? Frw.

$$\text{Principal} = \frac{\text{Simple interest} \times 100}{\text{Rate} \times \text{Time}}$$

$$= \frac{10{,}000 \times \overset{5}{\cancel{100}}^{\cancel{25}}}{\underset{1}{\cancel{5}} \times \cancel{4}^{1}}$$

$$= 10{,}000 \times 5$$

$$= 50{,}000$$

Therefore, principal is 50,000 Frw.

Method 2

S.I = 10,000 Frw, R = 5% p.a,
T = 4 years, P = ? Frw.

$$\text{S.I} = P \times \frac{R}{100} \times T$$

Substitute in the value of S.I, R and T.

$$10{,}000 = P \times \frac{\overset{1}{\cancel{15}}}{\underset{5}{\cancel{100}_{\cancel{25}}}} \times \cancel{4}$$

Multiply by 5 both sides.

$$5 \times 10{,}000 = \frac{P}{\underset{1}{\cancel{5}}} \times \cancel{5}^{1}$$

$$50{,}000 = P$$

Therefore, the principal 50,000 Frw.

Study tip

- Principal is the money saved, borrowed or lent.
- Principal = $\frac{\text{Simple interest} \times 100}{\text{Rate} \times \text{Time}}$.



Application 9.6

Find the principal given:

1. Simple interest is 2,000 Frw, rate is 2% p.a and time is 2 years.
2. Simple interest is 45,000 Frw, rate is 3% and time is 9 months.
3. Rate is 4% p.a, time is 3 years and simple interest is 60,000 Frw.
4. Time is 5 years, simple interest is 72,000 Frw and rate is 20% p.a.
5. Simple interest is 15,000 Frw, rate is 3% p.a. and time is 2 years.
6. Rate is 12.5% p.a., simple interest is 10,000 Frw and time is 3 months.
7. Time is 6 months, rate is 60% p.a and simple interest is 24,600.
8. Simple interest is 14,400 Frw, rate is 12% per year and time is 12 months.

9.7 Solving problems involving principal

Activity

- Alpha deposited some money in the bank. If at the rate of 4% in 2 years he earned a simple interest of 5,000 Frw, what is the money he banked?
- Explain how you get it.

Example 1

Kankera banked money in a bank that gives simple interest at rate of 8% per year. She earned interest of 480,000 Frw in 3 years. How much did she bank?

Solution

Rate = 8%, Simple Interest = 480,000 Frw, Time = 3 years, Principal = ?

$$\text{Principal} = \frac{\text{Simple interest} \times 100}{\text{Rate} \times \text{Time}}$$

$$= \frac{\overset{20{,}000}{\cancel{480{,}000}} \times 100}{1\ \cancel{8} \times \cancel{3}_{1}}$$

$$= 20{,}000 \times 100$$

$$= 2{,}000{,}000 \text{ Frw}$$

Therefore, principal is 2,000,000 Frw.

Example 2

An amount of money gained an interest of 14,400 Frw. It was invested for 9 months. If the simple interest rate is 6%, calculate principal.

Solution

R = 6%, S.I = 14,400 Frw, T = 9 months or $\frac{9}{12}$, P = ?

$$\text{Principal} = \frac{\text{Simple interest} \times 100}{\text{Rate} \times \text{Time}}$$

$$= \frac{\overset{1{,}600}{\cancel{14{,}400}} \times \overset{2}{\cancel{12}} \times 100}{1\ \cancel{6} \times \cancel{9}_{1}}$$

$$= 1{,}600 \times 2 \times 100$$

$$= 320{,}000 \text{ Frw}$$

Therefore, principle is 320,000 Frw.

Study tip

- Read and interpret the question correctly.
- Principal is the money saved, borrowed or lent.
- Principal = $\frac{\text{Simple interest x 100}}{\text{Rate x Time}}$.



Application 9.7

Work out the following:

1. Mary banked money at 5% simple interest rate and earned an interest of 5,000 Frw in 5 years. How much did she bank?
2. Kambabazi borrowed money that earned her 45,600 Frw as interest in 4 years at a simple interest rate of 12%. Calculate the amount she borrowed.
3. Mutangana banked money in a bank that gives an interest rate of 10% per year. 4 years later, he earned an interest of 65,000 Frw. What was the principal?
4. Twagirimana lent money and earned an interest of 64,000 Frw at a rate of 10% per year in 2 years. How much did he lend out?
5. Kalisa borrowed money. He wanted to boost his grocery. After 6 months, he paid 87,000 Frw at a rate of 12% per year. How much did he borrow?
6. Kanyange banked money in Equity Bank. She was offered a simple interest rate of 24% per year. She withdrew it after 6 months. Calculate the amount she banked if she was given an interest of 96,000 Frw.
7. Mukandahiro save money in a SACCO. She was offered a simple interest rate of 20% per year. How much did she save if she was given 160,000 Frw as interest 4 years later?

9.8 Calculating the time

Activity

- Get a slip of paper.
- Multiply simple interest by 100 both sides.
- Now divide the product by the product of P and R both sides.
- What expression is formed?
- Equate T to the expression to form an equation.
- Present your result to the class.

Example

Find time given, the simple interest is 12,000 Frw, principal is 144,000 Frw and rate is 20% par annum (p.a).

Solution

Method 1

S.I = 12,000 Frw, P = 144,000 Frw, R = 20%, T = ?.

$$\text{Time} = \frac{\text{Simple interest} \times 100}{\text{Principle} \times \text{Rate}}$$

$$= \frac{12{,}000 \times 100}{144{,}000 \times 20}$$

$$= \frac{1 \times 5}{12 \times 1}$$

$$= \frac{5}{12}$$

Therefore, time is $\frac{5}{12}$ of a year or 5 months.

Method 2

S.I = 12,000 Frw, P = 144,000 Frw, R = 20%, T = ?.

$$\text{S.I} = \text{P} \times \frac{\text{R}}{100} \times \text{T}$$

Substitute in the value of S.I, R and P.

$$12{,}000 = 144{,}000 \times \frac{20}{100} \times \text{T}$$

$$12{,}000 = 1440 \times 20 \times \text{T}$$

Divide 28,800 both sides.

$$\frac{12{,}000}{28{,}800} = \frac{28{,}800\,\text{T}}{28{,}800}$$

$$\frac{5}{12} = \text{T}$$

Therefore, time is $\frac{5}{12}$ of a year or 5 months.

Study tip

- Time is the period in years that principal is invested.
- Time is calculated by substituting simple interest, principal and time.
- Time $= \frac{\text{Simple interest} \times 100}{\text{Principal} \times \text{Rate}}$.



Application 9.8

Calculate time given:

1. Simple Interest is 20,000 Frw, principal is 200,000 Frw and interest rate is 5% per year.
2. Principal is 100,000 Frw, simple interest is 10,000 Frw and interest rate is 10% p.a.
3. Interest rate is 5% p.a, principal is 1,200,000 Frw and simple interest is 40,000 Frw.
4. Simple interest is 20,000 Frw, interest rate is 10%, principal is 400,000 Frw.
5. Interest rate is 25%, simple interest is 1,000,000 Frw and principal is 4,000,000 Frw.

6. Principal is 6,000,000 Frw, interest rate is 4% p.a. and simple interest is 240,000 Frw.
7. Simple interest is 144,000 Frw, principal is 720,000 Frw rate is 2% p.a.
8. Interest rate is 10% p.a. principal is 2,480,000 Frw and S.I is 124,000 Frw.

9.9 Solving problems involving time

Activity

How long do you take to earn 4,000 Frw as simple interest at the rate of 4% p.a. when you deposit 50,000 Frw in a bank?
Explain your working out to the class.

Example 1

Eva deposited 15,000 Frw in a bank that offers a simple interest rate of 3% per year. If she got a simple interest of 1,800 Frw, how long had she banked the money?

Solution

Principle = 15,000 Frw, Rate = 3%

Interest = 1,800 Frw, Time = ?

$$\text{Time} = \frac{\text{Simple interest} \times 100}{\text{Principal} \times \text{Rate}}$$

$$= \frac{^{90}\cancel{1,800} \times \cancel{100}}{_{1}\cancel{15,000} \times \cancel{20}_{1}}$$

$$= \frac{6 \times 1}{1 \times 1}$$

$$= 6$$

Therefore, the time was 6 years.

Example 2

Kamanzi borrowed 1,200,000 Frw from Bank. The bank offered 6% per year. He paid interest of 240,000 Frw. How long did she use the money?

Solution

P = 12,000,000 Frw, R = 6%, S.I = 240,000 Frw, T = ?

$$\text{Time} = \frac{\text{Simple interest} \times 100}{\text{Principal} \times \text{Rate}}$$

$$= \frac{^{40}\cancel{240,000} \times \cancel{100}}{1,\cancel{200,000} \times \cancel{6}_{1}}$$

$$= \frac{^{10}\cancel{40} \times 1}{_{3}\cancel{12} \times 1}$$

$$= 3\frac{1}{3}$$

Therefore, the time was $3\frac{1}{3}$ years.

Study tip

- $\text{Time} = \frac{\text{Simple Interest} \times 100}{\text{Principal} \times \text{Rate}}$.
- Time is always expressed in years.



Application 9.9

Work out the following:

1. Kayitesi deposited 18,000 Frw in a bank that gives an interest rate of 10% per year. How long did it take her to get 1,800 Frw as simple interest?
2. Mr Ntwari borrowed 50,000 Frw from a bank that gives an interest rate of 10% p.a. How long did she use the money if she earned an interest of 25,000 Frw?
3. Nsenga got an interest of 70,000 Frw at a rate of 20% per year. How long did it take him if he deposited 350,000 Frw?
4. Munyandekwe lent Mukandori 240,000 Frw at an interest rate of 5% per year. After how long will he get interest of 30,000 Frw?
5. How long will 720,000 Frw take to amount to 960,000 Frw at a rate of 5%?
6. Muhakanizi deposited 1,000,000 Frw in KCB at an interest rate of 12% p.a. He earned 240,000 Frw as interest. How long was the money in the bank?

9.10 Calculating the amount

Activity

- Take a slip of paper.
- Write principle, rate and time values of your choice.
- Now find the sum of interest and principal.
- What is your answer?
- Explain your working out to your classmates.

Example

Calculate the amount given: principal 1,200,000 Frw, interest rate $12\frac{1}{2}$% p.a, and time is 6 months.

Solution

P = 200,000 Frw, R = $12\frac{1}{2}$% p.a., T = $\frac{5}{12}$ of a year, S.I = ?, A = ?

$$S.I = P \times \frac{R}{100} \times T$$

$$= 1{,}200{,}000 \times 12\frac{1}{2} \times \frac{6}{12}$$

$$= \cancel{1{,}200{,}000}^{3{,}000\ \cancel{6{,}000}} \times \frac{25}{\cancel{200}_{1}} \times \frac{\cancel{6}^{1}}{\cancel{12}_{2\ 1}}$$

$$= 3{,}000 \times 25 \times 1$$

$$= 75{,}000$$

Therefore, the simple interest is 75,000 Frw.

Add principal and interest

Amount = Principal + Interest

= 1,200,000 + 75,000 Frw

= 1,275,000 Frw

Therefore, the amount is 1,275,000 Frw.

Study tip

- Amount is the sum of principal and interest.
- To find amount, first calculate interest, then add it to principle.



Application 9.10

Find the amount given:

1. Principal is 1,000,000 Frw, rate is 20% p.a, time is 2 years.
2. Rate is 12% p.a, time is 4 years, principal is 1,400,000 Frw.
3. Time is 3 years, rate is 25% p.a, principal is 2,840,000 Frw.
4. Principal is 380,000 Frw, time is 1 year, rate is 15% p.a.
5. Rate is 18% p.a, time is 4 years, principal is 1,020,000 Frw.
6. Time is 8 months, principal is 2,250,000 Frw, interest rate is $12\frac{1}{2}$% p.a.
7. Principal is 1,962,000 Frw, time is 4 months, interest rate is $12\frac{1}{2}$% p.a.
8. Rate is $7\frac{1}{2}$% p.a, time is 4 months, principal is 1,248,000 Frw.
9. Time is 9 months, interest rate is 20% p.a, principal is 3,200,000 Frw.
10. Principal IS 960,000, time is $1\frac{1}{2}$ years, rate is 12% p.a.
11. Rate is $7\frac{1}{2}$%, time is 3 years, principal is 480,000 Frw.
12. Time is 5 months, rate is 15%, principal is 1,440,000 Frw.

9.11 Solving problems involving amount

Activity

- Suppose you are a manager of certain Bank.
- Role-play a situation of a person who came to apply for a loan in your Bank.
- Show how the loan is processed and then identify what the manager does in order to help the bank and the client to achieve their business objectives.
- Explain your point of view.
- Present your work.

Example

A school deposited 1,800,000 Frw on a fixed account in a Bank. The money took 3 months at an interest rate of 20% per year. How much money was on the account after 3 months?

Solution

P = 1,800,000 Frw, T = 3 months or $\frac{3}{12}$, rate = 20% p.a, S.I = ?, A = ?

$$S.I = P \times \frac{R}{100} \times T$$

$$= 1{,}800{,}000 \times \frac{20}{100} \times \frac{3}{12}$$

$$= \overset{15{,}000}{\cancel{1{,}800{,}000}} \times \frac{20}{\cancel{100}} \times \frac{3}{\overset{1}{\cancel{12}}}$$

$$= 15{,}000 \times 2 \times 3$$

$$= 90{,}000 \text{ Frw}$$

Simple interest is 90,000 Frw.

Add principal and interest

Amount = Principal + Interest

= 1,800,000 Frw + 90,000 Frw

= 1,890,000 Frw

Therefore, the amount is 1,890,000 Frw.

Study tip

- Amount is the total sum of money in the bank after the principle and the interest earned over a given period.
- Amount = Principle + interest earned.



Application 9.11

1. Uwacu borrowed 840,000 Frw from Equity Bank for one year. The interest rate is 18% per year. Calculate the amount she paid back.
2. A trader deposited 2,400,000 Frw in a bank. It offers an interest rate of 15% per year. What amount was on her account 6 months later?
3. Kibuye Dairy Cooperative society lent 1,500,000 Frw to each of its members for 2 years. They paid an interest of 5% per year. How much did every member pay back?
4. Katto borrowed 2,000,000 Frw from Jasmine for 8 months. He paid an interest of 12% per year. How much did he pay altogether?
5. Akim deposited 1,440,000 Frw in Coge Bank. He paid an interest rate of $12\frac{1}{2}$% per year for 6 months. Calculate the amount he paid back.

6. Kamana deposited 288,000 Frw from UMURENGE SACCO. He was offered an interest rate of 8% per year for 4 months. Find out how much he paid the bank altogether.
7. The cost of a car is 4,000,000 Frw. Ndahiro had 3,000,000 Frw and borrowed the rest from a bank. He was to pay in 2 years at an interest rate of 20%.
   (a) How much interest did he pay?
   (b) Calculate the amount the car cost him altogether.
8. Mukandahiro deposited 6,500,000 Frw on her fixed account in a bank. Her account balance accumulated for 4 years. Calculate the amount she had if the interest rate was $12\frac{1}{2}$% per year.

9.12 Saving money in the bank or putting it in investments

Activity

- You are aware that successful people, institutions and businesspeople save money in the banks or in investments.
- Suppose you are given 20,000 Frw, write down different ways of saving from it.
- Discuss and give an example of the following:
  (i) the advantage of saving money in banks.
  (ii) the advantages of putting money in investments.
- Present your working out to the class.

Example 1

Suppose you are given 10,000 Frw. Make a budget and show how you can save from it.

Solution

Budget: one dozen of books for 3,000 Frw, one graph book for 500 Frw, one handkerchief for 500 Frw, six pens 1,000 Frw, one Maths set 2,000 Frw

Expenditure = (3,000 + 500 + 500 + 1,000 + 2,000) Frw = 7,000 Frw

Expenditure = 7,000 Frw

Saving = Owned money – Expenditure

= (10,000 – 7,000) Frw

= 3,000 Frw

Example 2

A man invested 1,500,000 Frw for 2 years. He earned an interest of 10% p.a. How much did he find on his account? State the role of investment.

Solution

Simple Interest $= P \times \frac{R}{100} \times T$

$= 1{,}500{,}000 \times \frac{10}{100} \times 2$

$= 1{,}500{,}0\cancel{00} \times \frac{10}{1\cancel{00}} \times 2$

$= 15{,}000 \times 10 \times 2$

$= 300{,}000$ Frw

Amount = principal + simple interest

$= (15{,}000 + 300{,}000)$ Frw

$= 1{,}800{,}000$ Frw

Investment helps people to earn much money and increase their fortune.

Study tip

- Savings is the amount of money kept after spending some of it on basic needs. Savings is calculated by subtracting expenditure from the money owned.
- Savings = Income - Expenditure.
- Banks or investments give additional money to the saver or investor. It is because they use the saved money to earn more money.



Application 9.12

Work out the following:

1. Given 15,000 Frw, show how you can save from it.
2. A man saved 10,500 Frw in a bank. He was offered 3,500 Frw as monthly interest. How much had he after 1 year?
3. Uwera invested money in a business that returns 9% annual interest rate. as interest at the end of a year.
   (a) How much interest did she earn every month?
   (b) If she invested 10,000,000, how much did she have on her account after 8 months?

4. Find the total amount after earning 6,000 Frw monthly for $1\frac{1}{2}$ years if Zaninka invested 1,500,000 in a company.
5. Ashimwe invested 14,000,000 Frw in a company. It was offering 5% interest rate per year on each share. Each share was 100,000 Frw.
   (a) How much interest did he earn in $2\frac{1}{2}$ years?
   (b) Find the amount of money on his account after 4 years.
6. Amani bought shares of 50,000 Frw each. He invested 4,000,000 Frw in a company. Each share realised 5,000 Frw monthly.
   (a) How much interest did he earn after 1 year?
   (b) Calculate the amount he had on his account altogether.

9.13 Solving problems involving savings

Activity

- John wants to get an interest of 3,000 Frw in one year. How much must he invest for that period at 8%?
- Explain your working step.

Example 1

Gabriel saved money in a bank. He was offered 1,500 Frw monthly as interest. If he deposited 1,350,000 Frw. How much did he have on his account after 2 years?

Solution

1 year = 12 months

2 years = 2 x 12 = 24 months

In 1 month he got interest of 1,500 Frw

In 24 months he got interest of (1,500 x 12) = 180,000 Frw

Amount = Principal + Interest

= 1,350,000 + 180,000

= 1,530,000 Frw.

Gabriel had 1,530,000 Frw after two years.

Example 2

Kayitesi has been paid 4,000 Frw by one of her business partners. She wants to save this money.

(a) Advise her on the different methods of saving.

(b) Portray the importance of saving to Kayitesi.

Solution

(a)
- She can deposit the money in a bank to earn interest.
- She can invest the money by buying shares in different investments such as companies, co-operative societies, and others.
- If it is the only amount she has, she should spend some of it on needs, but not all. She must save some in the bank or SACCO.

(b)
- Kayitesi earns more money in form of interest.
- She will improve her standard of living by earning more money in future.
- She will always be having money available to sustain her needs.

Study tip

- Savings is the amount of money kept after spending some of it on basic needs. Savings is calculated by subtracting expenditure from the money owned.
- Savings = Income - Expenditure.
- Banks or investments give additional money to the saver or investor. It is because they use the saved money to earn more money.



Application 9.13

Work out the following:

1. Gasana invested 3,500 US dollars in a trading company offering an interest of 5 US dollars weekly.
   (a) How much interest will he have in 3 years?
   (b) What will be the account balance?
2. Mutesi earns 200,000 Frw monthly. She spends 172,000 Frw and saves the rest. Calculate the amount that she realises after $1\frac{1}{2}$ years.

3. Joseph deposited 100,000 Frw on a fixed saving account that earns 3% simple interest monthly.
   (a) Find the interest earned after 5 years.
   (b) Calculate the total amount on his account then.
4. By saving 40,000 Frw every week, a teacher was offered 10,000 Frw every month as interest. How much will be on her account 2 months later?
5. Dusabe invested 2,000,000 Frw in a company. He bought shares. He was offered 10,000 Frw as interest every month. How much will he earn in 5 months?
6. Uwera saves 64,000 Frw in the bank every month. She is offered 10,000 Frw as interest every month. How much account balance does she have in 10 months?
7. 2,800,000 Frw is invested in a roofings company by a construction company monthly. An interest of 45,000 Frw is gained monthly.
   (a) How much will the construction company realise as interest in 5 months?
   (b) Find the total amount on its account.
8. Mastula saved 1,050,000 Frw in a bank. She was offered 35,000 Frw as interest monthly. How much had she in $1\frac{1}{2}$ years?

End of unit 9 assessment

1. Find simple interest earned on 150,000 Frw at an interest rate of 30% per year.
2. Mushumba earned an interest of 4,500 Frw for 2 year at 5% interest rate per year. What was the principal?
3. How long will 700,000 Frw take to gain an interest of 140,000 Frw at an interest rate of 5%?
4. Calculate the simple interest on 25,000 Frw for 2 years at an interest rate of 10% per year.
5. How long will Gasana earn 24,000 Frw if she deposited 120,000 Frw in a bank which offer her an interest rate of 10% per year?
6. Find principal that will earn 10,000 Frw for Gihozo. The interest rate offered is 5% for 6 months.

7. Nkundimana invested 8,406,500 Frw. He realised a total amount of 9,026,300 Frw. How much interest did he gain?
8. Dusabe borrowed 1,200,000 Frw from a bank and invested it for 8 months. The interest rate offered was $6\frac{1}{2}$% per year. How much did he pay altogether?
9. 152,400 Frw was gained from 286,000 Frw by Moses. He had kept his money in a SACCO for 8 months. Calculate the interest rate.
10. Show how you would spend 25,000 Frw and remain with savings.
11. Umurisa invested money in a SACCO for a period of time. She had 9,600,000 Frw on her account. She was given 1,052,000 Frw as interest. How much did she invest?
12. 7,200,000 Frw was invested by a co-operative society. It was offered an interest rate of $12\frac{1}{9}$ % per year. The time for investing the money was 9 months. Calculate the total amount on the account.
13. Kamanzi deposited 1,280,000 Frw in a bank for 8 months. He realised an interest of 64,000 Frw. Calculate the interest rate the bank offered him.
14. Lucumu invested 8,400,000 Frw in a compnay. He was offered interest of 20,000 Frw.
    (a) How much interest had he after 2 years?
    (b) How much money did he have on the account after $3\frac{1}{4}$ years?
15. Irebe borrowed 480,000 from a SACCO for $1\frac{1}{2}$ years. The interest rate offered was $12\frac{1}{2}$ per year.
    (a) How much interest did she pay back?
    (b) What amount did she pay the SACCO?

Unit 10 Equivalent expressions and number sequences

Key unit competence: To be able to write sequences of whole numbers, fractions and decimals.

Introduction

Pour 3 litres of water in a small jerrycan and mark the point. Remove the water and then get a soda bottle of 300 ml. Put the water in a bottle at a time as you pour in the same jerrycan until you fill it to the same level. What do you observe?

(a) Find out the number of soda bottles to fill the jerrycan at the same level as for 3 litre bottles.

(b) After your experience, are 3 litters of water equivalent to 10 times the water in the bottle of soda containing 300 ml? Justify your answer.

10.1 Algebraic expressions

Activity

Write the following algebraic expressions.

1. Twice a number added to 4.
2. The product of 2 and a number minus 3.
3. The difference between a number and 5.
4. A number multiplied by 3 take away 1.

Example 1

Write this algebraic expression.

Add 7 to m, then subtract 3.

Solution

Add 7 to m then subtract 3.

= (m + 7) - 3

Example 2

Write this algebraic expression.

Twice x add thrice y.

Solution

Twice x add thrice y.

= 2x + 3y

Example 3

Write this algebraic expression: The difference between a and b divided by 4.

Solution

The difference between a and b divided by 4.

$= \frac{a - b}{4}$

Example 4

Write this algebraic expression.

The product of 2 and y added to the product of 3 and z.

Solution

It becomes 2y + 3z

Study tip

- Correctly interpret the operation terms used.
- The terms which relate to (+) are; add, sum, plus, more and increase.
- The terms which relate to (-) are; subtract, reduce, difference, take away, decrease, and less.
- The terms which relate to (x) are; multiply, product and times.
- The terms which relate to (÷) are; divide, share, average and distribute.



Application 10.1

Write the following algebraic expressions for these:

1. 3 times the difference between p and q.
2. Divide thrice the product of m and n by 7.
3. Divide the sum of y and 6 by 5.
4. Twice x take away 6, then multiply the result by 3.
5. Half of the product of x and y.
6. The sum of p and q multiplied by the product of r and s.

10.2 Equivalent expressions

Activity

- Collect 6 pens and 4 rubbers.
- Form 2 groups of each item.
- Write an algebraic expression relating the collected items to the grouped ones.

Example 1	Example 2
(3a + 6) = 3(a + 2)	(y - 4) = 2(y + 2)
Solution	**Solution**
Open brackets	Open brackets
3a + 6 = 3 x a + 3 x 2	y - 4 = 2y + 4
3a + 6 = 3a + 6	so y - 4 is not equivalent to 2y + 4
Therefore, 3a + 6 is equivalent to 3a + 6.	y - 4 ≠ 2y + 4

Study tip

- To find equivalent expressions, open brackets then simplify.
- The expression on the left has to be equivalent to the expression on the right.



Application 10.2

Find the equivalent expressions.

(a) 4a + 2b = 2(2a + b)

(b) 5(x - 5) = 5x - 5

(c) (3y + 4)3 = 9y + 12

(d) 6q - 4 = $(3q - 2)^2$

(e) 11(m - 2n) = 22m - 22

(f) 9(2x + 1) = 3(6x + 3)

(g) (y + 3) + 2(y - 1) = 3y + 2

(h) 13(9 - 2) + a - 3 = 14a - 29

(i) 4(x - 1) - x + 7 = 3(x + 1)

(j) 2(y - 4) + y - 2 = 3(y - 3) - 1

(k) 3(m + 5) = 5(m + 3)

(l) 5(x - 2) + 3 = 3 + (5x - 6) - 4

10.3 Finding the missing consecutive numbers

Activity

- Collect 100 straws.
- Distribute them to 5 learners in the order 1st to 5th.
- Give the 1st two straws. Five straws to the 2nd and 8 straws to the 3rd learner.
- Write the order of the given out straws on paper slips.
- How many straws should be given to the 4th and 5th learners?
- What is the order of increase? Explain your working out to the class.

Example

Find the missing numbers.

3, 7, 11, ..., ..., ..., ...

Solution

First find the order of increase.

This is done by finding the common difference.

So, (7 - 3) = 4, (11 - 7) = 4

The order of increase is adding 4 to the next number.

The 4^{th} number is (11 + 4) = 15

5^{th} number is; (15 + 4) = 19

6^{th} number is (19 + 4) = 23

7^{th} number is (23 + 4) = 27

So the linear sequence is; 3, 7, 11, 15, 19, 23, 27.

Study tip

- To get the next number in a linear sequence, work out the common difference.
- Then continue adding the common difference to find the missing numbers.



Application 10.3

Find the missing numbers in the sequences below;

(a) 3, 8, 13, 18, ___, ___, ___, ___,.

(b) 1, 4, 7, 10, ___, ___, ___, ___,.

(c) 2, 5, 8, 11, ___, ___, ___, ___,.

(d) 5, 9, 13, 17, ___, ___, ___, ___,.

(e) 1, 7, 13, 19, ___, ___, ___, ___,.

(f) 3, 14, 25, 36, ___, ___, ___, ___,.

(f) 4, 10, 16, 22, ___, ___, ___, ___.

(f) 9, 22, 35, 48, ___, ___, ___, ___,.

10.4 Finding the missing consecutive fractions and decimals

Activity

- Write $\frac{1}{3}$ on a piece of paper.
- Add $\frac{1}{3}$ to the first fraction and write the sum.
- Continue adding $\frac{1}{3}$ to find other six consecutive fractions.
- Form a linear sequence for the fractions worked out.
- Explain your working out to the class.

Example 1

Find the missing fractions:

$1, 1\frac{1}{2}, 2, 2\frac{1}{2}, ___, ___, ___, ___.$

Solution

First find the order of increase.

$\left(1\frac{1}{2} - 1 = \frac{1}{2}\right), \left(2 - \frac{1}{2} = 1\frac{1}{2}\right), \left(2\frac{1}{2} - 2 = \frac{1}{2}\right)$

The order of increase is adding $\frac{1}{2}$ to the next fraction.

The 4^{th} fraction = $\left(2\frac{1}{2} + \frac{1}{2}\right) = 3$

The 5^{th} fraction = $\left(3 + \frac{1}{2}\right) = 3\frac{1}{2}$

The 6^{th} fraction = $\left(3\frac{1}{2} + \frac{1}{2}\right) = 4$

The 7^{th} fraction = $\left(4 + \frac{1}{2}\right) = 4\frac{1}{2}$

Therefore, the linear sequence is $1, 1\frac{1}{2}, 2, 2\frac{1}{2}, 3, 3\frac{1}{2}, 4, 4\frac{1}{2}$.

Example 2

Find the missing decimal.

0.2, 0.5, 0.8, 1.1, ___, ___, ___, ___.

Solution

First find the order of increase.

(0.5 - 0.2 = 0.3), (0.8 - 0.5 = 0.3),

(0.11 - 0.8 = 0.3)

The order of increase is adding 0.3.

The 5th decimal = (1.1 + 0.3) = 1.4

The 6th decimal = (1.4 + 0.3) = 1.7

The 7th decimal = (1.7 + 0.3) = 2.0

The 8th decimal = (2.0 + 0.3) = 2.3

Therefore, the linear sequence is:

0.2, 0.5, 0.8, 1.1, 1.4, 1.7, 2.0, 2.3.

Study tip

- To get the next fraction or decimal in a linear sequence, work out the common difference.
- Then continue adding the common difference to find the missing fraction or decimal.

Application 10.4

Find the missing fraction or decimal in the sequences below;

(a) $1, 2\frac{1}{2}, 4, 5\frac{1}{4}$, ___, ___, ___.

(b) $2, 3\frac{1}{4}, 4\frac{1}{2}, 5\frac{3}{4}$ ___, ___, ___.

(c) $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}$, ___, ___, ___.

(d) $\frac{2}{3}, 1\frac{1}{3}, 2, 2\frac{2}{3}, 3\frac{1}{3}$, ___, ___, ___

(e) $\frac{3}{4}, 1\frac{1}{12}, 1\frac{5}{12}, 1\frac{3}{4}, 2\frac{1}{12}$, ___, ___, ___.

(f) $1, 1\frac{4}{5}, 2\frac{3}{5}, 3\frac{2}{5}$, ___, ___, ___.

(g) 0.1, 0.5, 0.9, 1.3, ___, ___, ___.

(h) 0.2, 0.7, 1.2, 1.7, ___, ___, ___

(i) 0.10, 0.13, 0.16, 0.19, ___, ___, ___.

(j) 2.03, 2.08, 2.13, 2.18, ___, ___, ___

(k) 0.01, 0.11, 0.21, 0.31, ___, ___, ___.

(l) 0.20, 0.35, 0.50, 0.65, ___, ___, ___

10.5 Finding the general term/rule of a linear sequence

Activity

- Write the linear sequence; 2, 5, 8, 11, ___, ___, ___, ___.
- What is the order of increase, that is term to term?
- Order the terms as 1st, 2nd, 3rd, 4th, etc.
- Multiply each position by the increasing term.
- What number did you combine with the above to satisfy the value of the number in that position?
- What do you realise?
- Make a presentation of your working out in class.

Example

Find the general rule for the linear sequence below:

7, 13, 19, 25, 31, ___, ___, ___, ___.

Solution

First get the common difference.

(13 – 7 = 6), (19 – 13 = 6), (25 – 19 = 6), (31 – 25 = 6)

Let the next term be n.

Order of term	1st	2nd	3rd	4th	5th	nth
Term	7	13	19	25	31	n
	(1 x 6) + 1	(2 x 6) + 1	(3 x 6) + 1	(4 x 6) + 1	(5 x 6) + 1	(n x 6) +1

(1 x 6) + ? = 7, (2 x 6) + ? = 13, (3 x 6) + ? = 19, (4 x 6) + ? = 25, (5 x 6) + ? = 31, (n x 6) + ?

The number added in the order is 1.

So, (n x 6) + 1 = 6n + 1.

Therefore, the general rule is 6n + 1.

Study tip

To find the general rule for a sequence, multiply the positive (n) by the common difference, then add or subtract a constant.



Application 10.5

Find the general term/rule for the linear sequences below.

(a) 5, 9, 13, 17, ____, ____, ____
(b) 2, 4, 6, 8, ____, ____, ____
(c) 3, 11, 19, 27, 35,____, ____, ____
(d) 7, 12, 17, 22, 27, ____, ____, ____
(e) 6, 10, 14, 18, ____, ____, ____
(f) 3, 6, 9, 12, ____, ____, ____
(g) 1, 4, 7, 10, ____, ____, ____
(h) 3, 8, 13, 18, ____, ____, ____
(i) 2, 8, 14, 20, ____, ____, ____
(j) 1, 3, 5, 7, ____, ____, ____
(k) 2, 5, 8, 11, ____, ____, ____
(l) 1, 5, 9, 13, ____, ____, ____

10.6 Finding the general term/rule of linear sequence for fractions and decimals

Activity

- Write 1, $1\frac{1}{2}$, 2, $2\frac{1}{2}$, 3, ____, ____, ____, on slips of paper.
- Work out the order of increase.
- Find the missing terms.
- Applying the concept of finding the general term, find the same for the linear sequence you listed.

Example 1

Find the general rule for the linear sequence below:

$\frac{3}{4}$, $1\frac{1}{2}$, $2\frac{1}{4}$, 3, ___, ___, ___, ___.

Show all the necessary working.

Solution

Let the next term be n.

Order of term	1st	2nd	3rd	4th	nth
Term	$\frac{3}{4}$	$1\frac{1}{2}$	$2\frac{1}{4}$	3	n
	$1 \times \frac{3}{4}$	$2 \times \frac{3}{4}$	$3 \times \frac{3}{4}$	$4 \times \frac{3}{4}$	$n \times \frac{3}{4}$

Multiply the order of term by the common difference. Then find the constant to be added or subtracted.

$\left(1 \times \frac{3}{4} = \frac{3}{4}\right)$, $\left(2 \times \frac{3}{4} = 1\frac{1}{2}\right)$, $\left(3 \times \frac{3}{4} = \frac{9}{4} = 2\frac{1}{4}\right)$, $\left(4 \times \frac{3}{4} = 3\right)$, $\left(n \times \frac{3}{4} = \frac{3}{4}n\right)$

Therefore, the general rule is: $\frac{3}{4}n$. Values of n are 1, 2, 3, etc.

Example 2

Find the general rule for the linear sequence below:

0.1. 0.4, 0.7, 1.0, ___, ___, ___, ___.

Solution

Let the next term be n.

Order of term	1st	2nd	3rd	4th	nth
Term	0.1	0.4	0.7	1.0	n
	1 x 0.3 - ?	2 x 0.3 - ?	3 x 0.3 - ?	4 x 0.3 - ?	n x 0.3 - ?
	1 x 0.3 - 0.2 = 0.1	2 x 0.3 - 0.2 = 0.4	3 x 0.3 - 0.2 = 0.7	4 x 0.3 - 1.0 = 1.0	4 x 0.3 - 0.2 = 0.3n - 0.2

- Multiply the order of term by the common difference. Then find the constant to be subtracted,
- The common difference is 0.3, the constant to be subtracted is 0.2.
- Therefore, the general rule is 0.3n - 0.2.

Study tip

- First find the common difference, then multiply it by the order of term.
- If the product is not equal to the term, add or subtract a constant.

Application 10.6

Find the general term/rule for the linear sequences below.

(a) $1\frac{1}{5}, 1\frac{2}{5}, 1\frac{3}{5}, 1\frac{4}{5}$, ___, ___, ___.

(b) $2\frac{2}{3}, 3\frac{1}{3}, 4, 4\frac{2}{3}$ ___, ___, ___.

(c) $3\frac{1}{4}, 3\frac{1}{2}, 3\frac{3}{4}, 4$, ___, ___, ___.

(d) $\frac{2}{3}, 1\frac{1}{3}, 2, 2\frac{2}{3}$ ___, ___, ___.

(e) $\frac{5}{6}, 1\frac{2}{3}, 2\frac{1}{2}, 3\frac{1}{3}$, ___, ___, ___.

(f) $5\frac{2}{7}, 5\frac{4}{7}, 5\frac{6}{7}, 6\frac{1}{7}$, ___, ___, ___.

(g) 0.2, 0.7, 1.2, 1.7 ___, ___, ___.

(h) 2.03, 2.08, 2.13, 2.18, ___, ___, ___

(i) 0.1, 0.5, 0.9, 1.3, ___, ___, ___.

(j) 0.20, 0.35, 0.50, ___, ___, ___.

10.7 Finding the missing number or n^{th} term in a linear sequence

Activity

- Find the general rule for the linear sequence: 1, 3, 5, 7, ___, ___, ___
- Now, taking n to be the order of term, calculate to find the 12^{th} term.
- Explain to the class.

Example

Find the 20^{th} term in the sequence: 2, 4, 6, 8, 10, 12, ___, ___, ___

Solution

Find the common difference:

(4 – 2) = 2, (6 – 4) = 2, (10 – 8) = 2, (12 – 10) = 2

Order the terms:

1^{st},	2^{nd},	3^{rd},	4^{th},	5^{th},	6^{th},	n^{th}
2	4	6	8	10	12	n

Find the general rule:

(1 x 2) = 2, (2 x 2) = 4, (3 x 2) = 6, (4 x 2) = 8, (5 x 2) = 10, (6 x 2) = 12, (n x 2) = $2n$

The general term is 2n, n is the term.

The 20^{th} term is; 2 x 20 = 40.

The 20^{th} term is 40.

Study tip

- To find the n^{th} term in a given sequence, first find the general rule.
- Multiply the position by the common difference and decide on how to get each term including the n^{th} term.
- Use the general rule to find the n^{th} term, by substitution.



Application 10.7

Find the missing number

(a) Find the 15th term in the sequence: 5, 8, 11, 14, 17, ___, ___, ___, ___.

(b) Find the 19th term in the sequence: 1, 3, 5, 7, 9, ___, ___, ___, ___.

(c) Find the 20th term in the sequence: 2, 7, 12, 17, 22, ___, ___, ___, ___.

(d) What is the 30th term in the sequence: 3, 7, 11, 15, 19, ___, ___, ___, ___.

(e) What is the 26th term in the sequence: 2, 4, 6, 8, ___, ___, ___, ___.

(f) Find the 40th term in the sequence: 2, 9, 16, 23, 30, ___, ___, ___, ___.

(g) What is the 99th term in the sequence: 4, 7, 10, 13, 16, ___, ___, ___, ___.

(h) Find the 100th term in the sequence: 7, 13, 19, 25, ___, ___, ___, ___.

10.8 Finding the missing fraction or nth term in a linear sequence

Activity

- Write the sequence: $\frac{1}{4}, \frac{3}{4}, 1\frac{1}{4}, 1\frac{3}{4}$, ___, ___, ___, ___.
- Find the common difference.
- Write out the general rule for the fractional sequence.
- Try to find the 10th and 15th terms using the general rule, but not by listing.
- What do you observe?
- Explain your procedure to the class.

Example

Find the 12th term in the sequence: $2\frac{3}{5}, 3\frac{1}{5}, 3\frac{4}{5}, 4\frac{2}{5}$, ___, ___, ___, ___.

Solution

Find the order of the increase.

$\left(3\frac{1}{5} - 2\frac{3}{5} = \frac{16}{5} - \frac{13}{5} = \frac{3}{5}\right)$, $\left(3\frac{4}{5} - 3\frac{1}{5} = \frac{19}{5} - \frac{16}{5} = \frac{3}{5}\right)$, $\left(4\frac{2}{5} - 3\frac{4}{5} = \frac{22}{5} - \frac{19}{5} = \frac{3}{5}\right)$

Order the terms:

1st,	2nd,	3rd,	4th,	nth
$2\frac{3}{5}$	$3\frac{1}{5}$	$3\frac{4}{5}$	$4\frac{2}{5}$	n

Find the general rule:

$\left(1 \times \frac{3}{5} + ? = 2\frac{3}{5}\right)$, $\left(2 \times \frac{3}{5} + ? = 3\frac{1}{5}\right)$, $\left(3 \times \frac{3}{5} + ? = 3\frac{4}{5}\right)$,

$\left(4 \times \frac{3}{5} + ? = 4\frac{2}{3}\right)$, $\left(n \times \frac{3}{5} + ?\right)$

The constant added is 2.

The general rule is: $\left(n \times \frac{3}{5} + 2\right) = \frac{3}{5}n + 2$. Values of n are 1, 2, 3, etc.

The 12th term $= \frac{3}{5}n + 2 = \left(\frac{3}{5} \times 12\right) + 2 = \frac{36}{5} + 2$

$= 7\frac{1}{5} + 2 = 9\frac{1}{5}$

The 12th term is $9\frac{1}{5}$.

Study tip

- Find the general rule for the linear sequence.
- Substitute the value of n^{th} term for n, then solve the equation to get the n^{th} term.



Application 10.8

Find the missing term below:

(a) Find the 7th term in the sequence: $\frac{5}{6}, 1\frac{1}{3}, 1\frac{5}{6}, 2\frac{1}{3}$, ____, ____, ____, ____.

(b) Find the 11th term in the sequence: $\frac{2}{3}, 1, 1\frac{2}{3}, 2\frac{1}{3}$, ____, ____, ____, ____.

(c) Find the 14th term in the sequence: $1\frac{1}{3}, 2, 2\frac{2}{3}, 3\frac{1}{3}$, ____, ____, ____, ____.

(d) What is the 20th term in the sequence: $\frac{3}{4}, 1\frac{3}{4}, 2\frac{3}{4}, 3\frac{3}{4}$, ____, ____, ____, ____.

(e) What is the 24th term in the sequence: $3\frac{1}{2}, 4, 4\frac{1}{2}, 5$, ____, ____, ____, ____.

(f) Find the 30th term in the sequence: $2\frac{1}{8}, 2\frac{1}{4}, 2\frac{3}{8}, 2\frac{1}{2}$, ____, ____, ____, ____.

(g) What is the 50th term in the sequence: $5\frac{1}{10}, 5\frac{1}{2}, 5\frac{9}{10}, 6\frac{3}{10}$, ____, ____, ____, ____.

(h) Find the 100th term in the sequence: $\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 1\frac{1}{4}$ ____, ____, ____, ____.

10.9 Finding the number sequence using the general term/rule

Activity

- Write 4n - 1 on slips of paper.
- Substitute the order 1st, 2nd, 3rd, 4th and 5th for n to complete the table.

Order of term	1st	2nd	3rd	4th	5th
Solving for 4n - 1	_____	_____	_____	_____	_____
	_____	_____	_____	_____	_____

- Write the linear sequence worked out.
- Make presentation to the class.

Example 1

Given 6n + 1, find the linear sequence.

Solution

Write the order of terms: 1st, 2nd, 3rd, 4th.

Substitute the order of term for n, then solve.

1st order = 6n + 1 = 6 x n + 1 = 6 x 1 + 1 = 6 + 1 = 7

2nd order = 6n + 1 = 6 x n + 1 = 6 x 2 + 1 = 12 + 1 = 13

3rd order = 6n + 1 = 6 x n + 1 = 6 x 3 + 1 = 18 + 1 = 19

4th order = 6n + 1 = 6 x n + 1 = 6 x 4 + 1 = 24 + 1 = 25

Therefore, the linear sequence is: 7, 13, 19, 25, ____, ____, ____, ____.

Example 2

Given $\frac{1}{2}n + 8$, find the linear sequence.

Solution

Write the order of terms: 1st, 2nd, 3rd, 4th.

Substitute the order of term for n, then solve.

1st order = $\frac{1}{2}n + 8 = \frac{1}{2} \times n + 8 = \frac{1}{2} \times 1 + 8 = 8\frac{1}{2}$.

2nd order = $\frac{1}{2}n + 8 = \frac{1}{2} \times n + 8 = \frac{1}{\cancel{2}_{1}} \times \overset{1}{\cancel{2}} + 8 = 9$.

3rd order = $\frac{1}{2}n + 8 = \frac{1}{2} \times n + 8 = \frac{1}{2} \times 3 + 8 = 9\frac{1}{2}$.

3rd order = $\frac{1}{2}n + 8 = \frac{1}{2} \times n + 8 = \frac{1}{\cancel{2}_{1}} \times \overset{2}{\cancel{4}} + 8 = 10$.

Therefore, the linear sequence is: $8\frac{1}{2}$, 9, $9\frac{1}{2}$, 10, ____, ____, ____, ____.

Study tip

To find the linear sequence, substitute the order of term for n in the general term/rule. Then solve.



Application 10.9

Find the number sequences for the general rules below:

(a) $2n$	(b) $2n + 3$	(c) $3n + 2$	(d) $3n - 1$
(e) $5n - 2$	(f) $4n$	(g) $n + 5$	(h) $4n - 3$
(i) $\frac{3}{4}n$	(j) $\frac{1}{2}n$	(k) $\frac{5}{6}n + 1$	(l) $\frac{7}{10}n + 3$
(m) $10n$	(n) $4n + 1$	(o) $8n + 4$	(q) $7n$
(r) $8n + 3$	(s) $n + 4$	(t) $n + 10$	(u) $5n - 2$
(v) $\frac{3}{4}n + 2$	(w) $\frac{3}{5}n$	(x) $\frac{1}{2}n - 1$	(y) $\frac{1}{3}n + 4$

End of unit 10 assessment

1. Write the following algebraic expression:
   (a) Subtract 6 from n, then multiply by 2.
   (b) The sum of m and n divided by 4.
   (c) Double x and add 8.
   (d) A quarter the product of a and b.
2. Find the equivalent expressions.
   (a) $3(2x - 4) = 2(3x - 6)$
   (b) $\frac{1}{2}(4y + 2) = 2y + 1$
   (c) $14(2k - 1) = 7(4k - 2)$
   (d) $(x + 3) + 2(x - 1) = 3x + 2$

(e) $169m - 13n = 13(13m - n)$

(f) $\frac{1}{4}(28x - 8) = \frac{1}{2}(14x - 4)$

3. Find the missing consecutive numbers.

(a) 1, 5, 9, 13, ___, ___, ___, ___.

(b) 3, 7, 11, 15, ___, ___, ___, ___.

(c) 4, 9, 14, 19, ___, ___, ___, ___.

(d) 3, 6, 9, 12, ___, ___, ___, ___.

(e) 1, 6, 11, 16, ___, ___, ___, ___.

(f) 2, 6, 10, 14, ___, ___, ___, ___.

4. Find the general rule for the linear sequences below;

(a) 1, 4, 7, 10, 13, ___, ___, ___, ___.

(b) 2, 6, 10, 14, ___, ___, ___, ___.

(c) 5, 11, 17, 23, ___, ___, ___, ___.

(d) 7, 16, 25, 34, ___, ___, ___, ___.

5. (a) Find the 8^{th} term in the sequence; 3, 7, 11, 15, ___, ___, ___, ___.

(b) What is the 39^{th} term in the sequence; 1, 6, 11, 16, ___, ___, ___, ___.

(c) Find the 87^{th} term in the sequence; 4, 10, 16, 22, ___, ___, ___, ___.

(d) What is the 101^{st} term in the sequence; 2, 5, 8, 11, ___, ___, ___, ___.

6. Find the missing fractions in the sequences below:

(a) $\frac{1}{2}, \frac{5}{6}, 1\frac{1}{2}, 2\frac{1}{2}$, ___, ___, ___, ___.

(b) $1, 1\frac{4}{5}, 2\frac{3}{5}, 3\frac{2}{5}$, ___, ___, ___, ___.

(c) $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}$, ___, ___, ___, ___.

(d) $\frac{2}{3}, 1\frac{1}{3}, 2, 2\frac{2}{3}, 3\frac{1}{3}$, ___, ___, ___, ___.

Unit 11 Solving simple algebraic equations and inequalities

Key unit competence: To be able to form and solve simple algebraic equations and inequalities.

Introduction

In daily life situations, people are faced mathematical problems that require simple and complex calculations. In most cases, different techniques are used to solve those problems. Some people prefer to use only numbers (arithmetic methods) while others prefer to use numbers mixed with variables/letters (algebraic methods).

Consider the following situation and answer the related questions:

In a certain classroom, x represents the number of boys while the number of girls is 2 times the number of boys. The total number of students is 30.

(a) How do you think the above mathematical problem can be solved? Using arithmetic method or algebraic method?

(b) Try to change the above mathematical problem into algebraic equation and find out the answer.

11.1 Like terms of algebraic expressions

Activity

Write this expression on paper slips: 7x + 6x + 3a + a + 11x + 2a

- Group the same terms together.
- Then add them.
- What do you notice?
- Give your observation to the class.

Example 1

Simplify: 6abp – 7ab – 6abp + 8ab =

Solution

First collect like terms and simplify

= (6abp – 6abp) + (–7ab + 8ab)

= (0abp) + (1ab)

= ab

Example 2

Simplify: 8mn + 6pq – 5mn + 3pq =

Solution

First collect like terms and simplify

= 8mn + 6pq – 5mn + 3pq

= (8mn – 5mn) +(6pq + 3pq)

= 3mn + 9pq

Study tip

- Like terms have similar items or expressions.
- First collect like terms then simplify the expressions.
- Collecting like terms involves application of integers.



Application 11.1

Simplify these expressions:

(a) $4x + 5x =$

(b) $2y + 4y - 2y =$

(c) $2p + 4q + 3p - 2p =$

(d) $9mno - 6mno + 3xy - 2ab + 3xy + 6ab =$

(e) $5xyz - 3xyz + 2xzy =$

(f) A rectangle has length $2a - d$ and width $4a - 3d$. Find its perimeter.

(g) A square has sides of $3x + 6m$. Find its perimeter.

11.2 Unlike terms of algebraic expressions

Activity

- There are 40 cattle (c), 16 sheep (s) and 32 goats (g) on a farm.
- Write down an algebraic expression for the animals above.
- Is there any way of grouping same animals together?
- Discuss, then make a presentation to class.

Example 1

Simplify: $5x + 4y + 10b - r + r$

Solution

First collect like terms and simplify

$= 5x + 4y + 10b - r + r$

$= 5x + 4y + 10b$

Example 2

Simplify: $cd + 5cd - fk - 4s$

Solution

First collect like terms and simplify

$= cd + 5cd - fk - 4s$

$= 6cd - fk - 4s$

Study tip

- Unlike terms have expressions or items which are not similar.
- When simplifying algebraic expressions, unlike terms are not simplified because they do not have similar terms or expressions that can be collected together. Unlike terms differ letters and powers but coefficients may be the same.

Application 11.2

Simplify the following:

(a) $3xyz - 3xyr + 4xyz + abc$

(b) $12x^2y - 4xy^2 - 5a - 19b - a$

(c) $4x - xz - x + 6yx + 4tz$

(d) $17abfg - 3abcd - 5acd - 10defg$

(e) $5abc - 4bca + 2acd - abc$

(f) $fgh - fjk + 3fkj + 2fjk$

(g) $10pqr + 9prq - 4rxy - 6pqr$

(h) $stu - 4tuv + 7rst + 2tuv$

11.3 Substituting algebraic expressions with addition and subtraction

Activity

- If $x = 5$ and $y = 4$, evaluate: $x + 6y$
- If $w = 5$, $x = 3$ and $y = 6$, find the value of wxy.
- Explain your answers to the class.

Example 1

If $x = 2$, $y = 1$ and $z = 7$, simplify: $y + z - x$.

Solution

Substitute then simplify:

$y + z - x$

$= 1 + 7 - 2$

$= 8 - 2$

$= 6$

Example 2

If $x = 2$, $y = 1$ and $z = 7$, simplify $y + x + z$.

Solution

Substitute then simplify:

$y + x + z$

$= 1 + 2 + 7$

$= 3 + 7$

$= 10$

Example 3

If $x = 2$, $y = 1$ and $z = 7$, simplify: $z + x - y$

Solution

Substitute then simplify:

$z + x - y$

$= 7 + 2 - 1$

$= 9 - 1$

$= 8$

Example 4

If $x = 2$, $y = 1$, $z = 7$, simplify $x + z - y$.

Solution

Substitute then simplify:

$x - y + z$

$= 2 - 1 + 7$

$= 1 + 7$

$= 8$

Study tip

To substitute is to replace. To work out algebraic expressions, replace the letters with the given numbers and simplify.



Application 11.3

Work out the following:

If x = 10, y = 2 and t = 5, p = 6, q = 8, r = 4; evaluate the following:

(a)	x – 3	(b)	t + y	(c)	y + q – t
(d)	q + r – p	(e)	p + x + r – t	(f)	p + q + y
(g)	x + y + t	(h)	q + r – y	(i)	x + r + p
(j)	q + p – r	(k)	t + x + p – r	(l)	y + p + t
(m)	x + r + p	(n)	y + r – p	(o)	x + r – p

11.4 Substituting algebraic expressions involving multiplication

Activity

- Write 4 letters and equate them to values of your choice.
- Form multiplication statements for two or more letters.
- Substitute their values, then work out the products.
- Explain your working out to the class.

Example 1

If a = 3, b = 4, c = 5 and d = 6

Simplify ab + cd.

Solution

Expand, substitute then simplify:

ab + cd

= (a x b) + (c x d)

= (3 x 4) + (5 x 6)

= 12 + 30

= 42

Example 2

If a = 3, b = 4, c = 5 and d = 6

Simplify abd

Solution

Expand, substitute then simplify:

abd

= (a x b x d)

= (3 x 4 x 6)

= 12 x 6

= 72

Example 3

If a = 3, b = 4, c = 5 and d = 6

Simplify 2bc – 3ab

Solution

Expand, substitute then simplify:

2bc - 3ab

= (2 x b x c) – (3 x a x b)

= (2 x 4 x 5) – (3x 3 x 4)

= 40 – 36

= 4

Example 4

If a = 3, b = 4, c = 5 and d = 6

Simplify $\frac{3}{4}bd - \frac{2}{5}ac$

Solution

Expand, substitute then simplify:

$\frac{3}{4}bd - \frac{2}{5}ac$

$= \frac{3}{4} \times (4 \times 6) - \frac{2}{5} \times 3 \times 5$

$= \frac{3}{\cancel{4}_{1}} \times \cancel{24}^{6} - \frac{2}{\cancel{5}_{1}} \times \cancel{15}^{3}$

= 18 – 6

= 12

Study tip

- To work out algebraic expressions with multiplication, expand then substitute the values.
- Simply to obtain the answer.



Application 11.4

Work out the following:

If p = 6, q = 8, r = –4, s = 10 and t = –3, evaluate:

(a)	pqs	(b)	$q^2 + 2p$	(c)	st – 2r
(d)	prt	(e)	rs – 4t	(f)	pr + qt
(g)	5r – 4s	(h)	7qr – 3st	(i)	9(2p + 3r)
(j)	$\frac{2}{3}st - \frac{1}{4}(2r)$	(k)	$\frac{3}{10}qs + \frac{5}{6}pr$	(l)	$r^2 + 2t^3$
(m)	$\frac{1}{3}(st)^2$	(n)	$\frac{5}{6}r + st$	(o)	$\frac{1}{2}r^2 + 2t^2$
(p)	$\frac{1}{8}q^2 - t$	(q)	$\frac{4}{16}ps - q$	(r)	$\frac{1}{2}st^2 + q^3$
(s)	3s + tr	(t)	$2tq - r^2$	(u)	$r^3 + tsp$

11.5 Substituting algebraic expressions involving division

Activity

- Study the cards below:

$\frac{q}{r}$ $\frac{st}{r}$ $\frac{2m}{3} \div n$ $\frac{14qr}{3}$

- Select numbers of your choice, substitute and work out.
- Share your working out with classmates.

Example 1

If k = 4, l = 6, m = –3, n = 10 and p = –5

Simplify $\frac{kl}{m}$

Solution

Expand, substitute then divide.

$= \frac{k \times l}{m}$

$= \frac{4 \times 6}{-3}$

$= 24 \div -3$

$= -8$

Example 2

If k = 4, l = 6, m = –3, n = 10 and p = –5

Simplify $\frac{2mn}{p}$

Solution

Expand, substitute then divide.

$= \frac{2 \times (m \times n)}{p}$

$= \frac{2 \times (-3 \times 10)}{-5}$

$= (2 \times -30) \div -5$

$= -60 \div -5 = +12$

Study tip

- Dividing algebraic expressions involves applying the concept of dividing integers.
- To work out substituting algebraic expressions involving division, substitute into the equation, simply then divide.

Application 11.5

Work out the following:

If v = 2, w = 14, x = -4, y = 18 and z = -9.

(a) $\frac{y}{x}$ (b) $\frac{w}{v}$ (c) $\frac{y}{z}$

(d) $\frac{wx}{z}$ (e) $\frac{2vy}{x}$ (f) $\frac{xy}{2z}$

(g)	$\frac{3vw}{2w}$	(h)	$\frac{1}{4}(2y) \div 2z$	(i)	$x^3 \div v^2$
(j)	$(2vy) \div z^2$	(k)	$\frac{3}{10}vx$	(l)	$\frac{2}{8}(3vx) \div 8y$
(m)	$\frac{1}{3}(yv)$	(n)	$\frac{5}{6}y + xz$	(o)	$\frac{1}{2}v^2 + 2x^2$

11.6 Simple algebraic equations with one unknown

Activity

An equation consists two parts which are equal to one another as shown below.

- Get a beam balance.
- Place small stones on one side and some fruits like oranges or mangoes on the other.
- Do they balance?
- If not, remove some items on the heavier side or add some items on the lighter side to balance the two sides.
- Now, given x + 2 = 18, what number must be put in place of x so that the sides are equal?
- Explain how you get the number.
- Present your working out to the class.

Example 1

Solve 6y + 12 = 0

Solution

6y + 12 = 0

Collect like terms.

When a positive crosses the = sign, it changes to a negative.

Subtract 12 both sides (-12) because it is the inverse of +12.

$6y + 12 - 12 = 0 - 12$

$6y = -12$

Divide by 6 both sides.

$\frac{{}^{1}\cancel{6}y}{{}_{1}\cancel{6}} = \frac{\cancel{-12}^{2}}{\cancel{6}_{1}}$

Therefore y = –2

Example 2

Solve z – 4 = 6

Solution

z – 4 = 6

Collect like terms.

When a negative crosses the = sign, it changes to a positive.

Add +4 both sides because it is the inverse of – 4.

$z - 4 + 4 = 6 + 4$

$z = 10$

Example 3

Solve: $2x - 12 = 0$

Solution

$2x - 12 = 0$

Collect like terms.

When a negative crosses the = sign, it changes to a positive.

Add +12 both sides because it is the inverse of – 12.

$2x - 12 + 12 = 0 + 12$

$2x = 12$

Divide by 6 both sides.

$\frac{{}^{1}\cancel{2}x}{{}_{1}\cancel{2}} = \frac{\cancel{12}^{6}}{\cancel{2}_{1}} = 6$

Therefore x = 6

Example 4

Solve: $(x - 3) - 5(x + 1) = 0$

Solution

$(x - 3) - 5(x + 1) = 0$

Collect like terms.

When a negative crosses the = sign, it changes to a positive.

$(x - 3) - 5(x + 1) = 0$

Open brackets and collect like terms

$x - 3 - 5x - 5 = 0$

$x - 5x - 3 - 5 = 0$

Add +8 both sides because it is the inverse of – 8.

$-4x - 8 + 8 = 0 + 8$

$-4x = 8$

Divide by –4 both sides.

$\frac{\cancel{-4x}^{1}}{\cancel{-4}_{1}} = \frac{{}^{2}\cancel{8}}{{}_{1}\cancel{-4}} = -2$

Therefore x = –2.

Study tip

- To solve an equation involving brackets, remove brackets first, then collect like terms. Finally solve for the unknown.
- Add or subtract the same inverse both sides of the equation in order to eliminate a number.
- Multiply or divide by the same number both sides to get the answer.
- The digit on an unknown is called a co-efficient.

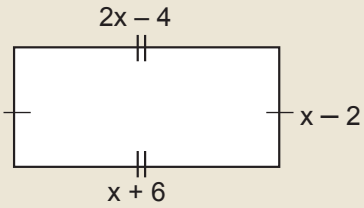
Application 11.6

Solve the following equations:

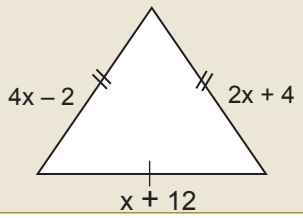
1. (a) $x - 6 = 8$ (b) $4r - 16 = 0$ (c) $9y + 3 = -6$
   (d) $y - 5 = 0$ (e) $4(z - 5) + 3(z + 2) = 0$ (f) $3m - 4 = 11$
   (g) $10p + 9 = 49$ (h) $12q - 11 = 109$ (i) $\frac{2x}{3} = 6$

2. Study the diagram below. Find the value of x.

(a)



(b)



11.7 Solving fractional algebraic equations

Activity

- Write a letter of your choice on a slip of paper.
- Multiply it by $\frac{1}{2}$.
- Add 4 to the product. The result is 16.
- Write an equation relating to the information above.
- Solve for the unknown.
- What is your answer?
- Make a presentation of your working out to the class.

Example 1

Solve: $\frac{1}{2}(x + 4) = 8$

Solution

Method 1

Open brackets

$$\frac{1}{2} \times x + \frac{1}{\cancel{2}_1} \times \cancel{4}^{2} = 8$$

$$\frac{1}{2}x + 2 = 8$$

Subtract the inverse of +2 which is -2 both sides.

$$\frac{1}{2}x + 2 - 2 = 8 - 2$$

$$\frac{1}{2}x = 6$$

Multiply 2 both sides to eliminate the denominator.

$$\cancel{2}^{1} \times \frac{1}{\cancel{2}_1}x = 6 \times 2$$

$$x = 12$$

Method 2

$$\frac{1}{2}(x + 4) = 8$$

Multiply by 2 both sides to eliminate the denominator.

$$\cancel{2}^{1} \times \frac{1}{\cancel{2}_1}(x + 4) = 8 \times 2$$

$$x + 4 = 16$$

Subtract the inverse of +4 which is -4 both sides.

$$x + 4 - 4 = 16 - 4$$

$$x = 12$$

Example 2

Solve: $\frac{1}{4}(4a - 8) = \frac{2}{3}(6a + 3)$

Solution

$\frac{1}{4}(4a - 8) = \frac{2}{3}(6a + 3)$

Open brackets

$\frac{1}{\cancel{4}_1} \times \cancel{4}^{1}a - \frac{1}{\cancel{4}_1} \times \cancel{8}^{2} = \frac{2}{\cancel{3}_1} \times \cancel{6}^{2}a + \frac{2}{\cancel{3}_1} \times \cancel{3}^{1}$

$(a - 2) = 4a + 2$

Collect like terms on same sides.

$-2 - 2 = 4a - a$

Negative subtract negative gives negative.

$-4 = 3a$

Divide both sides by 3

$\frac{-4}{3} = \frac{\cancel{3}^{1}a}{\cancel{3}_1}$

$a = -1\frac{1}{3}$

Therefore, $a = -1\frac{1}{3}$

Study tip

- When solving equations, apply the concept of operations on integers.
- Open brackets, then simplify the equation.
- The number or fraction on an unknown is called a co-efficient.
- Multiply the co-efficient by all the terms inside the brackets.



Application 11.7

Solve the following equations:

(a) $\frac{1}{4}(8x - 4) = 6$ (b) $\frac{2}{3}(9p + 6) = 16$ (c) $\frac{3}{4}(12m - 8) = 12$

(d) $\frac{9}{10}(20a + 40) = 54$ (e) $\frac{3}{5}(x - 4) = 6$ (f) $\frac{1}{6}(m - 2) = 3$

(g) $\frac{13}{15}(15x + 30) = 65$ (h) $\frac{7}{12}(48p - 96) = 112$ (i) $\frac{1}{4}(4a - 8) = \frac{2}{3}(6a + 3)$

(j) $\frac{3}{4}(12x - 4) = 24$ (k) $\frac{2}{5}(10m - 5) = 14$ (l) $\frac{11}{10}(40p - 20) = 33$

(m) $\frac{9}{15}(15a - 60) = 108$ (n) $\frac{5}{8}(64q - 48) = 90$ (o) $\frac{6}{7}(21x - 28) = 138$

(p) $\frac{9}{10}(20y - 10) = 117$ (q) $\frac{7}{16}(64p - 128) = 280$ (r) $\frac{11}{20}(40r + 80) = 88$

11.8 Solving problems involving equations

Activity

- Write an unknown on a slip of paper.
- Multiply it by 14.
- The result is 64.
- Form an equation for the information above.
- Try to find the value of the unknown.
- Show your working out to the class.

Example 1

Father is 20 years older than his son. In 10 years, father will be twice as old as the son.

(a) How old is the son?

(b) Find the father's age in 10 years.

Solution

Let son be x years old.

Father be (x + 20) years old.

In 10 years time:

Son will be: 2(x + 10) years old.

Father will be: (x + 20) + 10 years old

To find the son's age, we write an algebraic equation.

Son		Father
2(x + 10)	=	(x + 20 + 10)
2x + 20	=	x + 30

Subtract 20 from both sides.

2x + 20 - 20	=	x + 30 - 20
2x	=	x + 10

Subtract x from both sides.

2x - x	=	x - x + 10
x	=	10

(a) Son is 10 years old.

(b) Father will be: = (x + 20) + 10 years

= (10 + 20) + 10 years

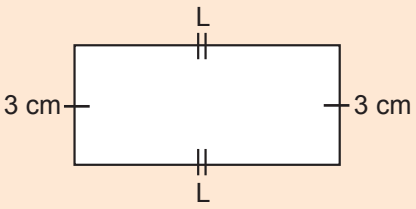
= 30 + 10

= 40 years old

Example 2

The perimeter of a rectangle is 24 cm. Its width is 3 cm. Find the length.

Solution



Perimeter = 2(L + W)

Substitute the values.

24 cm = 2(L + 3) cm

24 cm = 2(L + 3)

24 cm = 2 x L + 2 x 3

24 cm = 2L + 6

Subtract 6 from both sides.

24 - 6 = 2L + 6 - 6

18 = 2L

Divide 2 both sides.

$\frac{\cancel{18}^{9}}{\cancel{2}^{1}} = \frac{2L}{2}$

9 = L

Therefore, length is 9 cm.

Study tip

- Read and understand the word problems in order to form a correct equation.
- Apply the correct operations when solving the equation.

Application 11.8

1. The product of 2 numbers is 48. One of them is 6. What is the other number?
2. The product of 2 numbers is 96. One of them is 8. Find the other number.
3. The perimeter of a hexagon is 60 cm. What is its side?
4. The area of a triangle is 96 cm^2. Its height is 16 cm. Find its base.
5. Father is 30 years older than his son. In 12 years time the father will be twice as old as the son.
   (a) How old is the son?
   (b) How old is the father?
   (c) How old will the son be in 12 years' time?
   (d) Find the father's age in 12 years' time.
6. The sum of 3 consecutive counting is 36. Find the numbers.
7. The sum of 4 consecutive counting number is 98. What are numbers?

11.9 Solving algebraic inequalities with one unknown

Activity

Solve and discuss the following and show your working steps.

$2y + 26 > 6$

Example 1

Solve: $x - 1 < 0$

Solution

$x - 1 + 1 < 0 + 1$

Add 1 on both sides to eliminate it.

$x < 1$

Example 2

Solve: $-6 < 3x < 12$

Solution

$-6 < 3x < 12$

Divide by every term by 3.

$\frac{-\cancel{6}^{2}}{\cancel{3}_{1}} < \frac{\cancel{3}x^{1}}{1\cancel{3}} < \frac{^{4}\cancel{12}}{1\cancel{3}}$

$-2 < x < 4$

Example 3

Solve: $3x + 4 > x$

Solution

$3x + 4 > x$

Subtract 3x both sides to eliminate 3x on the left side.

$3x - 3x + 4 < x - 3x$

$\frac{\cancel{4}^{2}}{\cancel{-2}_{1}} < \frac{\cancel{-2x}^{1}}{1\cancel{-2}}$

$-2 < x$ or $x > -2$

Example 4

Solve: $-3y > 12$

Solution

$-3y > 12$

Coefficient -3 divide by both sides.

$\frac{^{1}\cancel{-3}y}{1\cancel{-3}} > \frac{^{4}\cancel{12}x}{1\cancel{-3}}$

$y < -4$ (Change of inequality sign because we devide both sides by a negative number)

Study tip

- Solve inequalities in the same way you solve equations. But instead of writing the equal sign, write the inequality sign.
- When you divide or multiply by a negative number, the < changes to > and > changes to <.

Application 11.9

Solve the following:

(a) $4x + 2 > x - 1$

(b) $(2x - 1) + 3(x - 2) > 12$

(c) $-2(x - 4) + 3(x - 1) > 4$

(d) $4y - 18 < 0$

(e) $y + 7 > 14$

(f) $^{-}4 < 2x < 12$

(g) $\frac{6x}{25} \leq 12$

(h) $4(w + 2) + 2(w - 3) > -8$

(i) $-6x < 24$

(j) $5x + 6 > 2x$

11.10 Finding the solution set

Activity

- Write an unknown. Let it be greater than 8.
- Multiply it by 2 and add 4.
- Solve the inequality formed.
- Which numbers are greater than the value of the unknown?
- Can you list all of them?
- Explain your working out to the class.

Example 1

Solve $3y - 5 < 7$ and find the solution set.

Solution

$3y - 5 < 7$

Add 5 both sides to eliminate it.

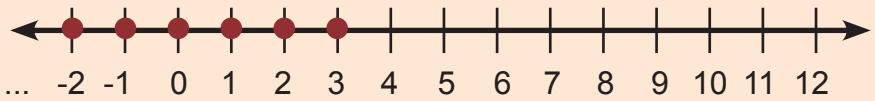
$3y - 5 + 5 < 7 + 5$

$3y < 12$

Divide every term by the co-efficient.

$\frac{1}{1}\frac{3y}{3} < \frac{4}{1}\frac{12x}{3}$

$y < 4$



List possible values that are less than 4. That is the solution set.

Therefore, the solution set is: y = {3, 2, 1, 0, -1, -2, ___, ___, ___}.

Example 2

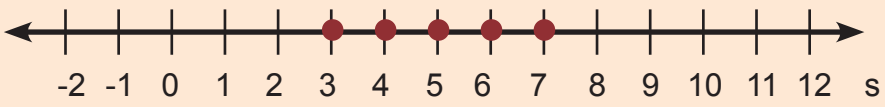
Solve: 32 > 4x > 8.

Solution

32 > 4x > 8

Divide every term by the co-efficient.

$\frac{8\ 32}{1\ 4} > \frac{1\ 4x}{1\ 4} > \frac{2\ 8}{1\ 4}$ = 8 > x > 2



The values of x range between 8 and 2.

Therefore, the solution set is: {7, 6, 5, 4, 3}.

Study tip

- Solution set is a set of numbers/integers which when substituted in the unknown, make the inequality statement true.
- An inequality with one "greater than" or "less than" symbol, gives a set of infinite values to the left or to the right.
- Inequalities with two "greater than" or "less than" symbols, give finite sets of values between the two limits.

Application 11.10

Solve and find the solution set:

(a)	x > 5	(b)	y > 3	(c)	2x + 3 < 11
(d)	m ≤ 4	(e)	x ≥ 2	(f)	3n - 3 > 6
(g)	4q + 11 ≤ 23	(h)	24 > 4y > 8	(i)	6 < 2x < 16
(j)	28 ≥ 4m ≥ 16	(k)	6 ≤ 6x < 6	(l)	96 > 8y ≥ 24

11.11 Solving problems involving simple algebraic equations and inequalities

Activity

- Twice a number plus eight gives twenty.
- What is the number?
- What do you notice?
- Discuss and show your working out.

Example 1

Think of a number and take away twelve, the answer is fourteen. What is the number?

Solution

Let the number be x

$x - 12 = 14$

$x - 12 + 12 = 14 + 12$

$x = 26$

Therefore the number is 26.

Example 2

Gitego added 8 to a certain number. His answer was greater than 12. Find the possible number.

Solution

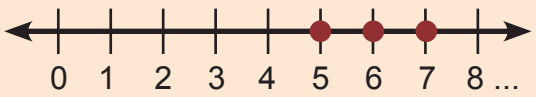
Let the number be y.

$y + 8 > 12$

Subtract (-8) the inverse of 8.

$y + 8 - 8 > 12 - 8$

$y > 4$



The solution set is: $y = \{5, 6, 7, ...\}$

Study tip

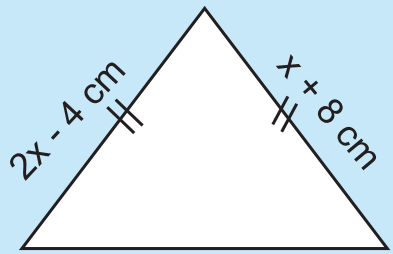
- To solve problems involving simple equations or inequalities, read the information carefully and interpret it correctly.
- Let the unknown number be represented by a letter. Finally, calculate for the value of the letter. Then give the solution set.

Application 11.11

1. I double a number and add six. My answer is less than twelve. Find the number.
2. I multiply three by a number. I get twenty four. What is the number?
3. If one is added to a number the answer is zero. What is the number?
4. If I add 7 to a number, I get ten. What is the number?
5. Think of a number, divide it by two. The answer is four. What is that number?
6. When 6 is added to x, the sum is less than 9. What are the possible values of x?
7. Aidah bought glasses. She broke 5. The remaining glasses are more than 9. What is the possible number of glasses she bought?
8. When a number is multiplied by 5, and reduced by 2, the answer is less than 8. What are the possible values of the number?

End of unit 11 assessment

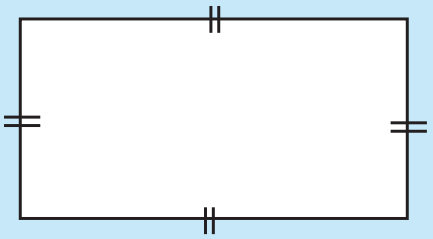
1. Simplify the following
   (a) $10x - 5 + 3x - 4b + 6$ (b) $-2bcdef - 5bcdef + 12bdecf$
   (c) $2xz + 5xyz - 2yz + 4xz$ (d) $y + 7 > 14$
2. When I divide a number by five, I get eight.
   What is the number?
3. If twenty five is subtracted from a number, one is the answer,.
   What is the number?
4. Find the perimeter of the triangle below:



(3x + 11) cm

5. The figure below is a rectangle. Study it and answer the questions that follow.

(5x + 5) cm



(2x) cm

(x + 27) cm

   (a) Find the value of x.
   (b) Find the length.
   (c) Find the width.
6. James needs a driver who is above 35 years. He should not be above 40 years.
   (a) Write an inequality to describe the age limits.
   (b) Write the required ages for the job.

7. Solve the following:

   (a) $-3 < x < 4$

   (b) $2x < 8x + 6$

   (c) $12x - 3 > 5 + 8x$

   (d) $2x + 14 = 0$

   (e) $4x + 7 = 2x + 5$

   (f) $2x + 1 = x + 1$

8. A pen costs 3 times as much as a pencil and a book costs 150 Frw more than a pencil. Their total costs is 1,200 Frw. Find the cost of the pencil.

9. Find the number of books that can be shared equally among 6 classes such that each class gets 18 books.

10. Given that; $p = 6$, $q = 4$, $r = 10$, $s = 3$, evaluate:

    (a) $2p + 3s$

    (b) $6r - 4q$

    (c) $11q + 3p - 2s$

    (d) $\frac{2}{5}r - \frac{1}{4}q - s$

11. Given that $x = \frac{1}{4}$, $y = -6$, $z = \frac{1}{2}$, evaluate:

    (a) $8x - 2y$

    (b) xz

    (c) $4y \div 2z$

    (d) $\frac{yz}{2y}$

Unit 12 Regular polygons and bearings

Key unit competence: To be able to use the bearings and compass points and understanding the relationship between them.

To use the angle sum of a triangle to determine the interior angles of regular polygons.

Introduction

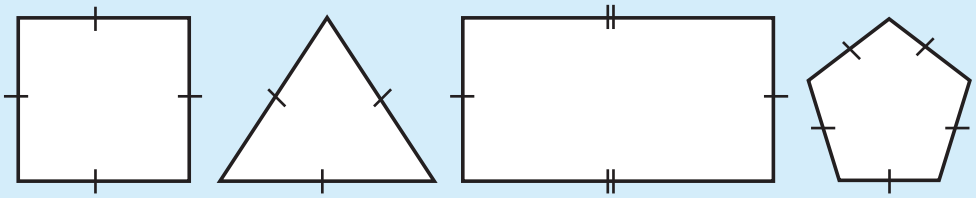
Polygons are plane shapes with many sides, angles and vertices. Some have same size of sides and angles while others don't. The concept of polygon is used for making different objects. Observe the classroom and identify polygons shaped objects and try to name them basing on the number of sides and angles.

(a) Is there in your class any object of 3 sides and 3 angles?
(b) Is there any object of 4 sides and 4 angles?
(c) Is there any object of 5 angles?
(d) How are the sides of the observed polygons? Find out if they are all equal and name that regular polygon.

12.1 Definition of Polygon and their Examples

Activity

1. Name the shapes below.



2. Observe the above shapes then define them.
3. Explain the differences and similarities between the polygons.

Study tip

- A regular polygon has equal sides and equal angles, otherwise it is not regular, it is irregular.
- We name a polygon basing on its number of sides.
- A circle is not a polygon because it is curved.

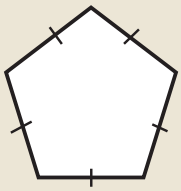
Example

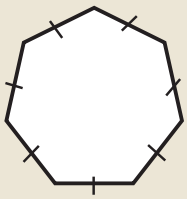
A polygon is any 2 dimensional shape formed with straight lines. The table shows the names of polygons and their corresponding number of sides.

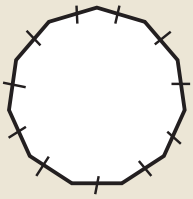
Number of sides	Name of polygon
3	Triangle
4	Square or rectangle
5	Pentagon
6	Hexagon
7	Heptagon or septagon
8	Octagon
9	Nonagon
10	Decagon

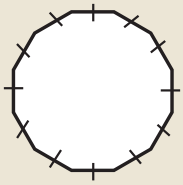
Application 12.1

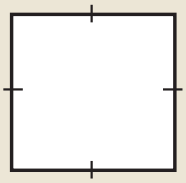
1. Name the following regular polygons.

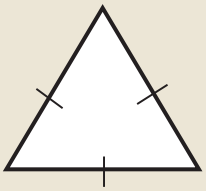
(a) 

(b) 

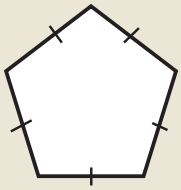
(c) 

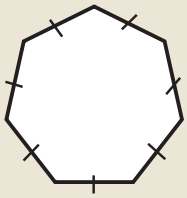
(d) 

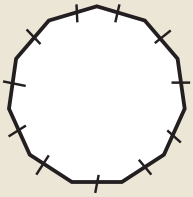
(e) 

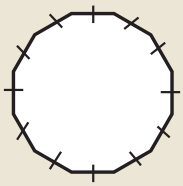
(f) 

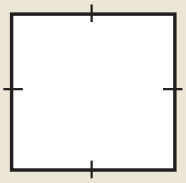
2. State the number of sides of the following polygons:

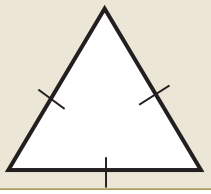
(a) 

(b) 

(c) 

(d) 

(e) 

(f) 

12.2 Investigating the interior and exterior angles of a polygon

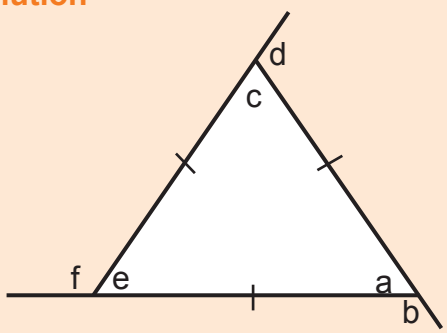
Activity

- Draw a triangle on a sheet of paper.
- Extend the edges from the vertices with straight lines using a ruler.
- Measure the inside angle and its adjacent outside angles using a protractor. What names do you give the inside angle and the outside angle?
- Find the sum of the two angles you measured?
- Now draw a rectangle, pentagon and a hexagon.
- Carry out the same steps you carried on a triangle.
- What is your observation?
- Demonstrate and make a presentation to the class.

Example 1

Draw an equilateral triangle. Measure the interior and exterior angles using a protractor.

Solution



$\angle a = 60^o$, $\angle b = 120^o$

$\angle a + \angle b = 60 + 120^o = 180^o$

$\angle c = 60^o$, $\angle d = 120^o$

$\angle c + \angle d = 60^o + 120^o = 180^o$

$\angle e = 60^o$, $\angle f = 120^o$

$\angle e + \angle f = 60^o + 120^o = 180^o$

$\angle a$ and $\angle b$ supplement each other.

$\angle c$ and $\angle d$ supplement each other.

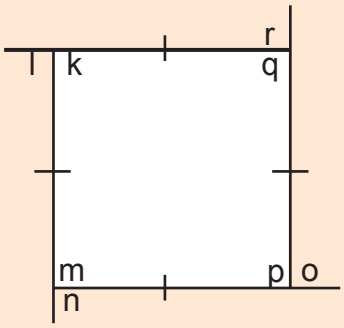
$\angle e$ and $\angle f$ supplement each other.

Therefore, the interior and its supplement exterior angles of a triangle add up to 180^o.

Example 2

Draw a square using a protractor. Measure the interior and exterior angles.

Solution



$\angle q = 90^o$, $\angle r = 90^o$

$\angle q + \angle r = 90^o + 90^o = 180^o$

$\angle p = 90^o$, $\angle o = 90^o$

$\angle p + \angle o = 90^o + 90^o = 180^o$

$\angle k = 90^o$, $\angle l = 90^o$

$\angle k + \angle l = 90^o + 90^o = 180^o$

$\angle m = 90^o$, $\angle n = 90^o$

$\angle m + \angle n = 90^o + 90^o = 180^o$

$\angle q$ and $\angle r$ supplement each other.

$\angle p$ and $\angle o$ supplement each other.

$\angle k$ and $\angle l$ supplement each other.

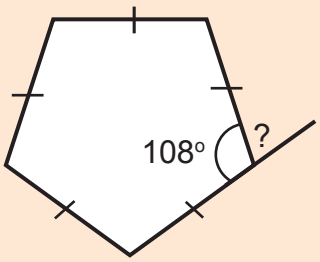
$\angle m$ and $\angle n$ supplement each other.

Therefore, the interior and its supplement exterior angle of a square add upto 180^o.

Example 3

The interior angle of a regular pentagon is 108°. Find its exterior angle.

Solution



Interior angle + exterior angle = 180°

108° + exterior angle = 180°

Subtract 108° from both sides

108 – 108 + exterior angle = 180 – 108°

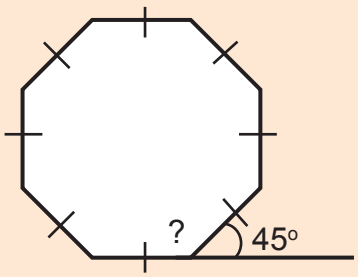
Exterior angle = 72°

Therefore, the exterior angle is 72°.

Example 4

The exterior angle of a regular octagon is 45°. Find its interior angle.

Solution



Interior angle + exterior angle = 180°

Interior angle + 45° = 180°

Subtract 45° from both sides

Interior angle + 45° – 45° = 180° – 45°

Interior angle = 135°

Therefore, the exterior angle is 135°.

Study tip

- The inside angle of a polygon is called interior angle.
- The outside angle of a polygon is called exterior angle.
- The interior angle and exterior angles of a polygon are adjacent to each other.
- The interior and exterior angles of a polygon are supplementary angles (add up to 180°).

Application 12.2

Use the polygon cards and sketches. Draw and measure to find the interior and exterior angles of these regular polygons.

(a)	Pentagon	(b)	Septagon	(c)	Nonagon
(d)	Hexagon	(e)	Duo-decagon	(f)	Heptagon
(g)	Decagon	(h)	Rectangle	(i)	Equilateral triangle
(j)	Square	(k)	Nuo-decagon	(l)	Octagon

12.3 Investigating the sum of interior and exterior angles of a regular polygon

Activity

- Get polygon cards.
- Trace the edges of a regular pentagon and octagon on sheets of paper.
- Extend the edges with a ruler and a pencil to form a straight line.
- Using a protractor, measure the interior angles, then add to get their sum.
- Then measure the exterior angles, add them to find their sum
- What did you find out?
- What is the objective of the activity?
- Make a presentation to the class.

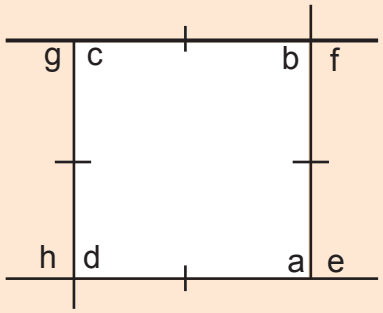
Example 1

Use a polygon card of a square and measure using a protractor to find its:

(a) Sum of the interior angles.

(b) Sum of the exterior angles.

Solution



(a) The interior angles are a, b, c, d.

$\angle a = 90^o$, $\angle b = 90^o$, $\angle c = 90^o$, $\angle d = 90^o$

The sum of the interior angles is:

$= \angle a + \angle b + \angle c + \angle d$

$= 90^o + 90^o + 90^o + 90^o$

$= 360^o$

Therefore, the sum of interior angles of a regular quadrilateral (square) is 360^o.

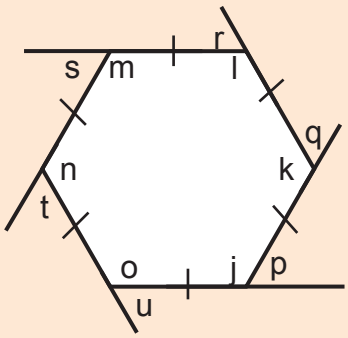
Example 2

Trace from a regular polygon card. Measure using a protractor to find its:

(a) Sum of the interior angles.

(b) Sum of its exterior angles.

Solution



(a) The interior angles are j, k, l, m, n, o.

$\angle j = 120^o$, $\angle k$ 120^o, $\angle l = 120^o$, $\angle m = 120^o$, $\angle n = 120^o$ and $\angle^o = 120^o$.

The sum of the interior angles is:

$= \angle j + \angle k + \angle l + \angle m + \angle n + \angle o$

$= 120^o + 120^o + 120^o + 120^o + 120^o + 120^o$

$= 720^o$

Therefore, the sum of interior angles of a regular hexagon is 720^o.

(b) The exterior angles are e, f, g, h.

= ∠e + ∠f + ∠g + ∠h

= $90^o + 90^o + 90^o + 90^o$

= 360^o

Therefore, the sum of exterior angles of a regular quadrilateral (square) is 360^o.

(b) The exterior angles are p, q, r, s, t, u.

= ∠p + ∠q + ∠r + ∠s + ∠t + ∠u

= $60^o + 60^o + 60^o + 60^o + 60^o + 60^o$

= 360^o

Therefore, the sum of exterior angles of a regular hexagon is 360^o.

Study tip

- The sum of the interior angles of a regular polygon is found by adding all the interior angles.
- All the interior angles of a regular polygon are equal.
- The sum of the exterior angles of a regular polygon is found by adding all the exterior angles.
- All the exterior angles of a regular polygon are equal.
- The sum of all exterior angles of a regular polygon is 360^o.

Application 12.3

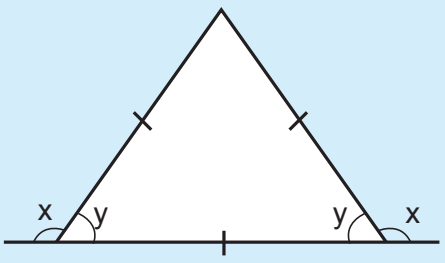
Using the polygon cards, draw and measure to find the sum of the interior angles and the sum of the exterior angles of the regular polygons below:

(a)	pentagon	(b)	septagon	(c)	nonagon
(d)	hexagon	(e)	duo-decagon	(f)	heptagon
(g)	decagon	(h)	rectangle	(i)	equilateral triangle
(j)	square	(k)	nuo-decagon	(l)	octagon

12.4 Finding the interior and exterior angles of a regular polygon

Activity

- Study the sketch of a triangle.
- Angle x is 120^o, what is angle y?
- If y is 60^o, what is angle x?
- Show your working out.
- Present your findings to the class.



Example 1

The exterior angle of a regular polygon is 72^o. Find its interior angle.

Solution

Exterior angle + interior angle = 180^o

72^o + interior angle = 180^o

$72^o - 72^o$ + interior angle = $180^o - 72^o$

Interior angle 108^o

Example 2

The interior angle of a regular polygon is 60^o. Find its exterior angle.

Solution

Exterior angle + interior angle = 180^o

Exterior angle + 60^o = 180^o

Exterior angle + $60^o - 60^o$ = $180^o - 60^o$

Exterior angle 120^o

Study tip

Interior and exterior angles of a regular polygon are supplementary. That is, exterior angle + interior angle = 180^o.

Application 12.4

1. Find the interior angle given the following exterior angles:
   (a) 76^o (b) 125^o (c) 105^o (d) 90^o
   (e) 140^o (f) 65^o (g) 109^o (h) 123^o
2. Find the exterior angle given the following interior angles:
   (a) 120^o (b) 45^o (c) 58^o (d) 135^o
   (e) 65^o (f) 125^o (g) 85^o (h) 144^o

12.5 Finding the sum of interior angles of a regular polygon

Activity

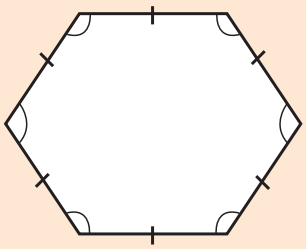
- Draw a regular polygon of four sides.
- Measure its interior angles using a protractor.
- Record your findings.
- Try to measure its exterior angles.
- Suggest how you can find the exterior angles while you have interior angles.
- Now draw a regular pentagon.
- Mark one vertex.
- Draw straight lines connecting to other vertices.
- What do you observe?

Example 1

Find the sum of interior angles of a regular hexagon.

Solution

Method 1



A hexagon has 6 sides. So, n = 6

Sum of interior angles

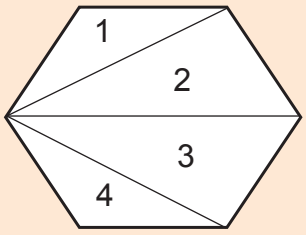
$= (n - 2) \times 180^o$

$= (6 - 2) \times 180^o$

$= 4 \times 180^o$

$= 720^o$

Method 2 (triangular method)



1 triangle $= 180^o$

4 triangles $= 4 \times 180^o$

$= 720^o$

The sum of interior angles of a regular polygon is 720^o.

Method 3

Each exterior angle $= \frac{360^o}{6}$

$= 60^o$

Interior angle + exterior angle = 180^o

Each interior angle $= 180^o$ – exterior angle

$= 180^o - 60^o$

$= 120^o$

A hexagon has 6 interior angles.

$= 6 \times 120^o$

$= 720^o$

Example 2

Find the number of sides of a polygon whose interior angle sum is 360^o

Solution

Sum of interior angles $= (n - 2) \times 180^o$

$360^o = (n - 2) \times 180^o$

$360^o = 180^o n - 360^o$

$360^o + 360^o = 180^o n$

$720^o = 180^o n$

$\frac{720^o}{180^o} = \frac{180^o}{180^o} n$

$4 = n$

Therefore the polygon has 4 sides.

Study tip

- Sum of interior angle is equal to each interior angle multiplied by the number of interior angles.
- 2 is subtracted from the number of sides because 2 sides do not form a triangle.
- Sum of interior angle = $(n - 2) \times 180^{o}$ where (n) is the number of sides.
- Interior angle sum of each triangle is 180^{o}. Number of triangles in a polygon x 180^{o} = interior angle sum of a polygon.

Application 12.5

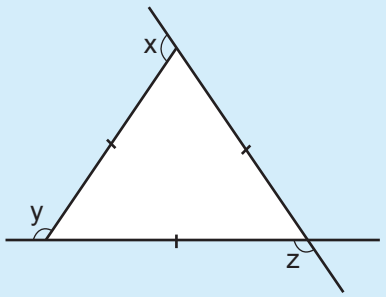
1. Find the sum of interior angles of:
   (a) Octagon (b) Heptagon (c) Nonagon
   (d) Decagon (e) Equilaterial Triangle (f) Rectangle
   (g) Square (h) Hexagon (i) Pentagon
2. Find the sum of interior angles of polygons whose number of sides are:
   (a) 11 sides (b) 15 sides (c) 17 sides
   (d) 8 sides (e) 10 sides (f) 9 sides
   (g) 14 sides (h) 4 sides (i) 20 sides
3. Find the number of sides of polygons whose interior angles sum is:
   (a) 360^{o} (b) 540^{o} (c) 720^{o}
   (d) 900^{o} (e) $1,260^{o}$ (f) $1,440^{o}$

12.6 Exterior angles of regular polygons and their sum

Activity

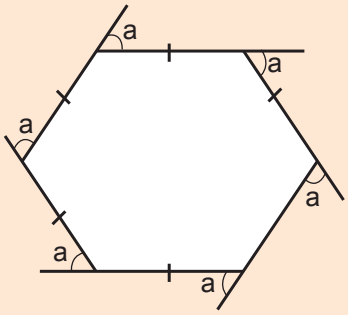
Use a protractor to measure angles x, y, and z.

- Find the sum of the exterior angles.
- What do you get?
- Present your findings to the class.



Example 1

Work out the value of a.



Solution

Sum of exterior angles is 360°

$$a + a + a + a + a + a = 360^\circ$$
$$6a = 360^\circ$$
$$\frac{6a}{6} = \frac{360^\circ}{6}$$
$$a = 60^\circ$$

Example 2

Find the number of sides of a regular polygon with an exterior angle of 45°. Name the polygon.

Solution

Each exterior angle = 45°.

$$\text{No. of sides} = \frac{\text{Exterior angle sum}}{\text{Each exterior angle}}$$
$$= \frac{360^\circ}{45^\circ}$$
$$= 8 \text{ sides}$$

The polygon is an octagon.

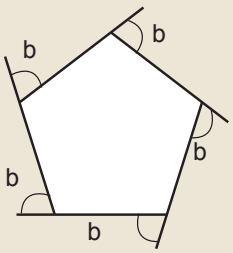
Study tip

- The sum of exterior angles of a regular polygon is 360°.
- Number of sides = 360° divided by one exterior angle.

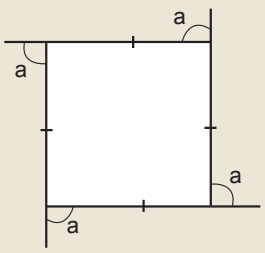
Application 12.6

1. Find the size of the exterior angles of the following regular polygons:

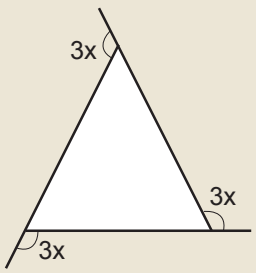
(a)



(b)



(c)



2. Find the number of sides of a regular polygon whose exterior angle is 60°.
3. Find the exterior angle of each of the following regular polygons:
   (a) Octagon (b) Decagon (c) Pentagon
   (d) Nonagon (e) duo-decagon

4. The size of each interior angle of a regular polygon is twice its exterior angle.
   (a) Find the exterior angle and interior angle.
   (b) Name the polygon.
5. Find the exterior angle of polygons whose interior angles are:
   (a) 120^o (b) 135^o

12.7 Finding sides and apothem

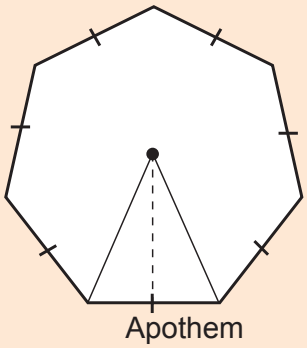
Activity

- Draw a regular hexagon and mark its centre.
- Locate the middle of one side of the polygon.
- Draw a straight line from the centre to the point you located on the side
- Take a ruler and measure the line.
- Explain the procedure you have used and record the result obtained.

Example 1

Draw a regular heptagon and show the apothem

Solution



Example 2

Find the apothem of an equilateral triangle whose side is 10 cm and the area is 60 cm^2.

Solution

Side = 10 cm

Area = 60 cm^2

Apothem = ? cm

$$\text{Apothem} = \frac{2 \times \text{Area}}{\text{Perimeter}}$$

Perimeter = Sum of all sides = 3 sides

$$= \frac{2 \times 60}{10 \times 3} \text{ cm}^2$$

$$= 4 \text{ cm}$$

Study tip

- Apothem is the length or distance from the centre of a polygon perpendicular to the side of a polygon.
- Apothem = $\frac{2 \times \text{Area}}{\text{Perimeter}}$.

Application 12.7

1. Differentiate between apothem and side of a regular polygon.
2. Draw a regular nonagon and show the apothem.
3. Draw a regular hexagon and show the apothem.
4. Find the apothem of a pentagon whose side is 8 cm and area is 120 cm^2.
5. Find the apothem of a hexagon whose side is 12 cm and area is 144 cm^2.
6. Find the apothem of a octagon whose side is 10 cm and area is 160 cm^2.
7. What is the apothem of a regular decagon with a side of 8 cm and area of 240 cm^2.

12.8 Finding perimeter of regular polygons

Activity

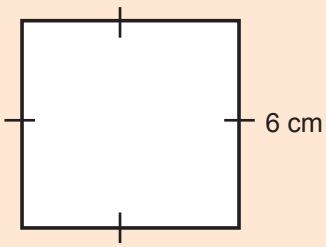
- Take a metre ruler.
- Measure the length of the sides of the classroom.
- Find its perimeter.
- State the formula for finding perimeter.
- Present your findings to the class.

Example 1

Find the perimeter of a square whose side is 6 cm.

Solution

To find the perimeter (p) add all sides of the given polygon.



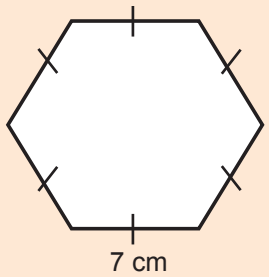
P = sum of all sides
P = 6 + 6 + 6 + 6 cm
= 24 cm

Example 2

Find the perimeter of a hexagon whose side is 7 cm.

Solution

To find the perimeter (p) add all sides of the given polygon.



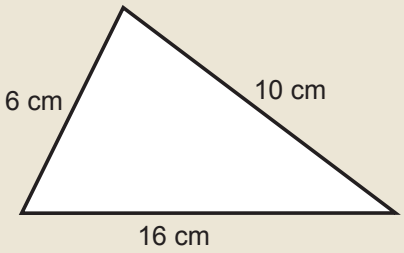
P = sum of all sides
P = 7 + 7 + 7 + 7 + 7 + 7 cm
= 42 cm

Study tip

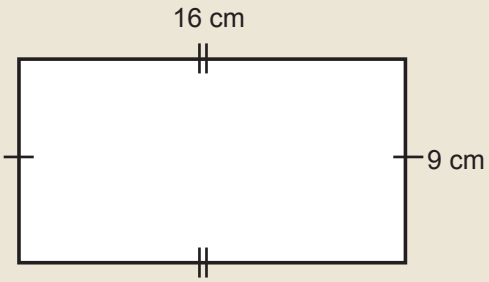
- To find perimeter of regular polygons, add the lengths of its sides.
- Perimeter is the distance round a shape or polygon.

Application 12.8

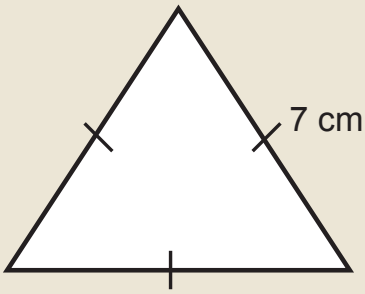
1. Find the perimeter of a triangle with sides 6 cm, 10 cm and 16 cm.



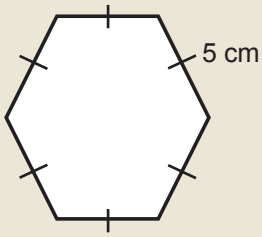
2. A rectangle has length of 16 cm and width of 9 cm. Find the perimeter.



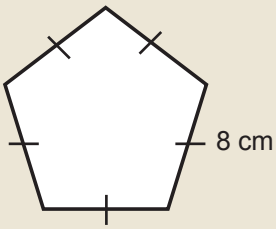
3. Find the perimeter of an equilateral triangle with sides of 7 cm.



4. Find the perimeter of a regular hexagon of sides 5 cm.



5. Find the perimeter of a regular pentagon of sides 8 cm.



6. Calculate the perimeter of a regular octagon with a side of 4.5 cm.
7. Find the perimeter of a regular duo-decagon with one of the sides 10 cm.
8. What is the perimeter of a regular 15 sided polygon with one of the sides 6.2 cm.
9. Find the perimeter of a regular nonagon with side 12.8 cm.
10. Find the perimeter of a regular triangle with side 4 cm.

12.9 Finding area of regular polygons

Activity

- Take a piece of paper and fold it.
- Cut it to make 2 equal parts but make sure that the 2 parts are triangles.
- Using a ruler, measure their sides.
- Find the area of each triangle.
- Discuss how you get the area of a regular polygon.

Example 1

Find the area of a regular pentagon whose side is 6 cm and apothem is 4 cm.

Solution

$$\text{Area} = \frac{\text{Apothem x Perimeter}}{2}$$

$$= \text{Apothem} = 4 \text{ cm}$$

$$\text{Side} = 6 \text{ cm}$$

$$\text{Perimeter} = 6 + 6 + 6 + 6 + 6 = 30 \text{ cm}$$

$$\text{Area} = \frac{\cancel{4}^{2} \times 30}{\cancel{2}_{1}}$$

$$= 2 \text{ cm} \times 30 \text{ cm}$$

$$= 60 \text{ cm}^2$$

Example 2

The side of a regular nonagon is 10 cm. Its apothem is 7 cm. Find its area.

Solution

A nonagon has 9 sides

$$\text{Perimeter} = 9 \times \text{sides}$$

$$= 9 \times 10 \text{ cm}$$

$$= 90 \text{ cm}$$

$$\text{Area} = \frac{1}{2} \times \text{apothem} \times \text{perimeter}$$

$$= \frac{1}{\cancel{2}_{1}} \times 7 \text{ cm} \times \cancel{90}^{45} \text{ cm}$$

$$= 7 \times 45 \text{ cm}$$

$$= 315 \text{ cm}^2$$

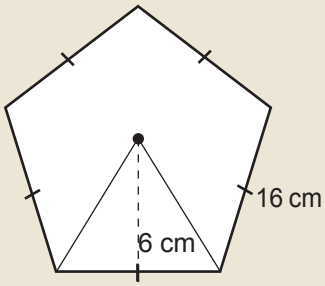
Study tip

- Area of a regular polygon consists of how many squares it is made of. We calculate it basing on the formula of the squares.
- The formula is: Area = $\frac{\text{Apothem x Perimeter}}{2}$.
- OR area = $\frac{1}{2}$ × apothem × perimeter

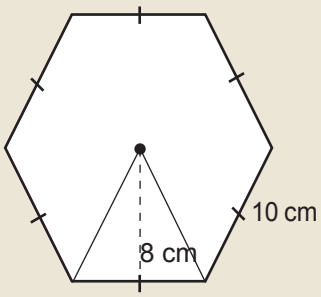
Application 12.9

1. Find area of an equilateral triangle whose side is 10 cm and apothem is 4 cm.
2. What is the area of a square garden with a perimeter of 24 m and apothem is 6 cm?
3. Mugisha wants to plant a forest of 20,000 trees. If he wants to plant 4 trees on 4 m^2, on which area will he plant his forest?
4. ABC is an equilateral triangle which has a side of 12 cm and apothem of 6 cm. Find its area.
5. Find the area of the following polygons:

(a)



(b)

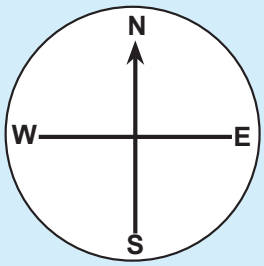


6. A regular polygon has 12 sides. Each is 10 cm. It has apothem of 8 cm. Calculate its area.
7. Find the area of a regular octagon with sides 12 cm and apothem of 10 cm.
8. A regular polygon has 15 sides. Each side is 18 cm. It has apothem of 14 cm. Calculate its area.

12.10 Finding bearings and compass points

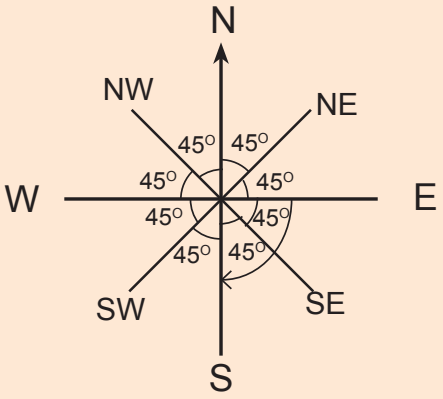
Activity

- Get a pair of compasses, a ruler and pencil.
- Draw a circle.
- Draw in the circle a compass direction.
- Name the 4 cardinal points.
- Measure the angles between the lines of compass direction.



Example 1

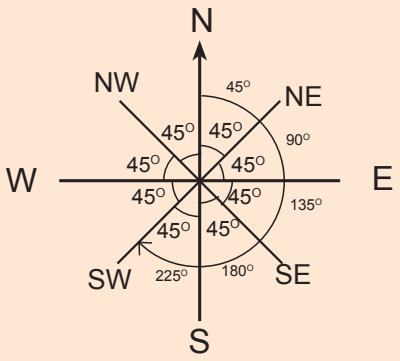
Face East and turn clockwise through 90°. What is the new direction?



The new direction is **South**.

Example 2

Agaba was facing in the North and turned clockwise through 225°. What is the new direction?



Agaba's new direction is **South west**.

Study tip

- A compass shows directions.
- The cardinal points of a compass have 90° between them.
- A compass has angle sum of 360°. It has 8 angles.
- Each angle of a compass $= \frac{360^\circ}{8} = 45^\circ$.
- Clockwise turn is a right turn and anti-clockwise turn is the left turn.
- To find direction of one point from another, the angle of turn is given.
- When the point is in North, start from North. That is, N angle E or N angle S.
- If the point is in South, start from South. That is S angle E or S angle W.

Application 12.10

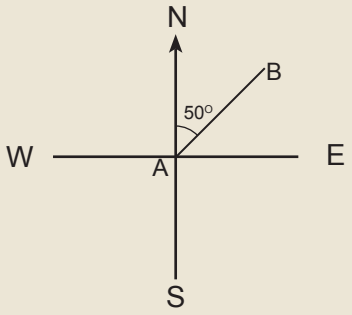
1. Find the new direction.
   (a) Clockwise 135° from West.
   (b) Clockwise 180° from North-East.
   (c) Stella turned clockwise from North through 135°. She then turned anti-clowise through 225°. Find her final direction.
   (d) Anti-clockwise 90° from East.

(e) Anti-clockwise 45° from North.

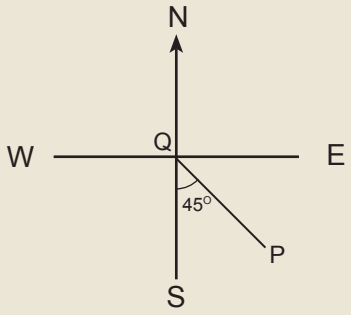
(f) Anti-clockwise 105° from West.

2. Find the direction of:

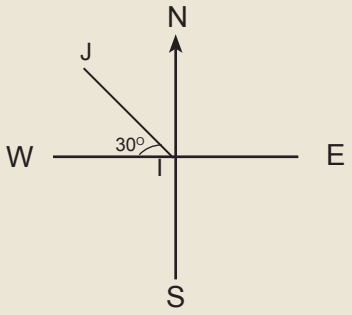
(a) B from A



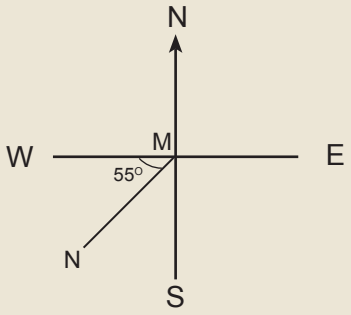
(b) P from Q



(c) J from I



(d) N from M

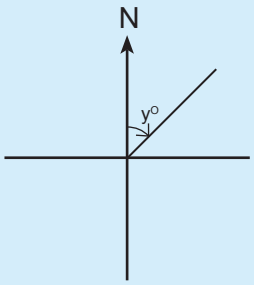


12.11 Finding the bearing

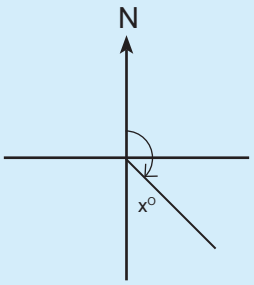
Activity

- Measure the following angles using a protractor

(a)



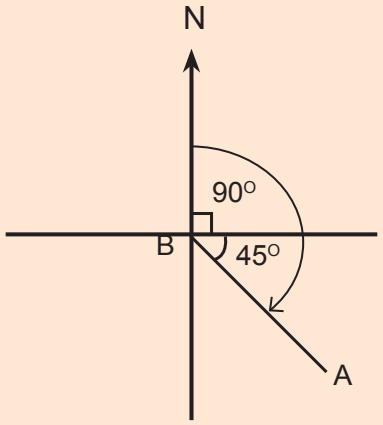
(b)



- Record and discuss your findings to the class.

Example 1

Find bearing of town A from town B.

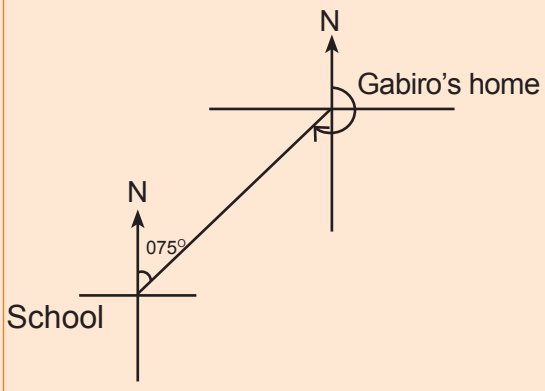


Bearing = 90 + 45 = 135°.

The bearing of town A from B is 135°

Example 2

The bearing of Gabiro's home from school is 075°. Find the bearing of the school from Gabiro's home.



Bearing of school from Gabiro's home

= 90° + 90° + 75° = 255°

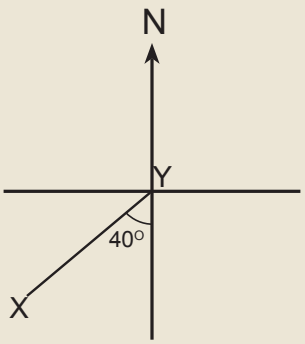
Study tip

- To get the bearing, face North and only turn clockwise. Find the total angle of turn.
- The angle is written with 3 digits. For example, 10° = 010°.
- Degrees of bearing do not involve cardinal points, e.g., 060° not N60°E.

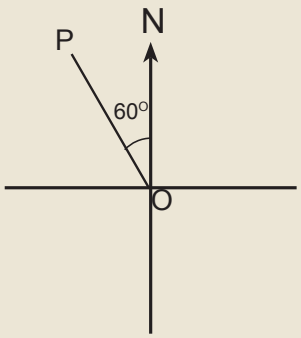
Application 12.11

1. Find the bearing of:

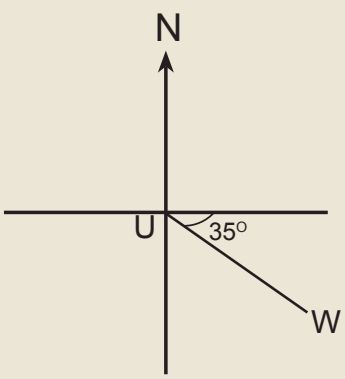
(a) X from Y



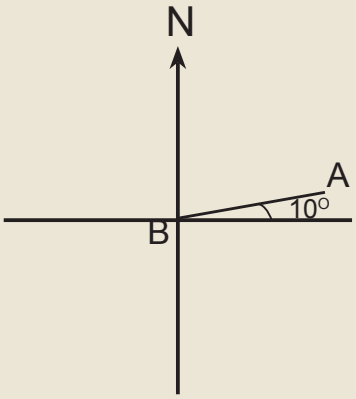
(b) P from O



(c) W from U

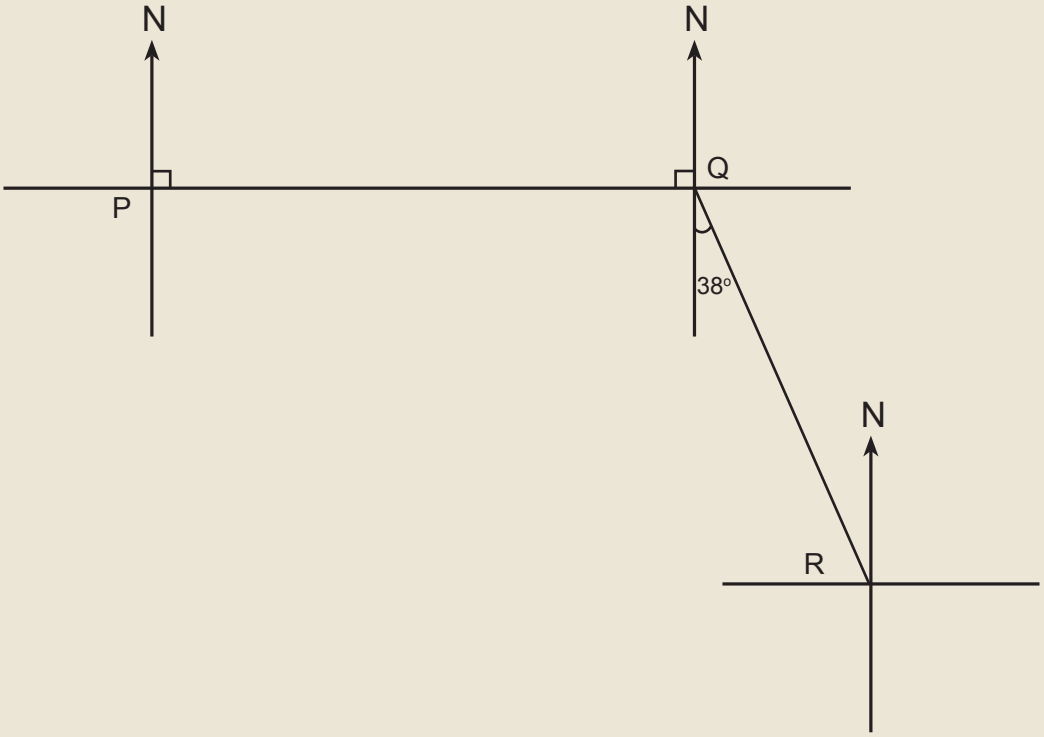


(d) A from B



2. The bearing of town A town from town B is 082°. Find the bearing of town B from town A.

3.



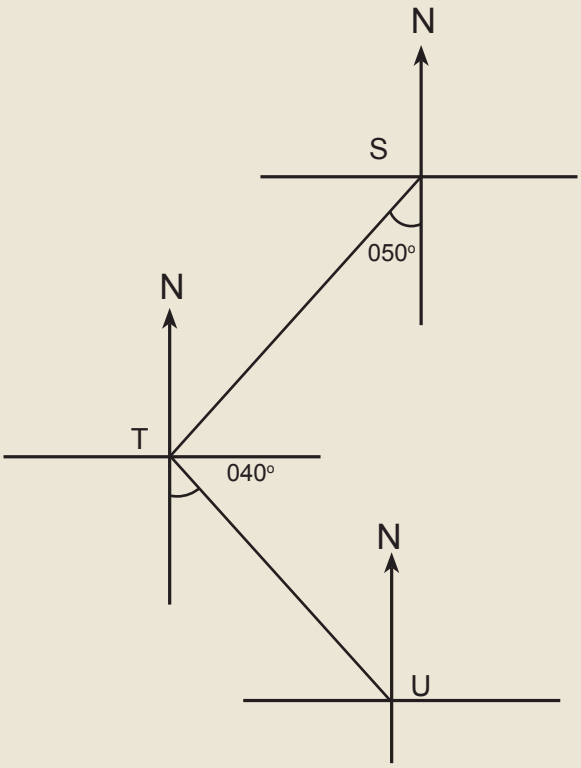
(a) Find the bearing of Q from P.

(b) What is the bearing of P from Q?

(c) Find the bearing of R from Q.

(d) What is the bearing of Q from R?

4.



(a) Find the bearing of T from S.
(b) What is the bearing of S from T?
(c) Find the bearing of U from T.
(d) What is the bearing of T from U?

12.12 Tiling/Construction

Activity

- Get sheets of paper.
- Use different polygon cards prepared for the lesson.
- Using glue, fix suitable polygon cards in a pattern such that no gaps are left in between.
- Name the activity you are carrying out.
- Which polygon cards can be used to tile the plane? (Rectangular sheet)
- How is tiling important in daily life?
- Present your working out to class.

Example 1

A square floor measures 9 m by 9 m. A builder tiles the floor with square tiles of sides 30 cm each.

(a) Explain how he would carry out tiling.

(b) How many tiles does he use?

(c) How does he use the interior angle?

Solution

(a) He should lay the tiles in straight lines across the floor. He should fit the edge of one tile exactly to fit the edge of the other tile, along all edges. The floor should have a squared pattern/design.

(b) Along one side of the room he uses:

$$= \frac{\text{Length of room}}{\text{Length of tile}}$$

$$= \frac{3\cancel{9} \times 100\ \cancel{cm}}{1\ \cancel{30}\ \cancel{cm}}$$

$$= 30 \text{ tiles}$$

Along the other side of the room he lays the same number of tiles, i.e, 30 tiles.

Altogether the tiles used are: side x side = (30 x 30) tiles = 900 tiles.

(c) The interior angles help the builder to fix the joint perfectly. So the tiles are fixed onto each other firmly.

Study tip

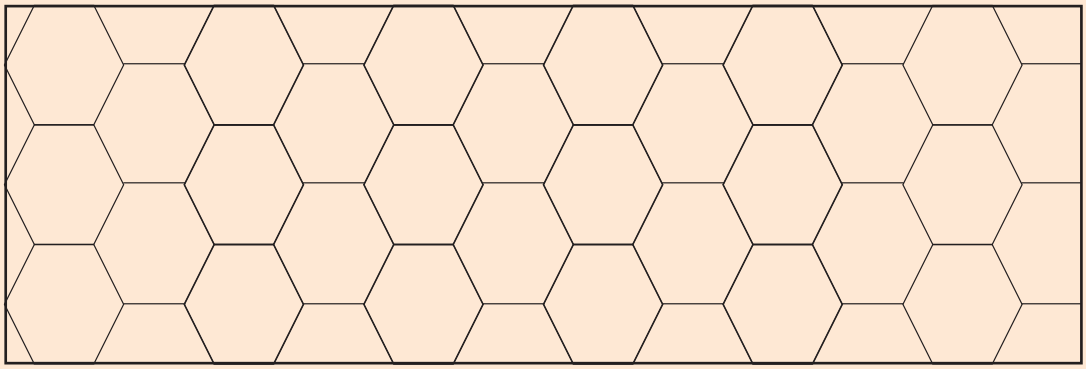
- Tiling is done to cover planes like floors, walls, compounds to form beautiful designs. Square, equilateral triangular, regular pentagonal, hexagonal and other shapes of tiles are used for tiling.
- Tiling a plane requires the dimensions of the tiles to fit perfectly the area of the plane.
- Designers measure the sides of a plane to calculate the number of tiles to be used.

Example 2

Draw a tiling pattern for regular hexagonal pavers each of sides 1 cm and interior angle 120°.

Solution

- Use a ruler to measure 1 cm sides and protractor to measure interior angle of 120°.
- Fix the hexagons firmly along the 120° vertices.

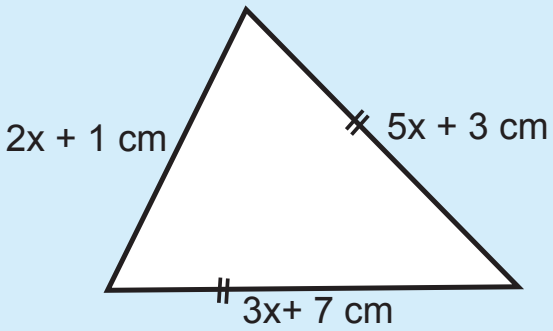


Application 12.12

1. Use regular pentagonal cards of sides 3 cm to tile a plane sheet of paper. How many cards do you use?
2. Use regular hexagonal cards of sides 2 cm. Tile your table top or desktop.
3. Visit a construction side or buildings. Study the tiling.
   (a) What polygons were used?
   (b) Copy the tiling design in your books.
4. A room had a floor measuring 9 m by 6 m. A builder laid tiles on the floor measuring 30 cm by 30 cm. How many tiles did the builder lay?
5. A manson laid hexagonal pavers on a compound. Each paver was 40 cm across. The compound was 40 m by 20 m.
   (a) Draw a tiling pattern for the above.
   (b) How many pavers were laid on the compound?
6. Tiling of square tiles was done on a floor. Each tile had a side of 25 cm. The floor measured 12 m by 10 m.
   (a) How many tiles were laid on the floor?
   (b) Draw a tiling pattern for the above.

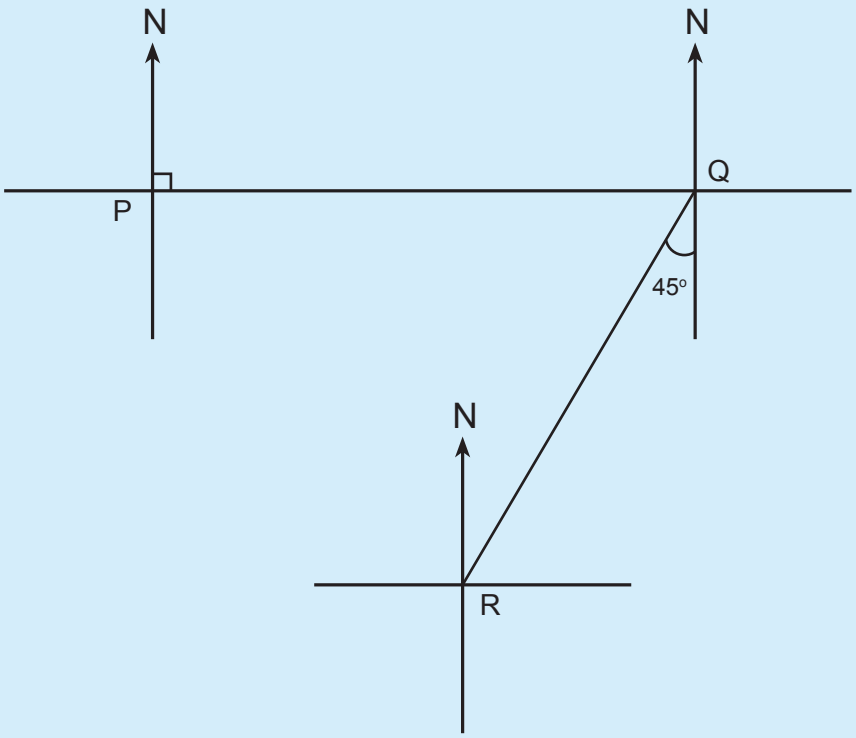
End of unit 12 assessment

1. Find the sum of interior angles of the following and show your working out.
   (a) Decagon (b) heptagon (c) Octagon
   (d) Pentagon (e) triangle (f) hexagon
2. Find the number of sides of regular polygons whose interior angle sums are:
   (a) 540^o (b) $1{,}620^o$ (c) 720^o
3. The exterior angles of a pentagon are: $(k + 5^o)$, $(2k + 3)^o$, $(3k + 2)^o$, $(4k + 1)^o$ and , $(5k + 4)^o$ respectively. Find the measure of each angle.
4. The perimeter of a regular pentagon is 120 cm. How long is its side?
5. Find the area of this figure if the apothem is 4 cm.



6. What is the angle between North and North-east?
7. A regular decagon of a side 12 cm has apothem of 18 cm. Calculate its area.
8. The bearing of Butare from Byumba is 270^o. Find the bearing of Byumba from Butare town.
9. Calculate the perimeter of a regular decagon of side 4.5 cm.
10. The interior angle sum of regular polygon is 180^O. Find its number of sides.
11. The bearing of town A from town B is 090^O. Find the bearing of town B from A.
12. A square room has sides 4 cm. Square tiles of sides 40 cm were laid on its floor.
    (a) Show the tiling pattern in your book.
    (b) How many tiles were laid?

13.



(a) What is the bearing of Q from P?

(b) Find the bearing of R from Q.

(c) What is the bearing of Q from R?

(d) Find the bearing of P from Q.

14. The bearing of Kabarore from Kigali is 035°. What is the bearing of Kigali from Kabarore?

15. Kibeho is on a bearing of 220° from Busasamana. Find the bearing of Busasamana from Kibeho?

Unit 13 Construction of polygons and nets for cuboids and prisms

Key unit competence: To be able to construct polygons using a protractor, a ruler and a pair of compasses. Design nets to make cuboids and prisms.

Introduction

Boxes, rooms, bars of soap, etc are all cuboids with faces and edges. Some of these cuboids have volume or space to be used as storage of things. For example houses are built with rooms to accommodate people. To build a room of a house, the builders measure the sides of the floor and height of walls suitable to the owner.

Can you think on how the area of the faces of one room is measured?

To be concrete, use a box and try to calculate the total surface area of all 6 sides of a box (remember that one face is a rectangular shape). What do you notice?

13.1 Drawing triangles using a protractor and ruler

Activity

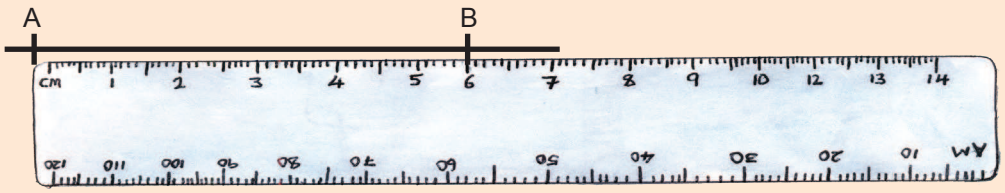
- Using a protractor and ruler, draw a right angle, angle 65° and angle 120°.
- Explain your working steps.
- Present your work to the rest of the class.

Example 1

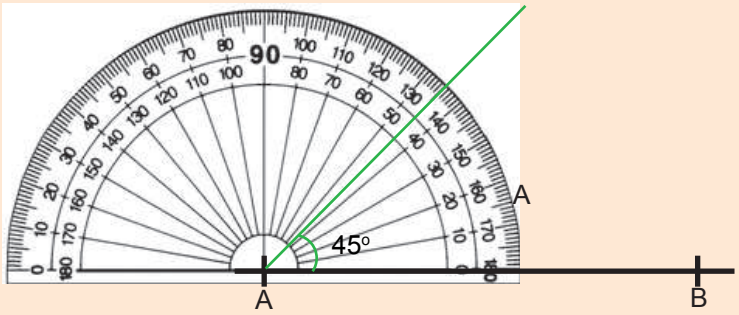
Using a protractor and ruler, draw a triangle ABC with AB = 6 cm, AC = 4 cm and angle BAC = 45°.

Solution

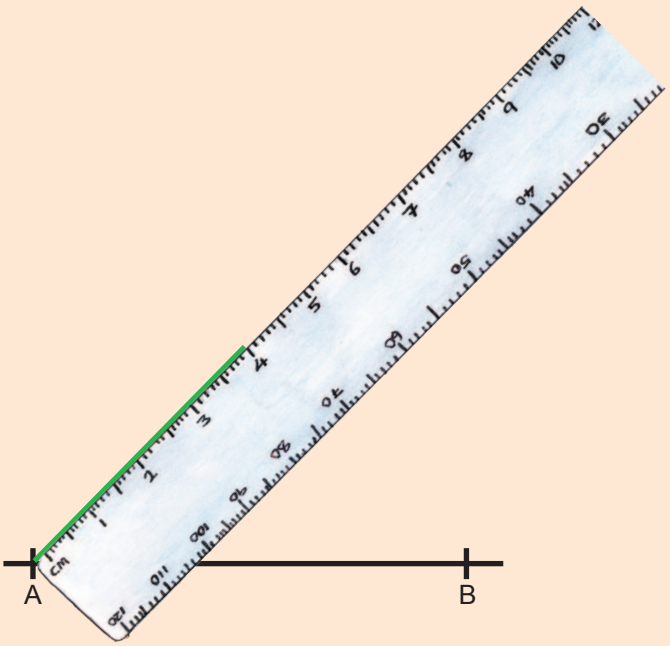
Step 1: Draw a baseline and mark on it line segment AB (6 cm) apart using a ruler.



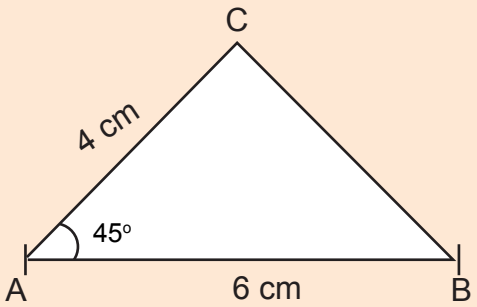
Step 2: Measure angle BAC = 45°. Use a protractor.



Step 3: Measure 4 cm of the 45° line from A. Use a ruler and label it C.



Step 4: Join B to C. Measure BC and name the triangle ABC.



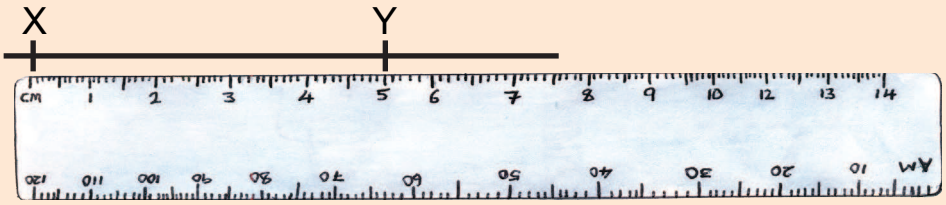
BC = 4.2 cm.

Example 2

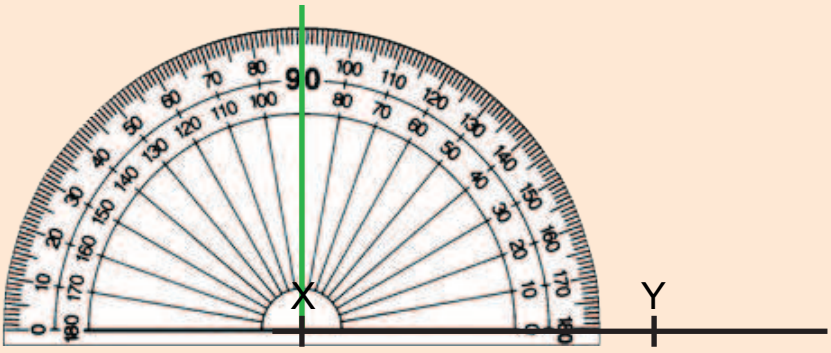
Draw triangle XYZ with XY = 5 cm, ∠ZXY = 90° and ∠XYZ= 60°. Use a ruler and a protractor.

Solution

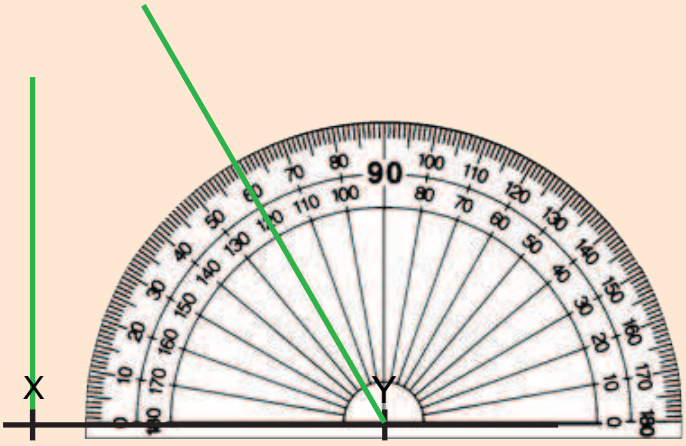
Step 1: Use ruler to draw a baseline and mark on it line segment XY = 5 cm apart.



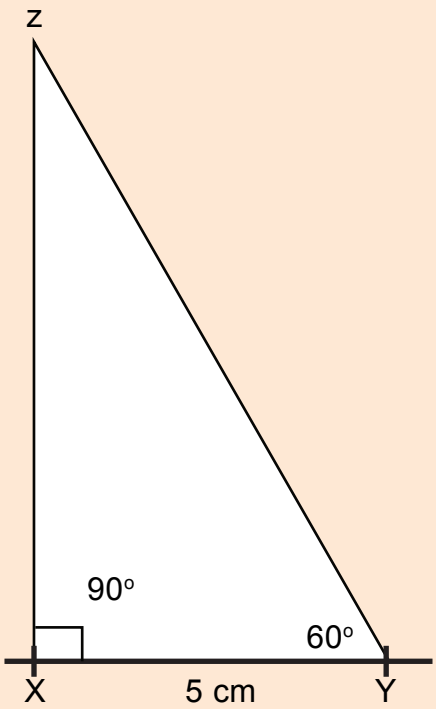
Step 2: Use a protractor to measure ∠90° at X.



Step 3: Use a protractor to measure ∠60° at Y.



Step 4: Join the 90° line to 60° line. Label the meeting point Z. Name the △XYZ.



Study tip

- Polygons are figures which are made up of more than two lines. For example: triangle, rectangle, square, pentagon and so on.
- Polygons are closed figures with straight sides.
- Use ruler to draw lines and protractor to draw angles.
- The symbol for angle is ∠ and for triangle is △.

Application 13.1

Using ruler, protractor and a pencil, construct the following triangles:

1. ABC with AB = 6 cm, ∠ABC = 60° and BC = 4 cm.
2. PQR with P = 50°, Q = 30° and PQ = 7 cm.
3. MNO with ∠M = ∠N = 50° and MN = 5 cm.
4. ABC with AB = 7.2 cm, AC = 5.5 cm and ∠CAB 75°.
5. BCD with BC = 5 cm, ∠BCD = 80° and BD = 6 cm.
6. KLM with ∠X = 80°, KL = 5.5 cm and ∠L = 35°

13.2 Drawing a square using a protractor and ruler

Activity

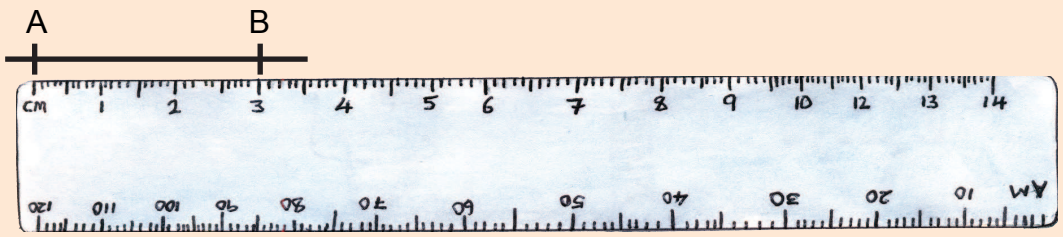
- Get a piece of paper.
- Draw the ground floor of the classroom.
- Name the shape drawn.
- Use a protractor to check that all angles are 90°.

Example

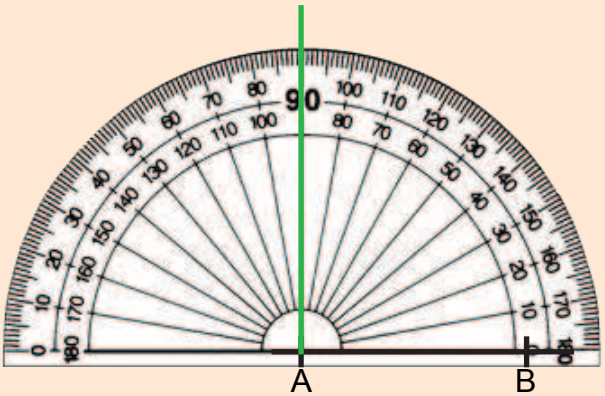
Using ruler and a protractor, draw a square ABCD with sides 3 cm.

Solution

Step 1: Use ruler to draw a baseline AB 3 cm apart.

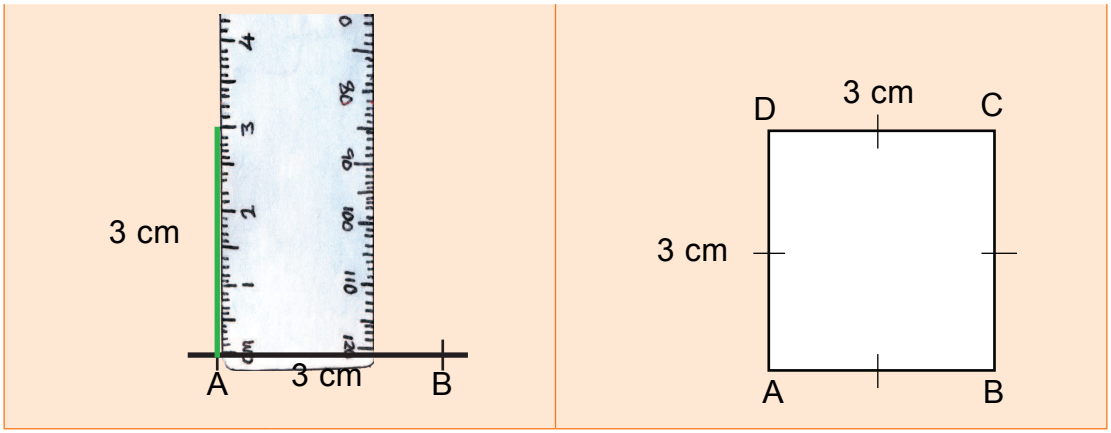


Step 2: Using protractor, measure ∠90° at A and B.



Step 3: From A, measure 3 cm along the 90° line. Label it D. Do the same at B and label it C.

Step 4: Join D to C and name the square ABCD.



Study tip

- A square has 4 equal sides.
- Angles of a square measure 90°.
- Use a sharp pencil to draw neat and accurate squares.



Application 13.2

Use ruler and protractor to draw the following:

(a) Square PQRS of side 5.5 cm.

(b) Square ABCD of side 4 cm.

(c) Square DEFG of side 3.8 cm.

(d) Square ABCD of side 4.2.

(e) Square WXYZ of side 7 cm.

(f) Square LKMN of side 9 cm.

13.3 Drawing a rectangle using a protractor and ruler

Activity

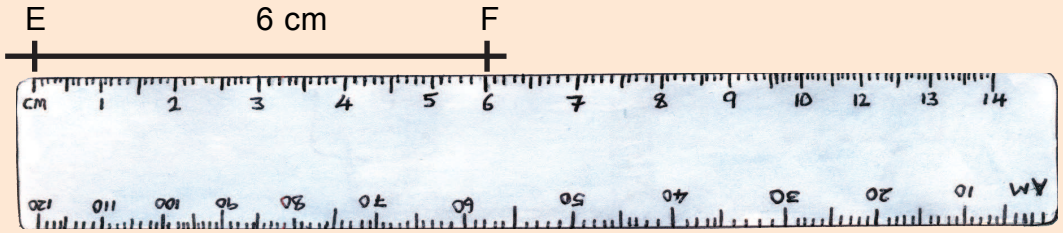
- Pick a sheet of paper.
- Using a protractor, measure the four angles.
- Write down the results.
- Measure the four sides.
- What do you observe?
- Make a class presentation.

Example

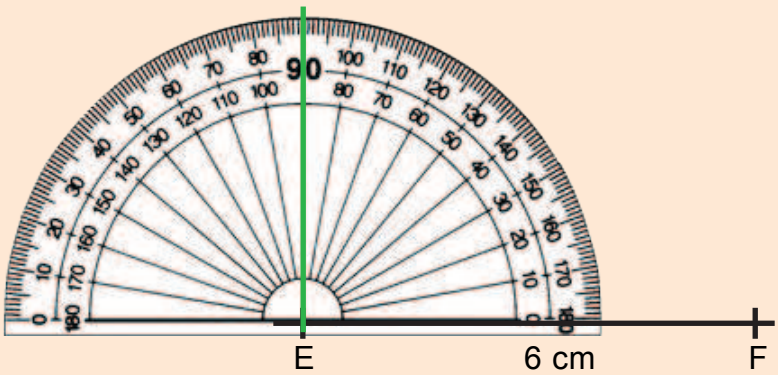
Draw a rectangle EFGH with length 6 cm and width 4 cm.

Solution

Step 1: Use ruler to draw a baseline EF 6 cm apart.

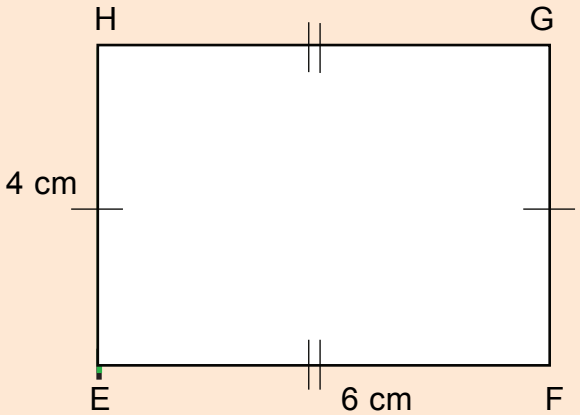


Step 2: Using protractor, measure ∠90° at E and F. Draw a straight line from E through the 90° mark.



Step 3: From E, measure 4 cm along the 90° line. Label it H. Do the same at F and label it G.

Step 4: Join H to G with a straight line.



Measure ∠H and ∠G. Each must be equal to 90°.

Measure line HG. It must be equal to 6 cm.

Denote the angles with 90° symbol (⌐).

Denote equal opposite sides with equal marks.

Name the rectangle EFGH.

Study tip

- Two opposite sides of a rectangle are equal.
- Angles of a rectangle measure 90° each.
- Use a sharp pencil and protractor to draw an accurate rectangle.



Application 13.3

Use ruler and protractor to draw the following:

(a) Rectangle ABCD with sides AB = 5.7 cm and BC = 3 cm.

(b) Rectangle DEFG with sides DE = 6.2 cm and EF = 3 cm.

(c) Rectangle HIJK with sides HI = 5.6 cm and IJ = 3 cm.

(d) Rectangle LMNO with sides LM = 6 cm and MN = 4.2 cm.

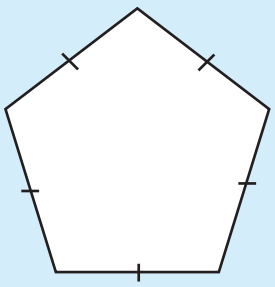
(e) Rectangle PQRS with sides PQ = 7.2 cm and QR = 4.8 cm.

(f) Rectangle TUVW with sides TU = 7 cm and UV = 4.4 cm.

13.4 Drawing a regular pentagon using a protractor and ruler

Activity

- Study the figure shown and answer questions that follow.
- Name the figure
- Measure its exterior angle.
- Measure its interior angle
- Present your work to the class.



Example 1

Use a protractor and ruler to draw a regular pentagon PQRST with side 3 cm.

Solution

Step 1: Find the interior angle. To get the interior angle, first find the exterior angle.

$$\angle\text{exterior} = \frac{360^\circ}{\text{Number of sides}}$$

$$= \frac{\cancel{360^\circ}^{60}}{1\ \cancel{6}} = 72^\circ$$

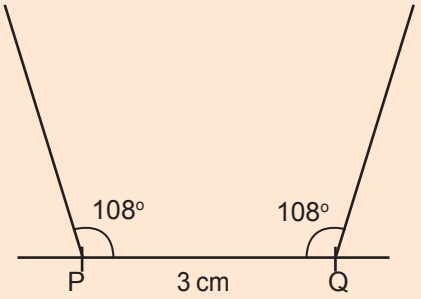
$$= 72^\circ$$

$$\angle\text{interior} = 180^\circ - \text{exterior angle}$$

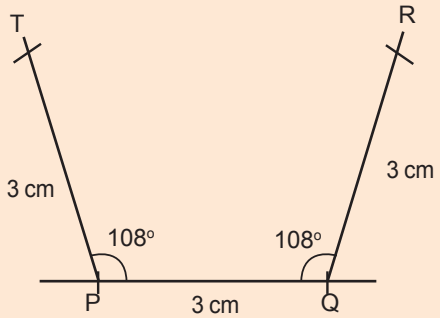
$$= 180^\circ - 72^\circ$$

$$= 108^\circ$$

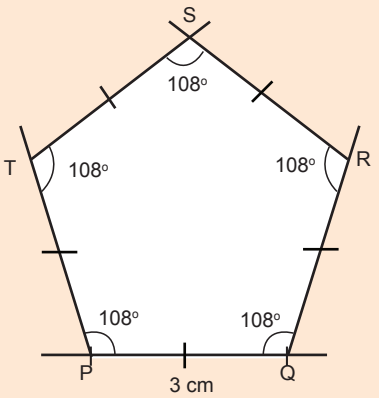
Step 2: Draw the baseline and mark off PQ (3 cm) apart. Use a protractor and draw 108° at P and Q.



Step 3: From P, measure 3 cm along the 108° line. Label it T. Do the same at Q and label it R.



Step 4: Draw 108° at T and R. Label the point where the two line meet S. Name the pentagon PQRST. Then indicate that all sides are equal.



Study tip

- To construct a pentagon, hexagon and other regular polygons, first find the exterior angle then the interior angle.
- Always start with the baseline.
- Draw equal sides using ruler.



Application 13.4

Using a protractor and a ruler, draw the following regular pentagons:

(a) Pentagon ABCDE of side 5 cm.
(b) Pentagon BCDEF of side 5.5 cm.
(c) Pentagon PQRST of side 6.5 cm
(d) Pentagon ABCDE of side 4 cm.
(e) Pentagon PQRST of side 6.8 cm.
(f) Pentagon FGHIJ of side 8.2 cm.

13.5 Drawing a regular hexagon

Activity

- Pick a pencil, ruler and a sheet of paper.
- Without measuring, draw a six sided figure. Name it.
- Measure the sides and angles.
- What do you observe?
- What should you do to draw a regular six sided figure?
- Discuss and share with other groups.

Example 1

Using a protractor and ruler, draw a regular hexagon STUVWX of side 3.5 cm.

Solution

Step 1: Find the interior angle. To get the interior angle, first find the exterior angle.

$$\angle\text{exterior} = \frac{360^\circ}{\text{Number of sides}}$$

$$= \frac{\cancel{360^\circ}^{60}}{1\ \cancel{6}} = 60^\circ$$

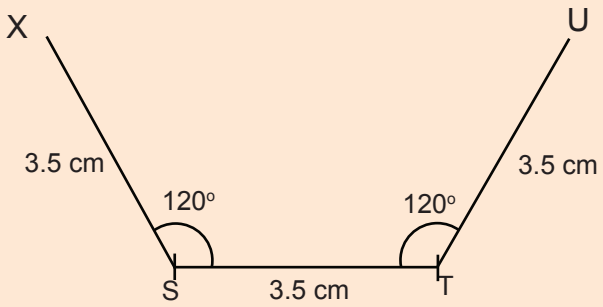
$$= 60^\circ$$

$$\angle\text{interior} = 180^\circ - \text{exterior angle}$$

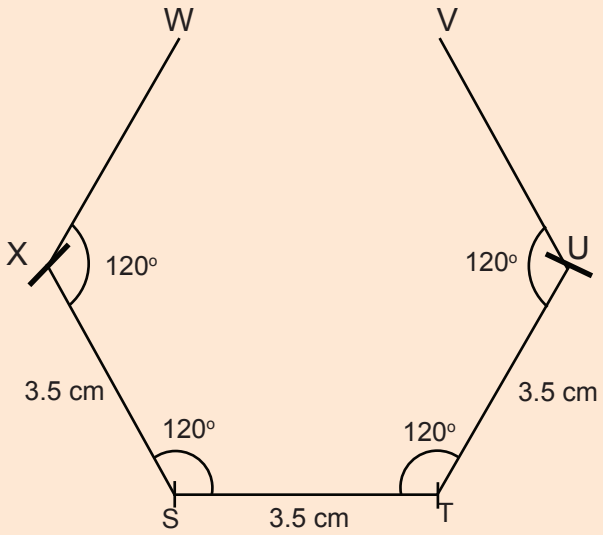
$$= 180^\circ - 60^\circ$$

$$= 120^\circ$$

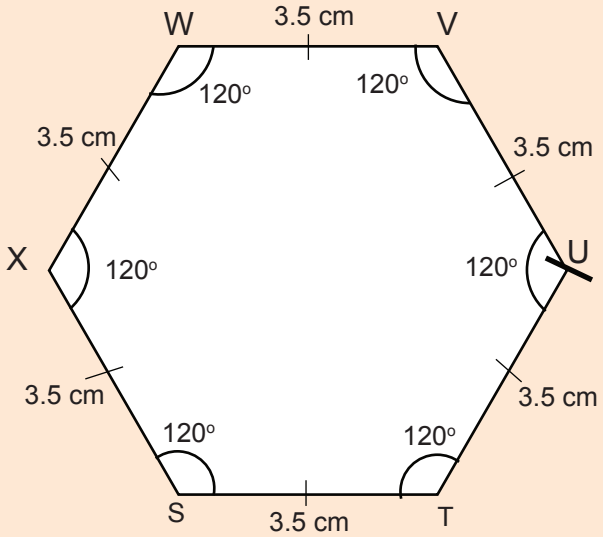
Step 2: Draw baseline ST 3.5 cm apart.
Use a protractor to draw 120° at S and T.
Measure 3.5 cm from S and T along the 120° lines. Label the point X and U.



Step 3: Draw 120° at X and U. Measure 3.5 cm from X and U along the 120° line. Label the points W and V.



Step 4: Draw a straight line joining W and V. Measure ∠W and ∠V to confirm they are 120° each. Indicate that all sides are equal. Name the polygon.



Study tip

- A regular hexagon has 6 equal sides.
- A regular hexagon has 6 interior and 6 exterior equal angles.
- Each exterior angle of a regular hexagon is 60°.
- Each interior angle of a regular hexagon is 120°.



Application 13.5

Using a protractor and ruler, draw the following regular hexagons:

(a) Hexgon ABCDEF of side 4 cm.

(b) Hexgon GHIJKL of side 4.2 cm.

(c) Hexgon KLMNOP of side 5 cm.

(d) Hexgon PQRSTU of side 5.3 cm.

(e) Hexgon DEFGHI of side 3.8 cm.

(f) Hexgon STUVWX of side 4.5 cm.

13.6 Constructing triangles using a pair of compasses and a ruler

Activity

- Whats is an equilateral triangle?
- Using ruler and compasses only, construct an equilateral triangle with side 3 cm.
- Present your working out to the class.

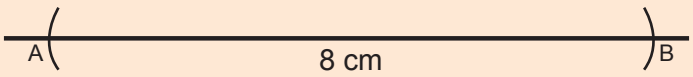
Example 1

Construct triangle ABC with AB = 8 cm, BC = 7 cm and AC = 5 cm. Use ruler and compasses only.

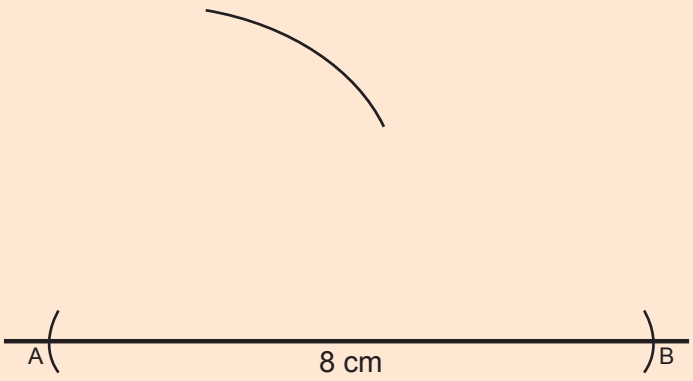
Solution

Step 1: Draw a baseline. Using a compass, measure 8 cm from ruler.

- Place a compass point at one side of the baseline.
- Make an arc along the baseline.
- Place the compass point at the drawn arc.
- Without adjusting the radius, draw an arc along the base line to the other side. Label the arcs A and B.

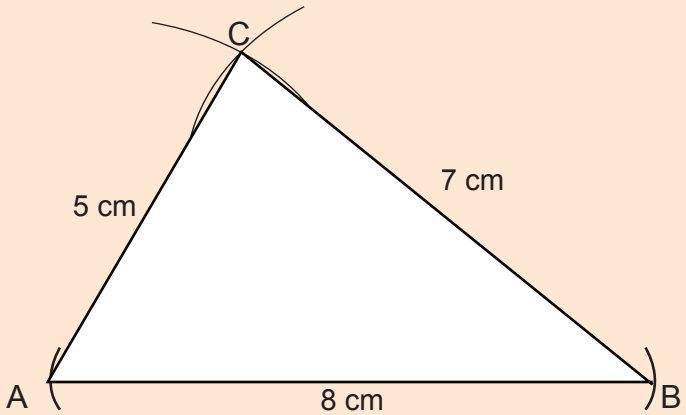


Step 2: Measure 5 cm from ruler. Place the compass point at A and make an arc above line AB.



Step 3: Measure 7 cm from ruler. Place the compass point at B and make an arc above AB to cross the arc down first.

Step 4: Label the point where the arcs cross each other C. Join A to C and B to C. Name the triangle ABC.

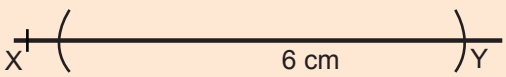


Example 2

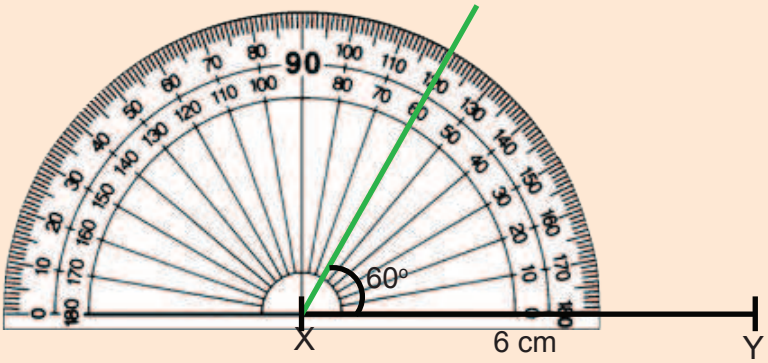
Construct triangle XYZ with XY = 6 cm, $\angle X = 60^{\circ}$ and XZ = 3 cm.

Solution

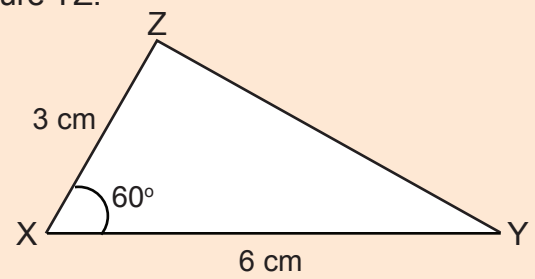
Step 1: Draw a baseline. Using a compass, measure 6 cm from the ruler. Place a compass point at one side of the baseline. Make an arc at the other side. Place the compass point at the drawn arc. Without adjusting the radius, draw an arc at the other side. Label the arcs X and Y.



Step 2: Construct ∠60° at X. Measure 3 cm from the ruler. Place the compass point at X and make an arc on the 60° line.



Step 3: Join Y to the marked arc on the 60° line. Label the point Z. Name the triangle XYZ. Measure YZ.



Study tip

- Use a sharp pencil to construct neat and accurate diagrams.
- To construct an equilateral triangle, measure and draw the baseline, then mark the same distances from both ends of the baseline to the top common vertex.
- To construct a triangle with all the sides given, measure and draw the baseline accurately. Construct the other two sides using a pair of compasses.
- To construct a triangle with 2 sides and 1 angle, construct in the order side, angle, side (SAS).
- To construct a triangle with 1 side and 2 angles, construct in the order side, angle, angle (SAA).

Application 13.6

Using compasses and ruler only, construct the following triangles:

1. Triangle ABC of sides 4.5 cm.
2. Triangle PQR with PQ = 6 cm, QR = 4 cm and PR = 3 cm.
3. Triangle XYZ with XY = 5.5 cm, ∠Y = 80° and YZ = 3.5 cm.

4. Triangle CDE with CD = 7.5 cm, angle C = 90° and angle CDE = 45°.
5. Triangle JKL with JK = 7 cm, KL = 6 cm and JL = 5 cm.
6. Triangle ABC with AB = 7 cm, AC = 6 cm angle A = 60°.
7. Triangle EFG with ∠GEF = 45°, ∠EFG = 60° and EF = 6.7 cm. Measure EG.

13.7 Constructing a rectangle using a pair of compasses and a ruler

Activity

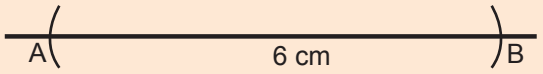
- Draw line segment XY of 5 cm.
- Construct 90° at X and another at Y.
- Measure 2 cm from X along the 90° line. Label it point D.
- Measure 2 cm from Y along the 90° line. Label it C.
- Draw the top side of 5 cm connecting D to C. Name the figure formed.

Example

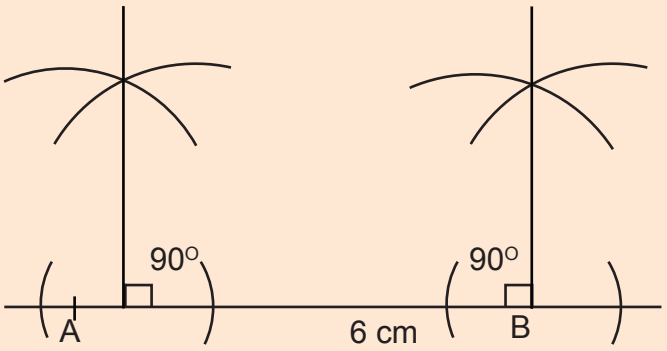
Construct a rectangle ABCD of AB = 6 cm and BC = 4 cm.
Use ruler and compasses.

Solution

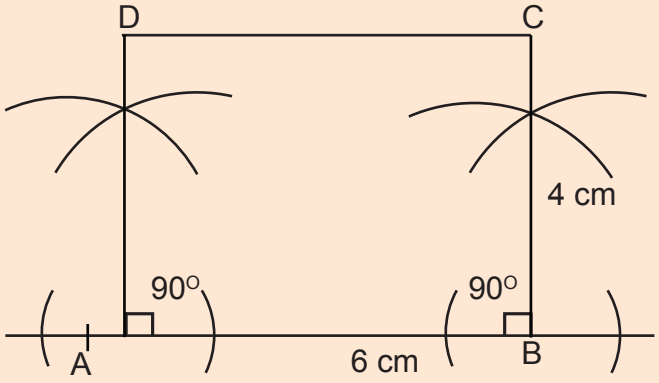
Step 1: Draw a baseline. Mark AB, 6 cm using compasses.



Step 2: Construct 90° at A and B. Now use a ruler to draw lines starting from A passing through the arcs. Draw another line from B through the arcs from B.



Step 3: Use a compass to measure and label points D and C from A and B of 4 cm respectively. Join D to C and name the rectangle ABCD.



Study tip

- To construct a rectangle, draw the baseline, construct the perpendicular widths and finally draw the opposite length.
- Use a sharp pencil and a properly fixed compass.



Application 13.7

Using compasses and ruler only, construct the following rectangles:

(a) Rectangle ABCD with length 5 cm and width 4 cm.

(b) Rectangle WXYZ with length 8 cm and width 5 cm.

(c) Rectangle ABCD with sides 5 cm, and 3.5 cm.

(d) Rectangle PQRS with sides 6.5 cm, and 4.2 cm.

(e) Rectangle CDEF with CD = 7.5 cm and DE = 2 cm

13.8 Constructing a square using a pair of compasses and a ruler

Activity

- Draw a circle on a sheet of paper. Mark the centre.
- Cut out the circle from the sheet of paper.
- Fold it into 2 equal parts through the centre.
- Draw a dotted line along the crease.
- Then fold the shape again into 2 equal parts, making quarters.
- Open and draw a dotted line along the crease.

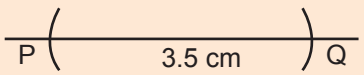
- Draw straight lines connecting the edges where the dotted lines touch the circle.
- Cut out the shape formed and measure the sides.
- Name the shape you have formed.
- Display your findings to the class and make a class presentation. Then poster the cut outs.

Example

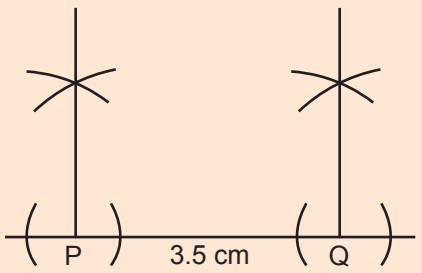
Construct a square of side 3.5 cm.

Solution

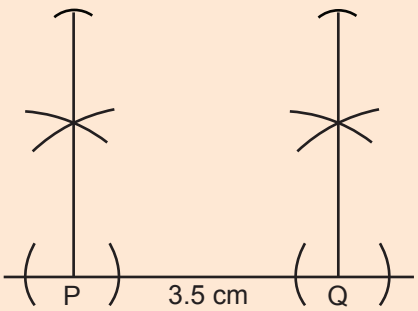
Step 1: Draw a line.
Measure 3.5 cm from the ruler.
Mark off two points 3.5 cm apart. Name the points P and Q.



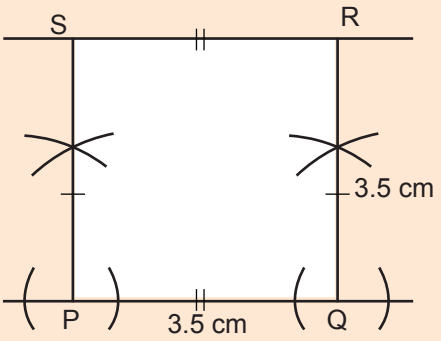
Step 2: Construct perpendicular lines at P, then at Q.



Step 3: Measure 3.5 cm from the ruler using compasses and a pencil. Mark PS 3.5 cm and QR 3.5 cm.



Step 4: Join R to S.
Name points R and S.



Study tip

- A square has 4 equal sides.
- Use a well sharpened pencil and a properly fixed pair of compasses.
- Measure to the accurate cm or mm.
- A square has 4 right angles.

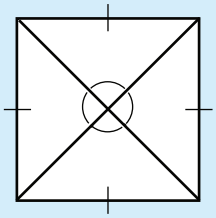
Application 13.8

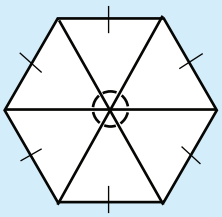
1. Construct squares with radii given below:
   (a) 3 cm (b) 4 cm (c) 5.5 cm (d) 7.2 cm
2. Construct a square with a side of 5.6 cm.
3. Construct a square with a side of 5.6 cm.
4. Construct a square with a side of 4.8 cm.

13.9 Finding the centre angle of a regular polygon

Activity

- Draw a square ABCD.
- Draw diagonals AC and BD. Where do they bisect each other?





- How many angles are formed at the centre. Name them.
- Measure them. What is their measure?
- Try with a regular hexagon. Carry out the same procedure.
- Draw lines connecting vertices through the centre. Count and measure them.
- How do the angles relate to the number of sides.
- Share the procedure with classmates and come up with a conclusion.

Example 1

Find the centre angle of a regular pentagon.

Solution

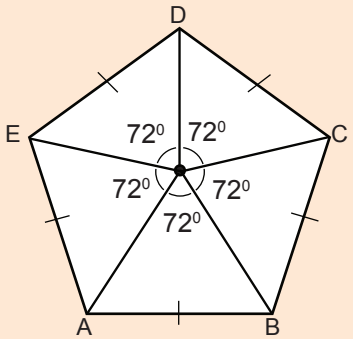
Angles at a point add up to 360°.

A pentagon has 5 centre angles.

$$\text{Each centre angle} = \frac{\text{Sum of centre angles}}{\text{Number of centre angles}}$$

$$= \frac{\cancel{360^\circ}^{\,60}}{\cancel{6}_{\,1}} = 72^\circ$$

The centre angle of a regular pentagon is 72°.



Example 1

Find the centre angle of a regular octagon.

Solution

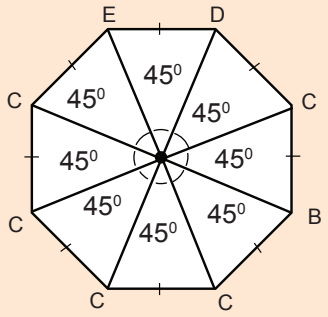
An octagon has 8 centre angles.

Angles at a point add upto 360°.

$$\text{Each centre angle} = \frac{\text{Sum of centre angles}}{\text{Number of centre angles}}$$

$$= \frac{\overset{45}{\cancel{360^\circ}}}{\underset{1}{\cancel{8}}} = 45^\circ$$

The centre angle of a regular octagon is 45°.



Study tip

- The centre angle of a regular polygon is equal to the sum of centre angles divided by the number of centre angles.
- The centre angle sum of a polygon is 360°.
- Number of centre angles = Number of exterior angles = Number of sides of a polygon.
- Centre angle = exterior angle of a regular polygon.
- Centre angle = $\frac{360^\circ}{\text{Number of centre angles}}$.



Application 13.9

Find the centre angles of the regular polygons below:

(a)	Equilateral triangle	(b)	Septagon	(c)	Nonagon
(d)	Decagon	(e)	Duo-decagon	(f)	Hexagon

13.10 Constructing a regular pentagon and regular hexagon

Activity

- Draw a circle on a sheet of paper.
- Mark the centre then draw its radius.
- Measure an angle of 72° at the centre.
- Draw another radius connecting to the circle. Name the point B.
- Place the compass needle at point A and the pencil at B. Using the same length, place at B to make an arc at C.
- From C, make an arc at D, then from D, make an arc to make point E.
- Using a ruler, connect the points A to B, B to C, C to D, D to E and E to A.

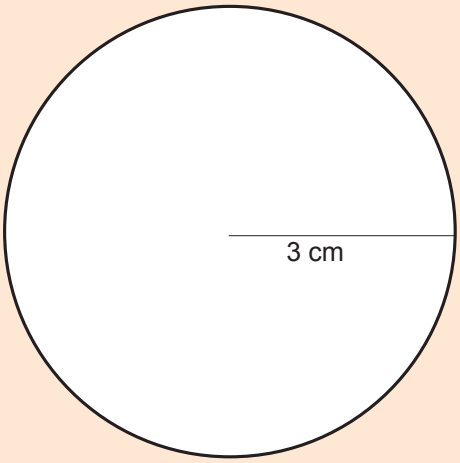
- How many sides has the polygon? Name the polygon.
- Measure each side. What is the measurement?
- Demonstrate and present to the class your procedure and findings.
- Carry out the same steps for constructing a regular hexagon but this time use the radius distance to make 6 arcs.
- Demonstrate and make a presentation to class.

Example 1

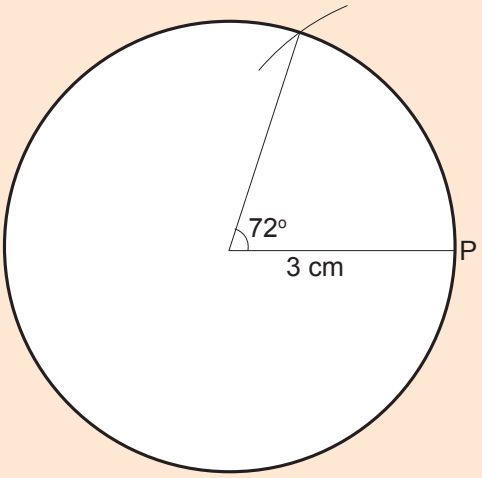
Construct a regular pentagon of radius 3 cm.

Solution

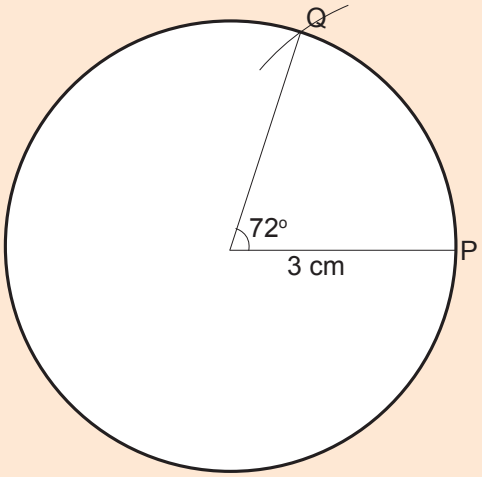
Step 1: Draw a circle of radius 3 cm.



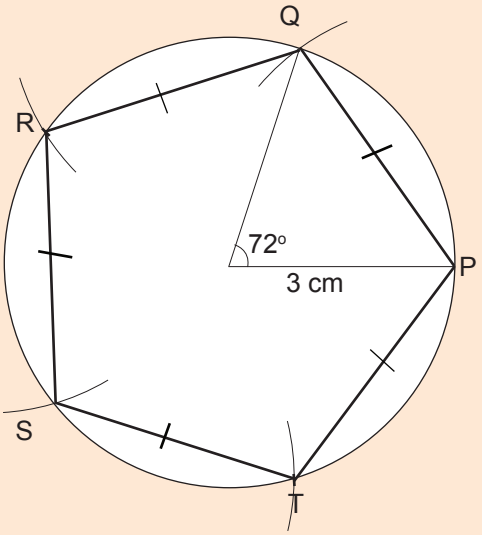
Step 2: Draw a centre angle of 72°.



Step 3: Place the compass needle at point P and the pencil point at point Q using the length AB, make arcs of equal length along the circle.



Step 4: Using a ruler, draw straight lines connecting the arcs to form edges. Name the vertices R, S and T.

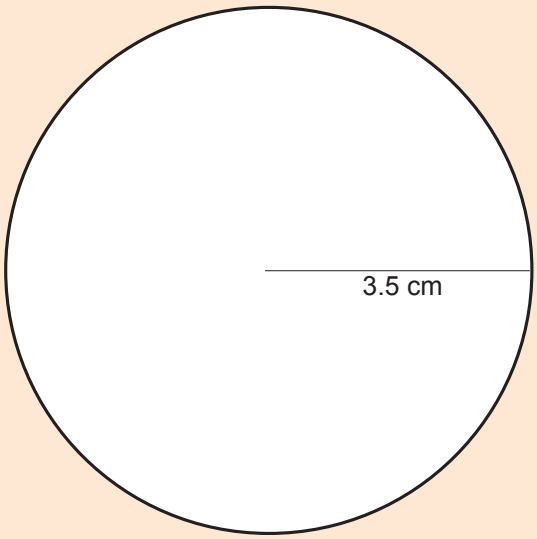


Example 2

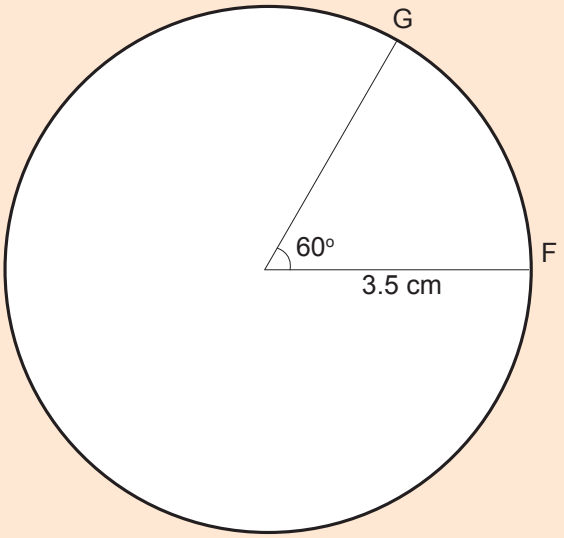
Construct a regular hexagon of radius 3.5 cm.

Solution

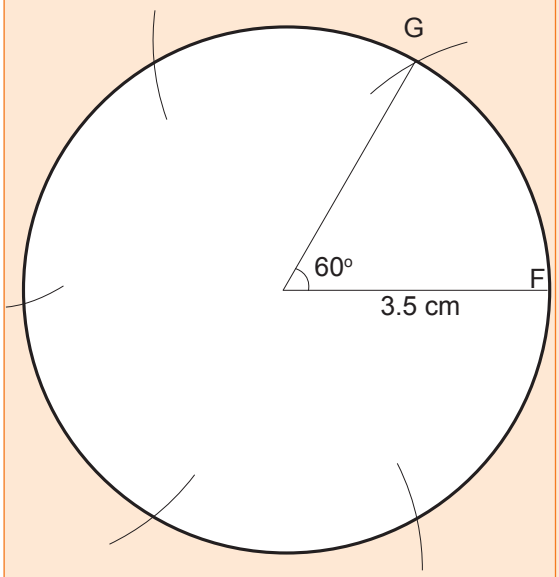
Step 1: Draw a circle of radius 3.5 cm.



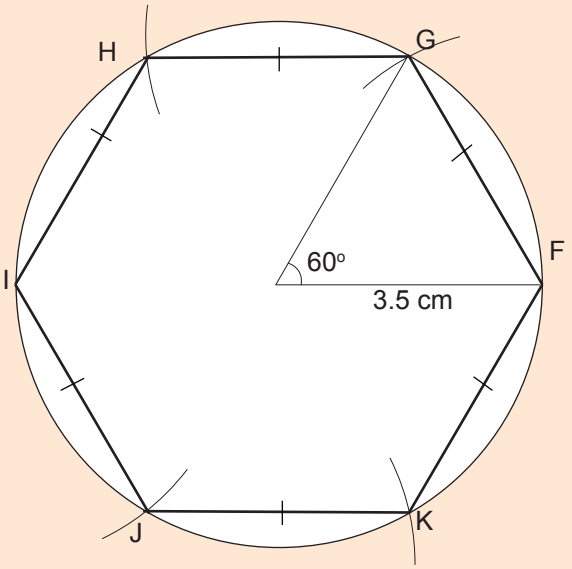
Step 2: Draw a centre angle of 60°.



Step 3: Place the compass needle at point F and the pencil point at G. Using the length FG, make arcs of equal length along the circle.



Step 4: Using a ruler, draw straight lines connecting the arcs to form edges. Name the vertices H, I, J, K.



Study tip

- The word pentagon means "a 5 sided figure".
- An irregular pentagon is a 5 sided figure with different measurements of its sides and centre angles.
- A regular pentagon has equal sides and equal centre angles of 72^{o}.
- The word hexagon means "a 6 sides figure".
- An irregular hexagon is a 6 sided figure with different measurements of its sides and centre angles.
- A regular hexagon has equal sides and equal centre angles of 60^{o}.
- Centre angle = exterior angle of a regular polygon.
- Centre angle = $\frac{360^{o}}{\text{Number of centre angles}}$.



Application 13.10

1. Construct regular pentagons with radii:
   (a) 3.5 cm (b) 4 cm (c) 4.8 cm
   (d) 5 cm (e) 5.5 cm (f) 7 cm
2. Construct regular hexagons with radii:
   (a) 3.6 cm (b) 4.2 cm (c) 4.5 cm
   (d) 5 cm (e) 5.6 cm (f) 8 cm

13.11 Constructing a regular septagon and a regular octagon

Activity

- Draw a circle on a sheet of paper.
- Mark the centre then draw its radius.
- Using a protractor, draw an angle of 45^{o} at the centre.
- Using compasses and pencil, measure the length of the arc along which the two radii from the centre connect.
- Make an arc where the 45^{o} line connects the circle.
- Do not change the radius of the compass and pencil.
- Place the compass point at the previous arc, and make another arc along the circle.
- Continue doing the same completely around the circle.
- Using a ruler, connect the arcs with straight lines.

- How many sides has the figure?
- Using a protractor, measure each interior angle. What do you notice?
- Write your observations and present to the class.

Example 1

Construct a regular septagon DEFGHIJ using radius 3.2 cm.

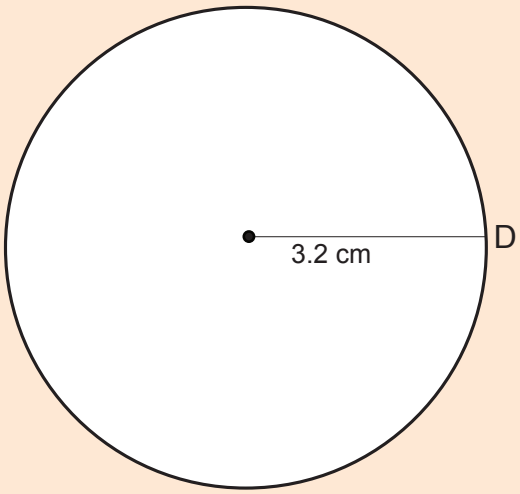
Solution

Step 1: Find the centre angle. A septagon has 7 equal sides. A septagon has 7 equal centre angles.

Each centre angle = $\frac{360^\circ}{7}$ 51.438 ..

Round it off to 51°.

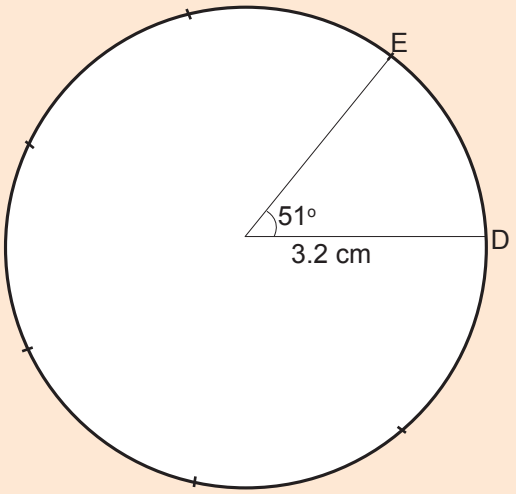
Step 2: Draw a circle of radius 3.2 cm. Name point D at the circle.



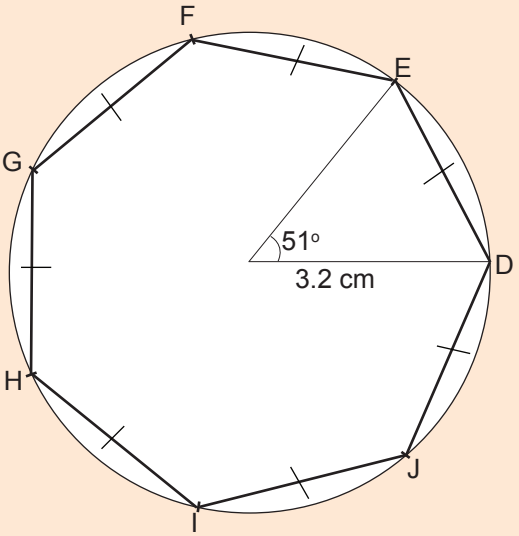
Step 3: Draw an angle of 51° at the centre. Name point E along the 51° line on the circle.

Step 4: Measure the length of the arc between the lines that form 51° along the circle. Make arc at C.

Without adjusting the radius between the compass point and pencil, make other arcs along the circle.



Step 5: Draw straight lines connecting the arcs along the circle.



Step 6: Label the remaining vertices FGHIJ and the sides with equal signs. The figure is a regular septagon.

Example 2

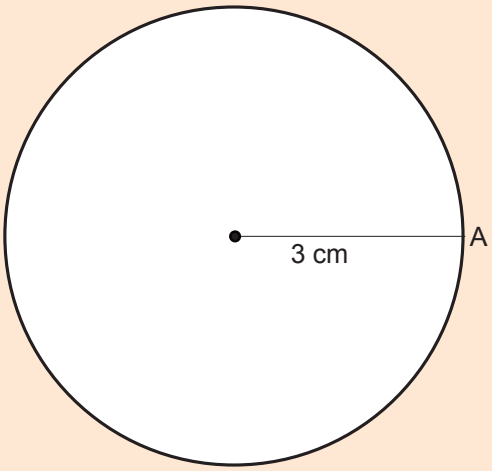
Construct a regular octagon ABCDEFGH using radius 3 cm.

Solution

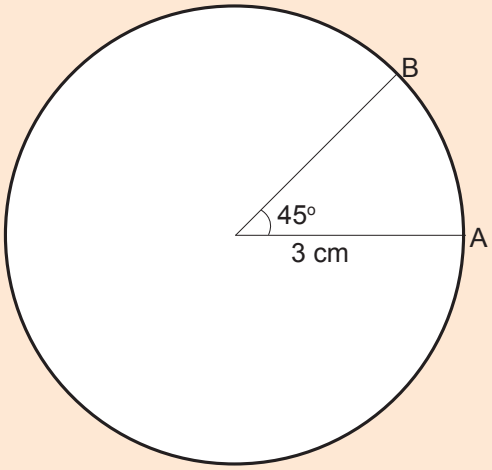
Step 1: Find the centre angle. An octagon has 8 centre angles.

Each centre angle = $\frac{360^{\circ}}{8} = 45^{\circ}$.

Step 2: Draw a circle of radius 3.5 cm. Name point A at the circle.

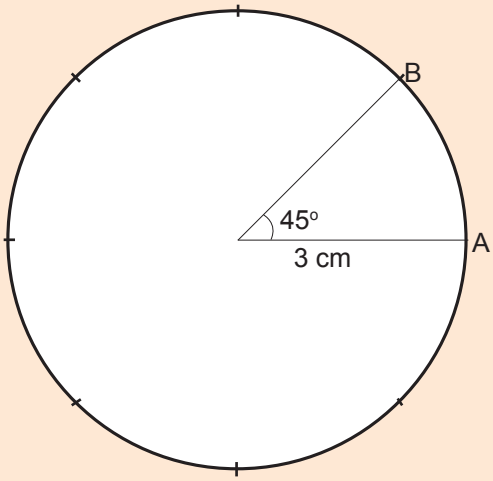


Step 3: Draw an angle of 45° at the centre. Name point B on the circle.

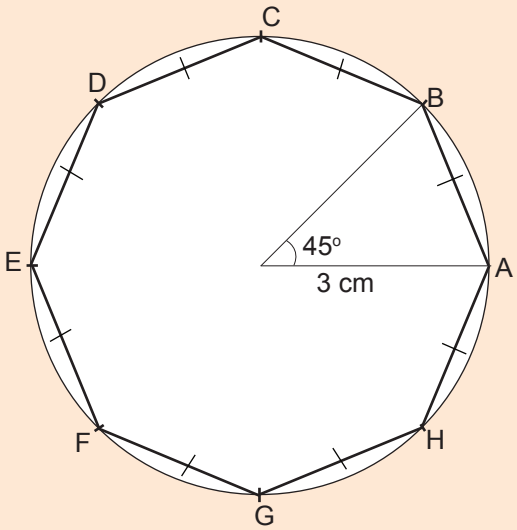


Step 4: Measure the length between radius and 45° line.

Make similar arcs along the circle without altering the length.



Step 4: Draw straight lines connecting the arcs along the circle. Label all the vertices with letters and equal marks.



The figure is an octagon.

Study tip

- The word septagon means “a 7 sided figure”.
- An irregular septagon is a 7 sided figure with different measurements of its sides and centre angles.
- A regular septangon has equal sides and equal centre angles of approximately 51^{o}.
- The word octagon means “an 8 sides figure”.
- An irregular octagon is an 8 sided figure with different measurements of its sides and centre angles.
- A regular octagon has equal sides and equal centre angles of 45^{o}.
- To construct regular septagon and octagon, first find the centre angle.
- Draw arcs at equal length along the circle, then join them with straight lines.
- Centre angle = exterior of a regular polygon.
- Centre angle = $\frac{360^{o}}{\text{Number of centre angles}}$.



Application 13.11

Using a ruler and compasses, construct the following regular polygons:

(a) Septagon of radius 3 cm.
(b) Septagon of radius 3.6 cm.
(c) Septagon of radius 4 cm.
(d) Octagon of radius 3.8 cm.
(e) Octagon of radius 4 cm.
(f) Octagon of radius 4.2 cm.

13.12 Constructing a regular nonagon and decagon

Activity

- On slips of paper, draw a circle of any radius.
- Measure a centre angle of 40^{o}.
- Using compasses and a pencil, measure the length of arc between the radius and 40^{o} line. Make an arc.
- Without adjusting the compass, draw similar arcs along the circle.
- Join the arcs with straight lines.
- How many sides has the figure?
- What do you observe about its sides?
- Measure the interior angles. What do you realise?
- Name the figure.

Example 1

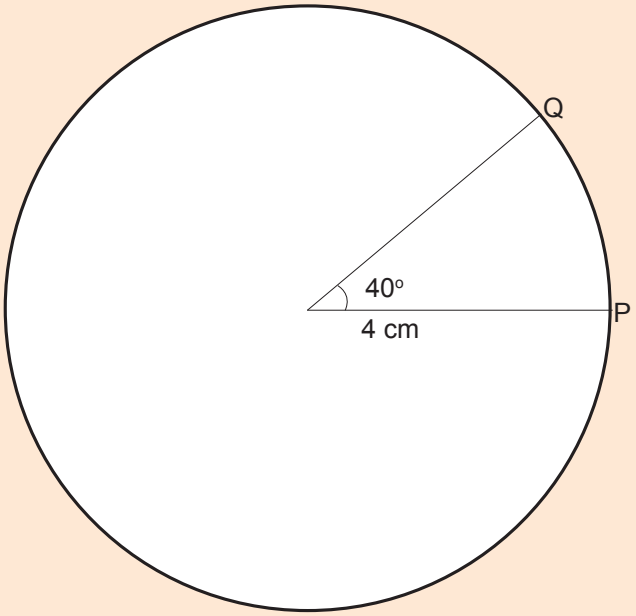
Using compasses and ruler, construct a regular nonagon PQRSTUVWX of radius 4 cm.

Solution

Step 1: Draw a circle of radius 4 cm. Name point P on the circle.

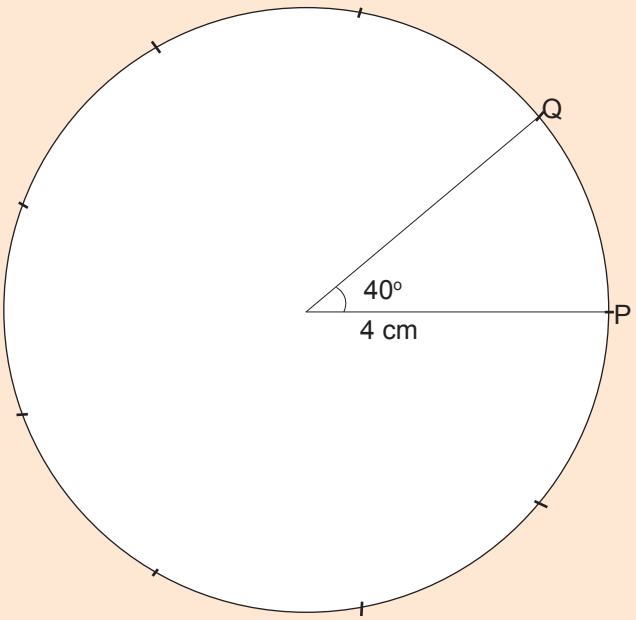
Step 2: Find the centre angle. Centre angle = $\frac{360^\circ}{9} = 40^\circ$.

Step 3: Draw a centre angle of 40° in the circle.

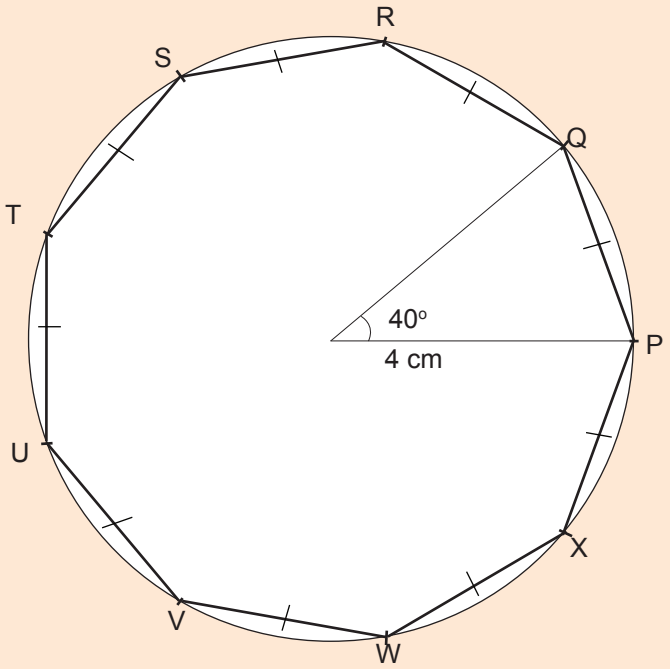


Step 4: Measure the distance between P and Q using compasses and a pencil. Draw an arc at Q.

Step 5: Without adjusting the compass, make other arcs along the circle.



Step 6: Using a ruler, draw straight lines connecting the arcs.



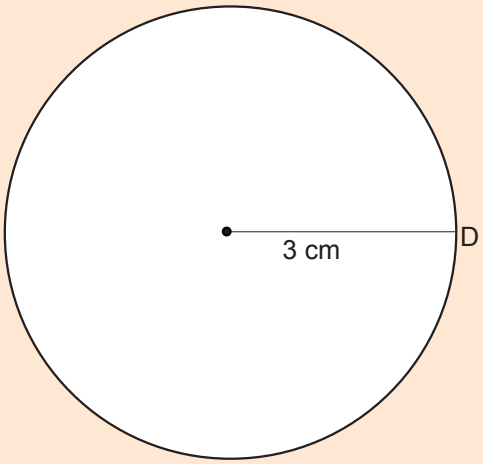
Step 7: Complete labelling the vertices. Mark sides with equal symbols. The regular polygon is a nonagon PQRSTUVWX.

Example 2

Construct a regular decagon of radius 3 cm.

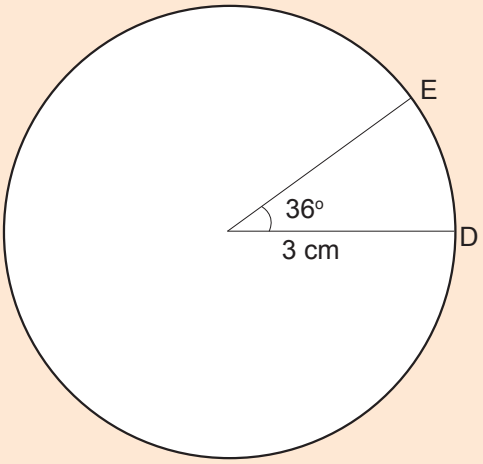
Solution

Step 1: Draw a circle of radius 3 cm.

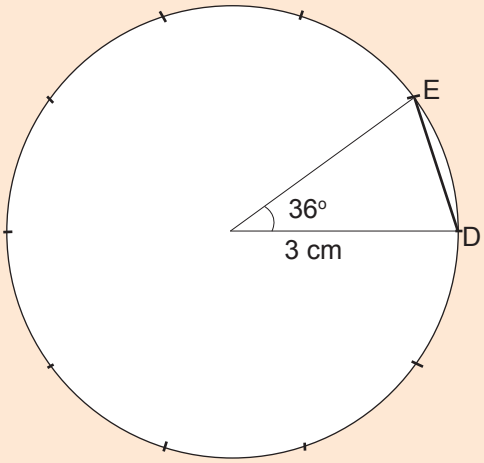


Step 2: Find the centre angle of a decagon. Centre angle = $\frac{360^{\circ}}{10} = 36^{\circ}$.

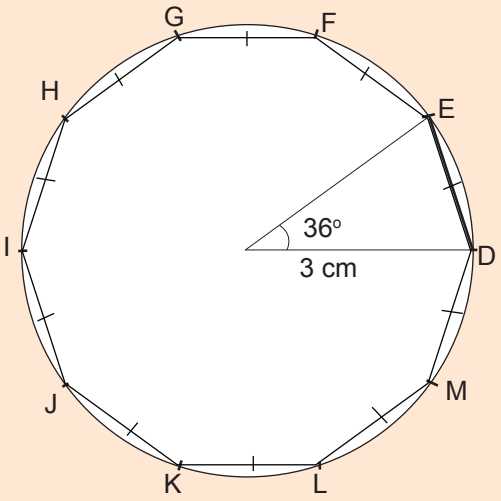
Draw an angle of 36° in the circle.



Step 3: Measure the length of arc DE. Using compasses and a pencil, make arcs along the circle.



Step 4: Join the arcs with straight lines.



Step 5: Complete labelling other vertices DEFGHIJKLM. Name the polygon. The polygon is a regular decagon.

Study tip

- The word nonagon means "a 9 sided figure".
- A regular nonagon has equal sides and equal centre angles of 40°.
- The word decagon means "a 10 sides figure".
- A regular octagon has equal sides and equal centre angles of 36°.
- To construct regular septagon and octagon, first find the centre angle.
- Draw arcs at equal length along the circle, then join them with straight lines.
- Centre angle = exterior of a regular polygon.
- Centre angle = $\frac{360^\circ}{\text{Number of centre angles}}$.



Application 13.12

1. Construct regular nonagons with radii:
   (a) 3.2 cm (b) 3.5 cm (c) 4 cm
   (d) 4.3 cm (e) 5.8 cm (f) 8 cm
2. Construct regular decagons with radii:
   (a) 2.8 cm (b) 3.4 cm (c) 4.5 cm
   (d) 6 cm (e) 5.6 cm (f) 10 cm

13.13 Designing nets of cuboids, cubes and prisms

Activity

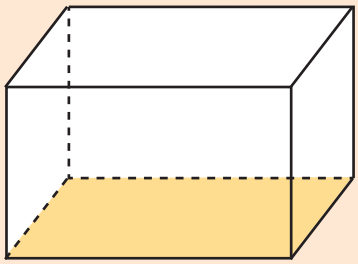
- Get an empty box of chalk and unfold it carefully.
- Display the flat shapes that were folded to form it.
- Draw the net of the flat shapes displayed.
- Present your working out to the rest of the class.

Study tip

- A net is a flat shape that folds up to make a prism.
- A prism is a three- dimensional figure with ends that are identical polygons.

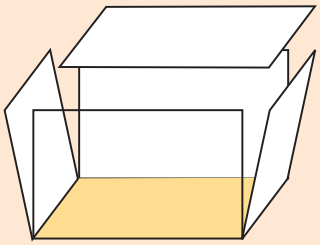
Example 1

Given the cuboid, display the flat shapes and draw its net.

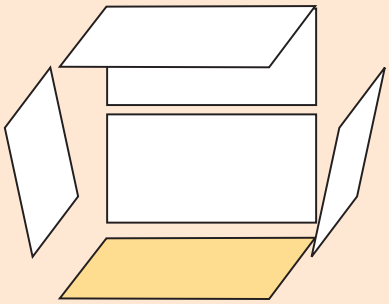


Solution

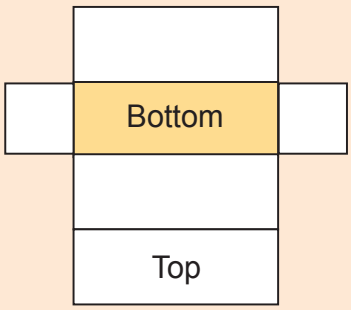
First unfold it to display the flat shapes.



Open the box



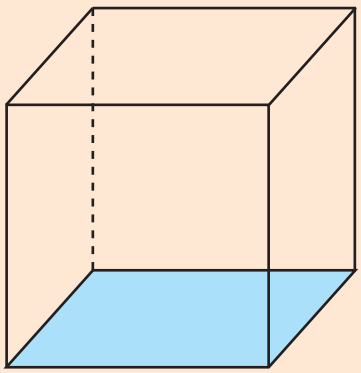
Next, cut the six faces

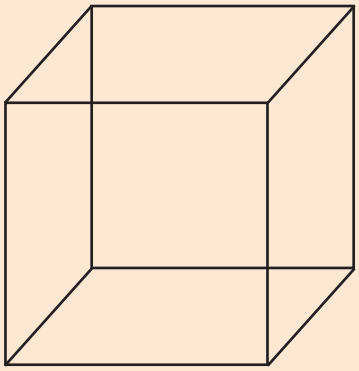


Finally, draw the net.

Example 2

Make a net for the cube.

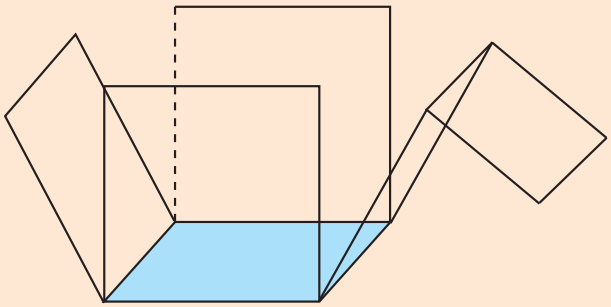




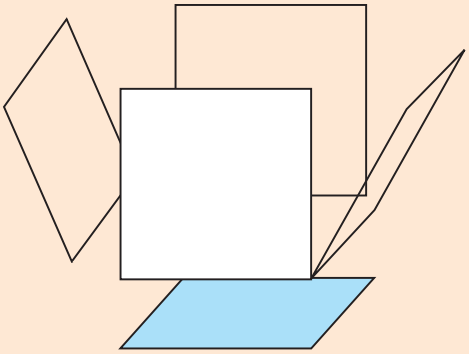
Solution

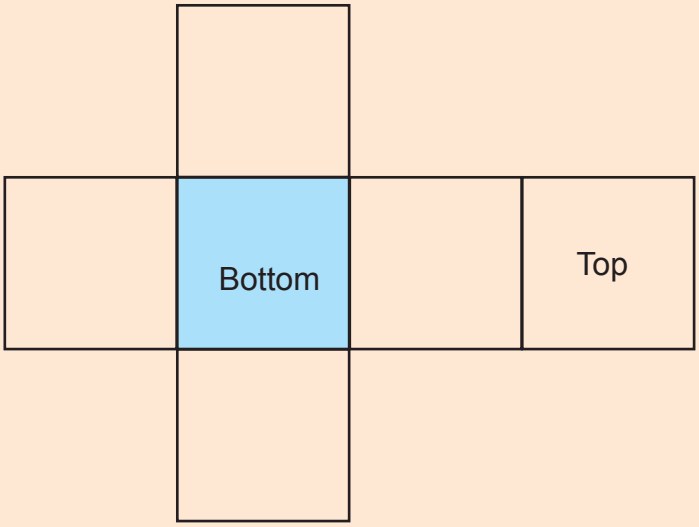
First unfold the cube to display the flat shapes.

Open the cube box.



Cut out the faces.



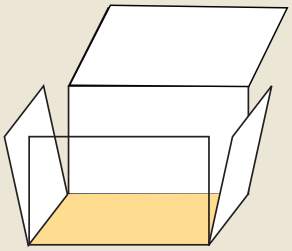


Finally, draw the net.

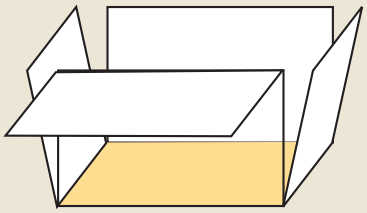
Application 13.13

1. Design nets for the following:

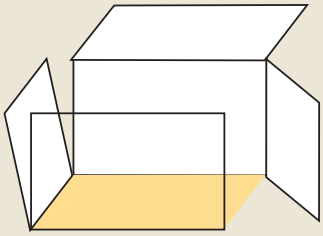
(a)



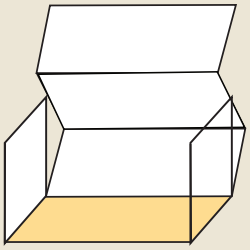
(b)



(c)

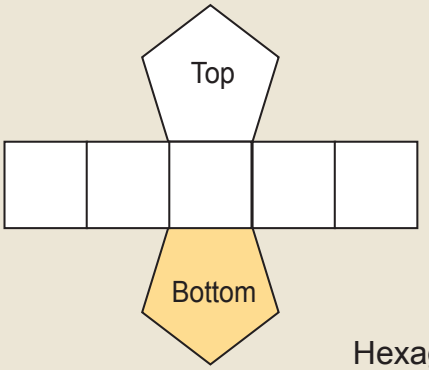


(d)

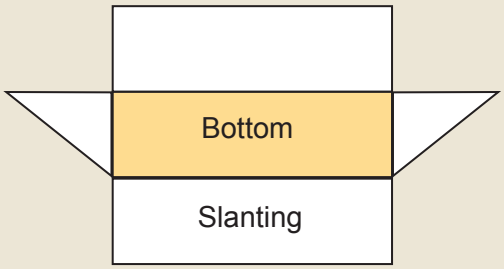


2. From the nets below, draw the prism.

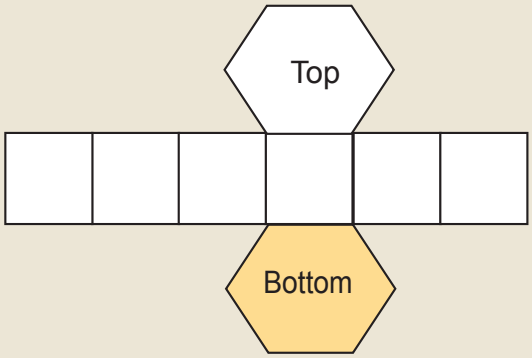
(a) Pentagonal prism



(b) Triangular prism

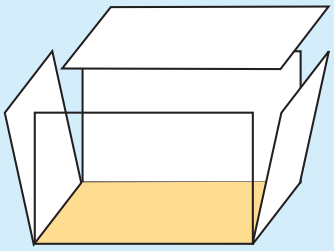


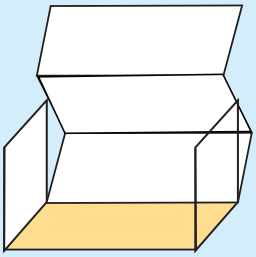
(c) Hexagonal prism



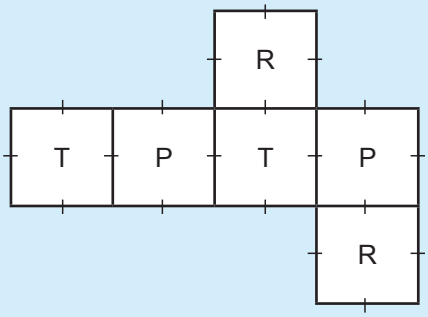
End of unit 13 assessment

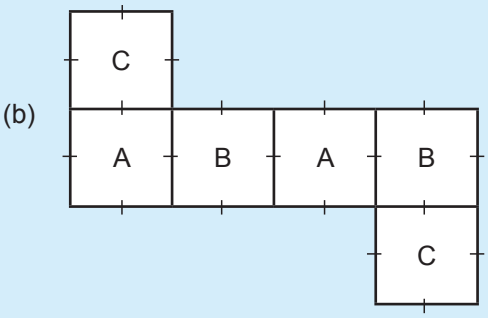
1. using a protractor and ruler, draw an equilateral triangle PQR of sides 5.5 cm
2. Draw a pentagon of sides 4.5 cm. Label it ABCDE. Use ruler and a protractor.
3. Using ruler and compasses only, construct the following polygons:
   (a) A square with sides 5.6 cm
   (b) A rectangle with length 6 cm and width 3 cm.
   (c) Triangle ABC with AB = 5.5 cm, angle A = 80^{O} and angle B = 50^{O}.
4. Copy and shade one pair of the bottom and top faces of the prisms below.

(a) 

(b) 

5. Draw the nets and cut their faces to make prisms

(a)


(b)


6. Construct Rectangle WXYZ where $\overline{WX} = \overline{ZY}$ = 8 cm and $\overline{WZ} = \overline{XY}$ = 6 cm. Measure diagonal WY.
7. Construct an equilateral triangle PQR of side 5.4 cm.
8. Use a potractor, ruler and pencil to construct triangle:
   (a) ABC with $\overline{AB}$ = 6 cm, $\angle A = 60^{O}$ and AC = 4 cm
   (b) XYZ with $\overline{ZX}$ = 6 cm, $\angle X = 40^{O}$ and XY = 8 cm.
   (c) KLM with $K = 85^{O}$, KL = 5.3 cm and $L = 40^{O}$.
9. Using ruler and compasses, contruct teh regular polygons below:
   (a) Hexagon ABCDEF with side 3.5 cm.
   (b) Octagon PQRSTUVW with side 2.8 cm.
   (c) Decagon ABCDEFGHIJ with side 5 cm.
   (d) Nonagon HIJKLMNOP with side 6 cm.

Unit 14

Area bounded by a circle, surface area of cuboid and volume of a cylinder

Key unit competence: To be able to calculate the area enclosed by a circle, surface area of cuboid and volume of a cylinder.

Introduction

Small squares combined form an area of a shape. The area of a circle is made up of small squares of any units of measure, for example, mm, cm etc. Move around noticing surfaces made up of small squares.

Designers, Carpenters, Architects all use the concept of area in their work.

Area depicts the size of space on a flat surface.

(a) Using a ruler, measure the sides of your Math exercise book and find out how many small squares of the units you used has the exercise book surface.

(b) Give examples of where the concept of area is used in real life.

14.1 Finding the area bounded by a circle

Activity

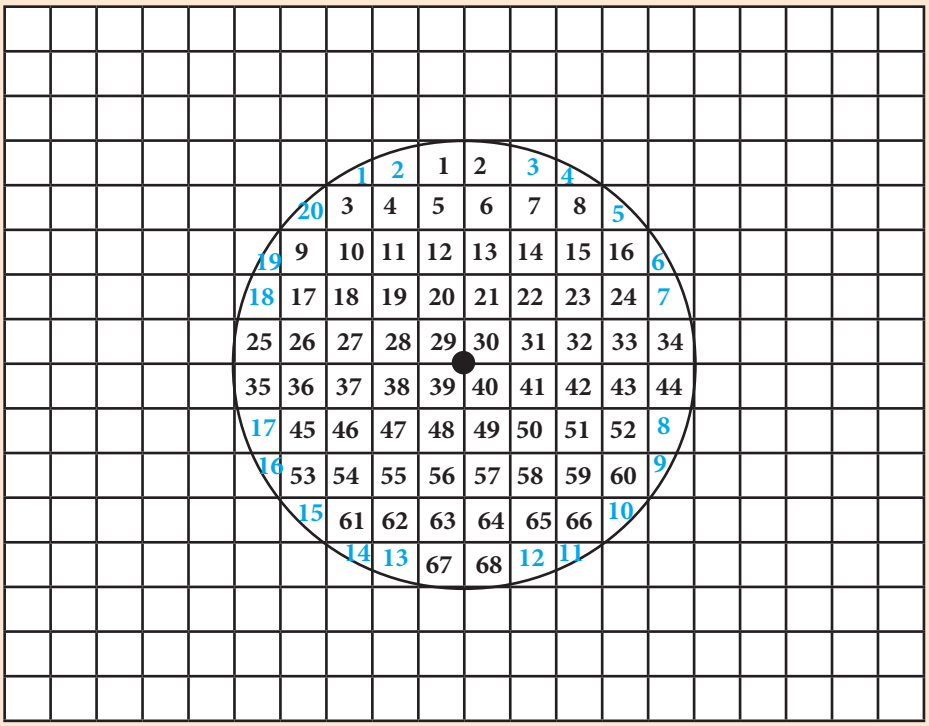
- Draw a circle of a radius of 3 squares on squared paper.
- Estimate the area enclosed by counting the squares.
- Add two parts of squares which do not form complete squares to form 1 square.
- Practice more with a radius of 2 squares, then 3 squares.
- Tabulate the results in the table below:

Radius (r)	r^2	Area

- What do you observe?
- Discuss the relationship in the tabulated values.
- Share the procedure and outcomes with other groups.

Example 1

Using squared paper, find the area of a circle of radius of 5 squares.



There are 68 whole squares + 20 portions of a square.

Area = 68 squares + 10 squares.

Area = 78 square units

Study tip

- Area of a circle is the region bounded inside it.
- Area is equal to the number of square units inside its boundary.
- Area divided by r^2 is equal to Pi(π) $\left(\frac{A}{r^2} = \pi\right)$.
- Pi(π) is constant.
- Pi(π) = 3.14 or $\frac{22}{7}$ or $3\frac{1}{7}$.

Application 14.1

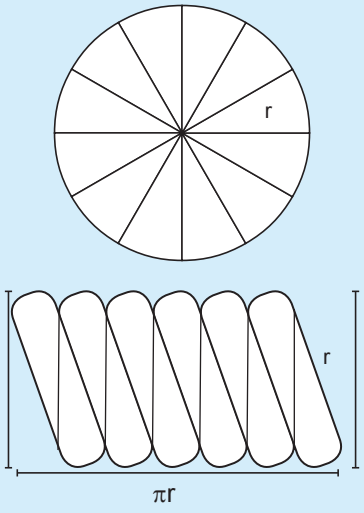
1. Using squared paper, find the area of a circle whose radius is:
   (a) 3 squares (b) 4 squares (c) 6 squares
   (d) 7 squares (e) 8 squares (f) 10 squares
   (g) 9 squares (h) 11 squares (i) 12 squares
2. Using the results in number 1 above, complete the table below:

Radius (r)		r^2	Area (A)
(a)			
(b)			
(c)			
(d)			
(e)			
(f)			

14.2 Calculating the area of a circle using radius

Activity

- Draw a circle on a paper and divide it into 12 segments as shown below.
- Now cut it out and cut it into 12 segments.
- Arrange the segments to approximate a parallelogram as shown.
- Link its dimensions to the circumference and radius. Establish the formula for calculating area of a circle. State the formula for calculating the area of the circle.



Parallelogram

Example 1

Calculate the area of a circle whose radius 5 cm.

Solution

radius = 5 cm, $\pi = 3.14$

Area of a circle $= \frac{1}{2}(2\pi r)r$

$= \pi r^2$

$= 3.14 \times 5^2$

$= 3.14 \times 5\,cm \times 5\,cm$

$= 3.14 \times 25\ cm^2$

$= 78.5\ cm^2$

Example 2

A saucepan has a circular bottom. Its radius is 0.7m. What is its area in cm^2.

Solution

Radius = 0.7m, $\pi = \frac{22}{7}$

Area of a circle $= \pi r^2$

$= \frac{22}{7} \times 0.7\ m \times 0.7\ m$

$= 22 \times 0.1 \times 0.7\ m^2$

$= 1.54\ m^2$

But 1 m $= 100$ cm

1 m^2 $= 10{,}000\ cm^2$

1.54 m^2 $= 1.54 \times 10{,}000 cm^2$

$= 15{,}400\ cm^2$

Study tip

- $\pi = 3.14$ or $\frac{22}{7}$ or $3\frac{1}{7}$
- Area is calculated in square units. For example, mm^2 cm^2, dm^2, dam^2, hm^2, m^2, km^2.
- Area is the region or space bounded or enclosed by a boundary.

Application 14.2

Calculate area of the circle with the following radii:

1. (a) 28 cm (b) 14 cm (c) 7 cm
   (d) 35 cm (e) 42 cm (f) 6.3 cm
2. A circular top of a dinning table has a radius 4.5 cm. Find its area. ($\pi = 3.14$ or $\frac{22}{7}$).
3. A Compact Disk has a radius of 7 cm. Find its area. ($\pi = 3.14$ or $\frac{22}{7}$)
4. A circular stool has a radius 15 cm. Find its area. ($\pi = 3.14$ or $\frac{22}{7}$)

14.3 Calculating the area of a circle given diameter

Activity

- Pick a circular object.
- Measure the diameter.
- How can you find the radius?
- Calculate area of the circular object.
- Share the procedure with your classmate.

Example 1

Find the area of a circle with a diameter of 28 cm.

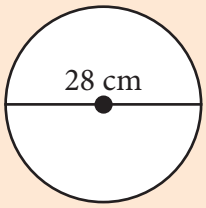
Solution

Find the radius:

$$\text{Radius} = \frac{\text{diameter (d)}}{2} = \frac{28\text{ cm}}{2} = 14$$

radius = 14 cm, $\pi = \frac{22}{7}$

Area of a circle $= \pi r^2$



$$= \frac{22}{\cancel{7}_{1}} \times 14\text{ cm} \times \cancel{14}^{2}\text{ cm}$$

$$= 22 \times 14\text{ cm} \times 2\text{ cm}$$

$$= 22 \times 28\text{ cm}^2$$

$$= 616\text{ cm}^2$$

Example 2

A saucepan has a circular bottom. Its diameter is 30 cm. What is its area in cm^2.

Solution

Find the radius:

$$\text{Radius} = \frac{\text{diameter (d)}}{2} = \frac{30\text{ cm}}{2} = 15$$

radius = 15 cm, $\pi = 3.14$

Area of a circle $= \pi r^2$

$$= 3.14 \times 15\text{ cm} \times 15\text{ cm}$$

$$= 3.14 \times 225\text{ cm}^2$$

$$= 706.5\text{ cm}^2$$

Study tip

- To get radius, divide the diameter by 2. $\left(r = \frac{d}{2}\right)$
- To get area when the diameter is given, first find the radius.
- If the radius is a multiple of 7, it is suitable to use $\pi = \frac{22}{7}$.
- If the radius is not a multiple of 7, it is suitable to use $\pi = 3.14$.
- Area can also be worked out by: $A = \pi \times \frac{d}{2} \times \frac{d}{2}$.

Application 14.3

1. Find the area of a circle whose diameter is 20 cm.
2. A circular garden has a diameter of 40 m. Find its area.
3. Find the area of the centre of a football play ground with a diameter of 4.2 m.
4. A circular billboard has a diameter of 280 cm. Find its area.
5. A tailor made a circular table cloth of a diameter of 1.4 m. Calculate the area.
6. A circular disc has a diameter of 30 cm. Calculate its area?
7. A circular floor has a diameter of 6 metres. What is its area?
8. The diameter of a circular bottom of a tin is 14 cm. Find its area.

14.4 Calculating area of a circle using circumference

Activity

- Pick a circular card.
- Using a thread or string, measure its circumference.
- Then measure the diameter.
- Divide the circumference by $\pi = 3.14$ or $\frac{22}{7}$. What do you get?
- Divide the result by 2. Write the answer down.
- Now use the answer for r, and work out the area.

Example 1

The circumference of a circular table is 132 cm. What is its area?

Solution

Circumference $= 2\pi r$

$132 = 2 \times \frac{22}{7} \times r$

$132 = \frac{44}{7}r$

$132 \times 7 = \frac{44 \times r}{\cancel{7}_1} \times \cancel{7}^{1} \times 7$

$924 = 44r$

$\frac{\cancel{924}^{21}}{\cancel{44}_1} = \frac{\cancel{144}^{1}}{\cancel{44}_1}r$

$21\text{ cm} = r$

$\therefore r = 21\text{ cm}$

Area of a circle $= \pi r^2$

$r = 21\text{cm}, \pi = \frac{22}{7}$

$= \frac{22}{7} \times (21\text{cm})^2$

$= \frac{22}{\cancel{7}_1} \times 21\text{ cm} \times \cancel{21}^{3}\text{cm}$

$= 22 \times 21\text{ cm} \times 3\text{ cm}$

$= 22 \times 63\text{ cm}^2$

$= 1{,}386\text{ cm}^2$

Example 2

The circumference of a circular garden is 125.6 m². Calculate its area.

Solution

Circumference = $2\pi r$

$125.6 = 2 \times 3.14 \times r$

$132 = 6.28\,r$

Divide both sides by 6.28

$$\frac{125.6}{6.28} = \frac{\overset{1}{\cancel{6.28}}\,r}{\underset{1}{\cancel{6.28}}}$$

$$\frac{125.6}{6.28} = \frac{1256}{10} \div \frac{628}{100}$$

$$r = \frac{\overset{2}{\cancel{1256}}}{10} \times \frac{100}{\underset{1}{\cancel{628}}}$$

$r = 2 \times 10$

$\therefore r = 20$ m

Area of a circle = πr^2

$= \pi \times r \times r$

$= 3.14 \times 20\text{ m} \times 20\text{ m}$

$= \frac{314}{\cancel{100}} \times \cancel{400}\text{ m}^2$

$= 314 \times 4\text{ m}^2$

$= 1256\text{ m}^2$

The area of the circular garden is 1256 m².

Study tip

- When given circumference to find area, first find radius then calculate area.
- Use suitable π. If the radius is a multiple of 7, you may use $\pi = \frac{22}{7}$, if not, you may use $\pi = 3.14$.
- Diameter = $2 \times$ radius.
- To get radius, divide the diameter by 2.



Application 14.4

1. The circumference of a circle is 88 cm. Calculate its area.
2. The circumference of a circular field is 628 cm. Calculate its area.
3. A circular plot of land has a circumference of 314 m. Calculate its area.
4. A circular stool has a circumference of 78.5 cm. Calculate its area.
5. The circumference of a circular disc is 0.88 cm. What is its area?
6. The circumference of a circular room is 17.6 m. Work out its area.

14.5 Finding the radius using area

Activity

- Pick circular cards with scribbled area.
- Divide area by π, either by $\frac{22}{7}$ or 3.14.
- Write the result.
- Try to calculate its square root. What do you get?
- Share your procedure with the class.

Example

Area of a circle is 1,386 cm^2. What is its radius?

Solution

Area $= \pi r^2$

$1{,}386 = \frac{22}{7} \times r^2$

$1{,}386 = \frac{22}{7} r^2$

$1{,}386 \times 7 = \frac{22 \times r^2}{\cancel{7}_{1}} \times \frac{1}{\cancel{7}}$

Divide 22 by both sides

$\frac{7 \times \cancel{1{,}386}^{63}}{\cancel{22}_{1}} = \frac{\cancel{122}^{1}}{\cancel{22}_{1}} r^2$

$7 \times 63 = r^2$

$441 = r^2$

Find the square root

$\sqrt{441} = \sqrt{r^2}$

Factorise to find the square root of 441.

$\sqrt{21 \times 21} = \sqrt{r \times r}$

$21 = r$

The radius is 21 cm.

Study tip

To find radius when area is given, substitute for A and π, then solve.

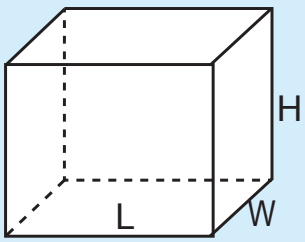
Application 14.5

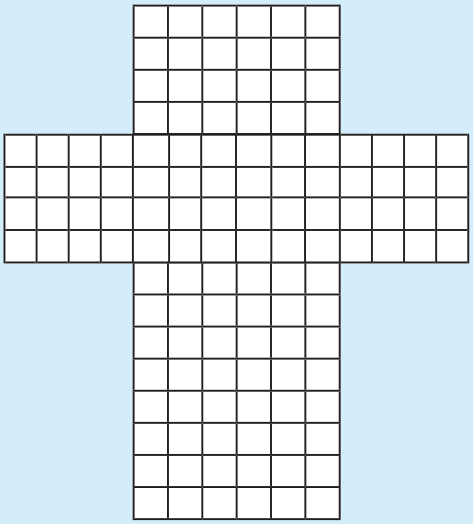
1. Find the radius of a circle with area of 154 cm^2.
2. What is the radius of a circle of area of 616 cm^2?
3. The area of a circular table is 6.16 m^2. Find the radius.
4. Calculate for the radius of a circle whose area is 1,256 cm^2.
5. Find the radius of a circular base of a tank whose area is 15,400 cm^2.
6. The area of a circular field is 7,850 m^2. Calculate for its radius.

14.6 Calculating the surface area of a cuboid

Activity

- Get a piece of squared paper and a pair of scissors.
- Cut the paper into a net similar to the one shown.
- Fold it to make the cuboid shown.
- Using masking tape, fix the edges onto each other firmly.





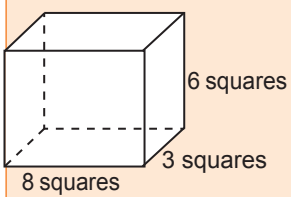
- How many equal faces does it have?
- How do you find the area of all the faces of the cuboid?

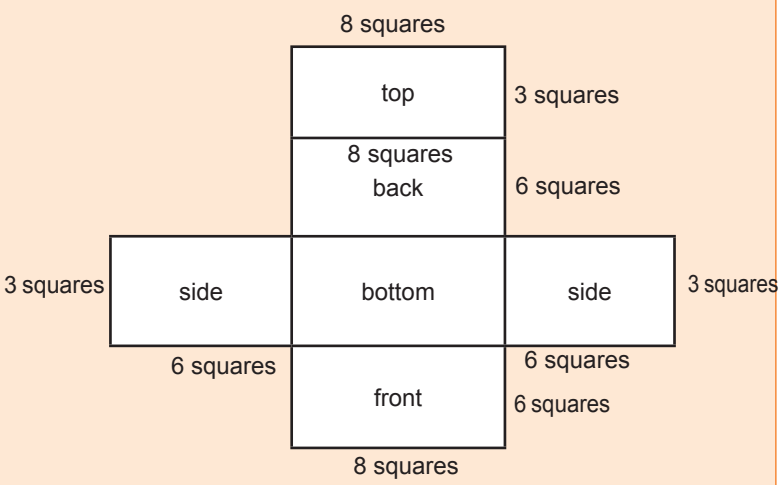
Example 1

Find the surface area of a cuboid of dimensions 8 squares by 3 squares by 6 squares.

Solution

Draw the cuboid.





There are 6 rectangles that is: Top = Bottom, Front = Back, Side = side

Surface area = $2(l \times w) + 2(l \times h) + 2(w \times l)$
= 2(8 × 3) squares + 2(8 × 6) squares + 2(3 × 6) squares
= (2 × 24) squares + (2 × 48) squares + (2 × 18) cm^2
= (48 + 96 + 36) squares
= 180 squares

Example 2

Calculate the surface area of a cuboid of 12 cm by 10 cm by 8 cm.

Solution

Every 2 opposite faces are equal. l = 12 cm, w = 10 cm and height = 8 cm.

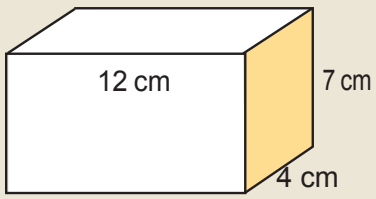
Surface area = $2(l \times w) + 2(l \times h) + 2(w \times h)$
= (2 x 12 cm x 10 cm) + (2 x 12 cm x 8 cm) + (2 x 10 cm x 8 cm)
= 240cm^2 + 192 cm^2 + 160 cm^2
= 592 cm^2

Study tip

- Surface area is the total area of all the faces of a 3-D shape.
- Area of a cuboid = $2lw + 2lh + 2wh$
- Area of a cube = 6(side × side). A cube has all the sides and faces equal.

Application 14.6

1. The length, width and height of cuboid are 10 cm, 8 cm and 7 cm respectively. Find the surface area of the cuboid.
2. The length ,width and height of a cuboid are 18 cm, 16 cm and 12 cm respectively. Find the surface area of the cuboid.
3. Find the surface area of the cuboid.



4. Find the surface area of a cuboid which has length 15 cm, width 11 cm and height 8 cm.
5. A factory packs bars of soap in boxes. Each box measures 20 cm long, 15 cm wide and 10 cm high. Calculate the surface area of each box.
6. Kamikazi packed cups in a box. Every edge of the box was 20 cm long. What is the surface area of the box?

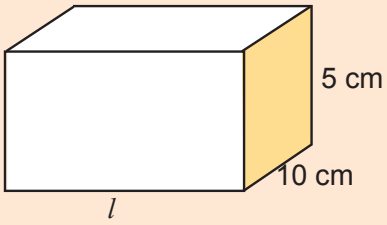
14.7 Finding the length of a cuboid

Activity

- Write w = 4, h = 5 and s = 148 on slips of paper.
- Try to solve for l in s = 2lw + 2lh + 2wh.
- What is the value of l?
- Try to formulate other values for w, h and s.
- What do you observe?

Example

The surface area of the figure below is 460 cm^2. Find its length.



Solution

Width (w) = 10 cm, height (h) = 5 cm, length (l) = ?

Surface area = 2(l x w) + 2(l x h) + 2(w x h)

460 cm^2 = 2(l x 10 cm) + 2(l x 5 cm) + 2(10 cm x 5 cm)

460 = 20 l + 10 l + 100

460 = 30 l + 100

460 - 100 = 30 l + 100 - 100 (subtract both sides)

360 = 30 l (Divide by 30 both sides)

$\frac{\overset{12}{\cancel{360}}}{\cancel{30}\,\overset{1}{}} = \frac{\overset{1}{\cancel{30}\,l}}{\cancel{30}\,\overset{1}{}}$ (Divide by 30 both sides)

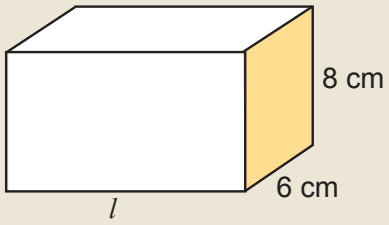
12 = l Therefore, the length is 12 cm.

Study tip

- To find length, substitute the width, height and surface area in the formula Surface area = 2(l x w) + 2(l x h) + 2(w x h).
- Then solve for length.

Application 14.7

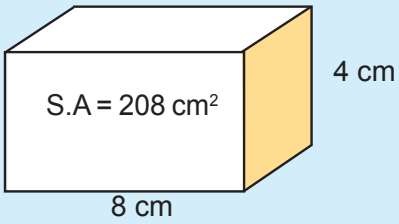
1. A rectangular box has surface area of 214 cm^2. If its width is 5 cm and height is 6 cm. Find its length.
2. A rectangular water tank has a surface area of 166 cm^2. Its width is 5 m and height is 4 m. What is the length?
3. The surface area of the box shown is 348 cm^2. Calculate its length.
4. A ware house has walls of 9 m wide and 7 m high. The surface area of the walls is 446 m^2. Calculate the length.
5. A small box has a width of 8 cm and height of 5 cm. It has a surface area of 262 cm^2. Find its length.
6. A septic tank has a surface area of 812 m^2. Its width is 10 m and height is 12 m. Calculate its length.



14.8 Finding the width of a cuboid

Activity

Study the box below.



- Which dimension is not given?
- Write the formula for finding surface area of the 3-D shape shown.
- Use it to find the missing dimension.
- Present your working out to class.

Example

A rectangular container has length of 6 m, height of 4 m and surface area of 148 m^2. Find its length.

Solution

Width (w) = ?, height (h) = 4 m, length (l) = 6 m

Surface area = $2(l \times w) + 2(l \times h) + 2(w \times h)$

$148 = 2(6 \times w) + 2(6 \times 4) + 2(w \times 4)$

$148 = 12w + 48 + 8w$

$148 = 20w + 48$

$148 - 48 = 20w + 48 - 48$ (subtract 48 both sides)

$100 = 20w$ (Divide by 20 both sides)

$\frac{\overset{5}{\cancel{100}}}{\cancel{20}_{1}} = \frac{\overset{1}{\cancel{20}}w}{\cancel{20}}$ (Divide by 20 both sides)

$5 = w$ Therefore, the width is 5 cm.

Study tip

- To find width, substitute the length, height and surface area in the formula Surface are $= 2(l \times w) + 2(l \times h) + 2(w \times h)$.
- Then solve for width.

Application 14.8

1. Surface area of a rectangular box is 94 cm^2. The length is 5 cm and the height is 3 cm. Calculate for its width.
2. What is the width of a cuboid whose surface area is 446 cm^2, length is 9 cm and height is 10 cm?
3. The length of a cuboid is 11 cm, the height is 9 cm and the surface area is 370 cm^2. Find its width.
4. The height of a rectangular container is 8 m. Its length is 12 m and its surface area is 592 m^2. Find its width.
5. Surface area of a rectangular container is 418 m^2. Its length is 11 m and its height is 8 m. What is its width?
6. The length of a wall at a hall is 12 m and the height is 10 m. Surface area of the wall is 724 m^2. Calculate the width.

14.9 Finding the height of a cuboid

Activity

- On slips of paper, write a = 6, b = 7 and y = 292.
- Try to solve the equation: y = 2ab + 2ac + 2bc.
- What is the value of c?
- Show your working out to the class.

Example

The surface area of a box is 38,200 cm^2. Its length is 90 cm and its width is 80 cm. Calculate its height.

Solution

Width (w) = 80 cm, length (l) = 90 cm, height (h) = ?

Surface area = $2(l \times w) + 2(l \times h) + 2(w \times h)$

38,200 = $2(90 \times 80) + 2(90 \times h) + 2(80 \times h)$

38,200 = $14{,}400 - 14{,}400 + 180h + 160h$

38,200 = $14{,}400 + 340h$

38,200 - 14,400 = $14{,}400 + 340h$ (subtract 14,400 both sides)

23,800 = $340h$ (Divide by 340 both sides)

$\frac{\overset{70}{\cancel{23{,}800}}}{\underset{1}{\cancel{340}}} = \frac{\overset{1}{\cancel{340}h}}{\cancel{340}_{1}}$ (Divide by 340 both sides)

70 = h Therefore, the height is 70 cm.

Study tip

- To find height, substitute the width, length and surface area in the formula Surface area = $2(l \times w) + 2(l \times h) + 2(w \times h)$.
- Then solve for height.

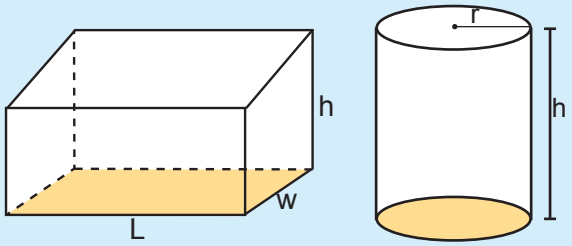
Application 14.9

1. Surface area of a box body of a track is 108 m^2. Its length is 6 m and width 3 m. Find the height.
2. A container measures 60 cm of length, 50 cm of width and its surface area is 14,800 cm^2. What is the height?
3. The length of a rectangular tank is 10 m. Its width is 6 m. Find its height is the surface area is 280 m^2.
4. A wardrobe has a length of 75 cm, width of 45 cm and surface area of 54,750 cm^2. What is its height?
5. A container's width is 7 m. length is 11 m and surface area is 334 m^2. Calculate for its height.
6. The walls of a classroom measure 12 m of length and 10 m of width. Surface area is 504 m^2. Find the height.

14.10 Finding volume of a cylinder

Activity

- Observe the cuboid and cylinder below:



- Name the figures forming their bases.
- State the formula for calculating the areas of their bases.
- Considering the formula of finding the volume of a cuboid, how can you find the volume of a cylinder?
- Defend your answer.

Example 1

A cylindrical fuel tank has a radius of 2 m. Its height is 6 m. Calculate its volume.

Solution

Volume = Base area × height

= $\pi r^2 \times h$

= $(3.14 \times 2 \times 2)\ m^2 \times 6\ m$

= $(3.14 \times 4)\ m^2 \times 6\ m$

= $12.56\ m^2 \times 6\ m$

= $75.36\ m^3$

Example 2

A gas cylinder has volume 1,540 cm^3. Its height is 10 cm. Find its radius.

Solution

Volume = Base area × height

$V = \pi r^2 \times h$

$1{,}540 = (\frac{22}{7} \times r^2) \times 10$

$1{,}540 = \frac{220}{7} r^2$

$\frac{7}{\cancel{220}_{1}} \times \cancel{1{,}540}^{7} = \frac{\cancel{220}^{1}}{\cancel{7}_{1}} r^2 \times \frac{\cancel{7}^{1}}{\cancel{220}_{1}}$

$= 7 \times 7 = r^2$

$\sqrt{(7 \times 7)} = \sqrt{(r \times r)}$

$7 = r$

$r = 7$ cm

The radius is 7 cm.

Study tip

- The formula for finding volume of a cylinder is $V = \pi r^2 h$. It means base area x height. The formula can be used to find height if the volume and radius are given. It can also be used to find radius if height and volume are given.
- Volume is calculated in cubic units, for example, mm^3, cm^3, dm^3, m^3, dam^3, hm^3.
- Mostly volume is calculated in cubic centimetres (c.c) or cm^3.

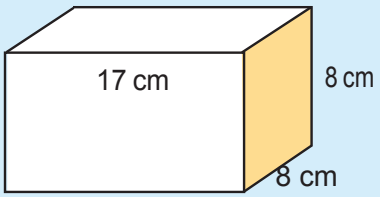
Application 14.10

1. Calculate the volume of a cylindrical tank whose radius is 70 cm and height 150 cm.
2. The height of a cylinder is 9 cm and volume 5,544 cm^3. Find the radius and base area.
3. In a cylinder, h = 10 m and base area = 154 m^2. Find the volume of the cylinder.
4. A cylinder has height of 15 m and base area of 616 m^2. Find the radius and volume of the cylinder.
5. A cylinder has a radius of 6 cm and height of 15 cm. What is the volume and base area of the cylinder?
6. A cylindrical juice can has radius 7 cm. Its height is 9 cm. Calculate its volume.

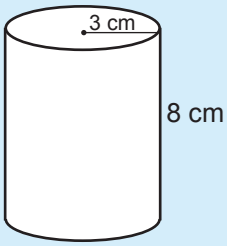
End of unit 14 assessment

1. Find the base area and volume of the following figures:

(a)



(b)

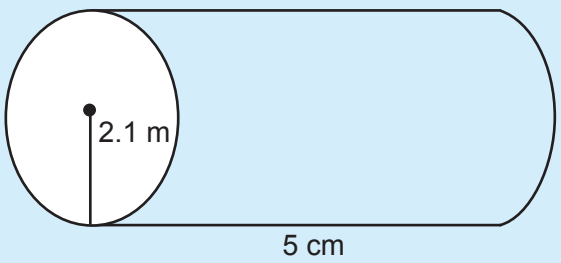


2. Find the area of the floor of a house with a circular foundation of a diameter of 12 m.
3. Find the volume of a cylindrical tank which has radius 2.5 m and a height of 3 m. $\left(\pi \text{ is } \frac{22}{7}\right)$

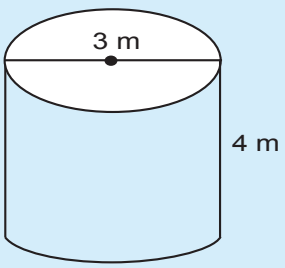
4. Calculate the volume of a cylindrical petrol barrel with a radius of 0.5 m and height of 3 m.
5. A rectangular prism has height of 6 dm, width 4 dm and the length of 10 dm. Find its surface area.
6. The length of a hollow calvert is 6 m. What is the volume of the material used to make it if the inner radius is 0.4 m and the outer diameter is 1.4 m?
7. Calculate the area with radii:

   a) 56 cm b) 21 cm c) 72 cm
8. The diameter of a cylindrical water tank is 300 cm. The height is 100 cm. Calculate the volume.
9. Calculate the volume of the shapes below in cubic centimetres.

(a)



(b)



Unit 15 Statistics

Key unit competence: To be able to extend methods for collecting data, and representing and interpreting it in order to answer the questions or explore hypothesis.

Introduction

Data are helpful in knowing how things are taking place. They are also helpful in good planning. With records, one is able to remember what took place in the past.

People are able to know things that happened long ago due to stored data or information. Business people, schools, government record data or information.

(a) How do you think collecting data and keeping them is important in daily life?

(b) Which kind of data can a person collect? Give an example of data which can be collected in the environment.

(c) Is correcting data, organizing data and interpreting them useful to make a family budget? What do you think about a national budget?

(d) Do you think statistics is important to you? Give 2 reasons.

15.1 Collecting the data to investigate a question

Activity

- List the marks obtained by every learner in your class in the end of month test.
- Group the same marks, and record their number.
- Use strokes to represent the number of marks recorded.
- Put these in a table form.
- Explain the data in the table to the class.

Example

Below is mass in kg of P. 6 learners who underwent a Body Mass Test.

28 30 28 33 35 40 40 28 30 30 42 40 35 40 40 40 33 30
28 30 30 30 33 40 33 33 35 40 35 40 35 35

(a) Use tally marks to organise the data above in the table

(b) How many learners underwent the Body Mass Test?

Solution

(a)

Mass in kg	Tally	Frequency (Number of learners)
28	||||	4
30	𝍸 ||	7
33	𝍸	5
35	𝍸 |	6
40	𝍸 ||||	9
42	|	1
Total		32

(b) 32 learners underwent the Body Mass Test.

Study tip

- Frequency is the number of occurrences.
- Collecting data using tables helps in grouping data.
- Investigating data helps in analysis and proper planning.



Application 15.1

1. The following marks were obtained by a P. 6 class in a Mathematics Examination.

55 68 66 66 83 57 70 73 70 73 55 66 68 83 66
73 72 57 73 83 57 71 65 78 74 56 68 71 83 85
83 68 70 71 55 70 61 63 67 72 70 74 59 60

(a) Use tallies to organise the data in a table.

(b) How many learners did the examination?

2. Kayitesi bought kg of maize flour for several days as shown below.

5 3 6 5 6 3 1 2 1 2 3 4 5 6 3 4 4 3 5 2 1 1 2 3 4 5 6 4 5 6
6 6 5 4 4 2 3 4 3 4 2 2 4 1 4 2 3 4 4 3 2 4 6 5 4 6 4 5 4 5

(a) Organise the data in a table.

(b) State the number of kg she bought altogether.

15.2 Interpreting the data in frequency tables

Activity

- Study the table below

Marks obtained in a Science test out of 50	Tally
10	|
20	𝍸 |
30	𝍸 |||
40	||||
Total	100

- Write the frequency of each mark obtained.
- If all learners did the test, how many learners are in the class?

Example

The table below shows the marks obtained by P.6 learners in a Mathematics exam by percentage in a school. Study it and answer the questions that follow.

Marks	**Tally**	**Frequency** (Number of learners)
96	||||	4
95	𝍸	5
93	𝍸 ||	7
84	𝍸 𝍸 ||	12
81	𝍸 𝍸 𝍸 |||	18
72	𝍸	5
70	|||	3
68	𝍸 |	6
Total		

(a) How many learners did the exam? 60 learners

(b) Which mark was scored more by learners? 81%

(c) What was the least mark scored? 68%

(d) How many learners scored 70%? 3 learners

(e) Find the number of learners who got above 80%. 46 learners

Study tip

- Frequency is the number of times a quantity appears.
- Tallies are written with strokes to show how many times a number appears.
- Tallies are also used to show the occurrences.



Application 15.2

1. Mr. Kagame planted different types of saplings. The data is shown in the table below. Study it and answer the questions that follow.

Saplings	**Tally**	**Frequency**
Eucalyptus	𝍸 𝍸 𝍸 𝍸	20
Pine	𝍸 𝍸 𝍸 |	16
Spruce	𝍸 𝍸|	11
Cypress	𝍸 𝍸 𝍸 𝍸	20
Podo	𝍸 𝍸 𝍸 𝍸|||	23
Cedar	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸|	31
Total		

   (a) How many cypress and podo saplings were planted?
   (b) Which type of saplings had the same number?
   (c) Find how many more cedar saplings than spruce saplings were planted?
   (d) How many saplings were planted altogether?

2. The table below shows the age of learners in P. 6 class. Use it to answer the questions that follow:

Age	Tally	Frequency
10 years	|||	
11 years	𝍸 𝍸 𝍸 𝍸||	
12 years	𝍸 𝍸 𝍸|||	
13 years	𝍸||	
Total		

   (a) How many learners are 12 years old?
   (b) How old are most of the learners?
   (c) How many learners are in the class?

15.3 Representing the data in a bar chart

Activity

- The table below shows items that a country exports as well as the contribution of each item to the total item value.

Export item	Tea	Sorghum	Flowers	Beans	Maize
Percentage of item	20	45	15	12	8

- Represent the above information in a bar graph.
- Make a presentation to the class.

Example

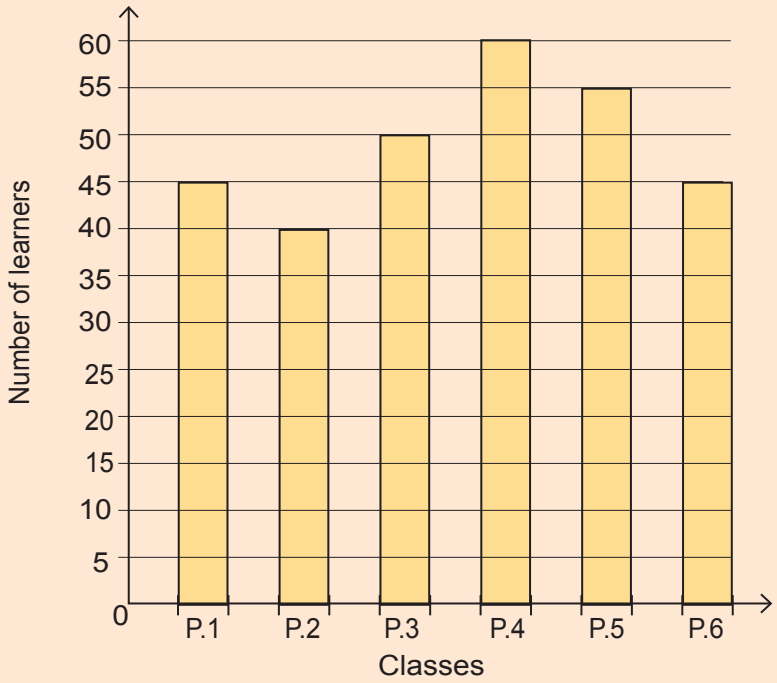
The table below shows number of learners per class in a school in Rwanda.

Class	P.1	P.2	P. 3	P. 4	P. 5	P. 6
Number of learners	45	40	50	60	55	45

(a) Represent the data in a bar chart.
(b) How many learners are in the whole school?

Solution

(a) A bar graph showing number of learners per class in a school.



(b) 45 + 40 + 50 + 60 + 55 + 45 = 295. The school has 295 learners.

Study tip

- A bar graph is a graph consisting of vertical or horizontal bars whose lengths are proportional to the amounts or quantities of statistical data represented.
- A bar graph is also called bar chart or bar diagram
- Always choose a suitable scale for the vertical axis.



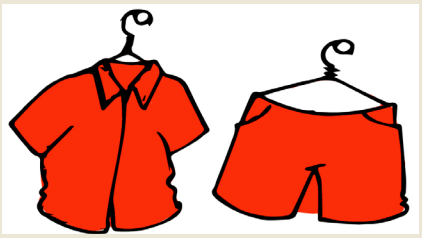
Application 15.3

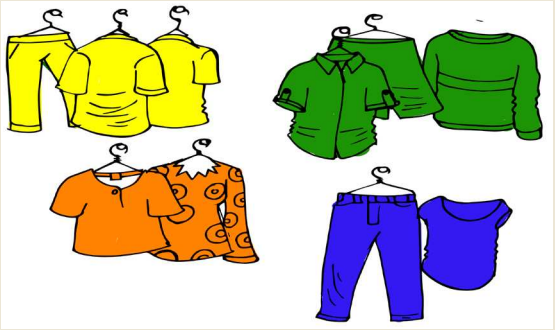
1. The table below shows the number of litres of milk collected on a farm in a week.

Day	Mon	Tue	Wed	Thurs	Fri	Sat	Sun
Milk in litres	50	60	45	75	40	25	65

   (a) Represent the above information in a bar graph.
   (b) How many litres of milk were produced in the week?

2. P. 6 learners were asked to pick clothes of their best colour and the results were recorded as follows: G = green, R = red, B = blue, O = orange and Y = yellow.





G Y R B G G R Y R O G R Y Y G G Y R B G G R Y

R O G R Y Y G G Y R B G G R Y R O G R Y

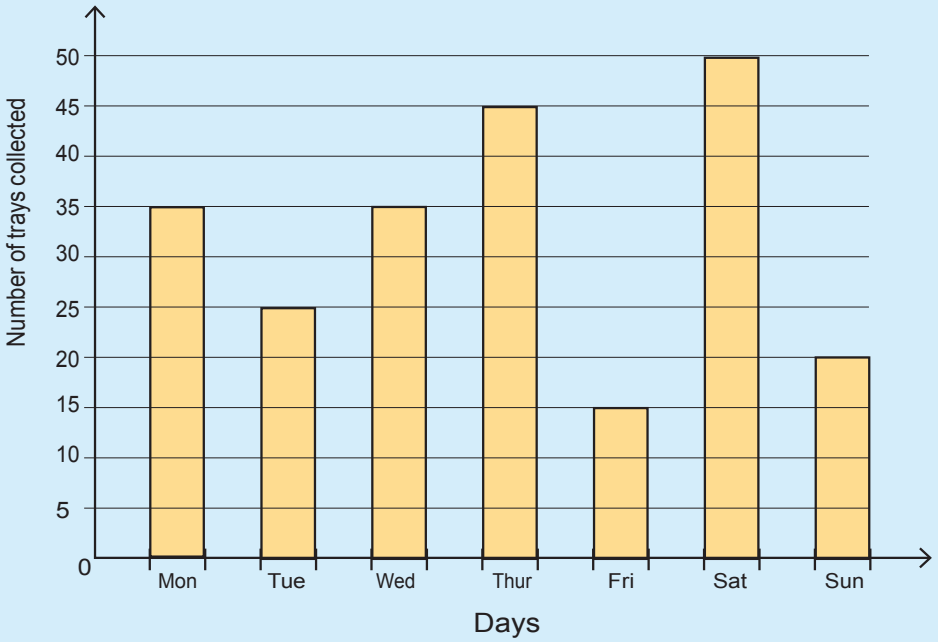
Y G Y R G Y G R R O O Y G O R R R.

   (a) Organise the data in a table.
   (b) Represent the data in a bar chart.

15.4 Interpreting the data in a bar chart

Activity

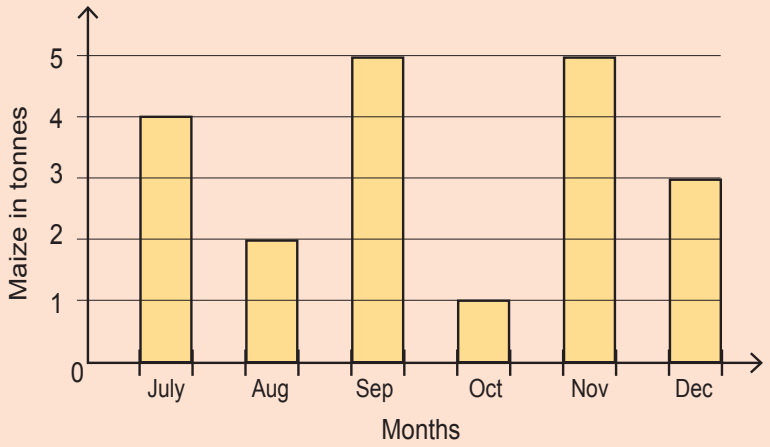
- Study the bar graph below and answer the questions that follow:



- How many days are shown on the graph?
- What is the scale on the vertical axis?
- Find the number of trays collected in the whole week.
- On which day did Mr. Uwera collect fewer trays of eggs?

Example

1. The bar graph shows Mr. Muhire's maize exports.



(a) In which months did Mr. Muhire export the same number of tonnes?
(b) How many more kg of maize were exported in November than in August?
(c) Find the total tonnes of maize exported in the last half of the year.

Solution

(a) In the months of September and November
(b) November = 5 tonnes and August = 2 tonnes
(5 − 2) tonnes = 3 tonnes.
but 1 tonne = 1,000 kg
3 tonnes = 3 × 1,000 kg = 3,000 kg
Therefore 3,000 kg more were exported in the month of November than in August.
(c) Total tonnes exported = (4 + 2 + 5 + 1 + 5 + 3) tonnes = 20 tonnes.

Study tip

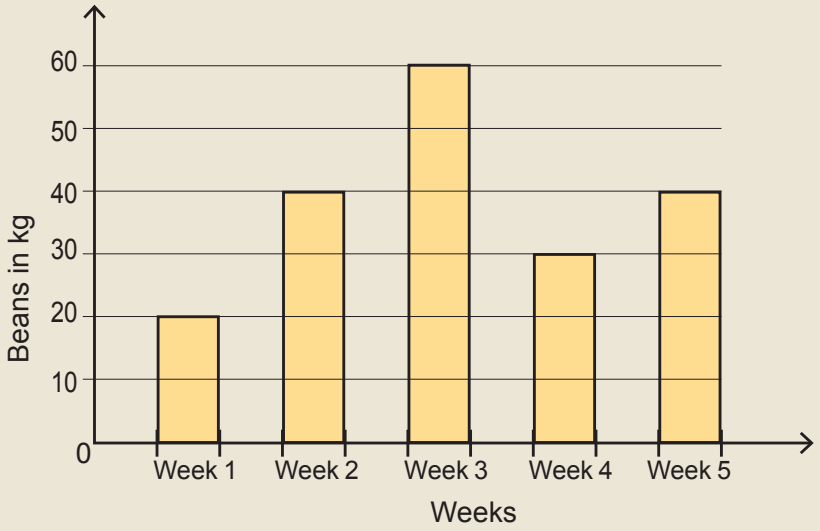
- The title tells the subject on which data was collected. Always take note of what is represented on the vertical and horizontal axes.
- The vertical axis is divided into equal parts.
- The reading on each part of the vertical axis is the vertical scale.



Application 15.4

Study the graph and answer the questions that follow:

1. Beans in kg sold at a shop in 5 weeks



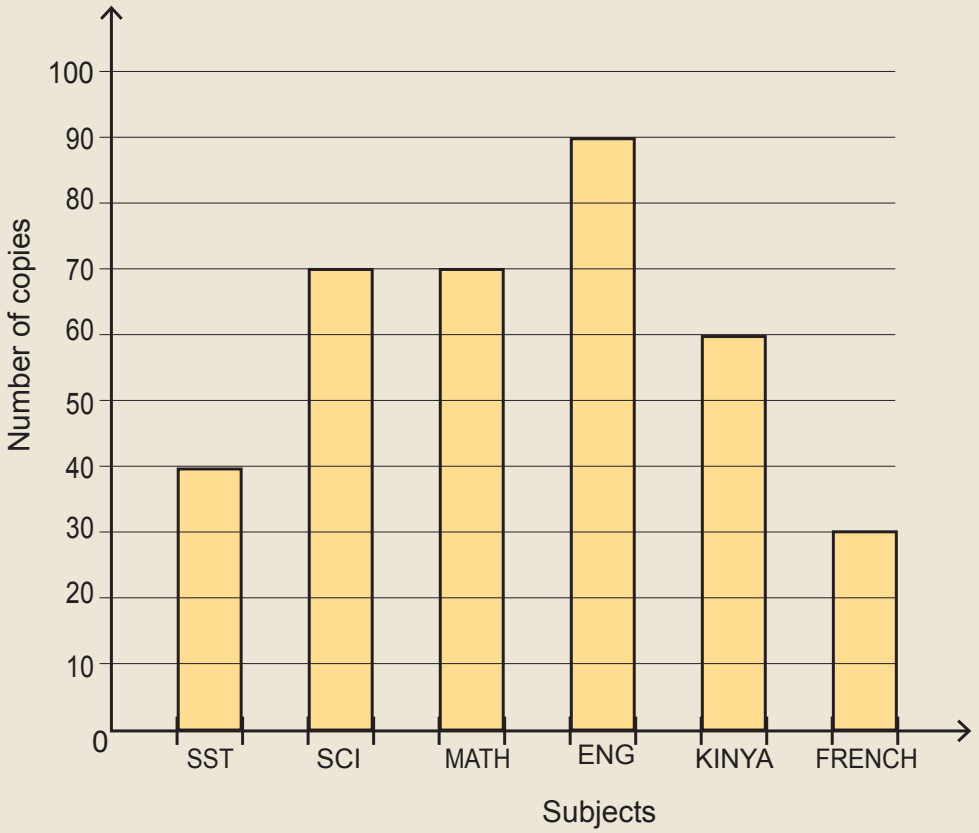
(a) In which weeks did the shopkeeper sell the same quantities of beans?

(b) In which week did the shopkeeper sell the highest quantity of beans?

(c) Find the total kilogram of beans sold in the 5 weeks

(d) If each kg of beans costs 700 Frw, how much did the shopkeeper get from the sale of beans in the five weeks?

2. The bar graph below shows the number of textbooks bought by a primary school in 2016. Study it and answer the questions that follow:



(a) Which subject has the highest number of copies?

(b) Which subjects have the same quantity?

(c) How many more English books were bought than Social Studies books?

(d) If each Mathematics book costs 5,500 Frw, how much money was spent on the Mathematics textbooks?

(e) How many textbooks were bought?

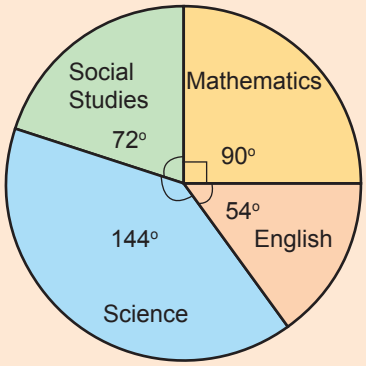
15.5 Representing Data in Pie Charts

Activity

- P. 6 learners at a school performed as follows in the end of year examination; Division I, 18 learners, Division II, 25 learners, Division III, 12 learners and Division IV, 5 learners.
- Get the fraction of each division.
- Multiply each fraction by 360^o to get the degrees of each division.
- Draw a circle and divide it into sectors by using the corresponding degrees.
- What special name is given to the above circle?
- Present your working out to the class.

Example 1

In a P. 6 class, 15 learners like Mathematics,12 learners like Social Studies, 24 learners like Science and 9 learners like English. Represent the information in a pie chart.



Solution

Total learners = (15 + 12 + 24 + 9) = 60 learners.

Find the sector of every subject. Use the table below.

Use a protractor to draw the angle sectors in the pie chart. Use the skill of drawing angles.

Subject	Number of learners	Fractions	Percentages	Degrees
Mathematics	15	$\frac{15}{60} = \frac{1}{4}$	$\frac{1}{4} \times 100 = 25\%$	$\frac{1}{4} \times 360^o = 90^o$
Social Studies	12	$\frac{12}{60} = \frac{1}{5}$	$\frac{1}{5} \times 100 = 20\%$	$\frac{1}{5} \times 360^o = 72^o$
Science	24	$\frac{24}{60} = \frac{2}{5}$	$\frac{2}{5} \times 100 = 40\%$	$\frac{2}{5} \times 360^o = 144^o$
English	9	$\frac{9}{60} = \frac{3}{20}$	$\frac{3}{20} \times 100 = 15\%$	$\frac{3}{20} \times 360^o = 54^o$
Total	**60**	**1**	**100%**	**360°**

Study tip

- A pie chart is one way of presenting data in a circular graph that is divided into sectors. Each part is represented in degrees.
- The quantity represented in each sector is corresponding to its fraction out of 360°. It is also called a circle graph.
- First find the total of items if it is not given. Write each item as a fraction of the total then multiply by 360°. Fractions add up to 1, degrees add up to 360° while percentages add up to 100%.



Application 15.5

1. The table below shows percentage expenditure of a publishing company. Study the data and represent it in a pie chart.

Item	Printing cost	Transportation cost	Paper cost	Binding	Royalty	Promotion cost
Percentage expenditure	20%	10%	25%	20%	15%	10%

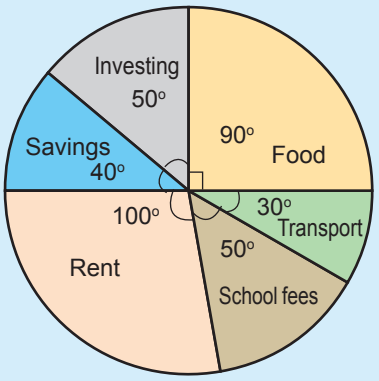
2. The table below shows how 120 learners participated in different games. Represent the information in a pie chart.

Game	**Football**	**Netball**	**Volleyball**	**Tennis**	**Rugby**
Number of pupils	40	15	20	10	35

15.6 Interpreting the data in pie charts to draw a conclusion

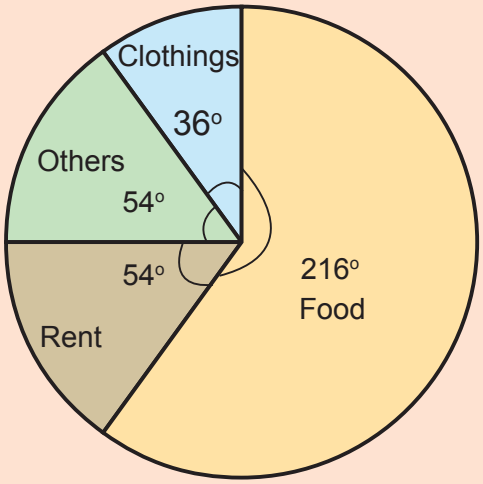
Activity

- The pie-chart represents the monthly expenditure of Mrs. Muziranenge's salary. Study it and answer the questions that follow.
- If 240,000 Frw is spent on transport, how much does she earn?
- How much more money is spent on rent than on savings?
- Present your working to the class.



Example

A teacher spends his salary of 43,000 Frw as shown in the pie chart. Study it and answer the questions that follow.



(a) How much is spent on food?

Money spent on food = $\frac{\cancel{216^\circ}^{6}}{\cancel{360^\circ}_{1}} \times 43{,}000$

= 6 x 4,300 = 25,800 Frw

(b) Which items does he spend on the same amount?

He spends the same amount on rent and others.

(c) What is the percentage of money spent on food?

Percentage spent on food = $\frac{\cancel{216^\circ}^{6}}{\cancel{360^\circ}_{1}} \times 100\%$ = 6 × 10% = 60%

(d) How much more money does he spend on food than on others?

Money spent on others $\frac{54^\circ}{360^\circ} \times 43{,}000$ Frw = 6,450 Frw

The difference = (25,800 - 6,450) Frw = 19,350 Frw

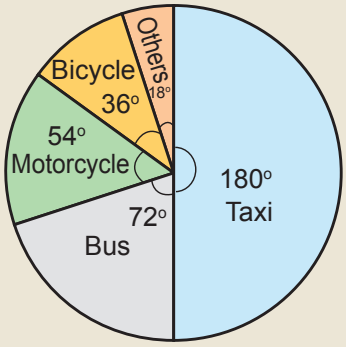
19,350 Frw more is spent on food than on other items.

Study tip

- To get the number of items for each sector, multiply the fraction for that sector by the total number.
- To get percentage for each sector, multiply the fraction for the sector by 100%.

Application 15.6

1. The pie chart shows the means of transport that people use in an area.

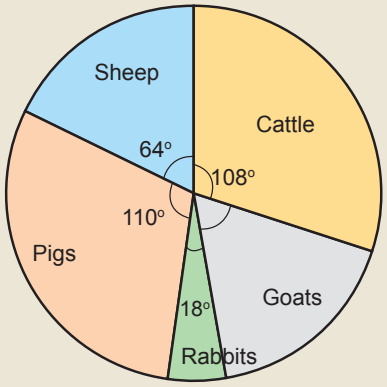


   (a) Which means of transport is used by most people in the area?

   (b) If 6,000 people use motorcycles, how many people are in the area?

   (c) How many more people use taxis than bicycles?

2. The pie chart below shows animals Mr. Mwebesa has on his farm.
   (a) Find the percentage of goats reared on the farm.
   (b) If there are 300 animals on the farm, how many pigs are there?
   (c) If each rabbit is sold at 15,000 Frw, how much can he get from selling all the rabbits on his farm?



End of unit 15 assessment

1. Below are heights in cm of P.6 learners in their study of measurement.

125	128	118	130	142	125	125	118	118	128
134	128	118	125	130	128	130	142	125	128
130	128	130	132	128	132	130	125	142	128
130	128	130	128	140	134	130	128	132	138

   (a) Use tally marks to organise the data above in a table.
   (b) How many learners were measured?
   (c) What was the modal height?

2. The following are age groups of people in a village on the Voter's register.

Age group	15-19	20-25	26-30	31-35	36-40	41-45	46-50	51-55	56 and above
Frequency	18	31	33	30	29	22	18	25	20

   Represent the data in a bar chart.

3. P.6 learners were asked to state the food type they like most. 9 learners like cassava, 10 learners like rice, 5 learners like Irish potato, 6 learners like banana. Represent the information in a pie chart.

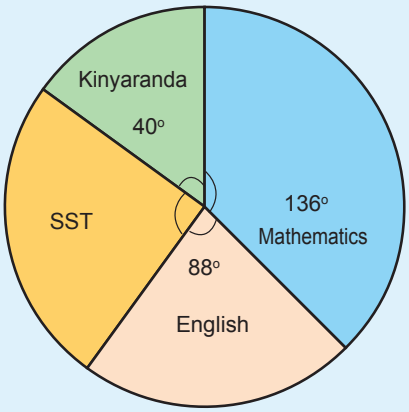
4. In 2016, some people were asked to predict the national team which would win the World Cup. Their predictions were as follow:

National team	**Number of predictions**
Brazil	13
German	9
France	8
Argentina	12
Portugal	10
Spain	8

   Represent the above information in a pie graph and in a bar graph.

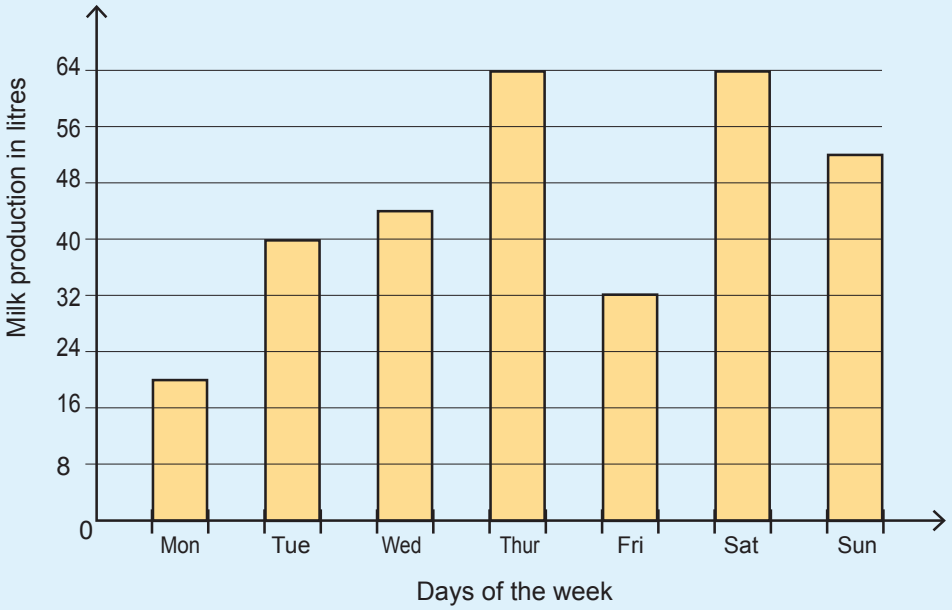
5. In a village, there are different games played. Some fans were asked which of these game they like most and the responses were recorded as follows: 60 like football, 22 like netball, 19 like tennis, 44 like basketball and 35 like volleyball. Represent the information in a pie chart?

6. 45 learners in a school were asked which subjects they like most and the responses recorded and represented in the pie chart below. Study it and answer the questions that follow.



   (a) Find the percentage of learners who like Social Studies.
   (b) How many learners like Mathematics?
   (c) How many more learners like Mathematics than Kinyarwanda?

7. The graph shows weekly milk production at Domina's diary farm.



   (a) What is the scale on the vertical axis?
   (b) On which day did Domina collect 56 litres?
   (c) Find the total of litres collected on Thursday and Friday?
   (d) On which days did Domina collect the same litres of milk?
   (e) If each litre costs 800 Frw, how much did Domina receive when she sold the milk she collected on Wednesday?

Unit 16



Key unit competence: To be able to order events in terms of likehood (impossible, equally likely, certain).

Introduction

Things in nature have no law of happening. Some things occur by chance. Some things must happen; some have even chance of happening, while others are practically impossible. For example, before any Football match starts, the spectators are not sure who will win. The chance of winning the game is equal at the begging. At the end of the game, different possible outcomes or results may occur.

(a) With support of examples could you predict how the different possible outcomes or results will look like?

(b) Do you think that to have the same score in a football match is certain to happen? Equally likely to happen? or impossible to happen? Justify your answer.

16.1 Vocabulary of chance: impossible, certain, equally likely, events, chance, unlikely, likely

Activity

Play a game of Bingo or any other game.

- Write down six numbers between 1 and 12 of your choice.
- Toss two dice. Strike out the number shown by the dice if it is in your chosen number list.
- If your chosen number list has all the strikes, call out Bingo.
- So, are some numbers easier to get than others? Why?
- Is the appearance of a number a chance? Discuss.

Example

Use "Yes" or No" to answer the questions below.

1. Is it possible to show a tail (T) when a coin is tossed?
2. Is it possible to show a head (H) when a coin is tossed?
3. Is it possible to show a tail and a head at the same time when a coin is tossed?
4. Does impossible mean that there is no chance of an event to happen?

Answer	1	2	3	4
	Yes	Yes	No	Yes

Study tip

- Probability is defined as the chance that something will happen.
- Probability is written as numbers between zero and one.
- If the probability is 1 then the event is certain. That is, it is going to happen. For example,when you toss a coin, a head or tail shows up.
- The probability of 0 means that an events is impossible. If you toss a coin you cannot get both a tail and a head at the same time, so this has 0 probability.
- Likely means that the probability of one event is higher than the probability of another event.
- Unlikely means that one event is less likely to happen.
- Equal probability means that the chance of each event to happen are equal.



Application 16.1

1. Discuss the likelihood of the following events:
   (a) Getting a head when you toss a coin.
   (b) It will rain in Kigali this year.
   (c) A woman will give birth to a boy.
   (d) The sun will rise tomorrow.
   (e) You will get a six when you throw a die.
   (f) You will get a total of 1 when you throw two dice.
   (g) Our teacher will become the president.
   (h) I was born yesterday.

2. Explain the following word used in probability:
   (a) Impossible
   (b) Certain
   (c) Equally
   (d) Likely
   (e) Unlikely
3. Create your own statements and associate them with vocabulary of chance.

16.2 Using data to decide how likely something is to happen

Activity

- Roll a die (remember there are 6 options, that is; (1, 2, 3, 4, 5, 6)
- How many ways are there of a 2 showing up?
- How many ways are there of a 7 showing up?
- Make a presentation to class.

Example

A die was rolled once. Find the probability that:

(a) 1 shows up

(b) 0 shows up

(c) 7 shows up

Solution

There are six options, that is; 1, 2, 3, 4, 5 or 6 showing up.

(a) There is 1 option out of 6 for 1 to show up.

Therefore probability = $\frac{1}{6}$

(b) There is 0 option so it cannot show up.

Therefore probability = $\frac{0}{6} = 0$

(c) There is no option of 7 showing up.

Therefore probability = $\frac{0}{6} = 0$

Study tip

- Probability is always greater than or equal to 0 and less than or equal to 1.
- Probability means how possible something may happen.
- Probability is how likely it is that something will happen.



Application 16.2

1. Which of these numbers is not a probability?

   (a) -0.001 (b) 0.5 (c) 1.001

   (d) 0 (e) 1 (f) 20%

   (g) $\frac{9}{10}$ (h) 4 (i) $\frac{3}{6}$

   (j) -8 (k) 50% (l) 100^{o}

2. A card is drawn at random from a deck of cards (52 cards) Find the probability of getting a queen.

3. A die is rolled once.

   (a) Find the probability that the number obtained is greater than 4.

   (b) Is it possible to get the number which is greater than 6? Justify your answer.

4. Two coins are tossed once. Find the probability that head only is obtained.

5. A card is drawn at random from a deck of cards. Find the probability of getting the king of a heart.

End of unit 16 assessment

1. What is the probability of the following events:

   (a) A coin tossed showing 6.

   (b) Raining today.

   (c) A mother giving birth to a baby boy or girl.

   (d) A cow eating grass.

2. The table below shows the learners enrolment in a school.

Year	2010	2011	2012	2013	2014	2015	2016	2017
Enrolment	252	302	290	354	370	406	432	458

Use the language of chance to define these statements.

(a) The school started in 2010.

(b) Enrolment in 2018 will increase.

(c) The learners in the school perform well.

3. Suppose you roll a die. Find the probability that the number obtained is less than 4.

4. A card is drawn at random from a deck of cards (52 cards). Find the probability of getting the king of diamond.

5. There are red, blue and black pens in a pack. What is the chance of picking at random:

(a) a black pen?

(b) a blue pen?

(c) black, blue or red pen?

(d) Order the likelihood of chances to happen.

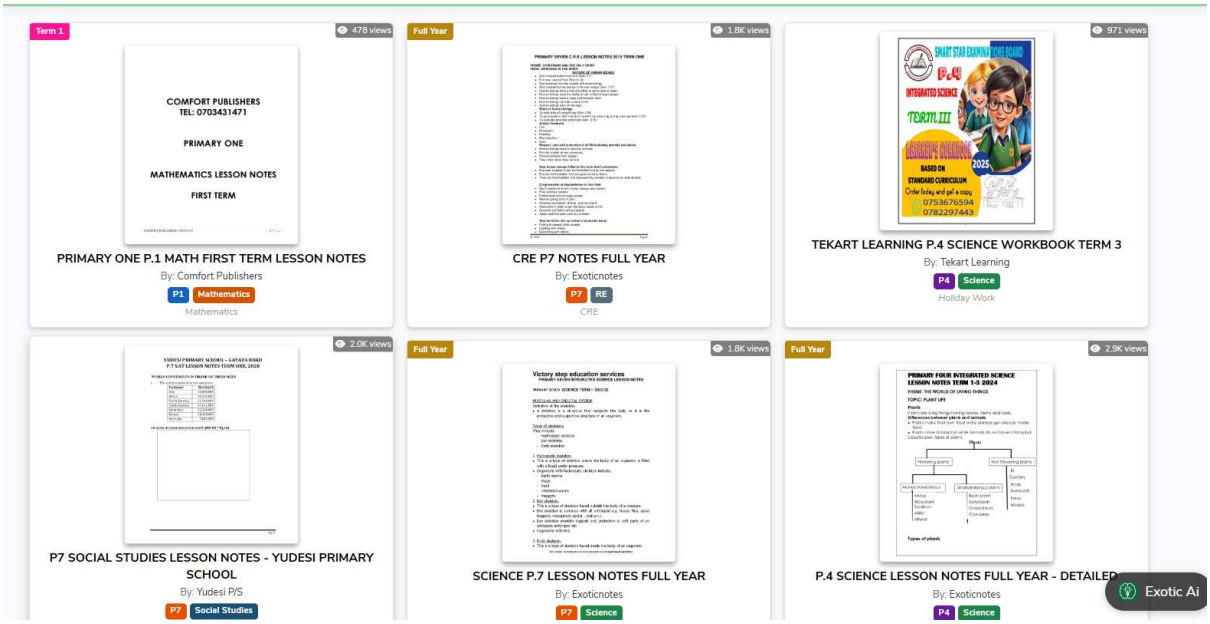
Glossary

- **An event**: This is any possible outcome from the sample space. An event is also a subset of a sample space.
- **Cube**: This is a solid bounded by six identical faces which are squares.
- **Cuboid**: This is a solid bound by three pairs of identical faces which are all rectangles.
- **Data**: This is a set of values and observations that gives raw information in a more organised form.
- **Direct proportion**: This is where the quantities are such that, when one quantity increases or decreases in the ratio a:b, the other quantity decreases or increases in the ratio b:a.
- **Interest**: This is the money paid for borrowing or depositing money for a specific period of time. It is also a fixed percentage charge on unpaid loans.
- **Loss**: This is the amount of money lost when a commodity is sold below the actual buying price.
- **Percentage**: This is the fraction whose denominator is 100.
- **Probability**: This is the likelihood/possibility of an occurrence of an event at a given period of time.
- **Profit:** This is the extra amount gained after selling a commodity at a price higher than the buying price.
- **Pyramid**: This is a solid figure formed with triangular slanting faces which meet at a vertex above a polygonal base.
- **Ratio**: This is a mathematical statement of how two or more quantities or numbers compare.
- **Statistics**: This is the study of collecting, organising, representing and displaying of numerical data. Examples of data include ages, mass, height of students.

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