CURRENT ELECTRICITY

Current flows when electric charge passes a given area of a conductor.

Terms used

(i) Charge (Q)

This is the quantity of electricity that passes a given point in a conductor at a given time.

Its *S.I unit* is the *coulomb(C).*

(ii) A coulomb (C)

A coulomb is the quantity of charge that passes a given section of a conductor when a steady current of one ampere (1A) flows in one second.

(iii) Current (I)

Current is the rate of flow of charge through a conductor.

$$I = \frac{Q}{t}$$

Its *S.I unit* is the *ampere (A)*

(iv) Ampere (A)

An ampere is the current flowing through a circuit or conductor when a charge of one coulomb passes any section of the circuit or conductor in one second.

Alternatively; It is the current flowing through a conductor when a charge of one coulomb flows through it per second.

(v) Potential difference (V)

This is the work done in moving a charge of one coulomb from one point to another in a circuit.

$$V = \frac{Work}{Charge} = \frac{W}{Q}$$

Its *S.I unit* is the *volt (V)*

(vi) Volt

This is the p.d between two points in a circuit in which one joule of work is done to transfer one coulomb of charge from one point to another.

Note

Whenever current flows, it does so because electric potential between two points is different. However if the electric potential is the same, current does not flow.

(vii) Electromotive force (emf)

This is the work done in transferring a charge of one coulomb around a complete circuit in which a battery or cell is connected.

Alternatively; It is the p.d across across the terminals of a cell when connected in an open circuit

Note:

An open circuit is a circuit in which no current is drawn to the external load.

(viii) Terminal p.d

It is the voltage across the terminals of a battery or cell when connected in a closed circuit.

Alternatively; It is the work done in transferring one coulomb of charge around a complete circuit in which a battery is connected or supplying current.

It is less than the emf of the source (cell) since when current is flowing, some voltage is lost due to the internal resistance of the source

(ix) Resistance (R)

This is the opposition to the flow of current in circuit.

$$R = \frac{V}{I}$$

Its *S.I unit* is the *ohm* (Ω)

***(x) Ohm* (Ω)**

This is the resistance of a conductor through which a current of one ampere flows when the p.d across its ends is one volt.

OHM'S LAW

It states that under constant physical conditions, the current flowing through an electric conductor is directly proportional to the p.d across its ends.

$$V \alpha I$$

$$V = RI$$

$$\boxed{V = IR}$$

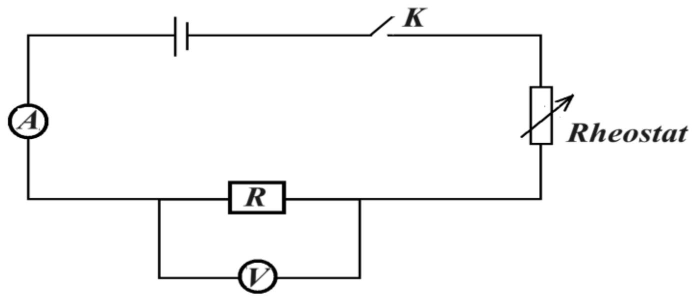
Where *V is potential difference, I is current and R is resistance of the conductor.*

Limitations of Ohm's law

(i) It does not apply for semi – conductors and gases

(ii) It is only obeyed if the physical conditions like temperature are constant

VERIFICATION OF OHM'S LAW



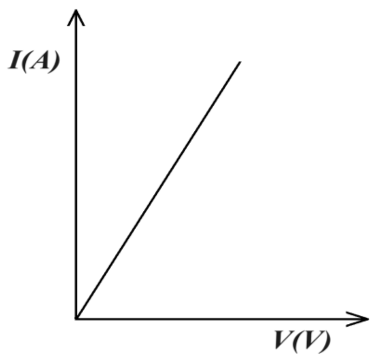
The apparatus is arranged as shown in the circuit above.

Switch, *K* is closed and the rheostat is set to give a suitable value of current *I* and the corresponding voltages, *V* is read from the voltmeter.

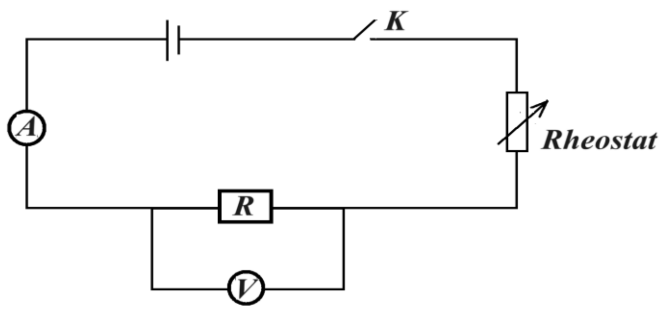
The experiment is repeated for different values of *I* and *V* read from the ammeter and voltmeter respectively.

The results are tabulated in a suitable table.

A graph of *I* against *V* is plotted and is a straight line through the origin. This verifies ohm's law.



AN EXPERIMENT TO DETERMINE RESISTANCE OF A CONDUCTOR



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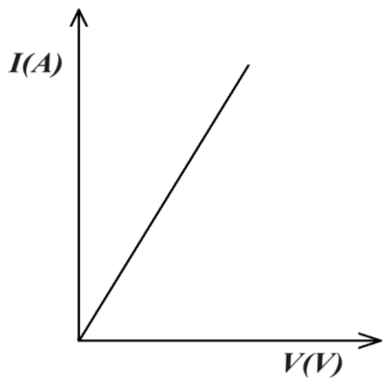
The results are tabulated in a suitable table.

A graph of *I* against *V* is plotted and is a straight line through the origin.



The slope, *S* of the graph is calculated and the resistance, *R* is calculated from $R = \frac{1}{S}$

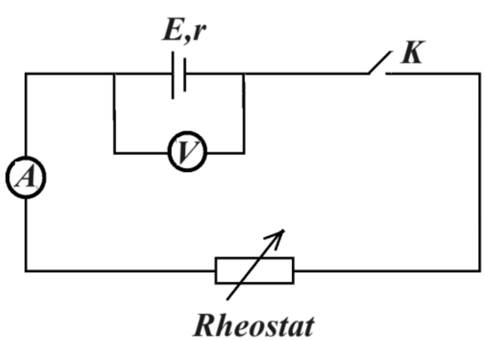
Note

If the graph plotted is of *V* against *I*, then the resistance, *R=slope*.

Revision question

Describe an experiment to determine the resistance of a conductor using the voltmeter – ammeter method.

AN EXPERIMENT TO DETERMINE EMF AND INTERNAL RESISTANCE OF A CELL



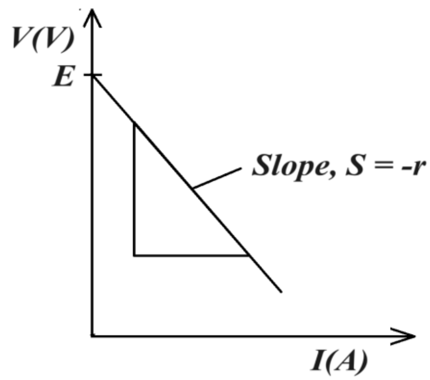
The apparatus is arranged as shown above.

The switch, *K* is closed and the rheostat is adjusted such that the ammeter, *A* gives the smallest possible value of current.

The values if current, *I* and p.d *V* are read from the ammeter and voltmeter respectively and recorded.

The rheostat is adjusted to give several values of *I* and *V*.

The values are tabulated and a graph of *V* against *I* is plotted.



The intercept on the *V- axis* is obtained and is the *emf* of the cell.

The slope, *S* is obtained and the internal resistance *r = -S.*

Theory of the experiment

From $E = I(R + r)$

$E = IR + Ir$

$E = V + Ir$

$$\boxed{V = -(r)I + E}$$

Comparing with the general equation of a straight line, y = mx+c

Slope, S = -r

Intercept, c = E

OHMIC AND NON – OHMIC CONDUCTORS

(a) OHMIC CONDUCTORS

These are conductors that obey ohm's law.

Examples

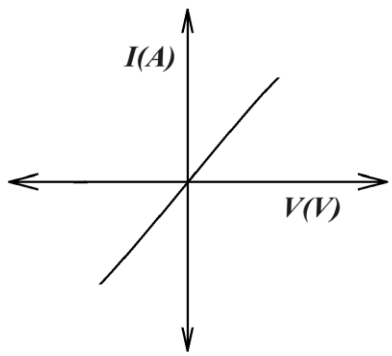
(i) All metals

(ii) Strong ionic solutions (strong electrolytes)

Characteristics of Ohmic conductors

(i) They obey ohm's law

(ii) Their current – voltage graph is a straight line through the origin.



(iii) When the p.d across the ends of such conductors is reversed, current is also reversed in direction but the magnitude remains constant.

(b) NON – OHMIC CONDUCTORS

These are conductors that do not obey Ohms law.

Examples

(i) Filament lamp (eg tungsten filament)

(ii) Gas at low pressure in a discharge tube

(iii) Semi – conductor diode (Junction diodes)

(iv) Weak electrolytes e.g dilute sulphuric acid (Copper II sulphate)

(v) Thermionic diode (vacuum diode)

(vi) Thermistors

(vii) Neon gas

Characteristics of Non Ohmic – conductors

(i) They do not obey ohm's law.

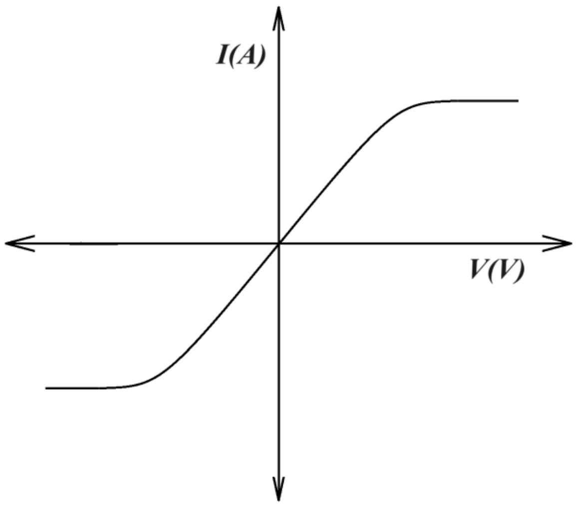
(ii) Their current – voltage graph is a curve instead of a straight line.

(iii) Their current – voltage graph may not pass through the origin.

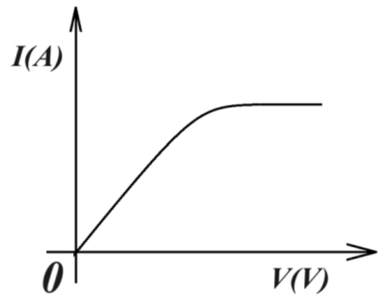
(iv) When the p.d across the ends of such conductors is reversed, they conduct very poorly or not at all.

CURRENT – VOLTAGE GRAPHS FOR NON – OHMIC CONDUCTORS

(i) Filament lamp(tungsten filament)



or

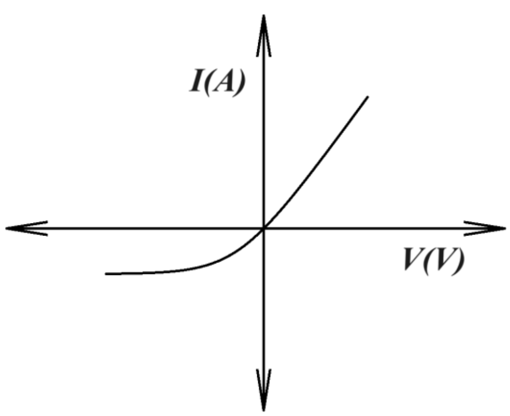


Explanation of the graph

Current is directly proportional to voltage for some values of voltage while the relationship is non – linear for bigger values of voltage.

Note: When the bulb has just been switched on, its resistance is very low and that is why a bulb can easily blow just as it is being switched on, since by then very high current flows through the filament. It is the high current which burns the filament.

(ii) Semi – conductor diode (Junction diodes)



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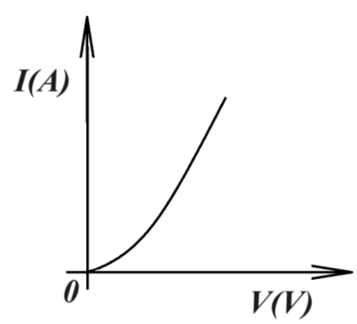
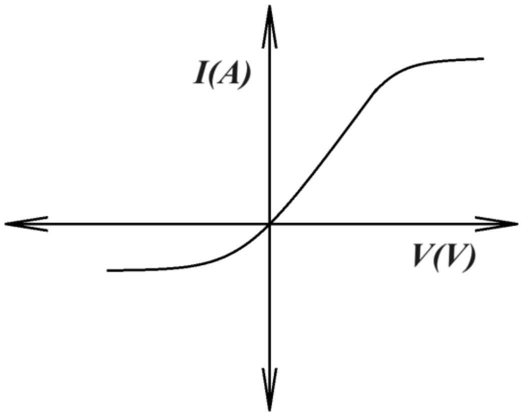


Explanation of the graph

When the diode is forward biased, a small increase in voltage produces a large increase in current. When it is reverse biased, a small reverse current is observed.

(iii) Thermionic diode (vacuum diode)



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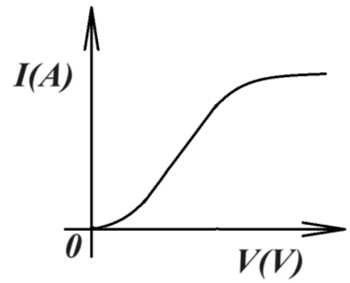
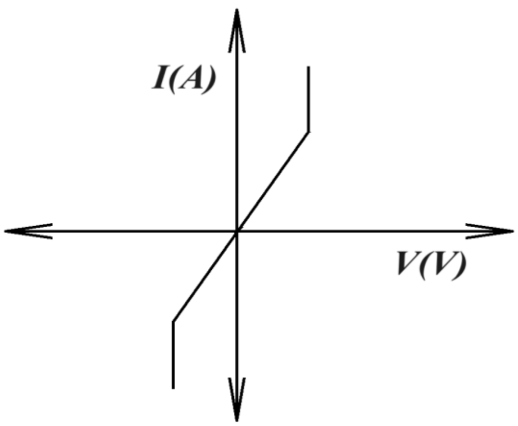


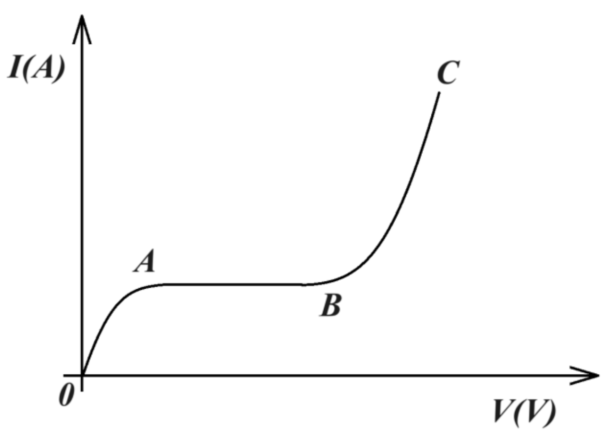
(iv) ***Thermistors***



Explanation of the graph

Thermistors are materials whose resistance decreases sharply as temperature increases.

(v) ***Gas at low pressure in a discharge tube (Neon gas)***



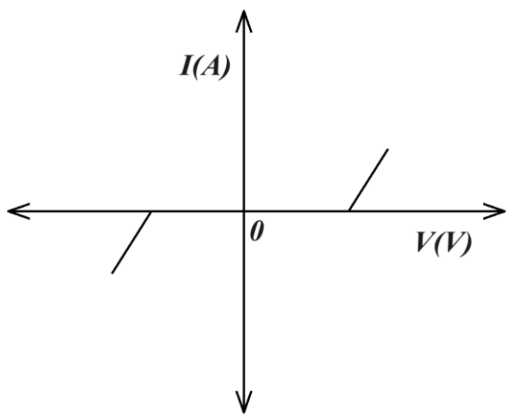
Explanation of the graph

Along *OA*, there is no glow observed and a very weak current detected due to photo electric effect on the cathode, releasing electrons.

Along *AB*, as voltage increases, the anode begins to attract electrons, thus increasing current slightly or very slowly.

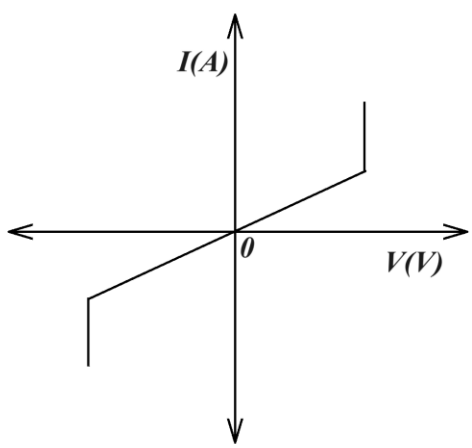
Along *BC*, electrons are accelerated by the anode and they collide with gas atoms, producing more electrons. Thus there is a rapid uncontrollable increase in current.

(vi) Weak electrolytes e.g dilute sulphuric acid (Copper II sulphate)



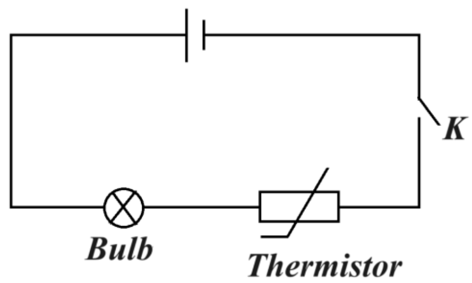
There is no current that flows for very low p.ds until after when the applied p.d is greater than a certain value. This is because the back emf in the electrolyte setup opposes the flow of current through the electrolyte, until the p.d applied is greater than the back emf.

(vii) Neon gas



Question

A dry cell, a bulb, a switch and a thermistor with a negative temperature coefficient of resistance are connected as shown below.



Explain what would happen when switch *K* is closed.

Approach

At room temperature, the resistance of the thermistor is high (due to a low temperature). Current flowing is thus small and the bulb lights dimly.

As current flows, the thermistor heats up (temperature increases) and the resistance decreases leading to an increase in current. Thus the brightness of the bulb increases.

MECHANISM OF ELECTRICAL CONDUCTION

Metals have electrons that are not bound to the nucleus of their atoms and so move freely and drift from one end to another in the metal lattice.

When a p.d is applied across its ends, an electric field is developed which accelerates the electrons.

The accelerated electrons collide with the atoms vibrating about their fixed positions and give out part of their kinetic energy to the atoms.

These electrons decelerate instantaneously after which they are accelerated by the same p.d in the same direction thus acquiring a constant average drift velocity in a direction from the negative terminal of the battery.

It is this drift of electrons that constitutes an electric current.

Note: During this process, the amplitude of vibration increases with increase in internal energy of the atoms as they rub against each other and the temperature of the atoms increases, hence the heating effect of an electric current.

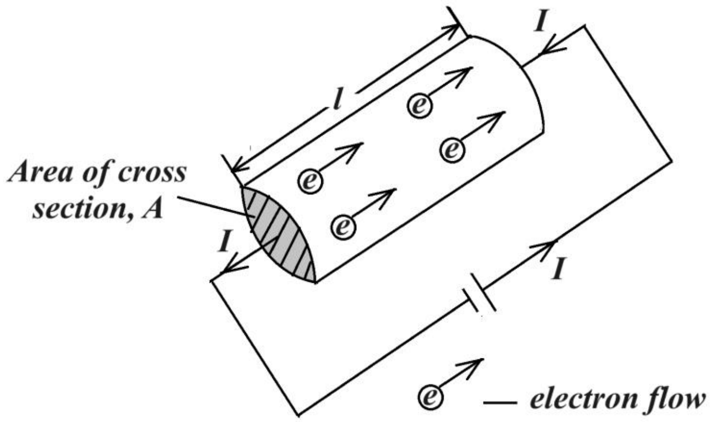
HEATING EFFECT OF AN ELECTRIC CURRENT

DRIFT VELOCITY

This is the average velocity with which the electrons travel through a conductor after acceleration by an electric field between the ends of a conductor.

Expression of the drift velocity

Consider a conductor of length, *l* and uniform cross sectional area, *A* having *n* free electrons per unit volume, each carrying a charge, *e*.



Volume of the conductor, Vol = Al

Total number of electrons = n. Vol = nAl

Total charge, Q = total number of electrons × electron charge

Total charge, Q = nAle

By definition, $I = \frac{Q}{t}$

$$I = \frac{nAle}{t} = nAe\left(\frac{l}{t}\right)$$

But $\frac{l}{t} = v$ *where* v *= drift velocity*

$$\Rightarrow \boxed{I = nevA}$$

$$\therefore \boxed{v = \frac{I}{neA}}$$

Note: It is observed from $I = nevA$ that $I \alpha A$ and this explains why resistance of a conductor increases when the cross sectional area is reduced.

Revision question

A conductor of length, *l* and cross sectional area, *A* has *n* free electrons per unit volume. If the drift velocity is *v* and each electron carries a charge, *e,* derive an expression for the current which flows.

What causes resistance?

When electrons flow through a conductor, they collide with the atoms of the conductor.

These collisions slow down the electrons thus reducing their drift velocity.

The number of collisions the electrons make with the atoms of the conductor depends on the arrangement of the atoms and determines the opposition to their flow and this causes resistance.

Explain how increase in temperature affects electrical resistance of;

(a) ***a conductor***

(b) ***semi – conductor***

Approach

(a) When temperature of a conductor is increased, the atoms of the conductor vibrate with larger amplitudes thus reducing the mean free path of the electrons.

Fewer electrons thus flow per second, implying a reduction in current. This implies that resistance increases with increase in temperature.

(b) For a semi – conductor, when the temperature is increased, the amplitude of vibration of the atoms increases but more electrons are also released for conduction. This increases the flow of electrons per second, implying an increase in current hence resistance reduces with increase in temperature.

CURRENT DENSITY (J)

This is the current flowing per unit cross sectional area of the conductor at right angles to the direction of flow.

$$J = \frac{I}{A}$$

$But\ \ I = nevA$

$$J = \frac{nevA}{A}$$

$$\boxed{J = nev}$$

The S.I unit for current density is *ampere per square metre* (Am^{-2})

EXAMPLES

1. A conductor of cross sectional area $4mm^2$ carries a current of $2mA$. If the conductor contains 10^8 *electrons* each carrying a charge of $1.6\times10^{-19}C$. Determine the;

   (a) drift velocity of the electrons.

   (b) current density of the conductor.

Solution

(a) $v = \frac{I}{neA}$

$$v = \frac{2\times10^{-3}}{10^{8}\times1.6\times10^{-19}\times4\times10^{-6}}$$

$v = 3125ms^{-1}$

(b) $J = nev$

$J = 10^{8}\times1.6\times10^{-19}\times3125$

$J = 500Am^{-2}$

Alternatively

$$J = \frac{I}{A} = \frac{2\times10^{-3}}{4\times10^{-6}}$$

$J = 500Am^{-2}$

2. A metal wire contains 5×10^{22} *electrons* per cubic metre and has a cross sectional area of $1mm^2$. If the electrons move along the wire with a mean drift velocity of $1mms^{-1}$.

(a) Calculate the current density

(b) Calculate the current flowing through the wire

Solution

(a) $J = nev$

$J = 5\times10^{22}\times1.6\times10^{-19}\times1\times10^{-3}$

$J = 8Am^{-2}$

(b) $J = \frac{I}{A}$

$\Rightarrow I = JA = 8\times1\times10^{-6}$

$I = 8\times10^{-6}A$

Alternatively

$I = nevA$

$I = 5\times10^{22}\times1.6\times10^{-19}\times1\times10^{-3}\times1\times10^{-6}$

$I = 8\times10^{-6}A$

3. A conductor contains 10^{12} *electrons* per cubic metre and has an electrical resistivity of $1.2\times10^{-8}\Omega m$. If the electrons drift with a speed of $1.25\times10^{-4}ms^{-1}$. Find the;

(a) current density

(b) electric field intensity

Solution

(a) $J = nev$

$J = 10^{12} \times 1.6 \times 10^{-19} \times 1.25 \times 10^{-8}$

$J = 2 \times 10^{-15} Am^{-2}$

(b) $E = \frac{V}{d} = \frac{IR}{d}$

$But\ R = \frac{\rho l}{A} = \frac{\rho d}{A}$

$\Rightarrow E = \frac{V}{d} = \frac{IR}{d} = \frac{I}{d}\frac{\rho d}{A} = \frac{I\rho}{A} = \left(\frac{I}{A}\right)\rho = J\rho$

$\Rightarrow E = 2 \times 10^{-11} \times 1.2 \times 10^{-8}$

$\therefore E = 2.4 \times 10^{-19} Vm^{-1}$

Assignment

1. A conductor of cross sectional area $4mm^2$ has a current density of $2.5mAm^{-2}$. The conductor contains 10^{12} *electrons per cubic metre*. Calculate the;

   (a) drift velocity of the electrons.

   (b) current flowing

2. A current density of $6\times10Acm^{-2}$ exists in an atmosphere where the electric field intensity is $100Vm^{-1}$. Calculate the electrical resistivity of the earth's atmosphere.

3. A conductor of uniform radius *1.2cm* carries a current of *3A* produced by an electric field of $120Vm^{-1}$. Find the electrical resistivity of the material.

FACTORS AFFECTING RESISTANCE OF A CONDUCTOR

The resistance of a conductor depends on the following factors.

(i) Length of the conductor

(ii) Cross sectional area of the conductor

(iii) Temperature of the conductor

Effect of length on resistance of the conductor

Increase in length of the conductor increases the resistance of the conductor.

This is because when the length of the conductor is increased, electrons move long distances and they make more collisions with the vibrating atoms about their mean positions which reduces the drift velocity of the electrons hence reducing the current flowing through the conductor.

According to ohm's law, a decrease in current is caused by an increase in resistance of the conductor and hence the longer the conductor, the higher the resistance.

$$\left(I\alpha\frac{1}{l} \text{ and } R\alpha\frac{1}{I} \Rightarrow R\alpha l \right)$$

Effect of cross sectional area on resistance of the conductor

Increase in temperature of the conductor increases the resistance of the conductor.

This is because when the cross sectional area is reduced, electrons are confined to move ina small space (volume) which increases the rate of collision with the atoms.

This therefore reduces the drift velocity of the electrons hence reducing the current flowing through the conductor.

According to ohm's law, since reducing cross sectional area reduces the amount of current flowing through the conductor, the resistance of the conductor therefore increases.

$$\left(I\alpha A \text{ and } R\alpha\frac{1}{I} \Rightarrow R\alpha\frac{1}{A} \right)$$

Effect of temperature on resistance of the conductor

Increase in temperature of a conductor increases the resistance of the conductor.

This is because increase in temperature increase the rate of vibration of the atoms about their mean positions.

The drift velocity of the electrons also decreases, which decreases their thermal (heat) energy.

Consequently, the number of collisions between the free electrons and the atoms increases and this further leads to a decrease in the drift velocity of the electrons.

This implies that the number of electrons passing a given point along the conductor per second decreases, hence a decrease in current.

According to ohm's law, a decrease in current implies an increase in resistance of the conductor.

$$(R\alpha T)$$

From the above factors,

$$R\alpha l \text{ and } R\alpha\frac{1}{A}$$

$\Rightarrow R \alpha \frac{l}{A}$

$\therefore \boxed{R = \rho \frac{l}{A}}$

Where ρ is the resistivity of the material of the conductor.

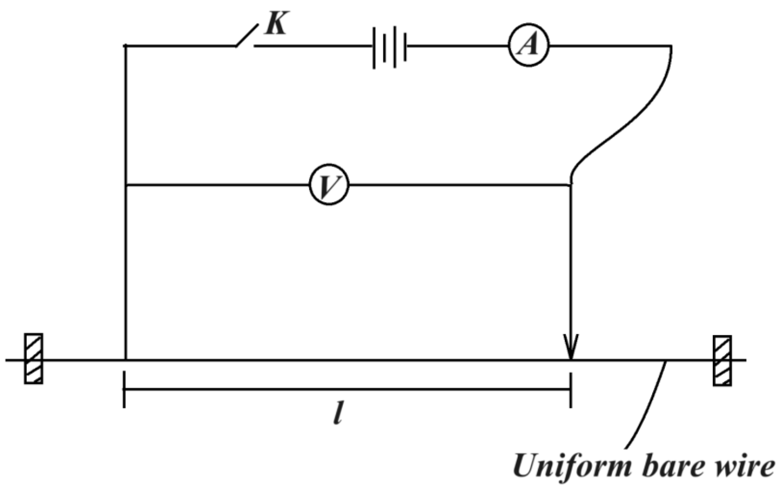
ELECTRICAL RESISTIVITY

This is the resistance between the opposite faces of a $1m^3$ of a material.

$$\boxed{\rho = R\frac{A}{l}}$$

Its *S.I unit* is *ohm metre (Ωm)*

An experiment to investigate the relationship between length and resistance of a conductor



The circuit is connected as shown above.

Starting with a known length, *l* of the wire, switch *K* is closed and the ammeter reading, *I* and voltmeter reading, *V* are noted and recorded.

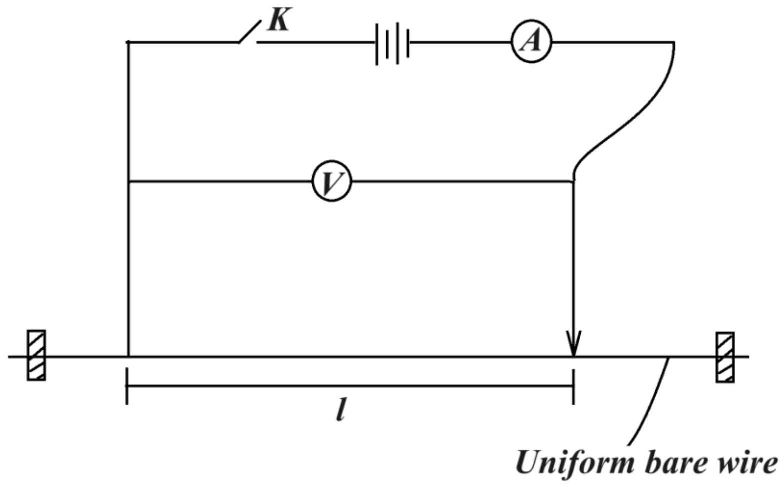
The experiment is repeated for different values of *l* and the corresponding ammeter and voltmeter readings, *I* and *V* respectively are noted.

Results are tabulated including values of $\frac{V}{I}$.

A graph of $\frac{V}{I}$ against *l* is plotted and this gives a straight line through the origin.

This implies that the resistance is directly proportional to the length of the conductor.

An experiment to determine the electrical resistivity of the material of a conductor using the voltmeter – ammeter method



The circuit is connected as shown above.

The diameter, d of the wire of the material whose resistivity is to be determined is measured using a micrometer screw gauge and recorded.

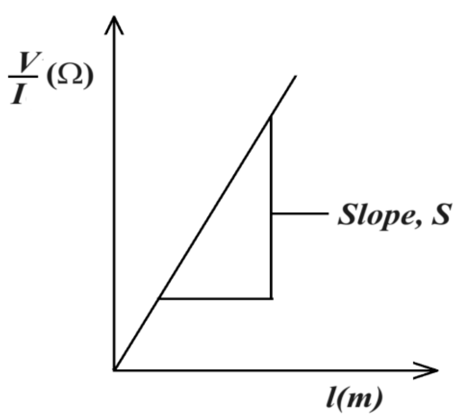
Starting with a known length, l of the wire, switch K is closed and the ammeter reading, I and voltmeter reading, V are noted and recorded.

The experiment is repeated for different values of l and the corresponding ammeter and voltmeter readings, I and V respectively are noted.

Results are tabulated including values of $\frac{V}{I} = R$.

A graph of R against l is plotted and this gives a straight line through the origin.

This implies that the resistance is directly proportional to the length of the conductor.



The slope, S is calculated and the resistivity determined from; $\rho = SA = S \times \frac{\pi d^2}{4}$

Examples

1. A steady uniform current of *5mA* flows along a conductor of cross sectional area $0.2mm^2$ and length *5m*. If the resistivity of the conductor is $3\times 10^{-5}\Omega m$, determine the p.d across the ends of the conductor.

Solution

$V = IR$

$But\ \ R = \rho\frac{l}{A}$

$\Rightarrow R = 3\times 10^{-5}\left(\frac{5}{0.2\times 10^{-6}}\right)$

$R = 750\Omega$

$\Rightarrow V = 5\times 10^{-3}\times 750$

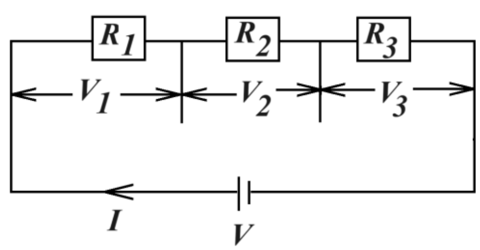
$V = 3.75V$

RESISTOR NETWORKS

Several resistors can be connected in different electrical circuits following either series or parallel arrangements

(a) Series arrangement

Consider three resistors R_1, R_2 and R_3 connected in series across a d.c supply of voltage, V as shown below.



In series arrangement, the current flowing through the resistors is the same but the p.d across each resistor is different.

$Total\ p.d,\ V = V_1 + V_2 + V_3 ------(1)$

From ohm's law,

$$\left.\begin{aligned} V_1 &= IR_1 \\ V_2 &= IR_2 \\ V_3 &= IR_3 \end{aligned}\right\} ------(2)$$

Sub (2) into (1)

$$V = IR_1 + IR_2 + IR_3$$
$$V = I(R_1 + R_2 + R_3)$$
$$\frac{V}{I} = R_1 + R_2 + R_3$$

But from ohm's law; $\frac{V}{I} = R$

$$\therefore \boxed{R = R_1 + R_2 + R_3}$$

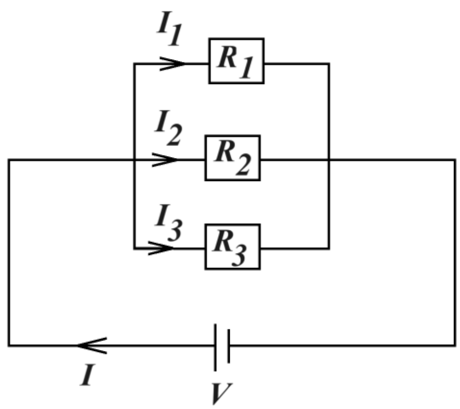
Note:

1. The effective (total) resistance of the resistors in series arrangement is the algebraic sum of the individual resistances.

2. The effective resistance is greater than the individual resistances.

(b) Parallel arrangement

Consider three resistors connected in parallel with a *d.c* supply of voltage, *V* as shown in the diagram below.



The p.d across the reisitors is the same but current flowing through each resistor is different.

Total current, $I = I_1 + I_2 + I_3 \text{------}(1)$

From ohm's law,

$$\left.\begin{aligned} I_1 &= \frac{V}{R_1} \\ I_2 &= \frac{V}{R_2} \\ I_3 &= \frac{V}{R_3} \end{aligned}\right\} \text{------}(2)$$

Sub (2) into (1)

$$I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$I = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$\frac{I}{V} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

But from ohm's law; $\frac{I}{V} = \frac{1}{R}$

$$\therefore \boxed{\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

For 2 resistors,

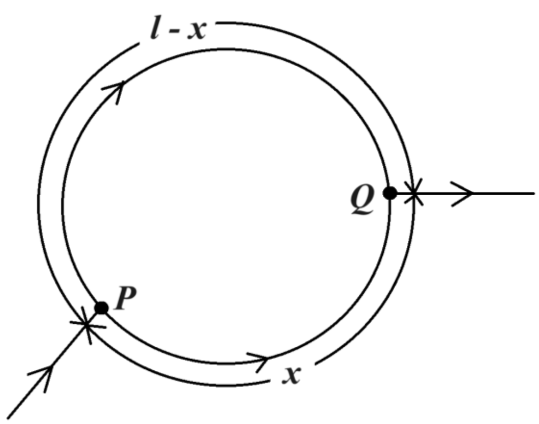
$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{R} = \frac{R_1 + R_2}{R_1 R_2}$$

$$\therefore \boxed{R = \frac{R_1 R_2}{R_1 + R_2} = \frac{\text{Product}}{Sum}}$$

Question

A wire of diameter *d*, length, *l* and resistivity, ρ has a current that enters and leaves the loop at *P* and *Q* respectively as shown in the diagram below.



Show that the resistance of the wire is given by $R = \frac{4\rho x(l - x)}{\pi d^2 l}$

Approach

The two resistors above are in parallel arrangement

Using $R = \rho \frac{l}{A}$

$$R_1 = \frac{\rho(l-x)}{A}$$

But $A = \pi r^2 = \frac{\pi d^2}{4}$

$$\Rightarrow R_1 = \frac{\rho(l-x)}{\left(\frac{\pi d^2}{4}\right)} = \frac{4\rho(l-x)}{\pi d^2} ------(1)$$

Similarly;

$$R_2 = \frac{\rho x}{\left(\frac{\pi d^2}{4}\right)} = \frac{4\rho x}{\pi d^2} ------(2)$$

For parallel arrangement

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{R} = \frac{1}{\left(\frac{4\rho(l-x)}{\pi d^2}\right)} + \frac{1}{\left(\frac{4\rho x}{\pi d^2}\right)}$$

$$\frac{1}{R} = \frac{\pi d^2}{4\rho(l-x)} + \frac{\pi d^2}{4\rho x}$$

$$\frac{1}{R} = \pi d^2\left(\frac{x+(l-x)}{4\rho x(l-x)}\right) = \frac{\pi d^2 l}{4\rho x(l-x)}$$

$$\therefore R = \frac{4\rho x(l-x)}{\pi d^2 l}$$

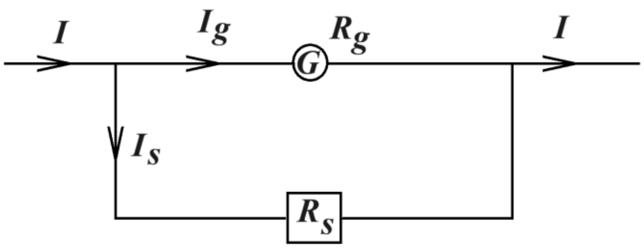
CONVERSION OF A MOVING COIL GALVANOMETER INTO AN AMMETER AND VOLTMETER

A moving coil galvanometer is also known as a milli ammeter and this instrument is used to measure very low currents and voltages of the order of *milli amperes (mA)* and *milli volts (mV)* respectively.

It can also be used to measure currents and voltages in *micro amperes(μA)* and *micro volts (μV)* respectively

(a) Conversion into an ammeter

In order to convert a galvanometer into an ammeter, a resistor of very low resistance called *a shunt* is connected in parallel with the galvanometer.



In this case, I_g is the full scale deflection of the galvanometer.

P.d across the galvanometer = P.d across the shunt

$$I_g R_g = I_s R_s$$

$$I_s = \frac{I_g R_g}{R_s} ------(1)$$

Total current to be measured, $I = I_g + I_s$

$$\Rightarrow I_s = I - I_g ------(2)$$

Substitute (2) into (1)

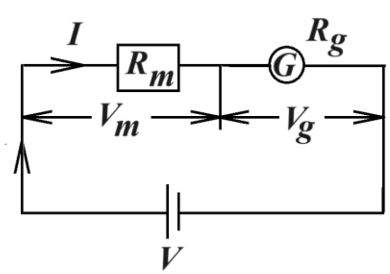
$$\frac{I_g R_g}{R_s} = I - I_g$$

$$\boxed{I_g R_g = (I - I_g)R_s} ------(*)$$

(b) Conversion into a voltmeter

In order to convert a galvanometer into a voltmeter, a resistor of very high resistance called *a multiplier* is connected in series with the galvanometer.

In this case, the current flowing through the galvanometer and the multiplier is the same and is the full scale deflection of the galvanometer.



Total P.d to be measured, $V = V_m + V_g$

By ohm's law,

$V = I_m R_m + I_g R_g$

But $I_m = I_g = I$

$$\boxed{V = I_g\left(R_m + R_g\right)}$$

Examples

1. A moving coil galvanometer of full scale deflection of *4mA* and has a resistance of 5Ω. How can it be converted into an ammeter reading up to *10A.*

Approach

$$I_g R_g = \left(I - I_g\right) R_s$$
$$R_s = \frac{I_g R}{I - I_g} = \frac{0.004 \times 5}{10 - 0.004}$$
$$R_s = 2 \times 10^{-3}\Omega$$

It can be converted into an ammeter by connecting a shunt of resistance $2 \times 10^{-3}\Omega$ in parallel with it.

2. A moving coil galvanometer has a full scale galvanometer deflection of *6mA* and has a resistance of *4Ω.* How can such an instrument be converted into a voltmeter measuring up to *20V.*

Approach

$$V = I_g\left(R_m + R_g\right)$$
$$20 = 60 \times 10^{-3}\left(R_m + 4\right)$$
$$R_m = \frac{20}{60 \times 10^{-3}} + 4$$
$$R_m = 3329.3\Omega$$

It can be converted to a voltmeter by connecting a multiplier of 3329.3Ω in series with it.

3. A moving coil galvanometer of resistance 5Ω sensitivity *2 divisions per milli ampere* gives a full scale deflection of *16 divisions.* Explain how it can be converted to;

(a) an ammeter reading up to 20A.

(b) a voltmeter that gives a full scale deflection if it reads *2 volts per division.*

Approach

(a) *Full scale deflection* $= \frac{16}{2} = 8mA$

$$I_g R_g = (I - I_g)R_s$$
$$8\times10^{-3}\times5 = (20 - 8\times10^{-3})R_s$$
$$R_s = \frac{8\times10^{-3}\times5}{19.992}$$
$$R_s = 2\times10^{-3}\Omega$$

It can be converted to an ammeter by connecting it in parallel with a shunt of resistance $2\times10^{-3}\Omega$.

(b) $V = I_g(R_m + R_g)$

$$V = 16\times2 = 32V$$

$$32 = 8\times10^{-3}(R_m + 5)$$
$$R_m = \frac{32}{8\times10^{-3}} - 5$$
$$R_m = 3995\Omega$$

It can be converted into a voltmeter by connecting it in series with a multiplier of resistance 3995Ω.

ELECTRICAL ENERGY AND POWER

Electrical power is the rate at which energy is converted from one form to another.

Or it is the energy liberated per second.

If a charge, *Q* is transferred from one point to another in a circuit in a time, *t* such that a current, *I* flows through the circuit.

By definition, $I = \frac{Q}{t}$

$$Q = It ------(1)$$

If the p.d between the points is *V*, then the work done, $W = VQ ------(2)$

Substitute (1) into (2)

$$W = VIt$$

This work is stored as electrical energy in the circuit and hence by definition of power,

$$Power = \frac{Work\ \ done}{time} = \frac{VIt}{t}$$

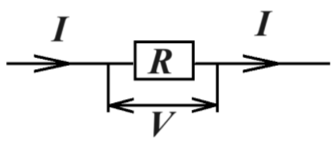
$\boxed{Power = IV}$

Electrical energy is the work done to transfer a charge from one point to another in an electrical circuit.

If all the electrical energy in a circuit is converted into heat energy by a device, then such a device is called ***a passive resistor.***

Definitions
1. Passive resistor
This is an electrical device that converts all electrical energy into heat energy,
Examples include; electric heaters, electric flat irons, etc.
2. Super conductors
These are materials whose resistance vanishes when cooled to a temperature of zero kelvins (0K) or -273°C.
They are often alloys.

Assuming that the resistance of the device is *R*,



$Electrical\ power = Power\ dissipated\ as\ heat$

$IV = P$

$But\ V = IR$

$\Rightarrow P = I(IR)$

$\therefore \boxed{P = I^2R}$

$Also,\ I = \frac{V}{R}$

$\Rightarrow P = \left(\frac{V}{R}\right)R$

$\therefore \boxed{P = \frac{V^2}{R}}$

ELECTROMOTIVE FORCE AND INTERNAL RESISTANCE OF A SOURCE

Electromotive force of the source is the energy converted into electrical energy when a charge of one coulomb passes through the source.

If a source of emf, *E* transfers a steady current, *I* within a time, *t*, then a charge, *Q* flowing through it is given by;

$$Q = It$$

But Electrical energy = Work done = $emf \times Q$

$$Electrical\,energy = E \times It$$

$$Electrical\ \ power = \frac{Electrical\ energy}{time}$$

$$P = \frac{EIt}{t}$$

$$\boxed{P = EI}$$

The *S.I* unit of emf is *volts (V)*

Internal resistance(r)

This is the opposition to the flow of electric current within a cell or battery due to its chemical composition.

The internal resistance of a cell accounts for the energy loses in it when delivering current.

Factors affecting internal resistance of a cell

(i) Surface area of the electrodes

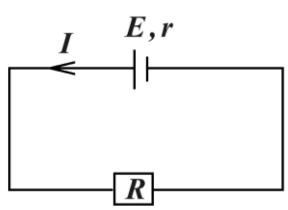
(ii) Nature of the electrodes

(iii) Distance of separation of the electrodes

(iv) Temperature and concentration of the electrolyte

The complete circuit equation

Consider a battery of emf, *E* and internal resistance, *r* joined to an external resistance, *R* such that a current, *I* flows through the circuit.



P.d across the resistor, $V = IR ------(1)$

From ohm's law, $E = I(R + r)$

$$I = \frac{E}{(R + r)} ------(2)$$

Substitute (2) into (1)

$$V=\left(\frac{E}{R+r}\right)R$$

$$V=\left(\frac{R}{R+r}\right)E$$

$$V(R+r)=ER$$

$$VR+Vr=ER$$

Dividing through by R

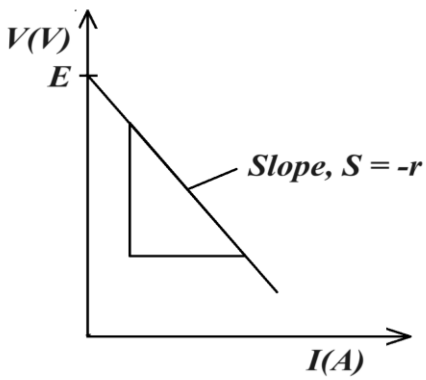
$$\frac{VR}{R}+\frac{Vr}{R}=\frac{ER}{R}$$

$$V+\left(\frac{V}{R}\right)r=E$$

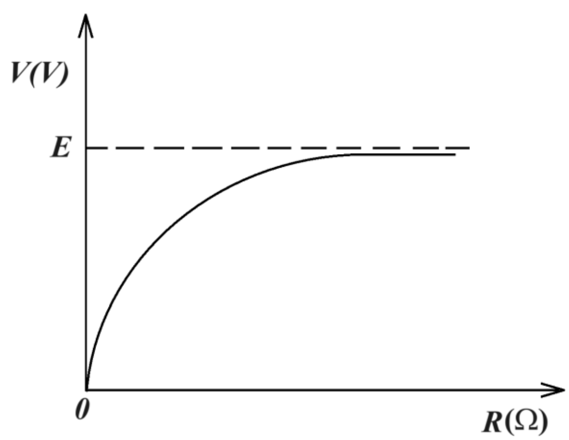
$$V+Ir=E$$

$$\boxed{V=(-r)I+E}$$

From the above equation, ***a graph of V against I*** will be of the form below.



However, ***a graph of terminal p.d, V against load resistance***, *R* has the shape below



As R becomes very large, less current flows from the source and the p.d across the terminals of the source (i.e terminal p.d) tends to the emf, E of the source.

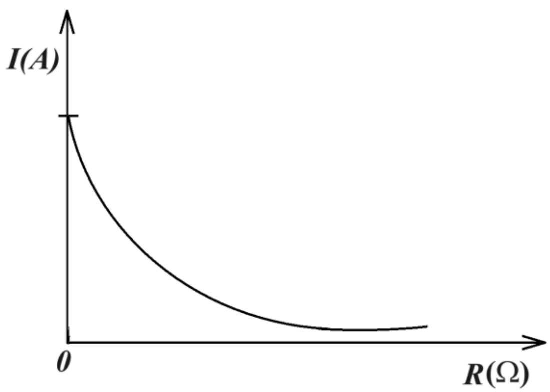
A graph of I against R is of the form below

From $E = I(R + r)$

$$\Rightarrow I = \frac{E}{R + r}$$

When $R = 0,\ I = \frac{E}{r}$

As $R \to \infty,\ I \to 0$



Definition of terms

1. ***Power input***

This is the power generated by a cell or battery.

It is given by $\boxed{P = EI}$

Where E is the emf of the source.

2. ***Power output***

This is the power dissipated in the electrical resistor or load or device.

It is given by;

$\boxed{P = I^2R}$ *where R is the resistance of the external resistor/ load.*

3. ***Power loss (Loss wattage)***

This is the difference between power input and power output of a device.

Power loss= Power input – Power output

$Power\ \ loss = IE - I^2R$

But $E = I(R + r)$

$$Power\ \ loss = I(I(R+r)) - I^2R$$
$$Power\ \ loss = I^2(R+r) - I^2R$$
$$Power\ \ loss = I^2R + I^2r - I^2R$$

$\boxed{Power\ \ loss = I^2r}$ *where r is the internal resistance of the cell or source*

4. Efficiency (η)

This is the ratio of power output to power input in an electric circuit.

It can also be expressed as a percentage.

$$\eta = \frac{Power\ \ output}{Power\ \ input} \times 100\%$$
$$\eta = \frac{I^2R}{IE} \times 100\%$$
$$But\ \ E = I(R+r)$$
$$\eta = \frac{I^2R}{I^2(R+r)} \times 100\%$$

$$\boxed{\eta = \left(\frac{R}{R+r}\right) \times 100\%}$$

OR

$$\eta = \left(\frac{R/R}{R/R + r/R}\right) \times 100\%$$

$$\boxed{\eta = \left(\frac{1}{1 + r/R}\right) \times 100\%}$$

$$If\ \ R = r$$

$$\eta = \left(\frac{r}{r+r}\right) \times 100\% = \left(\frac{r}{2r}\right) \times 100\% = \left(\frac{1}{2}\right) \times 100\%$$

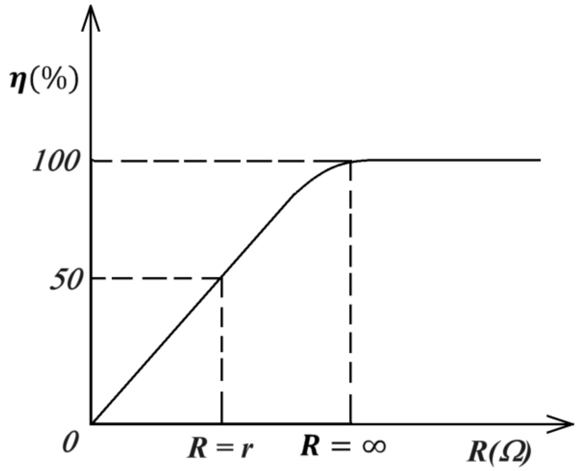
$$\boxed{\eta = 50\%}$$

$$As\ \ R \to \infty,\ \frac{r}{R} \to 0$$

$$\eta = \left(\frac{1}{1+0}\right) \times 100\%$$

$$\boxed{\eta = 100\%}$$

A graph of efficiency against R takes the shape below;

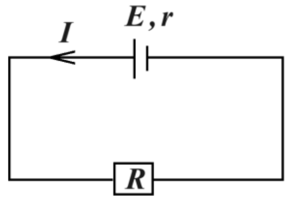


As the load resistance, R increases, the efficiency of the circuit or source increases i.e the efficiency tends to 100% as the load becomes very large.

Hence, for a high efficiency of the source, the resistance of the load should be many times higher than the internal resistance. This minimizes the lost wattage and hence maximizing the power output i.e most of the energy from the source is delivered to the external load.

MAXIMUM POWER THEORY

Consider a battery of *emf, E* and internal resistance, *r* connected to a resistor of variable resistance, *R* as shown below.



Power dissipated in the resistor, $P = I^2R ------(i)$

For a complete circuit, $E = I(R+r)$

$$I = \frac{E}{R+r} ------(ii)$$

Substitute (ii) into (i)

$$P = \left(\frac{E}{R+r}\right)^2 R = \frac{E^2R}{(R+r)^2}$$

$$P = (E^2R)\left((R+r)^{-2}\right) ------(*)$$

Let $u = E^2R$

$$\frac{du}{dR} = E^2$$

Let $v = (R+r)^{-2}$

$$\frac{dv}{dR} = -2(R+r)^{-3}$$

Using product rule,

$$\frac{dP}{dR} = v\frac{du}{dR} + u\frac{dv}{dR}$$

$$\frac{dP}{dR} = (R+r)^{-2}E^2 + E^2R\left(-2(R+r)^{-3}\right)$$

$$\frac{dP}{dR} = E^2(R+r)^{-2} - 2E^2R(R+r)^{-3}$$

But at maximum power, $\frac{dP}{dR} = 0$

$$\Rightarrow 0 = E^2(R+r)^{-2} - 2E^2R(R+r)^{-3}$$

$$2E^2R(R+r)^{-3} = E^2(R+r)^{-2}$$

$$2R(R+r)^{-1} = 1$$

$$\frac{2R}{R+r} = 1$$

$$2R = R+r$$

$$R = r ------(**)$$

Substitute (**) into (*)

$$P = E^2R(R+r)^{-2} = \frac{E^2R}{(R+r)^2}$$

$$P_{\max} = \frac{E^2r}{(r+r)^2} = \frac{E^2r}{(2r)^2} = \frac{E^2r}{4r^2}$$

$$\boxed{P_{\max} = \frac{E^2}{4r}}$$

OR

$$\boxed{P_{\max} = \frac{E^2}{4R}}$$

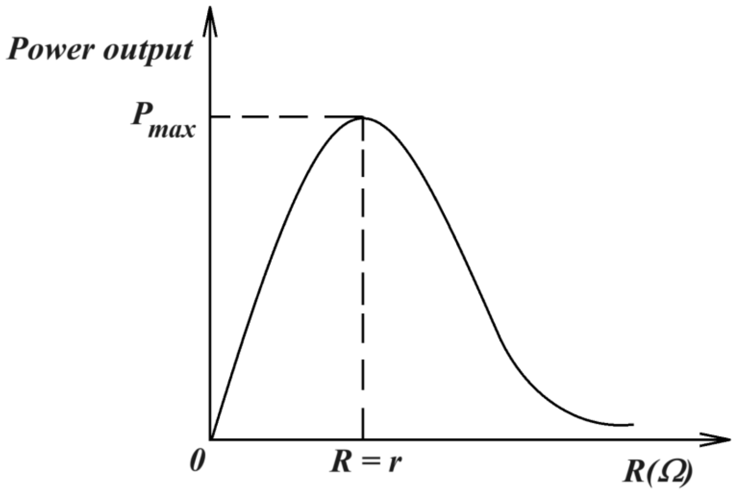
Conclusion

Power output is maximum when the load resistance is equal to the internal resistance of the emf source

Revision Question

A battery of *emf*, E and internal resistance, r is connected to a resistor of variable resistance, R. Obtain the expression for the maximum power dissipated in the resistor.

A graph of power output against load resistance, R



Examples

1. A battery of voltage *12V* and unknown internal resistance is connected in series with a resistor, *R.* A voltmeter connected across the resistor reads *11.4V* and the power dissipated in the battery is *0.653W.* Find the;

   (a) current flowing in the circuit

   (b) internal resistance of the battery

   (c) value of *R*

   (d) efficiency of the circuit.

   Solution

   (a) $E = I(R+r) = V + Ir$

   $12 = 11.4 + Ir$
   $Ir = 0.6$

   Power dissipated in the battery, $P = I^2 r$

   $P = I(Ir)$
   $0.653 = I \times 0.6$
   $I = 1.088A$

(b) $E = V + Ir$

$12 = 11.4 + 1.088r$

$0.6 = 1.088r$

$r = 0.55\Omega$

(c) *From ohm's law,* $V = IR$

$11.4 = 1.088 \times R$

$R = 10.478\Omega$

(d) $\eta = \left(\frac{R}{R+r}\right) \times 100\%$

$\eta = \left(\frac{10.478}{10.478 + 0.55}\right) \times 100\%$

$\eta = 95\%$

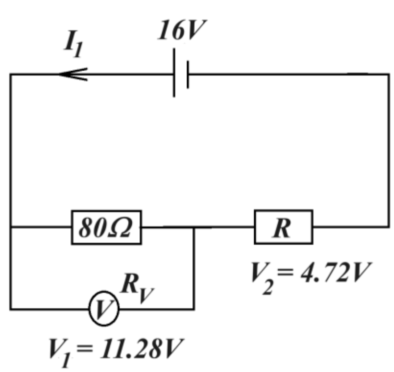
2. A battery of *emf 16V* and negligible internal resistance is connected in series with a resistor of resistance *80Ω* and another resistor of resistance *R*. When the voltmeter is connected across the *80Ω* resistor, it reads *11.28V*, while it reads *2.83V* when across *R*. Find the;

(a) resistance of the voltmeter

(b) value of *R*.

Solution

(a) ***Case 1***



For $V_1 = 11.28V$

$V_1 = I_1 R_P$

Where, $R_P = \frac{80R_V}{R_V + 80}$

$$11.28 = I_1\left(\frac{80R_V}{R_V + 80}\right) ------(i)$$

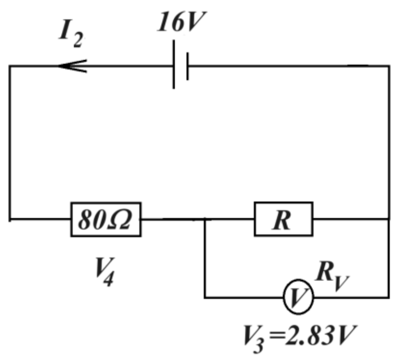
$For\ V_2 = 16 - V_1 = 16 - 11.28 = 4.72V$

$$V_2 = I_1R$$
$$4.72 = I_1R ------(ii)$$

$(i) \div (ii)$

$$\frac{11.28}{4.72} = \frac{80R_V}{(R_V + 80)R}$$
$$\frac{80R_V}{(R_V + 80)R} = 2.39 ------(*)$$

Case 2



$$V_3 = I_2\left(\frac{RR_V}{R + R_V}\right) ------(iii)$$

$But\ V_4 = V - V_3 = 16 - 2.83 = 13.17V$

$$V_4 = I_2 \times 80$$
$$13.17 = 80I_2$$
$$\Rightarrow I_2 = 0.1646A$$

Substitute for I_2 *into (iii)*

$$\frac{RR_V}{R + R_V} = \frac{2.83}{0.1646}$$
$$\frac{RR_V}{R + R_V} = 17.19$$

$$\frac{R + R_V}{RR_V} = \frac{1}{17.19}$$

$$\frac{1}{R}+\frac{1}{R_V}=\frac{1}{17.19}------(**)$$

From (*)

$$R=\frac{80R_V}{2.39(R_V+80)}$$

$$\frac{1}{R}=\frac{2.39(R_V+80)}{80R_V}$$

Substitute for $\frac{1}{R}$ *into (**)*

$$\frac{2.39(R_V+80)}{80R_V}+\frac{1}{R_V}=\frac{1}{17.19}$$

$$\frac{2.39(R_V+80)+80}{80R_V}=\frac{1}{17.19}$$

$$R_V=119.79\Omega$$

(b) *From (**)*

$$\frac{1}{R}+\frac{1}{R_V}=\frac{1}{17.19}$$

$$\frac{1}{R}=\frac{1}{17.19}-\frac{1}{119.79}$$

$$R=20.07\Omega$$

3. Two wires *A* and *B* have lengths which are in the ratio *2:1* and resistivity in the ration of *3:2.* If the wires are arranged in parallel and a current of *1A* flows through the combination, determine the;

(a) ratio of resistance of *A:B*

(b) current flowing through each wire

Solution

(a) $l_A:l_B=2:1$

Total ratio = 2+1 = 3

$$l_A=\frac{2}{3}l,\ and\quad l_B=\frac{1}{3}l$$

$$\rho_A:\rho_B=3:2$$

$$Total\ \ ratio=3+2=5$$

$$\rho_A = \frac{3}{5}\rho, \text{ and } \rho_B = \frac{2}{5}\rho$$

$$\text{Using } R = \rho\frac{l}{A}$$

$$\Rightarrow R_A = \rho_A\frac{l_A}{A} \text{ and } R_B = \rho_B\frac{l_B}{A}$$

$$\frac{R_A}{R_B} = \frac{\left(\rho_A {}^{l_A}\!/\!_A\right)}{\left(\rho_B {}^{l_B}\!/\!_A\right)} = \frac{\rho_A l_A}{\rho_B l_B}$$

$$\frac{R_A}{R_B} = \frac{\left({}^3\!/\!_5\,\rho\right)\left({}^2\!/\!_3\,l\right)}{\left({}^2\!/\!_5\,\rho\right)\left({}^1\!/\!_3\,l\right)} = \frac{{}^2\!/\!_5\,\rho l}{{}^2\!/\!_{15}\,\rho l} = 3$$

$$\frac{R_A}{R_B} = \frac{3}{1}$$

$$\therefore R_A : R_B 3:1$$

(b) *Since the wires are connected in parallel,*

$$V_A = V_B$$

$$I_A R_A = I_B R_B$$

$$\frac{I_B}{I_A} = \frac{R_A}{R_B} = \frac{3}{1}$$

$$I_B : I_A = 3:1$$

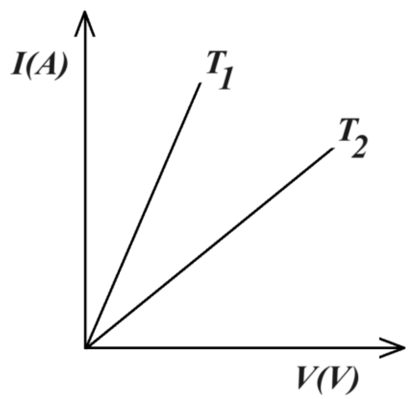
$$\Rightarrow I_B = \frac{3}{4}\times 1A$$

$$I_B = 0.75A$$

$$\text{Also } I_A = \frac{1}{4}\times 1A$$

$$I_A = 0.25A$$

4. The figure below shows the current – voltage graph of a metallic wire at two different temperatures T_1 and T_2. State which of the two temperatures is greater and explain your answer.



Approach

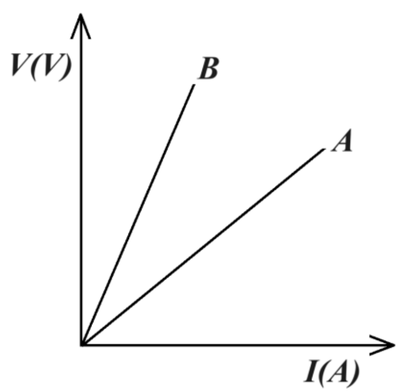
T_2 is greater than T_1 .

Reason

Since it is a current – voltage graph, its slope is the reciprocal of resistance.

At temperature T_1, the gradient or slope of the graph is greater than that at temperature T_2. This implies that the resistance at T_1 is lower than that at T_2 and thus since resistance is directly proportional to temperature, then T_2 is greater than T_1.

5. Voltage – current graphs for two wires *A* and *B* of the same material, radii and at the same temperature are shown in the figure below.



Account for the difference between the graphs.

Approach

Since it is a voltage – current graph, its slope gives the resistance of the material.

B has a steeper gradient than *A* and therefore *B* has a higher resistance than A.

Since resistance is directly proportional to length, it implies that *B* is longer than *A*. Hence the difference between the graphs.

Assignment

1. A battery of unknown emf and internal resistance is connected in series with a load *R*. If a very high resistance voltmeter is connected across the load, it reads *3.2V* and the power dissipated in the battery is *0.32W*. If the efficiency of the circuit is 50%, determine the;

   (a) current through the circuit

   (b) internal resistance of the battery.

   (c) resistance of the load

   (d) emf of the battery.

2. A battery of emf *20V* and internal resistance 4Ω is connected to a resistor of resistor 10Ω. Calculate the;

   (a) power generated

   (b) efficiency of the circuit

3. A power source of *emf 20V* and internal resistance 2Ω is connected to a resistance of 12Ω. Calculate;

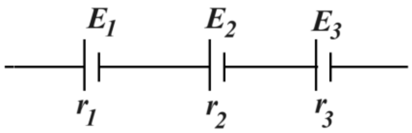
   (a) power generated by the source.

   (b) the efficiency of the circuit.

ARRANGEMENT OF CELLS

Cells can be arranged in series or in parallel with each other in the same circuit.

(a) Series arrangement



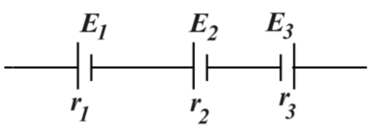
When three cells are connected in series as shown above, the effective emf in the circuit is the algebraic sum of the individual emfs of the cells, i.e.

Effective emf , $\boxed{E = E_1 + E_2 + E_3}$

The effective internal resistance is given by;

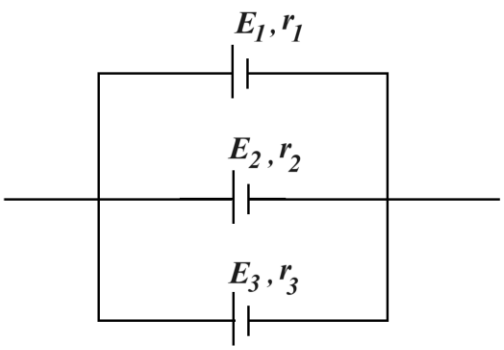
$$\boxed{r = r_1 + r_2 + r_3}$$

Note: If one of the cells is connected in opposition to the direction of flow of current, its emf is assigned a negative sign.



Effective emf, $\boxed{E = (E_1 + E_2) - E_3}$

(b) Parallel arrangement



Cells can only be connected in parallel if they have the ***same emf.***

In this arrangement,

Effective emf, $\boxed{E = E_1 = E_2 = E_3}$

The effective internal resistance however is given by;

$$\boxed{\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}}$$

For identical cells, the effective resistance is obtained as $\frac{1}{r} = \frac{n}{r_1}$

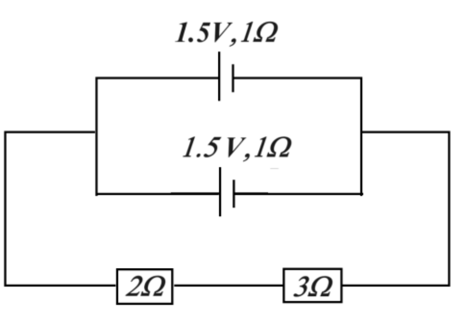
$$\boxed{r = \frac{r_1}{n}}$$

Where, n is the number of cells, r_1 is the internal resistance for one of the cells.

Examples

1. Two identical cells of emf *1.5V* and internal resistance *1Ω* each are connected in parallel and they are in series with resistors of *2Ω* and *3Ω.* Find the power dissipated in the *3Ω* resistor.

Solution



$R = 3 + 2 = 5\Omega$

Effective internal resistance, $r = \frac{1 \times 1}{1+1} = \frac{1}{2} = 0.5\Omega$

$E = I(R + r)$
$1.5 = I(5 + 0.5) = 0.5I$
$I = 0.273A$

Power dissipated, $P = I^2 R$

$P = 0.273^2 \times 3$
$P = 0.2236W$

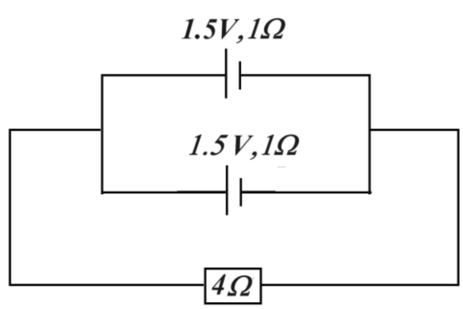
2. Two cells each having an *emf* of *1.5V* and internal resistance *1Ω* are connected to a *4Ω* resistor. Calculate the current in the resistor if the celss are;

(a) in parallel

(b) in series

Solution

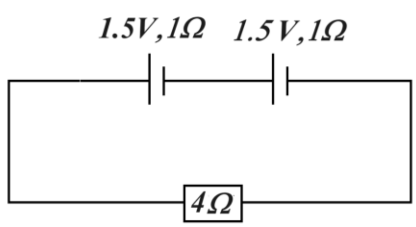
(a)



Effective internal resistance, $r = \frac{1 \times 1}{1+1} = \frac{1}{2} = 0.5\Omega$

$E = I(R + r)$
$1.5 = I(4 + 0.5)$
$1.5 = 4.5I$
$I = 0.33A$

(b)



$$r = 1 + 1 = 2\Omega$$
$$E = 1.5 + 1.5 = 3V$$
$$E = I(R + r)$$
$$3 = I(4 + 2)$$
$$I = \frac{3}{6}$$
$$I = 0.5A$$

Assignment

1. Two similar cells are connected in series and a 2Ω resistor connected across them such that a current of *0.25A* flows through the circuit. When a second resistor of 2Ω is connected in parallel with the first resistor, the current through the circuit increases to *0.3A.* Calculate the *emf* and internal resistance of each cell.

POTENTIAL DIVIDERS

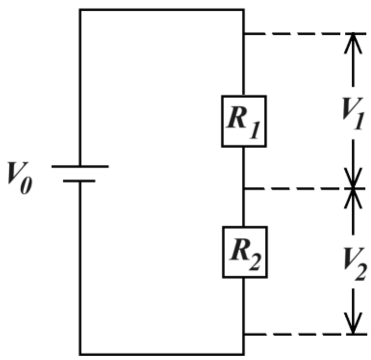
When resistors are connected in series, they form a potential divider. The purpose of this is to obtain small voltages.

Potential dividers consist of two types namely;

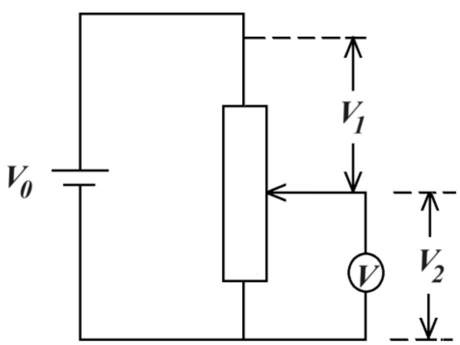
(i) Fixed potential dividers
(ii) Variable potential dividers

Note: A potential divider can be defined as a circuit arrangement that provides a method or means of tapping only the required voltage off a larger voltage.

Fixed potential divider



Variable potential divider



Consider two resistors R_1 and R_2 connected in series to a DC supply of voltage, V_O and negligible internal resistance as shown in the diagram above for a fixed potential divider.

Effective resistance, $R_E = R_1 + R_2$

Current flowing in the circuit,

$$I = \frac{V_0}{R_E}$$

$$I = \frac{V_0}{R_1 + R_2}$$

P.d across R_1, $V_1 = IR_1$

$$V_1 = \left(\frac{V_0}{R_1 + R_2}\right)R_1$$

$$\boxed{V_1 = \left(\frac{R_1}{R_1 + R_2}\right)V_0}$$

P.d across R_2, $V_2 = IR_2$

$$V_2 = \left(\frac{V_0}{R_1 + R_2}\right)R_2$$

$$\boxed{V_2 = \left(\frac{R_2}{R_1 + R_2}\right)V_0}$$

Generally: the p.d across the arrangement of resistors in series is given by;

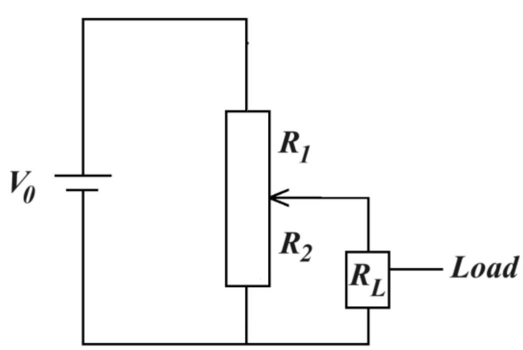
$$\boxed{V_2 = \left(\frac{R_i}{Effective\ \ resistance}\right)\text{supply p.d}}$$

Note:

In potential dividers, the load is connected across the section of the divider whose p.d is equal to its operating voltage.

The load might be a cooker, tv, radio etc. In this case, the resistance of the load will now be considered to be in parallel with resistance of that section of the divider.

Consider the variable potential divider shown below



R_2 and R_L are in parallel and so the effective resistance is given by;

$$R_E = \frac{R_2 R_L}{R_2 + R_L} + R_2$$

Pd across the parallel;

$$V_P = IR_P$$

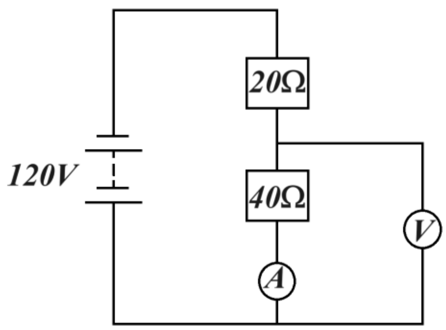
$$But\ \ I = \frac{V_0}{R_E}$$

$$\Rightarrow V_P = \left(\frac{V_0}{R_E}\right) R_P$$

$$\boxed{V_P = \left(\frac{R_P}{R_E}\right) V_0}$$

Examples

1. The figure below shows a potential divider. V is a very high resistance voltmeter and A is an accurate ammeter.



(i) Find the ammeter and a voltmeter reading.
(ii) If the voltmeter is replaced with another voltmeter of resistance 120Ω, what would be its new reading?
(iii) Find the percentage change in ammeter readings.
(iv) If the voltmeter above was replaced with a *CRO*, what would be its reading?

Solution

(i) *Effective resistance,* $R_E = 40 + 20 = 60\Omega$

For ammeter reading,

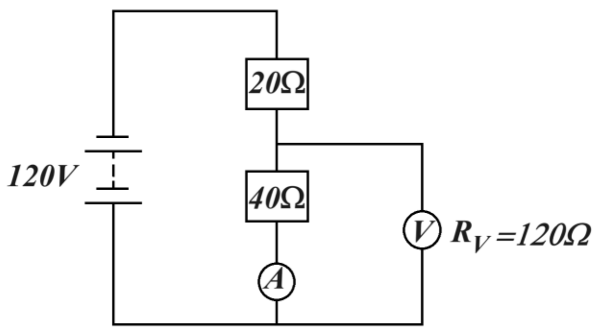
Using Ohm's law, $I = \frac{V}{R} = \frac{120}{60} = 2A$

For voltmeter reading,

Since voltmeter and 40Ω are in parallel, their p.d is the same therefore

Voltmeter reading, $V = IR = 40 \times 2 = 80V$

(ii)



For parallel arrangement, $R_P = \dfrac{40 \times 120}{40 + 120} = 30\Omega$

Effective resistance, $R_E = 30 + 20 = 50\Omega$

For a potential divider;

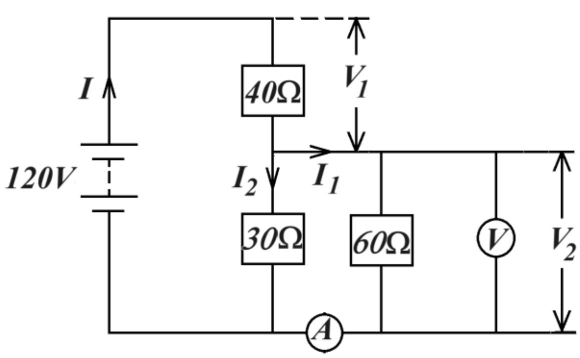
$$V_P = \left(\frac{R_P}{R_E}\right)V_0 = \left(\frac{30}{50}\right) \times 120 = 72V$$

New ammeter reading; $I^1 = \dfrac{V_P}{R_2} = \dfrac{72}{40} = 1.8A$

(iii) *Percentage change in ammeter reading* $= \left(\dfrac{2 - 1.8}{2}\right) \times 100\% = 10\%$

(iv) The voltmeter reading would be 80V because the CRO acts an ideal voltmeter (has an infinite resistance) which makes it very accurate hence its reading will be the same as that in (i) i.e $V = 80V$.

1. The figure below shows a potential divider. V is a very high resistance voltmeter and A is an accurate ammeter.



(i) Find the ammeter and voltmeter reading.

(ii) If the voltmeter was replaced with another voltmeter of resistance 120Ω, what would be its readings?

(iii) Find the percentage change in the ammeter reading

Solutions

(i) *For the parallel arrangement;*

$$R_P = \frac{30\times 60}{30+60} = \frac{1800}{90} = 20\Omega$$

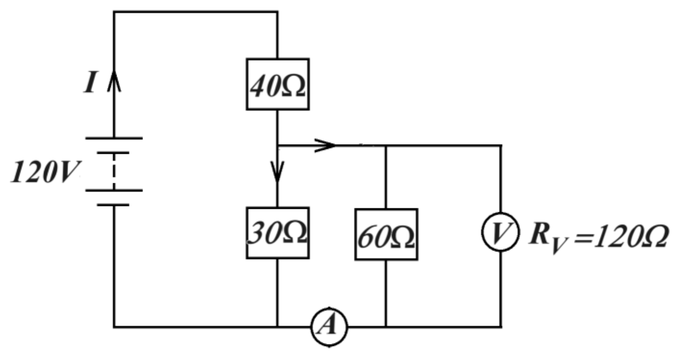
Effective resistance, $R_E = R_P + 40 = 20 + 40 = 60\Omega$

Using Ohm's $I = \frac{V_0}{R_E} = \frac{120}{60} = 2A$

Voltmeter reading, $V_P = IR_P = 20\times 20 = 40V$

Ammeter reading, $I_A = \frac{V_P}{R_{60}} = \frac{40}{60} = 0.667A$

(ii)



For parallel arrangement, $\frac{1}{R_p} = \frac{1}{30} + \frac{1}{60} + \frac{1}{120} = \frac{4+2+1}{120} = \frac{7}{120}$

$$R_P = \frac{120}{7}\Omega$$

Then effective resistance, $R_E = R_P + R_{40} = \frac{120}{7} + 40 = \frac{400}{7}\Omega$

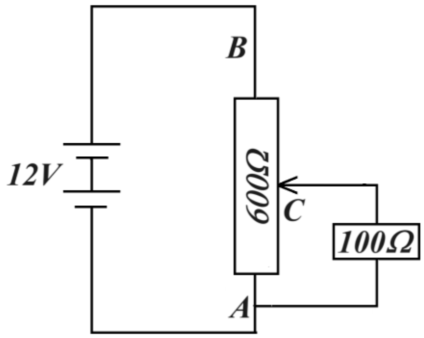
New voltmeter reading, $V_{new} = \left(\frac{R_P}{R_E}\right)V_0 = \frac{\left(\frac{120}{7}\right)}{\left(\frac{400}{7}\right)}\times 120 = 36V$

New ammeter reading, $I_{new} = \frac{V_{new}}{Effective\ \ of\ \ 60\Omega\ \ and\ \ 120\Omega}$

$$I_{new} = \frac{36}{\left(\frac{60 \times 120}{60 + 120}\right)} = 0.9A$$

(iii) $\% \ change \ in \ I = \frac{0.9 - 0.667}{0.667} \times 100\% = 35\%$

2. A 12V battery is connected across a potential divider of resistance 600Ω as shown below.



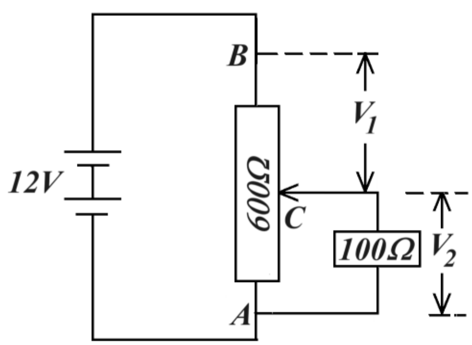
If a load of 100Ω is connected across the terminals of A and C when the slider is halfway up the divider, find the

(i) P.d across the load

(ii) P.d across A and C when the load is removed

Solutions

(i)



$$R_{AC} = \frac{1}{2} R_{AB} = \frac{1}{2} \times 600 = 300\Omega$$

For parallel arrangement, $R_P = \frac{300 \times 100}{300 + 100} = 75\Omega$

Effective resistance, $R_E = R_P + R_{BC} = 75 + 300 = 375\Omega$

Effective current flowing in the circuit, $I = \frac{V_0}{R_E} = \frac{12}{375} = 0.032A$

P.d across the load, $V_2 = IR_P = 0.032 \times 75 = 2.4V$

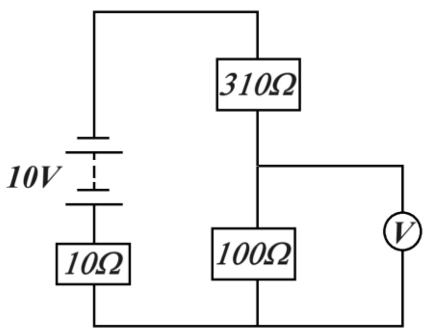
(ii) *When the load is removed, effective resistance,* $R_E = 600\Omega$

New current flowing in the circuit, $I_{new} = \frac{V_0}{New\ effective\ resistance}$

$I_{new} = \frac{12}{600} = 0.02A$

P.d across A and C, $V_{AC} = I_{new} \times R_{AC} = 0.02 \times 300 = 6V$

3. In the circuit below, a voltmeter, V has a resistance of 400Ω.



(i) Find the voltmeter reading

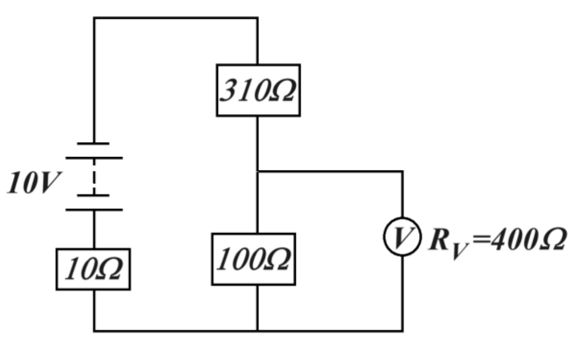
(ii) Calculate the power dissipated in the 100Ω resistor

(iii) What voltage would be obtained if the voltmeter is replaced with a CRO?

(iv) Explain the differences between the voltage obtained in (i) and (iii)

Solutions

(i)



Effective resistance, $R_E = 10 + \frac{100 \times 400}{100 + 400} + 310 = 400\Omega$

Using Ohm's law, $I = \frac{V}{R_E} = \frac{10}{400} = 0.025A$

Therefore, pd across voltmeter/100Ω, $V_P = IR_P = 0.025 \times \left(\frac{100 \times 400}{100 + 400}\right)$

$V_P = 2V$

(ii) $P = \frac{V_p^{\ 2}}{R} = \frac{2^2}{100} = 0.04W$

(iii) *If the voltmeter is replaced by CRO, effective resistance* R_E *will be,*

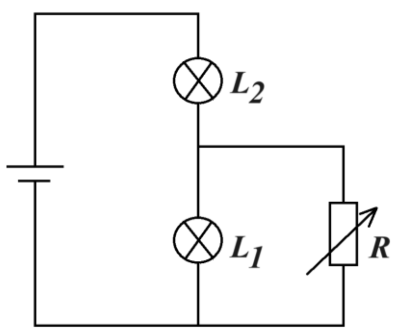
$R_E = 10 + 100 + 310 = 420\Omega$

Using Ohm's law, $I = \frac{V}{R_E} = \frac{10}{420} = 0.024A$

Therefore, pd across voltmeter/100Ω, $V_P^{\ 1} = IR_P = 0.024 \times 100 = 2.4W$

(iv) Using a CRO increases the effective resistance hence increasing the p.d since $V \alpha R$.
A CRO is an ideal voltmeter because it has an infinite resistance therefore the voltages obtained are different because in (iii) almost all current passes through the 100Ω resistor unlike in (i) where some current flows through the voltmeter.

4.



When two identical bulbs are connected in series to a battery of negligible internal resistance, they light normally. The variable resistor R is now connected across bulb L_2. Explain what happens to the brightness of the bulb.

When the variable resistor is set to its maximum resistance, there will be a very slight decrease in the brightness of bulb L_2. But as the resistance of the variable resistor is decreased, the brightness of the bulb continues to decrease until it becomes very dim.

CIRCUIT NETWORKS

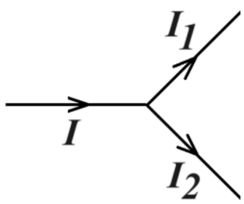
A circuit network is an arrangement of resistors and cells. In such networks, some resistors, current and emfs are given or known while others are unknown.

Solving circuit network problems in electricity involves the use of ***Kirchhoff's laws***

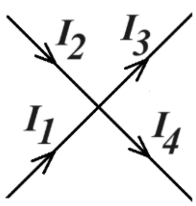
Kirchhoff's laws

Law 1

This states that the sum of currents entering a junction is equal to the sum of currents leaving the same junction. i.e, the algebraic sum of current at a junction is zero.



$$I = I_1 + I_2$$
$$I - I_1 - I_2 = 0$$
$$\therefore \boxed{\sum I = 0}$$



$$I_1 + I_2 = I_3 + I_4$$
$$I_1 + I_2 - I_3 - I_4 = 0$$
$$\therefore \boxed{\sum I = 0}$$

This law is sometimes referred to as the ***law of conservation of current at a point***.

Law 2

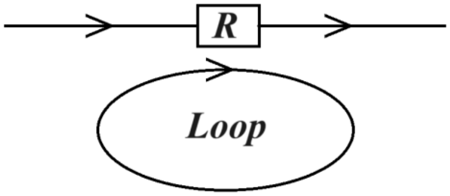
It states that in a closed loop, the algebraic sum of all potential differences is equal to the algebraic sum of all emf's source. i.e $\sum E = \sum V$.

This law is also referred to as ***closed loop equation*** and it is derived from the physical principle of conservation of energy.

SIGN ALLOCATION TO P.Ds

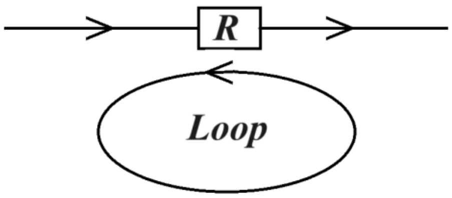
When allocating signs to p.d's across resistors in a closed loop, the direction of current flowing through the resistor is considered in relation to the direction of the loop.

The p.d across the resistor is considered positive if the direction of current through that resistor is the same as the direction of the loop.



$V = +IR$

The p.d across the resistor is considered negative if the direction of current through that resistor is opposite to the direction of the loop.

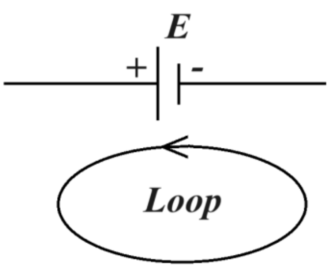


$V = -IR$

SIGN ALLOCATION TO EMFs

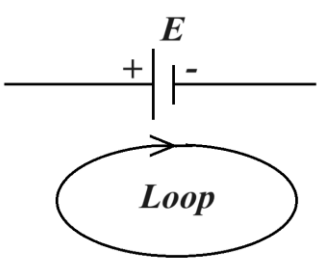
When allocating signs to emf's of a cell in a closed loop, the polarity of the cell is considered in relation to the direction of the loop.

The emf of the cell is considered positive if the loop moves from the negative to the positive terminal of the cell



$E = +ve$

The emf is considered to be negative if the loop moves from the positve terminal to the negative terminal of the cell

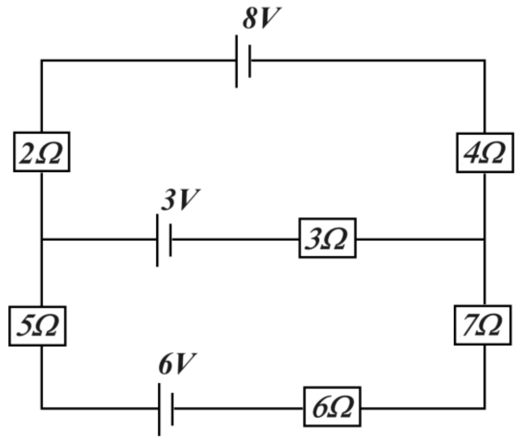


$E = -ve$

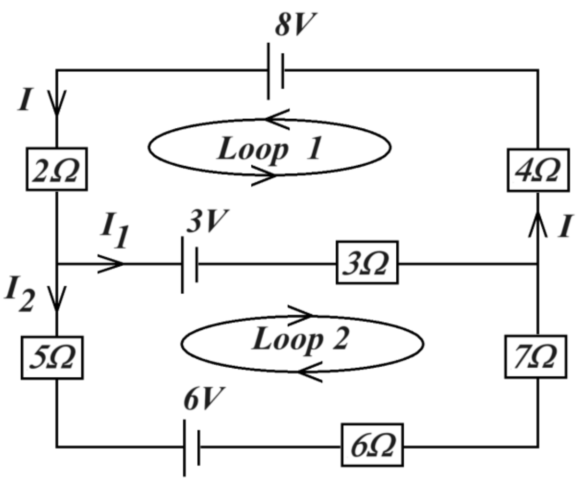
In some circuit networks, the direction of current may not be given. In such cases, you are free to choose any conventional direction for the flow of current. However, after calculations, a positive value of current will imply that the correction assumed and negative value will imply that current flows in the opposite direction.

Examples

1. Determine the current flowing through the 2Ω, 3Ω and 6Ω resistors in the diagram below.



Solutions



At point A

$I = I_1 + I_2 \dots\dots\dots\dots\dots\dots\dots.(i)$

Consider loop1

$\sum E = \sum V$

$8 - 3 = 2I + 3I_1 + 4I$

$5 = 6I + 3I_1 \dots\dots\dots\dots\dots\dots\dots.(ii)$

Consider loop2

$\sum E = \sum V$

$6 - 3 = 3_1 - 5I_2 - 6I_2 - 7I_2$

$3 = 3I_1 - 18I_2$

$1 = I_1 - 6I_2 \dots \dots \dots \dots \dots \dots \dots (iii)$

Substitute (i) int (ii)

$5 = 6(I_1 + I_2) + 3I_1$

$5 = 9_1 + 6_2 \dots \dots \dots \dots \dots \dots (iv)$

From (iii),

$I_1 = 1 + 6I_2$

Substitute for I_1 in (iv)

$5 = 9(1 + 6I_2) + 6I_2$

$5 = 9 + 54I_2 + 6I_2$

$-4 = 60I_2$

$I_2 = -\frac{1}{15}A$

$I_1 = 1 + 6I_2$

$I_1 = 1 + \left(6x\frac{-1}{15}\right)$

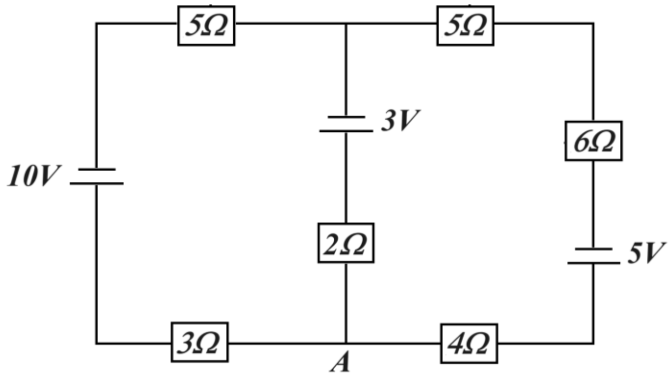
$I_1 = \frac{3}{5}A$

From (i) $I = \frac{3}{15} - \frac{1}{15}$

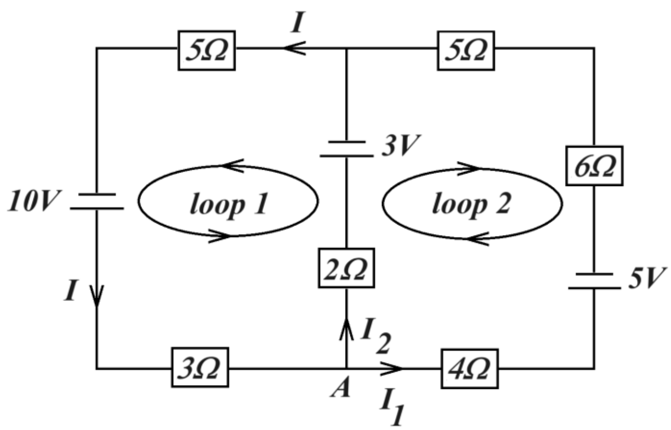
$I = \frac{8}{15}$A

The currents through the 2Ω, 3Ω and 6Ω resistors are $\frac{8}{15}$A, $\frac{3}{5}A$ and $\frac{1}{15}A$ respectively.

2. Find the current flowing through the 3Ω, 2Ω and 4Ω resistors.



Solution



At point A,

$I = I_1 + I_2 \ldots\ldots\ldots\ldots\ldots..(i)$

Considering loop 1,

$\sum E = \sum V$

$10 - 2 = 3I + 5I + 2I_2$

$8 = 8I + 2I_2 \ldots\ldots\ldots\ldots\ldots\ldots..(ii)$

Considering loop 2,

$\sum E = \sum V$

$5 - 3 = 2I_2 - 4I_1 - 6I_1 - 5I_1$

$2 = 2I_2 - 15I_1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots.(iii)$

Substitute (i) into (ii),

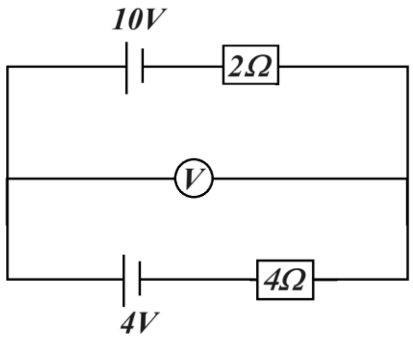
$8 = 8(I_1 + I_2) + 2I_1$

$8 = 8I_1 + 10I_2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(iv),$

On solving (iii) and (iv) simultaneously,

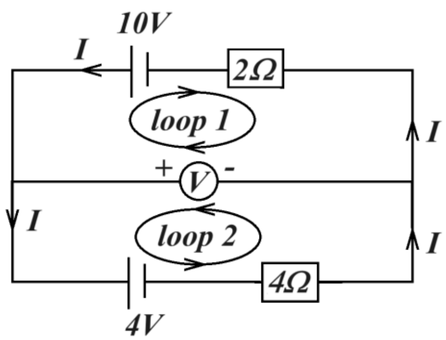
$I = \frac{66}{83}A,\ I_1 = \frac{-2}{83}A,\ I_2 = \frac{68}{83}A.$

3. The figure below shows a network of cells and resistors. V is a voltmeter of very high resistance



Determine the voltmeter reading.

Solutions



Consider loop 1,

$\sum E = \sum V$

$V + 2I = 10 \ldots\ldots\ldots\ldots\ldots\ldots.(i)$

Considering loop 2,

$\sum E = \sum V$

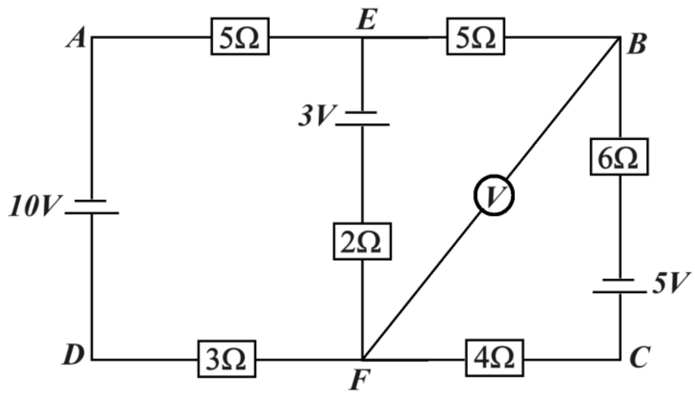
$V - 4I = 4 \ldots\ldots\ldots\ldots\ldots\ldots\ldots..(ii)$

On solving (i) and (ii),

$I = 1A \text{ and } V = 8V$

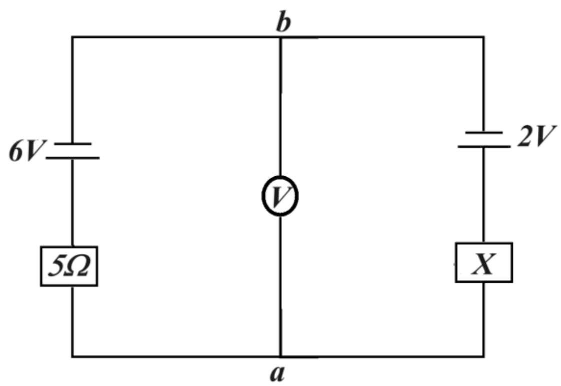
Trial Questions.

1.



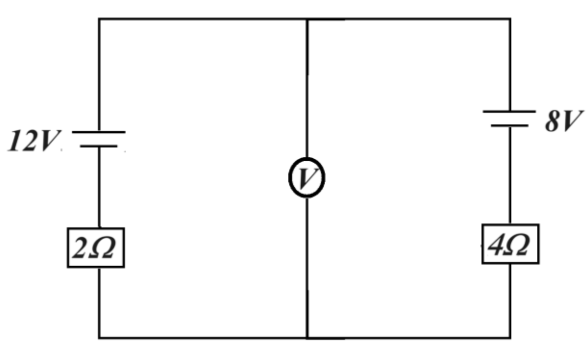
If a very high resistance voltmeter is connected across BF, what will be its reading

2. The figure below shows a very high resistance voltmeter connected across two points a and b of the circuit below.



If the voltmeter reads 3V, find the value of resistance x.

3. Resistors 2Ω and 4Ω are connected in series with power supplies of 12V and 8V as shown in the figure below.



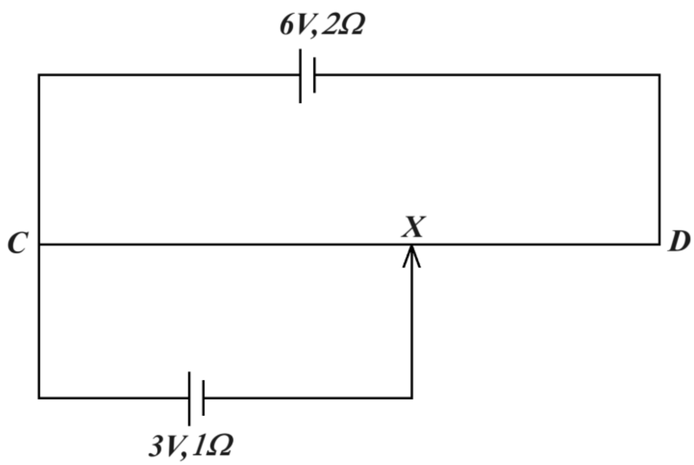
Calculate;

(i) The voltmeter reading.

(ii) The power dissipated in the 4Ω resistor.

2015 8(d)

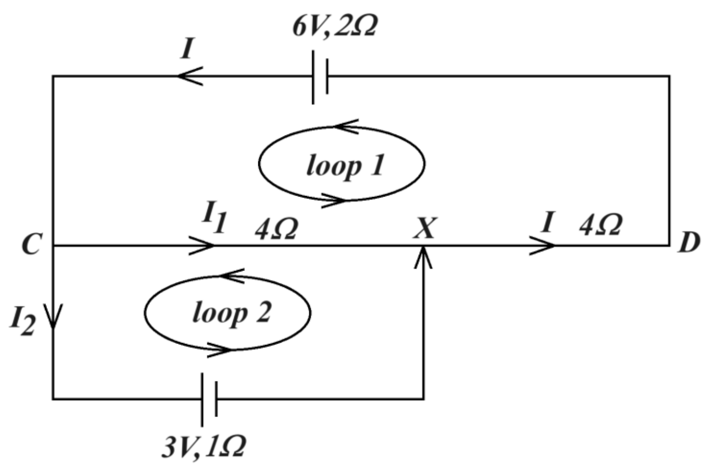
Two cells A of emf 6V and internal resistance 2Ω and B of emf 3V and internal resistance 1Ω respectively are connected across a uniform resistance wire CD of resistance 8Ω as shown in figure below.



If x is exactly in the middle of wire CD, calculate the;

(i) Power dissipated in CX
(ii) P.d across the terminal of cell A.

Solutions



At H;

$I = I_1 + I_2 \dots\dots\dots\dots\dots\dots. (i)$

Considering loop 1,

$\sum E = \sum V$

$6 = 2I + 4I_1 + 4I_1$

$2I + 8I_1 = 6 \ldots\ldots\ldots\ldots\ldots\ldots (ii)$

Considering loop 2,

$\sum E = \sum V$

$-3 = 1I_2 - 4I_1 \ldots\ldots\ldots\ldots\ldots\ldots.. (iii)$

On putting (i) into (ii), we get

$6 = 2I_2 + 10I_1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. (iv)$

On solving (iii) and (iv); $I_1 = \frac{-1}{3}A,\ I_2 = \frac{2}{3}A$

Power dissipated in cx, $P = I^2R = (\frac{2}{3})^2 x4 = 1.777778W\ or\ 1\frac{7}{9}W$

Pd across the terminals of the cell A.

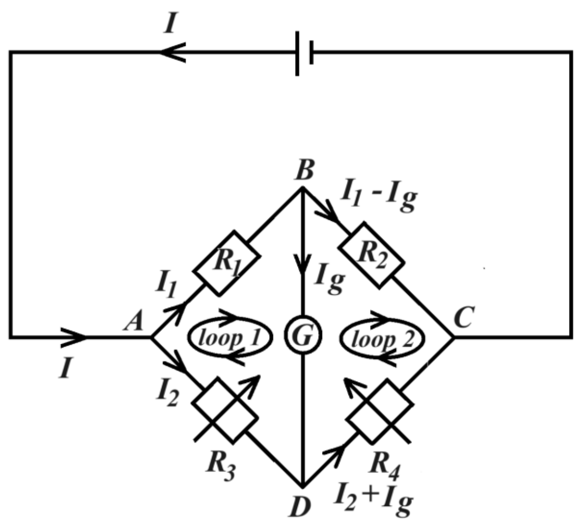
From $E = I(r + R) = Ir + IR$, where IR = V, this implies $E = V + Ir$

Therefore, $V = E - Ir$, but $I = I_1 + I_2 = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}A$,

$$V = 6 - \frac{1}{3}x2 = 5.33333V\ or\ 5\frac{1}{3}V$$

THE WHEATSTONE BRIDGE

This is a circuit arranged with four resistors $R_1,\ R_2,\ R_3$, and R_4 which gives a method of determining the unknown resistance.



In the circuit above, G is a center zero galvanometer.

Assuming the resistance of the galvanometer is R_g and R_3 and R_4 are variable resistors. In order to determine the value of unknown resistance, R_g and R_4 are adjusted until the galvanometer gives zero deflection. This implies that the current through the galvanometer, $I_g = 0$

Applying Kirchoff's laws;

For loop 1 (ABDA)

$\sum E = \sum V$

$0 = I_1R_1 + I_gR_g - I_2R_3$, but at balancing, $I_g = 0$,

This implies; $I_1R_1 = I_2R_3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (i)$

For loop 2 (BSDB)

$0 = I_1R_2 - I_2R_4$

$I_1R_2 = I_2R_4 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. (ii)$

Divide (i) and (ii), we get;

$\frac{R_1}{R_2} = \frac{R_3}{R_4}$........... This is the balancing condition (equation) for wheatstone bridge using Kirchoff's law.

Alternatively

We have noted that at balancing point, $I_g = 0$, and hence resistors R_1 and R_2, R_3 and R_4 are in series respectively. However, resistors R_1 and R_3, R_2 and R_4 are in parallel respectively,

$V_{AB} = V_{AD}$

This implies; $I_1R_1 = I_2R_3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (i)$

Also, $V_{BC} = V_{DC}$

This implies; $I_1R_2 = I_2R_4 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. (ii)$

On dividing (i) and (ii), we get

$\frac{R_1}{R_2} = \frac{R_3}{R_4}$............ This is the balancing condition (equation) for Wheatsone bridge using Ohm's law.

Assuming R_2, R_3 and R_4 are known, then the unknown resistance R_1 can be determined using the expression; $R_1 = (\frac{R_3}{R_4})R_2$

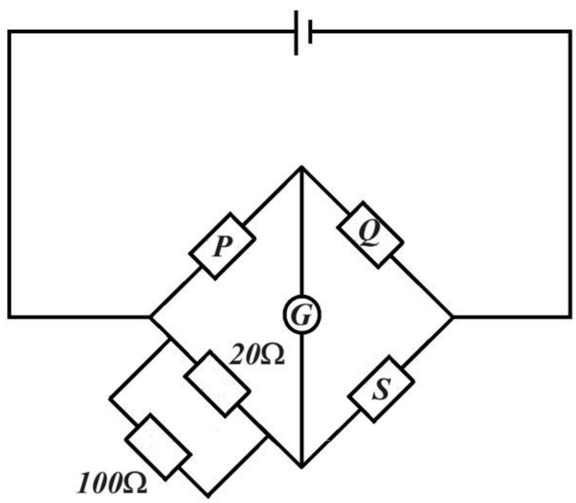
NOTE: The wheatstone bridge is not suitable for measuring (comparing) very low resistances and very high resistances.

Reason:

1. In case of very low resistances (less than one ohm), the resistance of the connecting wire becomes significant (big) in relation to the resistance of the specimen resistors and hence significant errors are obtained in the measurement or comparison.
2. In case of very high resistance, (>$10^6 \Omega$), the current flowing through the circuit will become very low such that the galvanometer becomes less sensitive and hence making it hard or impossible to obtain a balance point.

Examples

1. In the circuit below, no current flows through the galvanometer G, when the 20Ω resistor is shunted with a resistor of 100Ω. When p and q are interchanged, the shunt resistor has to be shunted to 50Ω for G to indicate zero deflection. Calculate the value of s and the ratio of p to q.



Case 1

$$R = \frac{20x100}{20 + 100} = 16.666667\Omega \ or \ 16\frac{2}{3}\Omega$$

Using the balancing condition,

$$\frac{P}{Q} = \frac{R}{S} = \frac{50}{3S}$$

$$\frac{P}{Q} = \frac{50}{3S} \dots\dots\dots\dots\dots\dots..(i)$$

Case 2: *(P and Q interchanged)*

$$R' = \frac{20x50}{20 + 50} = \frac{100}{7}\Omega$$

At balance point;

$$\frac{Q}{P} = \frac{R}{S}$$

$$\frac{Q}{P} = \frac{100}{7S} \dots\dots\dots\dots\dots\dots.(ii)$$

From (ii),

$$\frac{P}{Q} = \frac{7S}{100} \ldots\ldots\ldots\ldots\ldots..(x)$$

$(i) \div (x)$

$$\frac{50}{3S} = \frac{7S}{100}$$

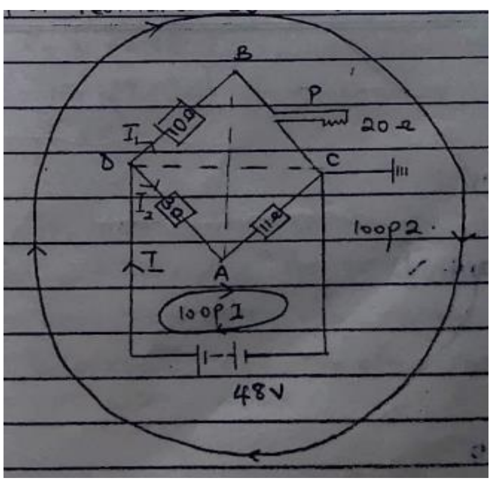
$$S = \sqrt{\frac{5000}{21}} = 15.43\Omega$$

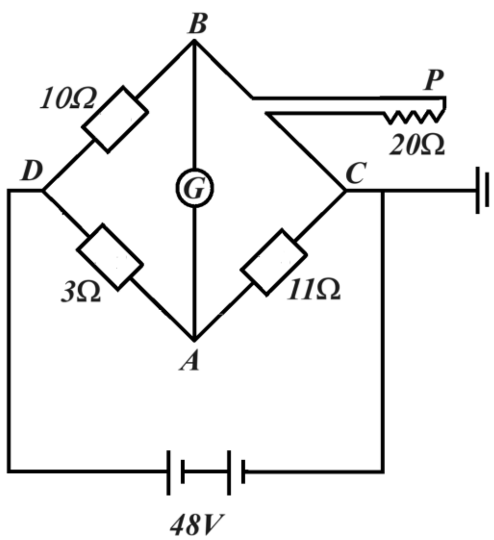
From (i), $\frac{P}{Q} = \frac{50}{3S}$

$$\frac{P}{Q} = \frac{50}{3x15.43} = 1.08$$

Question

The diagram below shows an unbalanced wheatstone bridge. P is a coil of resistance 20Ω at 0^0c.





(i) Calculate the current through the 10Ω resistor

(ii) Potentials at A and B

(iii) If a galvanometer G is now placed between A and B, what will be the direction o current through G.

(iv) If the temperature coefficient of resistance of the material of coil P is $4x10^{-3}K^{-1}$, to what temperature must the coil be raised for the bridge to balance?

Solutions

At point D;

$$I = I_1 + I_2 \ldots\ldots\ldots\ldots\ldots\ldots..(i)$$

From loop 1.

$$48 = 3I_2 + 11I_2$$

$I_2 = \frac{24}{7}A$

For loop 2,

$48 = \ 10I_1 + 20I_1$

$I_1 \ = 1.6A$

(ii)s

Potential at A;

$V = IR$

$= \frac{24x11}{7} = 37.714V$

Potential at B

$V = IR$

$= 1.6x20 = 32V$

(iii) Since current flows from the region of high potential to that of low potential, then the current flows through G from A to B.

TEMPERATURE COEFFICIENT OF RESISTANCE (α)

This is the fractional change in the resistance of a conductor at 0°C per degree celsius rise in temperature.

$$\alpha = \frac{fractional\ change\ in\ resistance\ at\ 0^0C}{change\ in\ temperature}$$

$$\alpha = (\frac{R_\theta - R_0}{R_0}) \div \Delta\theta$$

$$\alpha = \frac{R_\theta - R_0}{R_0\Delta\theta}$$

But $\Delta\theta = \theta = 0 = \theta$

$$\alpha = \frac{R_\theta - R_0}{R_0\theta}$$

$$R_\theta = R_0(1 + \alpha\theta)$$

The SI unit of temperature coefficient of resistance (α) per kelvin (K^{-1})

Note: For a conductor, its resistance increases with increase in temperature and therefore its TCR is positive.

For a semi-conductor, the resistance decreases with increase in temperature and therefore have negative TCR.

Qn. Explain why semi-conductor have a negative temperature coefficient of resistance.

For semi-conductor, increase in temperature leads to a decrease in resistance because they have few electrons in conduction band.

An increase in temperature increases the number of free electrons in the conduction band and this leads to an increase in the current flowing which implies a decrease in resistance hence a negative TCR.

Qn. Explain why conductors have a positive TCR.

Resistance of a conductor increases with increase in temperature.

An increase in temperature increases the kinetic energy of the atoms hence increasing their vibration at their mean positions and the rate of collision between the atoms and electrons, decreasing the drift velocity of the electrons hence a decrease in current which implies an increase in resistance, therefore a positive TCR.

Qn. 3 2022

A metal wire of length 100cm and cross-sectional are $1x10^{-6}m^2$ has a resistance of 0.2Ω at 0^oC. Given that its temperature coefficient of resistance is $6.2x10^{-3}K^{-1}$, calculate the resistivity of a metal at 500^0C

Solutions

$\alpha = 6.2x10^{-3}K^{-1}$ $\quad l = 100cm = 1m$

$\theta = 500^0C$ $\quad A = 1x10^{-6}m^2$

$R_0 = 0.2\Omega$ $\quad \rho_{500} = \ ??$

$$\rho_{500} = \frac{R_{500}A}{l} \ldots\ldots\ldots\ldots\ldots..(i)$$

$$R_{500} = R_0(1+\theta\alpha)$$

$$= 0.2(1+500x6.2x10^{-3})$$

$$= 0.82\Omega$$

Therefore; $\rho_{500} = \frac{0.82x1x10^{-6}}{1} = 8.2x10^{-7}\Omega m$

Qn. (4)

The temperature coefficient of resistance of a wire is $5x10^{-4}K^{-1}$. Find the length of a wire of diameter 1mm and resistivity $1x10^{-6}\Omega m$ at 25^0C needed to make a coil of resistance 6Ω at 95^0C

Solutions

$r = \frac{d}{2} = \frac{1x10^{-3}}{2} = 5x10^{-4}m$

$\rho_{25} = 1x10^{-6}\Omega m, \quad R_{95} = 6\Omega, \quad l = ??$

$R_\theta = R_0(1 + \theta\alpha)$

$R_{25} = R_0(1 + 25\alpha) \dots \dots \dots \dots \dots .. (i)$

$R_{95} = 6\Omega = R_0(1 + 95\alpha) \dots \dots \dots \dots \dots \dots .. (ii)$

$(i) \div (ii)$

$\frac{R_{25}}{6\Omega} = \frac{R_0(1 + 25\alpha)}{R_0(1 + 95\alpha)}$

$R_{25} = 5.8\Omega$

$A = \pi r^2 = \pi x(5x10^{-4})^2 = 7.85398x10^{-7}m^2$

$R = \frac{\rho l}{A},$

$l = \frac{5.8x7.85398x10^{-7}}{1x10^{-6}} = 4.555m$

Qn. 5

An electric heater contains 5m of nichrome wire of diameter 0.58mm. When connected to 240V supply, the heater dissipates 2.5Kw and the temperature of the heater is found to be 1020^0C. If the resistivity of nichrome at 10^0C is $10.2x10^{-7}\Omega m$. Calculate;

(i) Resistance of nichrome at 10^0C.
(ii) Mean temperature coefficient of resistance of nichrome between 10^0C and 1020^0C.

Qn. 6

A heating coil is to be made from nichrome wire which will operate a 12V supply and have a power of 36W when immersed in water at 100^0C. The wire available has an area of cross section of $0.10mm^2$

(a) Find the resistance of the coil at 100^0C.
(b) Calculate the length of the wire which would be required at standard conditions (Resistivity and TCR at 273 are $1.08x10^{-6}\Omega m$ and 8)

Qn. 7

The temperature coefficients of resistance of two wires A and B of diameters 1.2mm and 0.80mm, resistances are $0.004K^{-1}$ and $0.0003K^{-1}$ respectively. If the ratio of their resistances at 0^0C is 1.5. Calculate the;

(i) Ratio of their resistance at 100^0C.
(ii) Ratio of electrical resistivity at 100^0C, given that they have the same length.

Solutions

(i)

$d_A = 1.2mm, \quad d_A = 0.80mm, \quad \alpha_A = 0.0004K^{-1}, \quad \alpha_B = 0.0003K^{-1}$

$\frac{R_{0A}}{R_{0B}} = 1.5, \quad \frac{R_{100A}}{R_{100B}} = \quad ??$

For A, $\quad R_{100A} = R_{0A}(1 + 100\alpha_A) \dots \dots \dots \dots \dots (i)$

For B, $\quad R_{100B} = R_{0B}(1 + 100\alpha_B) \dots \dots \dots \dots \dots \dots (ii)$

$$\frac{R_{100A}}{R_{100B}} = \frac{R_{0A}(1 + 100\alpha_A)}{R_{0B}(1 + 100\alpha_B)}$$

$$= 1.5 \frac{(1+100x0.0004)}{(1+100x0.0003)} = 1.515$$

(ii) S

$$R_{100A} = \frac{\rho_{100A} l}{A_A} \dots \dots \dots \dots \dots \dots \dots (i)$$

$$R_{100B} = \frac{l\rho_{100B}}{A_B} \dots \dots \dots \dots \dots \dots (ii)$$

$(i) \div (ii)$

$$\frac{\rho_{100A}}{\rho_{100B}} = \frac{R_{100A} x A_A}{R_{100B} A_B} = (substitution) = 3.40875$$

5

(i)

$P = IV$

$2.5x1000 = 240I, I = 10.42A$

$$R_{1020} = \frac{240}{10.42} = 23.04\Omega$$

$$R_{10} = \frac{\rho l}{A} = \frac{10.2x10^{-7}x5}{\pi(2.9x10^\wedge 4)^2} = 19.302978\Omega$$

(ii) $R_{1020} = R_0(1 + \alpha\theta)$
$23.04 = R_0(1 + 1020\alpha) \dots \dots \dots \dots (i)$
$R_{10} = R_0(1 + \theta\alpha)$
$19.302978 = R_0(1 + 10\alpha) \dots \dots \dots \dots \dots \dots \dots (ii)$

$(i) \div (ii)$
$\frac{23.04}{19.302978} = \frac{R_0(1+1020\alpha)}{R_0(1+10\alpha)}$. On solving, $\alpha = 1.9x10^{-4}K^{-1}$

Try out Qn. 6

Qn.8

A temperature of wire of length 2.5m is varied and the corresponding resistances are obtained and recorded below.

$\Theta(^0C)$	75	120	150	250	300
$R_\theta(\Omega)$	103	103.8	104.4	105.9	106.8

Given that the diameter of the wire is 0.36mm;

Plot a suitable graph and use it to find the;

(i) Resistance and resistivity of the material of the wire at 0^0C.
(ii) Temperature coefficient of resistance of the material

Qn. 9

The table below shows the resistance of a wire at different temperatures.

$\Theta(^0C)$	30	50	70	90	110
$R_\theta(\Omega)$	103	107	111	115	119

Plot a graph of resistance against temperature and use it to find;

(i) Resistance of the wire at 0^0C.
(ii) TCR of the wire.

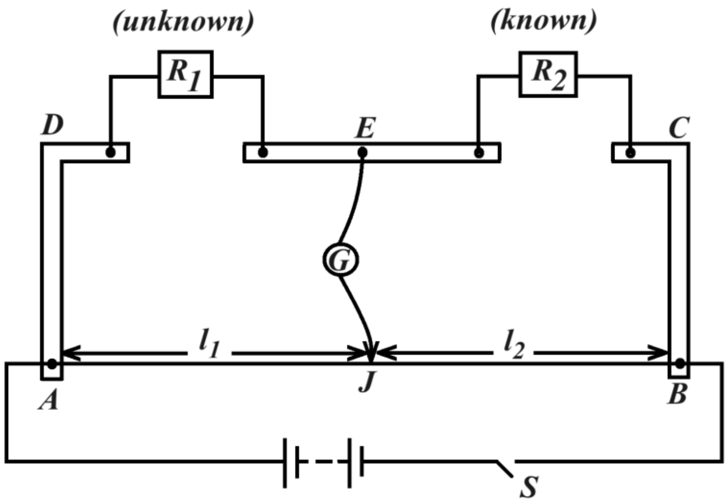
THE METER BRIDGE

This is the simplest practical form of a wheatsone bridge.

A simple meter bridge consists of a uniform resistance wire of a uniform cross-sectional area mounted on a meter rule.

Its major uses are;

- Measure and compare resistances;
- To determine the resistivity of a conductor eg a a wire.
- To measure the temperature coefficient of resistance of a conductor.



A meter bridge works on the same principle as the wheatstone bridge.

At the balance point, a galvanometer shows no deflection and hence the pd across DE is equal the pd across AJ.

$V_{DE} = V_{AJ}$

Using Ohm's law,

$I_1R_1 = I_2r'^{l_1} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(i)$

Also at balance point,

$V_{EC} = V_{JB}$.

$I_1R_2 = I_2r'^{l_2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(ii)$

$(i) \div (ii)$

$\frac{R_1}{R_2} = \frac{l_1}{l_2}$ … … … … … Balance condition for meter bridge.

Qn.

You are provided with a meter bridge, a galvanometer and a standard resistor of resistance R_s.

(a) Describe an experiment to determine the unknown resistance of a conductor using the above apparatus.

(b) Describe an experiment to compare resistance using a meter bridge.

END ERROR FOR METER BRIDGE

Due to imperfect electrical contacts at A and B, the contact resistances are equivalent to the extra lengths e_A and e_B of the wire at A and B respectively.

e_A and e_B are known as end errors or end corrections and therefore they should be added to the measured balance lengths l_1 and l_2 respectively to cater for the contact errors at A and B.

In practice, if we balance a meter rule using R_1 and R_2, the balance condition should be

$$\frac{R_1}{R_2} = \frac{l_1+e_A}{l_2+e_B} \dots\dots\dots\dots..(i)$$

In order to determine the end correction/error, the resistances R_1 and R_2 are interchanged and the new balance lengths are noted giving the new balance equation as,

$$\frac{R_2}{R_1} = \frac{l'_1+e_A}{l'_2+e_B} \dots\dots\dots\dots\dots\dots\dots (ii)$$

Solving (i) and (ii) simultaneously, would give the values of e_A and e_B.

Question 1

When resistors of resistances 4Ω and 8Ω ae connected respectively to the left-hand gap and right-hand gap of the meter bridge. The balance point is obtained at 32cm from the left end of the meter bridge. On interchanging the resistors, the balance point is obtained at 68cm from the left end of the meter bridge. If the resistance of the uniform wire of the meter bridge is 5Ω, calculate the end errors and the resistances corresponding to them.

Case 1,

$R_1 = 4\Omega, \quad R_2 = 8\Omega, \quad l_1 = 32cm, \quad l_2 = 68cm$

Using, $\frac{R_1}{R_2} = \frac{l_1+e_A}{l_2+e_B}$

$$\frac{4}{8} = \frac{32+e_A}{68+e_B}$$

$$2e_A - e_B = 4 \dots\dots\dots\dots..(i)$$

Case 2; On interchanging R_1 and R_2.

$R_1 = 8\Omega, \quad l_1 = 68cm, \quad R_1 = 4\Omega, \quad l_2 = 32\Omega$

$$\frac{8}{4} = \frac{68+e_A}{32+e_A}$$

$$2e_B - e_B = 4 \dots\dots\dots\dots\dots.(ii)$$

On solving (i) and (ii) simultaneously, $e_B = 4cm, \; e_A = 4cm$

Resistance corresponding to the end corrections, $R_{eA} = r'e_A$, where r' is the resistance per centimeter of the wire, and it $= \frac{5}{100} = 0.05\Omega cm^{-1}$

Therefore, $R_{eA} = R_{eA} = 0.05x4 = 0.2\Omega.$

Question 2

When resistors of 3Ω and 5Ω are connected to the left-hand gap and the right-hand gap respectively, a balance point is obtained at 37.4cm from the left-hand side.

When the resistors are interchanged, balance point is obtained at 62.8cm. If resistance of the of the slide wire is 10Ω.

Calculate the end corrections and the resistances corresponding to them.

Case 1.

$R_1 = 3\Omega,\ \ l_1 = 37.3cm,\ \ R_2 = 5\Omega,\ \ l_2 = 62.6cm$

$$\frac{R_1}{R_2} = \frac{l_1 + e_A}{l_2 + e_B}$$

$$\frac{3}{5} = \frac{37.4 + e_A}{62.6 + e_A}$$

$0.8 = 5e_A - 3e_B \ldots\ldots\ldots\ldots\ldots\ldots.(i)$

Case 2

$R_1 = 5\Omega,\qquad R_2 = 3\Omega,\ \ l_1 = 62.8cm,\ \ l_2 = 37.2cm$

$$\frac{5}{3} = \frac{62.6 + e_A}{37.4 + e_B}$$

$0.8 = 5e_B - 3e_A \ldots\ldots\ldots\ldots\ldots\ldots\ldots.(ii)$

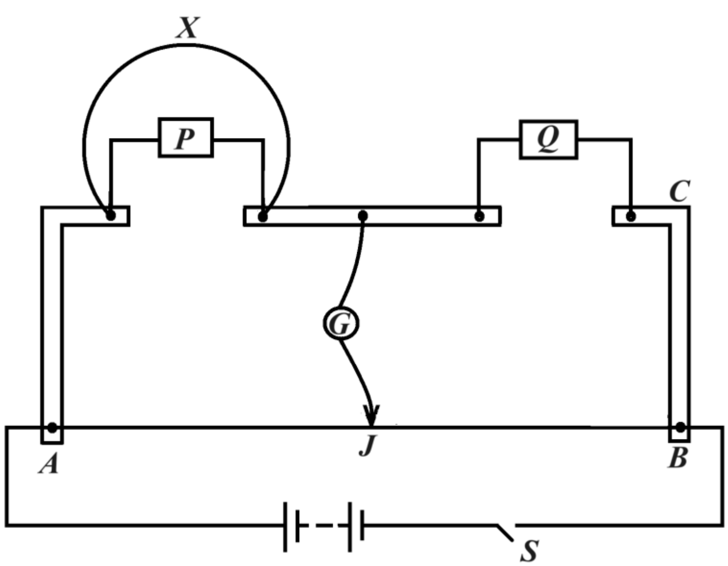
On solving (i) and (ii) simultaneously, $e_A = e_B = 0.4cm$

Resistance per centimeter, r' = $\frac{10}{100} = 0.1\Omega cm^{-1}$

$R_{eA} = r'e_A = 0.1 \times 0.4 = 0.04\Omega$

Question 3

The figure below shows two resistors P and Q of resistance 5Ω and 2Ω respectively connected in the two gaps of a meter bridge.



A resistance wire x of cross -sectional area 1mm^2 is connected across P so that the balance point is 66.7cm from A. If the resistivity of X is $1x10^{-5}\Omega m$ and the resistance wire AB of the meter bridge is 100cm, calculate the length of wire X.

Solutions

$A = 1mm^2$, $\rho = 1x10^{-5}\Omega m$, $l = ??$

From $R = \frac{\rho l}{A}$,

Therefore; $R_x = \frac{\rho_x l_x}{A_x}$……………………(x)

For the L.H.C, Effective resistance, $R_E = \left(\frac{5R_x}{5+R_x}\right)\Omega \ldots\ldots\ldots..(i)$

At balance point, $\frac{R_E}{Q} = \frac{66.7}{33.3}$, After substituting for Q, $R_E = 4.006\Omega$,

On substituting for R_E into (i), $R_x = 20.151\Omega$,

On substituting R_x into (x), $l_x = 2.0151m$

An experiment to determine the resistivity of a wire using a meter bridge