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Any author understands how hard it is to document a book while having a full time job and a family to attend to. For this reason, I dedicate this book to my wife Brenda Onyait, daughter Lorena Gillian Onyait and son Lincoln Edmond Onyait.

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SECTIONA: MECHANICS

CHAPTER1: DIMENSIONS OF A PHYSICAL QUANTITY

1.1.0: Fundamental quantities

These are quantities which can't be expressed in terms of any other quantities by using any mathematical equation. E.g.

Mass - M   Length - L   Time- T

1.1.1: Derived quantities

These are quantities which can be expressed in terms of the fundamental quantities of mass, length, and time e.g.

i) Pressure
ii) Acceleration
iii) Momentum
iv) Density

1.1.2: DIMENSIONS OF A PHYSICAL QUANTITY

This refers to the way a physical quantity is related to the three fundamental quantities of length, mass and time.

Or It refers to the power to which fundamental quantities are raised.

Symbol of dimensions is [ ]

Examples

[Area] $= L^2$

[Volume]$= L^3$

$[Density] = \frac{[Mass]}{[Volume]} = \frac{M}{L^3}$ =ML^{-3}

[Velocity]= $\frac{[Displacement]}{[Time]} = \frac{L}{T} = LT^{-1}$

[Acceleration]= $\frac{[Change\ in\ Velocity]}{[Time]} = \frac{LT^{-1}}{T} = LT^{-2}$

[Momentum]=[Mass][Velocity]=MLT^{-1}

[Weight]=[Mass][Gravitational acceleration]=MLT^{-2}

[Force]=[Mass][Acceleration]= MLT^{-2}

[Pressure]= $\frac{[Force]}{[Area]} = \frac{\mathrm{M}LT^{-2}}{L^2}$ =M$L^{-1}T^{-2}$

NB. Dimension less quantity has no dimensions and is described by a number which is independent of a unit of measurement chosen for the primary quantities

Examples of dimension less quantities

- Refractive index
- strain
- relative density
- all constants such as 2π, 2, π, 4 π, .

They are always given a dimension of one, (1)

1.1.3: USES OF DIMENSIONS

1. Used to check the validity of the equation or check whether the equation is dimensionally consistent or correct.
2. Used to derive equations

a) Checking validity of equations (dimensional homogenity)

When the dimensions on the L-H-S of the equations are equal to the dimensions on the R-H-S, then the equation is said to be dimensionally consistent.

Examples

1. The velocity V of a wave along a flat string is given by $V = \sqrt{\frac{TL}{M}}$

   T - Tension in the string
   L - Length of the string
   M - Mass of the string

Show that the formula is dimensionally correct.

Solution

$$V = \sqrt{\frac{TL}{M}}$$

L.H.S $[V] = LT^{-1}$

R.H.S $\left[\sqrt{\frac{TL}{M}}\right] = \left[\left(\frac{TL}{M}\right)^{\frac{1}{2}}\right] = \left(\frac{[T][L]}{[M]}\right)^{\frac{1}{2}}$

Tension (T) is a force therefore takes the dimensions of force.

$\left(\frac{MLT^{-2}L}{M}\right)^{\frac{1}{2}} = (L^2T^{-2})^{\frac{1}{2}}$

$= L^{2x\frac{1}{2}}T^{-2x\frac{1}{2}}$

$= LT^{-1}$

L.H.S = R.H.S

Since dimension on left are equal to dimensions on right then its correct

2. The period T, of a simple pendulum is given by $T = 2\pi\sqrt{\frac{l}{g}}$ Show that the equation is dimensionally correct.

Where 2 π = dimension less constant

l = length of pendulum

g = Acceleration due to gravity

Solution

L.H.S $[T] = T$

R.HS = $\left[2\pi\sqrt{\frac{l}{g}}\right] = \left[2\pi\left(\frac{l}{g}\right)^{\frac{1}{2}}\right] = [2\,\pi]\left(\frac{[l]}{[g]}\right)^{\frac{1}{2}}$

$= \left(\frac{L}{LT^{-2}}\right)^{\frac{1}{2}} = (T^2)^{1/2} = T$

Since the dimensions on the L.H.S are equal to the dimensions on the R.H.S then the equation is dimensionally consistent.

NB: Dimensions cannot be added or subtracted but for any equation to be added or subtracted then they must have the same dimensions.

Example

Show that the equation $v^2 = u^2 + 2as$ is dimensionally correct.

Solution

L.H.S $[v^2] = (LT^{-1})^2 = L^2T^{-2}$

R.H.S = $[u^2]$ = [2as]

$= (LT^{-1})^2 = LT^{-2}L$

$= L^2T^{-2} = L^2T^{-2}$

Since dimensions on the L.H.S are equal to dimensions on the R.H.S then the equation is dimensionally correct.

Exercise

1. Show that the following equations are dimensionally consistent wher symbols have their usual meanings

   i) $S = ut + \frac{1}{2}at^2$ ii) $v = ut + at$ iii) $Ft = mv - mu$

2. The frequency f of vibration of the drop of a liquid depends on surface tension, γ of the drop, its density, ρ and radius r of the drop. Show that $f = k\sqrt{\frac{\gamma}{\rho r^2}}$ where k is a non= dimensional constant

b) Deriving equations (dimensional analysis)

The method of dimension analysis is used to obtain an equation which is relating to relevant variables

Examples

1. Assume that the period (T) depend on the following
   i) Mass (m) of the bob
   ii) Length (l) of the pendulum
   iii) Acceleration due to gravity (g)

   Derive the relation between T, m, l, g

Solution

$T \propto m^x l^y g^z$

$T = K m^x l^y g^z$ x

Where K is a constant

If it's dimensionally consistent then

$[T] = [K][m]^x[l]^y[g]^z$

$T = M^x L^y (LT^{-2})^z$

$M^0L^0T = M^x L^y L^Z T^{-2Z}$

$M^0L^0T = M^x L^{y+Z} T^{-2Z}$

Powers of M; $x = 0$...............1

poweers of L; $y + z = 0$.........2

powers of T; $-2z = 1$ 3

$z = \frac{-1}{2}$

Put into (2); $y + \frac{-1}{2} = 0$

$y = \frac{1}{2}$

$x = 0, y = \frac{1}{2}, z = \frac{-1}{2}$

Since $T = K m^x l^y g^z$

$T = K m^0 l^{\frac{1}{2}} g^{\frac{-1}{2}}$

$T = K \frac{l^{\frac{1}{2}}}{g^{\frac{1}{2}}}$

$T = K\sqrt{\frac{l}{g}}$

2. Use dimensional analysis to show how the velocity of transverse vibrations of a stretched string depends on its length (l) mass (m) and the tension force (F) in the string.

solution

$V \propto l^x m^y F^z$

$V = K l^x m^y F^z$

$[V] = [K l^x m^y F^z]$

$[V] = [K][l^x][m^y][F^z]$

$LT^{-1} = L^x M^y (MLT^{-2})^z$

since $[K] = 1$

$MLT^{-1} = L^{x+z} M^{y+z} T^{-2z}$

Powers of M; $y + z = 0$..........(1)

Powers of L; M $x + z = 1$......(2)

Powers of T; $-2z = -1$...........(3)

$Z = \frac{1}{2}$

Put into (1); $y + z = 0$

$y + \frac{1}{2} = 0$

$y = \frac{-1}{2}$

Also for equation(2); $x + z = 1$

$x + \frac{1}{2} = 1 \therefore x = \frac{1}{2}$

but $V = K l^x m^y F^z$

$V = K l^{\frac{1}{2}} m^{\frac{-1}{2}} F^{\frac{1}{2}}$

$V = K\sqrt{\frac{l F}{m}}$

3. The viscous force (F) on a small sphere of radius (a) falling through a liquid of coefficient of viscosity η with a velocity V given by $F = K a^x \eta^y V^z$

   Use the method of dimensions to find the values of x, y, z (5marks)

Solution

$[\eta] = \frac{[Force]}{[Area]x[\ vel\ gradient]}$

[F] = MLT^{-2} and [A] = L^2

[Velocity gradient] = $\frac{[V_2 - V_1]}{[l]}$

[Velocity gradient]= $\frac{LT^{-1}}{L} = T^{-1}$

$[\eta] = \frac{MLT^{-2}}{L^2 T^{-1}}$

$[\eta] = M L^{-1} T^{-1}$

$[F] = [K][a^x][\eta^y][V^z]$

$MLT^{-2} = L^x (M L^{-1}T^{-1})^y (LT^{-1})^z$

$MLT^{-2} = M^y L^{x+z-y} T^{-y-z}$

For M : $y = 1$..........(1)

For L: $x + z - y = 1$.......(2)

For T: $-y - z = -2$........(3)

Put (1) into (3)

$-y - z = -2$

$-1 - z = -2$

$Z = 1$

Put into equation(2)

$x + z - y = 1$

$x + 1 - 1 = 1$

$x = 1$

$F = K a^x \eta^y V^z$

F = K a η V

UNEB 2016 NO 1 (a)

(i) Define dimensions of a physical quantity. (01mark)

(ii) In the gas equation

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT$$

Where P= pressure, V= volume, T=absolute temperature, and R= gas constant. What are the dimensions of the constants a and b. (04marks)

UNEB 2010 NO 4 (d)

The velocity V of a wave in a material of young modulus E and density ρ is given by $V = \sqrt{\left(\frac{E}{\rho}\right)}$

Shows that the relationship is dimensionally correct (03 marks)

UNEB2009 No 3b

A cylindrical vessel of cross sectional area, A contains air of volume V, at pressure p trapped by frictionless air tight piston of mass, M. The piston is pushed down and released.

i) If the piston oscillates with simple harmonic motion, shows that its frequency f is given

$f = \frac{A}{2\pi}\sqrt{\frac{\mathrm{p}}{\mathrm{MV}}}$ (06 marks)

ii) Show that the expression for f in b(i) is dimensionally correct (03 marks)

UNEB 2005 No1 b

The equation for the volume V of a liquid flowing through a pipe in time t under a steady flow is given by $\frac{V}{t} = \frac{\pi r^4 P}{8\eta l}$

Where r = radius of the pipe

ŋ = coefficient of viscosity of the liquid

P =pressure difference between the 2 ends

l =length of the pipe

Show that the equation is dimensionally consistent (3mks)

UNEB2003 No 1(a)

Distinguish between fundamental and derived physical quantities. Give two examples of each (04marks)

UNEB2002 No1

a) i)What is meant by the dimension of a physical quantity (01mark)

ii)For a stream line flow of a non-viscous, in compressible fluid, the pressure P at a point is related to the width h and the velocity v by the equation.

(P -a) = ρg(h-b) + ½ ρ (v^2-d) where a, b and d are constant and ρ is the density of the fluid and g is the acceleration due to gravity. Given that the equation is dimensionally consistent, find the dimensions of a, b and d (03 marks)

Solution

NB: We only add and subtract quantities which have the same dimensions.

$(P - a) = \rho g(h - b) + ½\rho(v^2 - d)$

LHS: $[P] = [a]$

$$[P] = \frac{[Force]}{[Area]} = \frac{MLT^{-2}}{L^2} = ML^{-1}T^{-2}$$

$[a] = ML^{-1}T^{-2}$

On the RHS: $[h] = [b]$

$[b] = L$

$[v^2] = [d]$

$(LT^{-1})^2 = [d]$

$[d] = L^2T^{-2}$

UNEB 2001 No 2 b

The velocity V of sound travelling along a rod made of a material of young's modulus y and density ρ is given by V=$\sqrt{\frac{Y}{\rho}}$ Show that the formula is dimensionally consistent (03 mks)

UNEB 1997 No 1

a) i) What is meant by dimensions of a physical quantity (1mk)

ii)The centripetal force required to keep a body of mass m moving it a circular path of radius r is given by $F = \frac{m\,v^2}{r}$ show that the formula is dimensionally consistent. (04 marks)

CHAPTER 2: MOTION

2.1.0: LINEAR MOTION

This is motion is a straight line

Distance

This is the length between 2 fixed point

Displacement

This is the distance covered in a specific direction

Speed

This is the rate of change of distance with time

OR It is the distance covered by an object per unit time.

The SI unit of speed is ms^{-1}

Velocity

It is the rate of change of displacement with time

OR It is the distance covered per unit time in a specific direction

The SI unit of velocity is ms^{-1}

Uniform velocity

Is the velocity of a body which covers equal displacement in equal time intervals.

Acceleration

It is the rate of change of velocity with time

It SI unit is ms^{-2}

Acceleration $= \frac{change\ in\ velocity}{time}$

$$a = \frac{v-u}{t}$$

Uniform acceleration

Constant rate of change of velocity.

Equations of uniform acceleration

1st equation

Suppose a body moving in a straight line with uniform acceleration a, increases its velocity from u to v in a time t, then from definition of acceleration

$$a = \frac{v-u}{t}$$

$$at = v-u$$

$$\boxed{v = u + a\,t}\text{................}1$$

2nd equation

Suppose an object with velocity u moves with uniform acceleration for a time t and attains a velocity v, the distance s travelled by the object is given by S = average velocity x time

$$S = \left(\frac{v+u}{2}\right)t$$

But $v = u + at$

$$S = \frac{(u+at+u)}{2}t$$

$$S = \frac{(2u+at)t}{2}$$

$$S = \frac{2ut+at^2}{2}$$

$$\boxed{S = ut + \frac{1}{2}at^2}\text{........................}2$$

3rd equation

S = average velocity x time

$$S = \left(\frac{v+u}{2}\right)t$$

But $t = \frac{v-u}{a}$

$$S = \left(\frac{v+u}{2}\right)\left(\frac{v-u}{a}\right)$$

$$S = \frac{(v+u)(v-u)}{2a}$$

$$S = \frac{v^2-u^2}{2a}$$

$$2as = v^2 - u^2$$

$$\boxed{v^2 = u^2 + 2as}\text{....................}3$$

Note

- The three equations apply only to uniformly accelerated motion
- When the object starts from rest then (u=0m/s) and when it comes to rest (v=0m/s)

- The acceleration can be positive or negative. When its negative, then it known as a retardation or deceleration

Examples

1) A car moving with a velocity of $10ms^{-1}$ accelerates uniformly at $1ms^{-2}$ until it reaches a velocity of $15ms^{-1}$. Calculate,
   i) Time taken
   ii) Distance traveled during the acceleration
   iii) The velocity reached 100m from the place where acceleration began.

Solution

i) $v = u + at$
u=10m/s, a=1m/s, v=$15ms^{-1}$,
$15 = 10 + t$
$t = 5s$
ii) $v^2 = u^2 + 2as$

$15^2 = 10^2 + 2x1xs$
$225 = 100 + 2s$
$S = 62.5m$
iii) $S = 100m$, v=? u=$10ms^{-1}$a=1

$v^2 = u^2 + 2as$
$v^2 = 10^2 + 2x1x100$
$v = 17.32m/s$

2) A particle moving in a straight line with a constant acceleration of $2ms^{-2}$ is initially at rest, find the distance covered by the particle in the 3rd second of its motion.

Solution

Using $S = ut + \frac{1}{2}at^2$
u=0m/s, t=2s and t=3s a= $2ms^{-2}$
t=2 : $s = 0x2 + ½ x2x2^2 = 4m$
When t=3: a=$2ms^{-2}$ u=0m/s
$s = 0x3 + ½ x2x3^2 = 9m$

Distance in 3rd
Distance for 3s – distance for 2s
$= 9 - 4 = 5m$
Distance in 3rds in 5m

3) A Travelling car A at a constant velocity of 25m/s overtake a stationery car B. 2s later car B sets off in pursuit , accelerating at a uniform rate of $6ms^{-2}$. How far does B travel before catching up with A

Solution

For A: $S_A = ut + \frac{1}{2}at^2$
Since it moves with a constant velocity a=0
S_A = 25t---------------(1)

For B: $S_B = ut + \frac{1}{2}at^2$

If B is to catch up with A then it must travel faster i.e it will take a time of (t-2)s
$S_B = 0x(t-2) + ½ x6(t-2)^2$
$S_B = 3t^2-12t+12$..............(2)

For B to catch A then
$S_A=S_B$
$25t = 3t^2 - 12t + 12$

$3t^2 - 37t + 12 = 0$
t= $\frac{37 \pm \sqrt{37^2 - 4x12x3}}{2x3}$
t=12s or t= $\frac{1}{3}$ s
Since the car leaves 2s later then time 12s is correct since it gives a positive value
S_B = 25x12
S_B = 300m

4) A train travelling at $72kmh^{-1}$under goes uniform deceleration of $2ms^{-2}$,when brakes are applied. Find the time taken to come to rest and the distance travelled from the place where brakes are applied.

Solution

u= $\frac{72x1000}{60x60}$ =$20ms^{-1}$
a=$-2ms^{-2}$, v=0 comes to rest

$v = u + at$
$0 = 20 - -2xt$
$t = 10s$
$s = ut + ½ at^2$

s= 20x10+1/2 x-2x10^2
s= 100m

EXERCISE:1

1. A particle is moving in a straight line with a constant acceleration of $6.0ms^{-2}$. As it pass a point A its sped is $20ms^{-1}$. What is its sped 10s after passing A **An[$80ms^{-1}$]**
2. A particle which is moving in a straight line with a velocity of $15ms^{-1}$ accelerates uniformly for 3.0s, increasing its velocity to $45ms^{-1}$. What distance does it travel while accelerating **An[90m]**
3. A car starts to accelerate at a constant rate of $0.80ms^{-2}$. It covers 400m while accelerating in the next 20s. what was the speed of the car when it started to accelerate **An[$12ms^{-1}$]**
4. A car moving at $30ms^{-1}$ is brought to rest with a constant retardation of $3.6ms^{-2}$. How far does it travel while coming to rest **An[125m]**

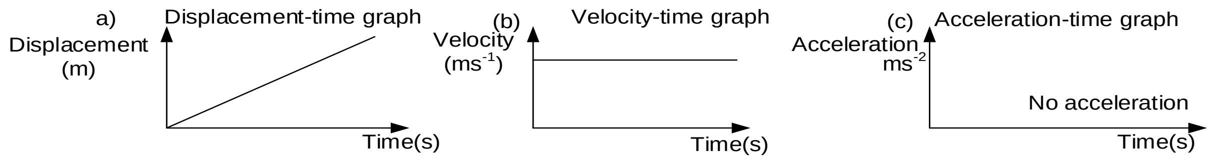
5. A car moving with a velocity of 54km/hr accelerates uniformly at a rate of 2ms^{-2}. Calculate the distance travelled from the place where acceleration began, given that final velocity reached is 72km/hr and find the time taken to cover this distance. **An [$43\frac{3}{4}m$, 2.5s]**

6. A bus travelling steadily at 30m/s along a straight road passes a stationary crab which, 5s later, begins to move with a uniform acceleration of 2ms^{-2} in the same direction as the bus
   (a) How long does it take the car to acquire the same speed as the bus
   (b) How far has the car travelled when it is level with the bus **An[15s, 1181m]**

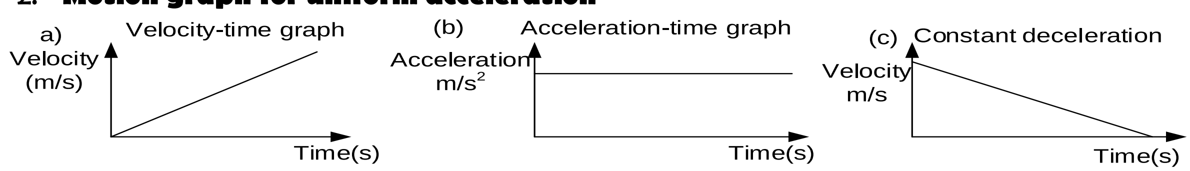
7. A body accelerates uniformly from rest at the rate of 6ms^{-2} for 15 seconds. Calculate
   i) velocity reached within 15 seconds
   ii) the distance covered within 15 seconds **An[90m/s, 675m]**

8. A particle moving on a straight line with constant acceleration has a velocity of 5ms^{-1} at one instant and 4s later it has a velocity of 15ms^{-1}. Find the acceleration and distance covered by particle.
An [a = 2.5ms^{-2}, s=40m]

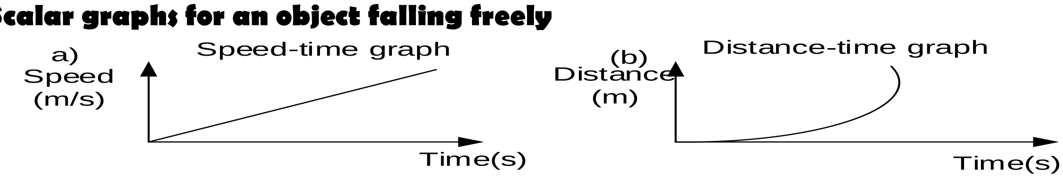
1. Motion graphs for uniform velocity



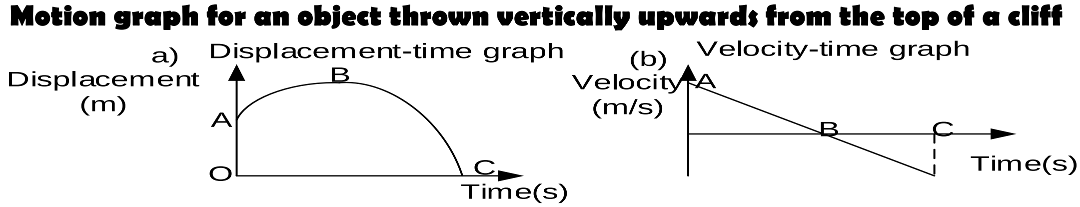
2. **Motion graph for uniform acceleration**



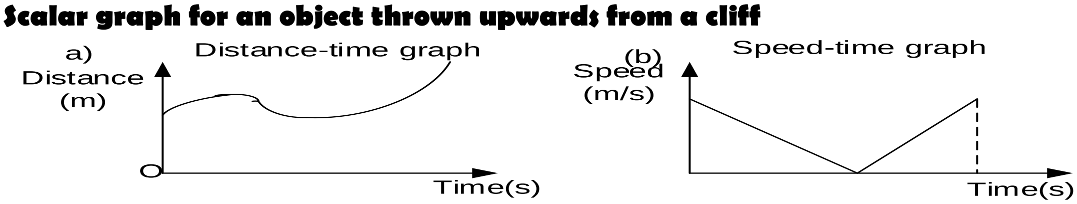
3. Scalar graphs for an object falling freely



4. Motion graph for an object thrown vertically upwards from the top of a cliff



5. Scalar graph for an object thrown upwards from a cliff



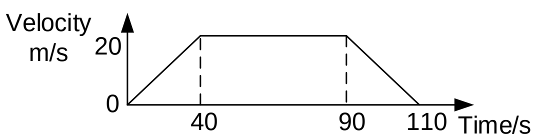
Note

For a body thrown vertically downwards,
$v = u + at$ becomes $v = u + gt$
$S = ut + \frac{1}{2}gt^2$ becomes $S = ut + \frac{1}{2}gt^2$
$v^2 = u^2 + 2as$ becomes $v^2 = u^2 + 2gh$

For a body projected vertically upwards
$v = u + at$ becomes $v = u - gt$
$S = ut + \frac{1}{2}gt^2$ becomes $S = ut - \frac{1}{2}gt^2$
$v^2 = u^2 + 2as$ becomes $v^2 = u^2 - 2gh$

Examples:

1. A car started from rest and attained a velocity of *20m/s* in *40s*. It then maintained the velocity attained for *50s*. After that it was brought to rest by a constant breaking force in *20s*.
   (i) Draw a velocity-time graph for the motion.
   (ii) Using the graph, find the total distance travelled by the car.
   (iii) What is the acceleration of the car?



Total distance = Total area using each part

$= {}^{1}/_{2}\, bh + LxW + {}^{1}/_{2}\, bh$

$= \left(\frac{1}{2} \text{ x } 40 \text{ x } 20\right) + (50\ x\ 20) + \left(\frac{1}{2} \text{x } 20 \text{ x } 20\right)$

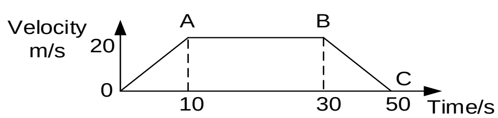
$= 400 + 1000 + 200$

$= 1600m$

2. A car from rest accelerates steadily for 10s up to a velocity of 20m/s. It continues with a uniform velocity for a further 20s and then decelerates so that it stops in 20s
   a) Draw a velocity-time graph to represent the motion
   b) Calculate;
      (i) Acceleration
      (ii) Deceleration
      (iii) Distance travelled
      (iv) Average speed

Solution

(a)



(i) Acceleration OA:

$$a = \frac{v-u}{t} = \frac{20-0}{10} = 2ms^{-2}$$

(ii) Deceleration BC:

$$a = \frac{v-u}{t} = \frac{0-20}{20} = -1\ ms^{-2}$$

$deceleration = 1\ ms^{-2}$

(iii) Distance =Area under graph

$$\left(\frac{1}{2}x10x20\right) + (20x20) + \left(\frac{1}{2}x20x20\right)$$

Distance $= 700m$

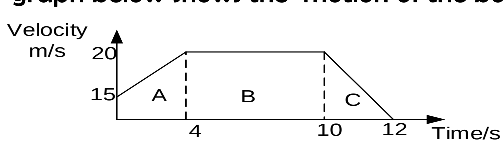
Method II(Area of a trapezium)

$A = {}^{1}/_{2}\, h(a+b)$

$= \frac{1}{2}x\ 20x(50+20) = 10\ (70) = 700m$

(iv) Average speed $= \frac{distance}{time} = \frac{700}{50} = 14m/s$

3. The graph below shows the motion of the body

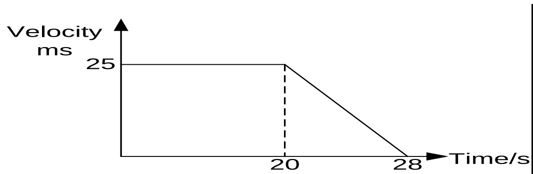


a) Describe the motion of the body
b) Calculate the total distance travelled

Solution

a) A body with initial velocity of 15m/s accelerates steadily to a velocity of 20m/s in 4s, it then continues with a uniform velocity for 6s and its brought to rest in 2s.
b) Distance travelled= $(4x15) + \left(\frac{1}{2}x4x5\right) + (20x6) + \left(\frac{1}{2}x20x2\right) = \quad 210m$

4. A car travelling at a speed of 90km/h for 20s and then brought to rest in 8s. Draw a velocity time graph and find the distance travelled.



Distance travelled;

$= (20x25) + \left(\frac{1}{2}x8x25\right)$

$= \ 600m$

2.1.2: MOTION UNDER GRAVITY

1. Vertical motion

a) When a body is projected vertically upwards it experiences a uniform deceleration of 9.81ms^{-2}. Its acceleration is given by a $= -$g=9.81ms^{-2}. The equations of motion become

$v = u - gt$ | $S = ut - \frac{1}{2}gt^2$ | $v^2 = u^2 - 2gs$

b) An object freely falling vertically downwards has an acceleration of $a = g$ = 9.81ms^{-2}.
The equations of motion become

$v = u + gt$ | $S = ut + \frac{1}{2}gt^2$ | $v^2 = u^2 + 2gs$

Definition

Acceleration due to gravity (g) is rate of change of velocity with time for an object falling freely under gravity.

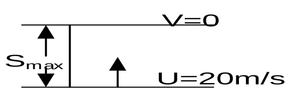
OR The force of attraction due to gravity exerted on a 1kg mass.

Free fall is motion resulting from a gravitational field that is not impeded by a medium that should provide a frictional retarding force or buoyancy.

Numerical examples

1. A ball is thrown vertically upwards with an initial speed of 20ms^{-1}. Calculate.
   i) Time taken to return to the thrower
   ii) Maximum height reached

solution



projected upwards; $v = u - gt$

At max height $v = 0$

$0 = 20 - 9.81t$

$t = 2.04s$

Time taken to reach maximum height $= 2.04s$

But the total time ta ken to return to the thrower $= 2t$

$= 2x2.04 = 4.08s$

$v^2 = u^2 - 2gs$

at max height v=0m/s, u=20m/s,

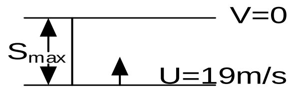
$g = 9.81$ms^{-2}

$0^2 = 20 - 2x9.81s_{max}$

$s_{max} = 20.39m$

2. A particle is projected vertically upwards with velocity of 19.6ms^{-1}. Find
   i) The greatest height attained
   ii) Time taken by the particle to reach maximum height
   iii) Time of flight

Solution



At greatest height $v = 0m/s$

$v^2 = u^2 - 2gs$

$0^2 = 19.6^2 - 2x9.81\ s_{max}$

$s_{max} = \frac{19.6^2}{2x9.81} = 19.58m$

ii) From $v = u - gt$

$u = 19.6, g =$ 9.81ms^{-2} $v = 0$ at max height

$0 = 19.6 - 9.81t$

$t = 1.998s$

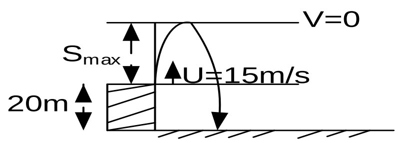
Time to maximum height $= 2.0s$

iii) Time of flight $= 2x$ time to max height

$= 2x2 = 4.0s$

3. A man stands on the edge of a cliff and throws a stone vertically upwards at 15ms^{-1}. After what time will the stone hit the ground 20m below the point of projection

Solution



v=0m/s at max height, s_{max}=? t=?

Method I: $v = u - gt$

$0 = 15 - 9.81t$

$t = 1.53s$

Time to maximum height $= 1.53s$

$v^2 = u^2 + 2gs$

$0 = 15^2 - 2x9.81s_{max}$

$$s_{max} = \frac{15^2}{2x9.81} = 11.47m$$

Maximum height $= 11.47m$

Total height $= (11.47 + 20) = 31.47m$

When the ball begins to return down from max height $u = 0m/s$

$$S = ut + \frac{1}{2}gt^2$$

$31.47 = 0xt + ½\,x9.81t^2$

$$t = \sqrt{\frac{31.47x2}{9.81}} = 2.53s$$

Total time $= (2.53 + 1.53) = 4.06s$

Time taken to hit the ground $= 4.06s$

Method II

The height of the cliff = 20m which is below the point of project therefore

$s = -2m\ u = 15m/s$

$$S = ut - \frac{1}{2}gt^2$$

$-20 = 15t - ½\,x9.81t^2$

$-20 = 15t - 4.905t^2$

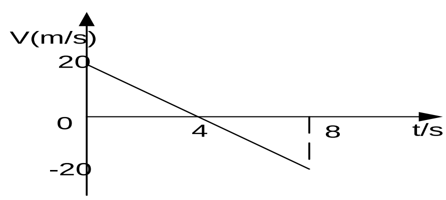
$t = 4.06s$

Time taken to hits the ground $= 4.06s$

4. A car decelerates uniformly from 20ms^{-1} to rest in 4s, then reverses with uniform acceleration back to it original starting point also in 4s
   a) Sketch the velocity-time graph for the motion, and use it to determine the displacement and average velocity
   b) Sketch the speed-time graph for the motion and use it to determine the total distance covered and the average speed.

Solution

Velocity-time graph



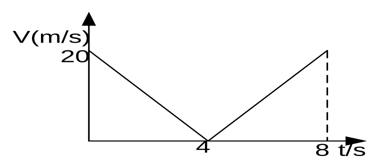
Displacement $s = ½\,bh + ½\,bh$

$= ½\,x4x20 + ½\,x4x(-20)$

$s = 40 - 4 = 0m$

Average velocity $= \frac{total\ displacement}{total\ time} = \frac{0}{8} = 0m/s$

Speed-time graph



Total distance $= ½\,x20x4 + ½\,x20x4$

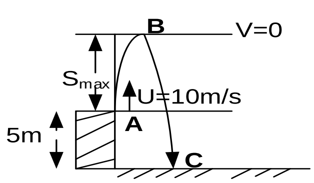
$= 80m$

Average speed $= \frac{total\ distance}{total\ time} = \frac{80}{8} =$ 10ms^{-1}

UNEB 2003 No 1 b(ii)

A ball is thrown vertically upwards with a velocity of 10ms^{-1}from a point 50m above the ground. Describe with the aid of a velocity - time graph, the subsequent motion of the ball. (10marks)

Solution



Time to reach max height v=0,

$v = u - gt$

$0 = 10 - 9.81t$

$t = 1.02s$

Time to reach maximum height is 1.02s

At Max height $v = 0$

$v^2 = u^2 - 2gs$

$0 = 10^2 - 2x9.81Smax$

$S_{max} = 5.1m$

Total height $= (5.1 + 5)$

$= 10.1m$

Time taken to move from max height to the ground is

t=?, u=0m/s g=9.81ms^{-2}

$$S = ut + \frac{1}{2}gt^2$$

$10.1 = 0xt + \frac{1}{2}x9.81t^2$

$$t = \sqrt{\frac{20.2}{9.81}} = 1.43s$$

Final velocity when the ball hits the ground v =?

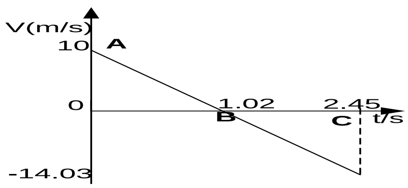
$u = 0, t = 1.43s, g = 9.81ms^{-1}$

$v = ut + gt$

$v = 0 + 9.81x1.43$

$= 14.03m/s$

Velocity-time graph



✓When the ball is thrown vertically upwards with a velocity of 10ms^{-1} it decelerates uniformly at 9.81ms^{-1} til its velocity reaches zero at B(maximum height).

✓The time taken to reach maximum height B is 1.02s and the maximum height is 5.1m

✓After reaching the maximum height, the ball begins to fall downwards with a uniform acceleration of 9.81m/s^2 but the direction is now opposite and therefore the velocity is negative until it reaches a final velocity of 14.03m/s in a time of 2.45s from the time of projection.

Exercise :2

1. A pebble is dropped from rest at the top of a cliff 125m high.
   (a) How long does it take to reach the foot of the cliff and with what speed does it hit the floor
   (b) With what speed must a second pebble be thrown vertically down wards from the cliff top if it is to reach the bottom in 4s . **An(5s, 50m/s, 11.25m/s)**
2. A stone is thrown horizontally from the top of a vertical cliff with velocity 15m/s is observed to strike the horizontal ground at a distance of 45m form the base of the cliff. What is;
   (a) The height of the cliff. **An(45m, 63.4°)**
   (b) The angle the path of the stone makes with the ground at the moment of impact
3. A ball is thrown vertically upwards and caught by the thrower on its return. Sketch a graph of velocity against time, neglecting air resistance
4. A ball is dropped from a cliff top and takes 3s to reach the beach below. Calculate
   a) The height of the cliff **An(44.1m)**
   b) Velocity acquired by the ball **An(29.4m/s)**
5. With what velocity must a ball be thrown upwards to reach a height of 15m **An(17.1ms^{-1})**
6. A stone is dropped from the top of a cliff which is 80m high. How long does it take to reach the bottom of the cliff **An(4.0s)**
7. A stone is fired vertically upwards from a catapult and lands 5.0s later.
   a) What was the initial velocity of the stone
   b) For how long was the stone at a height of 20m or more **An(25ms^{-1}, 3.0s)**
8. A stone is thrown vertically upwards at 10ms^{-1} from a bridge which is 15m above a river
   (a) What is the speed of the stone as it hits the river
   (b) With what speed would it hit the river if it were thrown downwards at 10ms^{-1} **An(20ms^{-1}, 20ms^{-1})**

UNEB 2014 No 1(c)

(i) State **Newton's laws of motion** (03marks)

(ii) Explain how a rocket is kept in motion (04marks)

(iii) Explain why passengers in a bus are thrown backwards when the bus suddenly starts moving. (03marks)

UNEB 2013 No 3(d)

(i) Define uniformly accelerated motion (03marks)

(ii) A train starts from rest at station **A** and accelerates at 1.25 m s^{-2} until it reaches a speed of 20 m s^{-1}. It then travels at this steady speed for a distance of 1.56km and then decelerates at 2 m s^{-2} to come to rest at station **B**. Find the distance from **A** and **B**

An (1 820m) (04marks)

UNEB 2011 No 1(a)

Define the following terms

(i) Uniform acceleration (01mark)

(ii) Angular velocity (01 mark)

UNEB 2010 No 1(d)

(i) Define uniform acceleration (01 mark)

(ii) With the aid of a vel-time graph, describe the motion of a body projected vertically upwards (03 marks)

UNEB 2009 No 2

a) Define the following terms

(i) Velocity

(ii) Moment of a force (02marks)

b) i) A ball is projected vertically upwards with a speed of $50ms^{-1}$, on return it passes the point of projection and falls 78m below. Calculate the total time taken **An(11.57s)** (05 marks)

UNEB 2008 No 1(a)

i) Define the terms velocity and displacement (02 marks)

ii) Sketch a graph of velocity against time for an object thrown vertically upwards (02 marks)

UNEB 2007 No 4(b)(i) What is meant by acceleration due to gravity

UNEB 2006 No 1

a) i) What is meant by uniformly accelerated motion (01 mark)

ii) Sketch the speed against time graph for a uniformly accelerated body (01 mark)

b) (i) Derive the expression $S = ut + \frac{1}{2} at^2$

For the distance S moved by a body which is initially travelling with speed u and is uniformly accelerated for time t (04 marks)

UNEB 1993 No 1

(a) Define the terms

(i) Displacement

(ii) Uniform acceleration

(b) i) A stone thrown vertically upwards from the top of a building with an initial velocity of 10m/s. the stone takes 2.5s to land on the ground.

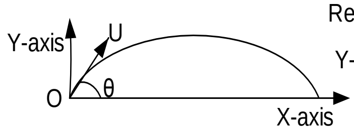
ii) Calculate the height of the building

(iii) State the energy changes that occurred during the motion of the stone (03 marks)

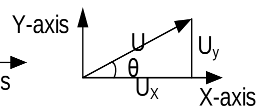
2. PROJECTILE MOTION

This is the motion of a body which after being given an initial velocity moves under the influence

Consider a ball projected at O with an initial velocity um/s at an angle θ to the horizontal.



Resolving initial velocity



$u_y = u\sin\theta$ -------------------(1)

Also: $\cos\theta = \frac{u_x}{u}$

$u_x = u\cos\theta$ -------------------(2)

From the figure: $\sin\theta = \frac{u_y}{u}$

Equation (1) is the initial vertical component of velocity

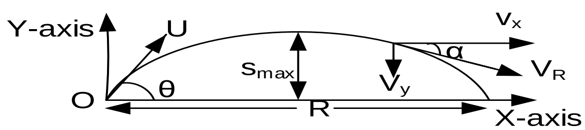
Equation (2) is the initial horizontal component of velocity

Note

The horizontal component of velocity $[u_x = u\cos\theta]$ is constant through the motion and therefore the acceleration is zero.

MATHEMATICAL FORMULAR IN PROJECTILES

All formulas in projectiles are derived from equations of linear motion



Finding velocity at any time t.

Horizontally: $v = u_x + at$

$u_x = u\cos\theta$,

$a = 0$ (constant velocity)

$\boxed{v_x = u\cos\theta}$

Vertically: $v = u_y + at$

$u_y = u\sin\theta$

a= −g

$\boxed{v_y = u\sin\theta - gt}$

Velocity at any time t

$\boxed{v = \sqrt{V_x^2 + V_y^2}}$

Direction of motion

$\boxed{\alpha = \tan^{-1}\left(\frac{V_y}{V_x}\right)}$ to the horizontal

Finding distances at any time t

horizontally : $s_x = u_x t + ½\, at^2$

$u_x = u\cos\theta$, a= 0

$\boxed{x = u\cos\theta\, t}$

Vertically: $s_y = u_y t + ½\, at^2$

$u_y = u\sin\theta$, $a = g$

$\boxed{y = u\sin\theta t - \frac{1}{2}gt^2}$

TERMS USED IN PROJECTILES

1. **MAXIMUM HEIGHT [GREATEST HEIGHT] [S_{max}]**

For vertical motion : at max height v=0,

$u_y = u\sin\theta$, $a = -g$, s= S_{max}

$v_y{}^2 = u_y{}^2 + 2gs$

$0 = (u\sin\theta)^2 - 2gS_{max}$

$2gS_{max} = u^2\sin^2\theta$

$\boxed{Smax = \frac{u^2\sin^2\theta}{2g}}$

Note : $\sin^2\theta = (\sin\theta)^2$ but $\sin^2\theta \neq \sin\theta^2$

2. **TIME TO REACH MAX HEIGHT [t]**

Vertically $v = u_y + at$ at max height v=0

$u_y = u\sin\theta$, $a = g$

$0 = u\sin\theta - gt$

$\boxed{t = \frac{u\sin\theta}{g}}$

3. TIME OF FLIGHT [T]

It refers to the total time taken by the projectile to move from the point of projection to the point where it lands on the horizontal plane through the point of projection.

Vertically: $S_y = u_y$ t + ½ at²

at point A when the projectile return to the plane S_y=0,

t=T(time of flight), $a = -g \ \ u_y = u \sin\theta$

$$0 = u\sin\theta T - \frac{gT^2}{2}$$

$$T\left(u\sin\theta - \frac{gT}{2}\right) = 0$$

Either $T = 0$ or $\left(u\sin\theta - \frac{gT}{2}\right) = 0$

$$\left(u\sin\theta - \frac{gT}{2}\right) = 0$$

$$u\sin\theta = \frac{gT}{2}$$

$$\boxed{T = \frac{2\,u\sin\theta}{g}}$$

Note: The time of flight is twice the time to maximum height

4. RANGE [R]

It refers to the horizontal distance from the point of projection to where the projectile lands along the horizontal plane through the point of projection.

Neglecting air resistance the horizontal component of velocity $\boldsymbol{u\cos\theta}$ remains constant during the flight

Horizontally : $S_x = u_x t + ½ at^2$

$u_x = u\cos\theta$, a=0 (constant velocity), t=T

$$R = u\cos\theta\, T + ½\, x 0 x T^2$$

$$R = u\cos\theta T$$

But $T = \frac{2\,u\sin\theta}{g}$

$$R = \frac{\boldsymbol{u^2\, 2\sin\theta\cos\theta}}{\boldsymbol{g}}$$

But from trigonometry $2\sin\theta\cos\theta = \sin 2\theta$

$$\boxed{R = \frac{u^2 \text{ Sin } 2\theta}{g}}$$

5. MAXIMUM RANGE [R_{max}]

For maximum range sin2θ =1, R=R_{max}

2 θ= $\sin^{-1}(1)$

$$2\,\theta = 90°$$

$$R_{max} = \frac{u^2\sin 90}{g}$$

$$\boxed{R_{max} = \frac{u^2}{g}}$$

6. EQUATION OF A TRAJECTORY

A trajectory is a path described by a projectile.

A trajectory is expressed in terms of horizontal distance x and vertical distance y.

For horizontal motion at any time t

$$x = u\cos\theta t$$

$t = \frac{x}{u\cos\theta}$ ----------------[1]

For vertical motion at any time t

$y = u\sin\theta t - ½\, gt^2$ -------------[2]

Putting t into equation [2]

$$y = u\sin\theta \frac{x}{u\cos\theta} - \frac{1}{2} g\left(\frac{x}{u\cos\theta}\right)^2$$

$$\boxed{\boldsymbol{y} = \boldsymbol{x\tan\theta} - \frac{gx^2}{2u^2\cos^2\theta}}$$

since $y = \mathrm{a}\,x - \mathrm{b}\,x^2$

the motion is parabolic

Either $\boxed{y = x\tan\theta - \frac{g\,x^2\,Sec^2\theta}{2\,u^2}}$

Or $\boxed{y = x\tan\theta - \frac{g\,x^2(1+\tan^2\theta)}{2\,u^2}}$

A. Objects projected upwards from the ground at an angle to the horizontal

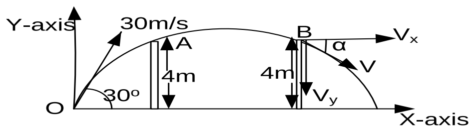
1. A Particle is projected with a velocity of 30ms⁻¹ at an angle of elevation of 30°. Find
   i) The greatest height reached
   ii) The time of flight
   iii) Horizontal range
   iv) The velocity and direction of motion at a height of 4m on its way downwards

Solution

(i) $Smax = \frac{u^2\sin^2\theta}{2g} = \frac{30^2\sin^2 30}{2x9.81} = 11.47m$

(ii) $\boldsymbol{T} = \frac{\boldsymbol{2\,u\sin\theta}}{\boldsymbol{g}} = \frac{2\,x\,30\,\sin 30}{9.81} = 3.06s$

(iii) R= $\frac{u^2\ sin2\theta}{g} = \frac{30^2\ sin2x30}{9.81} = 79.45m$



For vertical motion

$y = usin\theta t - ½\, gt^2$

$4 = 30sin30t - ½\, 9.81x\, t^2$

$4.905t^2 - 15t + 4 = 0$

$t = 2.76s$ or $t = 0.30s$

The value of $t = 0.30s$ is the correct time since it's the smaller value for which the body moves upwards.

$v_x = ucos\theta$

$v_x = 30cos\,30 = 25.98m/s$

$v_y = usin\theta - gt$

$v_y = 305x30 - 9.81x0.30 = 12.06m/s$

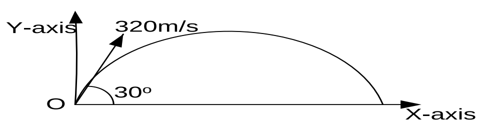
$v = \sqrt{V_x^2 + V_y^2} = \sqrt{25.98^2 + 12.06^2} = 28.64m/s$

Direction : $\alpha = tan^{-1}\frac{V_y}{V_x} = tan^{-1}\left(\frac{12.06}{25.98}\right)$ =**24.9°**

Velocity is 28.64m/s at 24.9° to horizontal

2. A projectile is fired with a velocity of 320m/s at an angle of 30° to the horizontal. Find
   (i) time to reach the greatest height
   (ii) its horizontal range
   (iii) maximum range

Solution



i) At max height $v = 0$,

$v = usin\theta - gt$

$0 = 320sin30 - 9.81t$

$t = \frac{320sin30}{9.81} = 16.31s$

ii) range $R = ucos\theta x$time of flight

Time of flight =twice time to max height

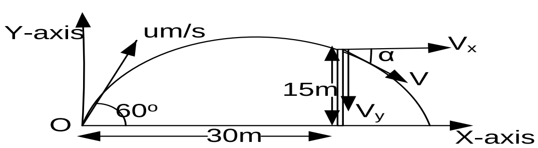
R= $320cos30x\,[2x16.31] = 9039.92m$

iii) max range

$R_{max} = \frac{u^2}{g} = \frac{320^2}{9.81} = 10438.33\text{m}$

3. A projectile fired at an angle of 60° above the horizontal strikes a building 30m away at a point 15m above the point of projection. Find
   (i) Speed of projection
   (ii) Velocity when it strikes a building

Solution



(i) Horizontal distance at time t : $x = ucos\theta t$

$30 = utcos60$

$t = \frac{60}{u}$

Also vertical distance at any time t

$y = usin\theta - \frac{1}{2}gt^2$

$15 = usin60x\frac{60}{u} - \frac{1}{2}x9.81\left(\frac{60}{u}\right)^2$

$15 = 51.96152423 - \frac{4.905\ x\ 3600}{u^2}$

$u = \sqrt{477.7400383} = 21.86m/s$

ii)but since $t = \frac{60}{u}$

$t = \frac{60}{21.86} = 2.75s$

$v_x = ucos\theta$

$v_x = 21.86cos60 = 10.93ms^{-1}$

$v_y = usin\theta - gt$

$v_y = 21.81sin60 - 9.81x2.75 = -8.09ms^{-1}$

velocity at any time

$v = \sqrt{V_x^2 + V_y^2} = \sqrt{10.93^2 + (-8.09)^2}$

$= 13.60ms^{-1}$

$\alpha = tan^{-1}\left(\frac{V_y}{V_x}\right) = tan^{-1}\left(\frac{8.09}{10.9}\right) = 36.6°$

The velocity is $13.60ms^{-1}$at $36.6°$to the horizontal

Alternatively

$y = xtan\theta - \frac{gx^2}{2u^2COS^2\theta}$

$y = 15m, x = 30m, \theta = 60°, u = ?$

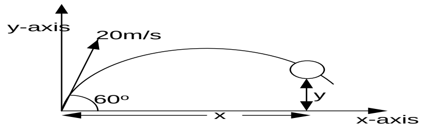
$15 = 30tan60 - \frac{9.81x30^2}{2u^2Cos^2 60}$

$15 = 51.96152423 - \frac{17658}{u^2}$

$u = \sqrt{477.7400383} = 21.86m/s$

4. A body is projected at an angle of 60° above horizontal and passes through a net after 10s. Find the horizontal and vertical distance moved by the body after it, was projected at a speed of 20m/s

Solution



Horizontal motion : $x = ucos\theta xt$

$x = 20cos60x10$

$x = 100m$

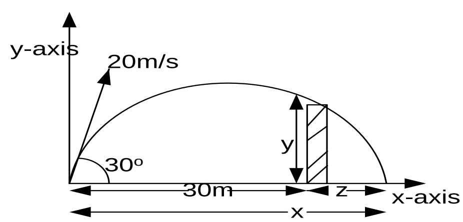
Vertical motion; $y = usin\theta t - ½\, gt^2$

$y = 20(sin60)x10 - ½\, x9.81x10^2$

$y = -317.29m$

5. A ball is kicked from the spot 30m from the goal post with a velocity of 20m/s at 30° to the horizontal. The ball just clears the horizontal bar of a goal post. Find;
   (i) Height of the goal post
   (ii) How far behind the goal post does the ball land

Solution



horizontal motion : $x = ucos\theta xt$

$30 = 20cos30xt$

$t = 1.732s$

For vertical motion: $y = usin\theta t - ½\, gt^2$

$y = (20sin30)x1.732 - \frac{1}{2}x9.81x(1.732)^2$

$y = 2.61m$

Height of the goal post = 2.61m

ii) Time of flight

$T = \frac{2\,u\,\sin\theta}{g} = \frac{2x20xsin30}{9.81} = 2.04s$

iii) Horizontal distance: $x = ucos\theta xt$

$x = 20cos30x2.04$ =35.33m

but $x = 20 + z$

$35.33 = 30 + z$

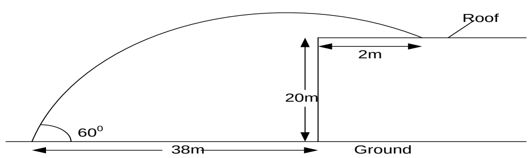
$z = 5.33m$ The ball 5.33m behind the goal

EXERCISE : 3

1. A particle is projected at an angle of 60° to the horizontal with a velocity of 20m/s. calculate the greatest height the particle attains **An[15.29m]**
2. A stone is projected at an angle of 60° to the horizontal with a velocity of 30m/s. calculate;
   (a) the highest point reached
   (b) Range
   (c) Time taken for flight
   (d) Height of the stone at the instant that the path makes an angle of 30° with the horizontal
   An[33.75m, 78m, 5.2s, 33.3m]
3. A particle is projected from level ground towards a vertical pole, 4m high and 30m away from the point of projection. It just passes the pole in one second. Find
   i) Its initial speed and angle of projection **An [39.29m/s, 16.5°]**
   ii) The distance beyond the pole where the particle will fall **An [24.42m]**
4. A particle is projected with a velocity of 30m/s at an angle of 40° above the horizontal plane. find ;
   a) The time for which the particle is in the air.
   b) The horizontal distance it travels **An [3.9s, 22.9m/s]**
5. A body is projected with a velocity of 200ms^{-1} at an angle of 30° above the horizontal. Calculate
   (a) Time taken to reach the maximum height
   (b) Its velocity after 16s **An [10.2s, 183m/s at 19.1°]**
6. A particle is projected from a level ground in such a way that its horizontal and vertical components of velocity are 20ms^{-1} and 10ms^{-1} respectively. Find
   (a) Maximum height of the particle
   (b) Its horizontal distance from the point of projection when it returns to the ground
   (c) The magnitude and direction of the velocity on landing **An [5.0m, 40m, 22.4m/s at 26.6° below horizontal]**
7. A particle is projected with a speed of 25ms^{-1} at 30° above the horizontal. Find;

(a) Time taken to reach the height point of trajectory
(b) The magnitude and direction of the velocity after 2.0s **An [1.25s, 22.9m/s at 19.1° below horizontal]**

8. A projectile is launched with a velocity of 1800m/s at an angle 60° with the horizontal. Determine the speed of the projectile at a height of 32km when falling downwards **An[1616.23m/s]**
9. A hammer thrown in athletics consists of a metal sphere of mass 7.26kg with a wire handle attached, the mass of which can be neglected. In a certain attempt it is thrown with an initial velocity which makes an angle of 45° with the horizontal and its flight takes 4.00s. stating any assumptions find;
   (i) The horizontal distance travelled
   (ii) Kinetic energy of the sphere just before it strikes the ground **An [80.0m, 2.90x10³J]**
10. A soft ball is thrown at an angle of 60 above the horizontal. It lands a distance 2m from the edge of a flat roof of height 20m. the edge of the roof is 38m horizontally from the thrower.



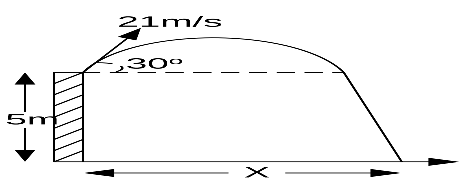
Calculate
   (i) The speed at which the ball was thrown **An ($25.4\ ms^{-1}$)**
   (ii) The velocity with which the ball strikes the roof **An ($15.64\ ms^{-1}$ $at\ 36.2°$ below the horizontal)**

11. A stone thrown upwards at an angle θ to the horizontal with speed ums^{-1} just clears a vertical wall 4m high and 10m from the point of projection when travelling horizontally. Find the angle of projection **An[38.66°]**

B. **Objects projected upwards from a point above the ground at an angle to the horizontal**

1. A particle is projected at an angle of elevation of 30° with a speed of 21m/s. If the point of projection is 5m above the horizontal ground, find the horizontal distance that the particle travels before striking the ground

Solution

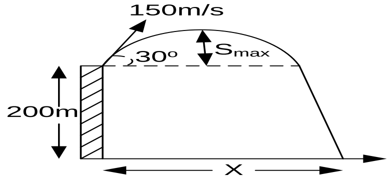


$u = -5m$ since it's below the point of projection
For vertical motion: $y = usin\theta t - ½\,gt^2$

$$-5 = 21sin30t - \frac{9.81t^2}{2}$$
$$4.905t^2 - 10.5t - 5 = 0$$
t= $2.54s$ or t = $-0.40s$
Time of flight $t = 2.54s$
For horizontal motion
$x = ucos\theta xt = 21(cos30)\,x2.54 = 46.19m$
The horizontal distance = $46.19m$

2. A bullet is fired from a gun placed at a height of 200m with a velocity of 150ms⁻¹ at an angle of 30° to the horizontal find
   i) Maximum height attained
   ii) Time taken for the bullet to hit the ground

Solution



i) $Smax = \frac{u^2\,sin^2\theta}{2g} = \frac{150^2\,sin^2 30}{2x9.81} = 286.70m$
The max height attained is 286.70m from the point of projection

ii) Time taken for the bullet to hit the ground
Vertical motion : $y = usin\theta t - ½\,gt^2$
$y = -200m$ since its below the point of projection
$$-200 = 150sin30t - ½\,x9.81t^2$$
$$-200 = 75t - 4.905t^2$$
$$t = 17.61s\ or\ t = -2.32s$$
Time taken is 17.61s

Trial :1

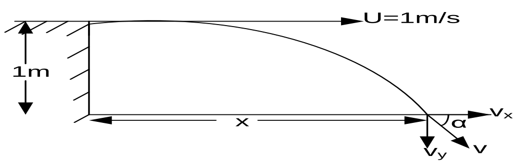
1. A particle is projected with a velocity of $10ms^{-1}$ at an angle of 45^0 to the horizontal, it hits the ground at a point which is 3m below its point of projection. Find the time fro which it is in the air and the horizontal distance covered by the particle in this time **An[1.76s, 12.42m]**
2. A pebble is thrown from the top of a cliff at a speed of 10m/s and at 30^0 above the horizontal. it hits the sea below the cliff 6.0s later , find;
   a) The height of the cliff . **An[150m, 52m]**
   b) The distance from the base of the cliff at which the pebble falls into the sea.

C. An object projected horizontally from a height above the ground

Examples

1. A ball rolls off the edge of a table top 1m high above the floor with a horizontal velocity $1ms^{-1}$. Find;
   i) The time it takes to hit the floor
   ii) The horizontal distance it covered
   iii) The velocity when it hits the floor

Solution



u=$1ms^{-1}$ θ=0^0 y=-1m below the point of projection

vertical motion: $y = usin\theta t - ½\, gt^2$

$-1 = 1xsin0xt - ½\, x9.81t^2$

$-1 = -4.905t^2$

$t = 0.45s$

ii) $x = ucos\theta t = 1xcos0x0.45 = 0.45m$

iii) velocity when it hits the ground

$v_x = ucos\theta = 1cos0 = 1m/s$

$v_y = usin\theta - gt$

$v_y = 1sin0 - 9.81x0.45 = -4.4m/s$

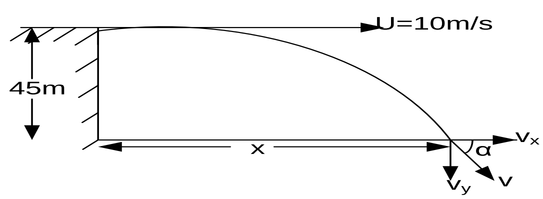
$\boldsymbol{v} = \sqrt{V_x^2 + V_y^2} = \sqrt{(1)^2 + (-4.4)^2} = 4.5ms^{-1}$

Direction: $\alpha = tan^{-1}\left(\frac{V_y}{V_x}\right) = tan^{-1}\left(\frac{4.4}{1}\right) = 77.2°$

The velocity is $4.5ms^{-1}$ at $77.2°$to the horizontal

2. A ball is thrown forward horizontally from the top of a cliff with a velocity of 10m/s. the height of a cliff above the ground is 45m. calculate
   i) Time to reach the ground
   ii) Distance from the cliff where the ball hits the ground
   iii) Direction of the ball just before it hits the ground

Solution



u=$10ms^{-1}$ θ=0^0 y=-45m below the point of projection

For vertical motion

$y = usin\theta t - ½\, gt^2$

$-45 = 10xsin0xt - ½\, x9.81t^2$

$t = 3.03s$

ii) $x = ucos\theta t = 10xcos0x3.03 = 30.3m$

iv) velocity when it hits the ground

$v_x = ucos\theta = 10cos0 = 10m/s$

$v_y = usin\theta - gt$

$v_y = 10sin0 - 9.81x3.03 = 29.72m/s$

$v = \sqrt{V_x^2 + V_y^2} = \sqrt{(10)^2 + (29.72)^2} = 31.36ms^{-1}$

$\alpha = tan^{-1}\left(\frac{V_y}{V_x}\right) = tan^{-1}\left(\frac{29.72}{10}\right) = 71.4°$

The velocity is $31.36ms^{-1}$ at $71.4°$to the horizontal

Trial:2

1. A pencil is accidentally knocked off the edge of a horizontal desktop. The height of the desk is 64.8cm and the pencil hits the floor a horizontal distance of 32.4cm from the edge of the desk, What was the speed of the pencil as it left the desk. **An[0.9ms^{-1}]**

2. An aero plane moving horizontally at 150ms^{-1} releases a bomb at a height of 500m. the hits the intended target. What was the horizontal distance of aero plane from the target when the bomb was released. **An(1500m)**

UNEB 2016 No1 (b)

A particle is projected from a point on a horizontal plane with a velocity, u, at an angle, θ, above the horizontal. Shwo that the maxmum horizontal range R_{max} is given by $R_{max} = \frac{u^2}{g}$ where g is acceleration due to gravity. (04marks)

UNEB 2014 No1 (a)

(i) What is a **projectile motion** (01marks)

(ii) A bomb is dropped from an aero plane when it is directly above a target at a height of 1402.5m. the aero plane is moving horizontally with a speed of 500kmh^{-1}. Determine whether the bomb will hit the target. **An (misses target by 2347.2m)** (05marks)

UNEB 2012 No 3 (d)

(i) Derive an expression for maximum horizontal distance travelled by a projectile in terms of the initial speed u and the angle of projection θ to the horizontal [02 marks]

(ii) Sketch a graph to show the relationship between kinetic energy and height above the ground in a projectile.

UNEB 2010 No (d)

iii) Calculate the range of a projectile which is fired at an angle of 45° to the horizontal with a speed of 20m/s **An [40.77m]**

UNEB 2009 No 1 (d)

A stone is projected at an angle of 20° to the horizontal and just clears a wall which is 10m high and 30m form the point of projection. Find the;

i) Speed of projection (04marks)

ii) Angle which the stone makes with the horizontal as it clears the wall (03marks)

An[73.78m/s, 16.9°]

UNEB 2006 No 1 (c)

A projectile is fired horizontally from the top of a cliff 250m high. The projectile lands 1.414x10^{3}m from the bottom of the cliff. Find the

i) Initial speed of the projectile (05 marks)

ii) Velocity of the projectile just before it hits the ground (05 marks)

An [198m/s, 210m/s at 19.5°]

UNEB 2000 No 3 (b)

(i) Define the terms time of flight and range as applied to projectile motion (02 marks)

(ii) A projectile is fired in air with a speed um/s at an angle θ to the horizontal. Find the time of flight of the projectile (02marks)

MARCH UNEB 1995 No 1

a) (i) write the equation of uniformly accelerated motion (03 marks)

(ii) Derive the expression for the maximum horizontal distance travelled by a projectile in terms of the initial speed u and the angle of projectile θ to horizontal (04 marks)

b) A bullet is fired from a gun placed a height of 200m with a velocity of 150m/s at an angle of 30° to the horizontal. Find

i) The maximum height attained

ii) The time for the bullet to hit the ground (07marks)

CHAPTER 3: COMPOSITION AND RESOLUTION OF VECTORS

3.1.0: VECTOR QUANTITY

It is a physical quantity with both magnitude and direction.
Example; displacement, velocity, acceleration, force, weight and momentum

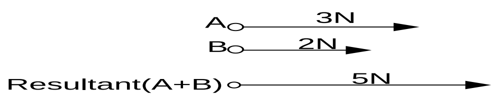
3.1.2: SCALAR QUANTITY

It is a physical quantity with only magnitude.
Example; distance, speed, time, temperature, mass and energy

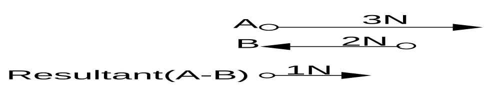
3.1.3: VECTOR ADDITION

A. Vectors acting in the same line

i) If vectors are acting in the same direction then resultant along that direction is just the sum of the two vectors.



ii) If they are moving in the opposite direction then, the resultant is difference of the vectors but along the direction of the bigger vector.



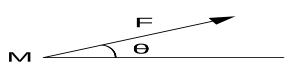
B . vectors acting at an angle

With vectors inclined at an angle to each other, a triangle of vectors is used to find the resultant. The resultant given by the line that completes the triangle.

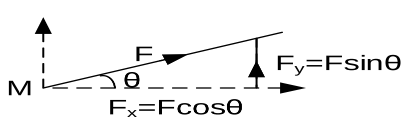
Components of a vector

The component of a vector is the effective value of a vector a long a particular direction. The component along any direction is the magnitude of a vector multiplied by the **cosine of the angle** between its direction and the direction of the component.

Suppose a force F pulls a body of mass m along a truck at an angle θ to the horizontal as shown below;



The effective force that makes the body move along the horizontal is the component of F along the horizontal



$Cos\theta \ = \frac{F_x}{F}$

$\boxed{F_x \ = \ Fcos\theta}$

$Sin\theta \ = \frac{F_y}{F}$

$\boxed{F_y = Fsin\theta}$

$\boxed{\text{Resultant vector } \boldsymbol{F_R} = \ \sqrt{\boldsymbol{F_x^2} + \ \boldsymbol{F_y^2}}}$

$\boxed{\text{Direction}\alpha = \ \boldsymbol{tan^{-1}}\left(\frac{\boldsymbol{F_y}}{\boldsymbol{F_x}}\right)}$

Hints

When a vector is inclined at an angle θ to the horizontal then;

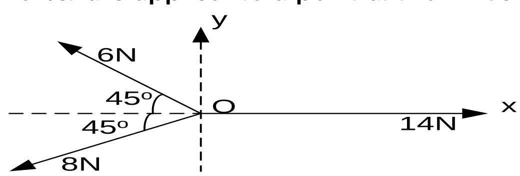
- Along the horizontal, the component of the vector is $cos\theta$
- Along the vertical, the component of the vector is $sin\ \theta$

When a vector is inclined at θ to the vertical then;

- Along the horizontal, the component of the vector is $sin\ \theta$
- Along the vertical, the component of the vector is $cos\theta$

Examples

1. Three forces are applied to a point as shown below



Calculate
a) The component in directions Ox and Oy respectively
b) Resultant force acting at O

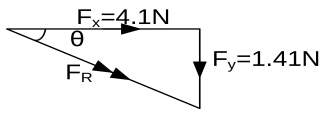
Solution

Components along Ox

$F_x = 14 - 6cos45 - 8cos45 = 4.10N$

Component along Oy

$F_y = 6sin45 - 8sin45 = -1.41N$



$F_R = \sqrt{F_x^2 + F_y^2} = \sqrt{4.1^2 + (-1.41)^2} = 4.34N$

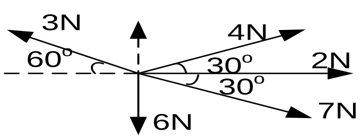
Direction $\theta = tan^{-1}\left(\frac{F_y}{F_x}\right) = tan^{-1}\left(\frac{1.41}{4.1}\right) = 19.0°$

Resultant force is $4.34N$ at $19.0°$ below the horizontal

2. Forces of 2N, 4N, 3N, 6N, and 7N act on a particle in the direction 0°, 30°, 120°, 270° and 330° respectively. Find the magnitude and direction of a single force represented by the above forces.

Solution

Note: the directions given involve 1,2 and 3 digits there fore they are angles and must be read anticlockwise starting from the positive x-axis

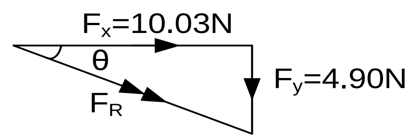


Resultant component along x-axis

$F_x = 2 + 4cos30 + 7cos30 - 3cos60 = 10.03N$

Resultant component along y-axis

$F_y = 4sin30 + 3sin60 - 7sin30 - 6 = -4.90N$



$F_R = \sqrt{10.03^2 + (-4.90)^2} = 11.16N$

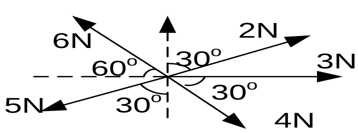
$\theta = tan^{-1}\left(\frac{F_y}{F_x}\right) = tan^{-1}\left(\frac{4.90}{10.03}\right) = 26.04°$

The resultant force is 11.16N at 26.04° below the horizontal.

3. Forces of 2N, 3N, 4N, 5N, and 6N act on a particle in the direction 030°, 090°, 120°, 210°, and 330° respectively. Find the resultant force.

Solution

Note: the directions given involve 3 digits there fore they are bearings and must be read clockwise starting from the positive y-axis



Resultant along the x-axis

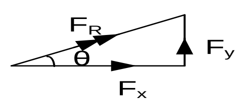
$F_x = 3 + 2sin30 + 4cos30 - 5cos30 - 6cos60$

$F_x = 1.964N$

Resultant along the y-axis

$F_y = 6sin60 + 2cos30 - 5cos30 - 4sin30$

$F_y = 0.598N$



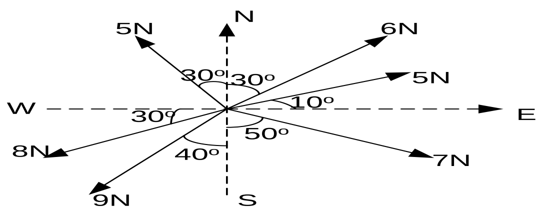
$F_R = \sqrt{F_x^2 + F_y^2} = \sqrt{1.964^2 + 0.598^2} = 2.053N$

Direction $\theta = tan^{-1}\left(\frac{F_y}{F_x}\right) = 16.9°$

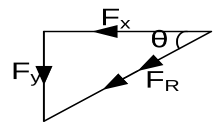
The resultant force is 2.053N at 16.9° above the horizontal

4. Forces of 6N, 5N, 7N, 8N, 5N, and 9N act pm a particle in the direction N30°E, N30°W, S50°E, N60°W, N80°E and S40°W, respectively. find the resultant force.

Solution



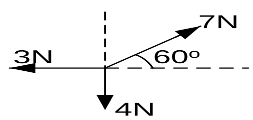
$$F_x = 5cos10 + 6sin30 + 7sin50 - 9sin40 - 8cos50 - 5sin30 = -1.927N$$
$$F_y = 5cos30 + 6cos30 + 5sin10 - 8sin30 - 9cos40 - 7cos50 = -4.999N$$



$$F_R = \sqrt{F_x^2 + F_y^2} = \sqrt{1.927^2 + 4.999^2} = 5.36N$$
$$\theta = tan^{-1}\left(\frac{F_y}{F_x}\right) = tan^{-1}\left(\frac{4.999}{1.927}\right) = 68.9°$$

Resultant force is 5.36N at 68.9° below horizontal

5. A particle at the origin O is acted upon by the three forces as shown below. Find the position of the particle after 2 seconds of its mass is 1kg.



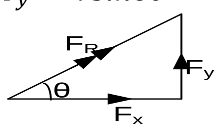
Solution

Resultant along horizontal

$$F_x = -3 + 7cos60 = 0.5N$$

Resultant along vertical

$$F_y = 7sin60 - 4 = 2.06N$$



$$F_R = \sqrt{F_x^2 + F_y^2} = \sqrt{0.5^2 + 2.06^2} = 2.12N$$

But $F_R = ma$

$2.12 = 1a$

$a = 2.12ms^{-2}$

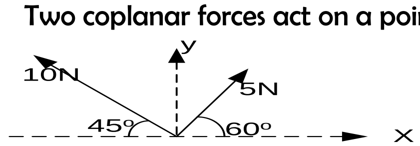
From $S = ut + ½\, at^2$

$u = 0 \quad t = 2s \quad a = 2.12ms^{-2}$

$S = 0x2 + ½\, x2.12x2^2 = 4.24m$

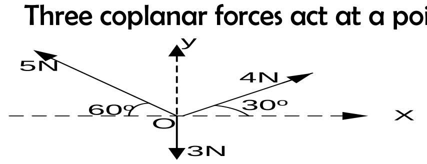
EXERCISE:4

1. A force of 3N acts at 60° to a force of 5N. find the magnitude and direction of their resultant
   An(7N at 21.8° to the 5N force)
2. A force of 3N act at 90° to a force of 4N. Find the magnitude and direction of their resultant
   An(5N at 37° to the 4N force)
3. Two coplanar forces act on a point O as shown below



Calculate the resultant force
An[12.3N at 68.0° above the horizontal]

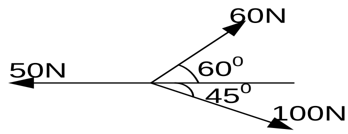
4. Three coplanar forces act at a point as shown below



Find the resultant force acting at O
An[3.4N at 73.1° above the horizontal]

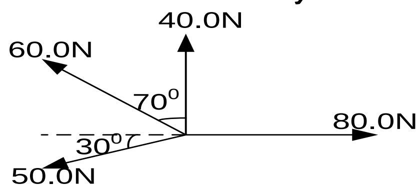
5. Forces of 2N, 1N, 3N and 4N act on a particle in the directions 0°, 90°, 270° and 330° respectively. Find the magnitude and direction of the resultant force.
   An[6.77N at 36.2° below the horizontal]

6. Forces of 7N, 2N, 2N, and 5N act on a particle in the direction 060°, 160°, 200° and 315° respectively. Find the resultant force. **An[4.14N at 52.36° below the horizontal]**
7. Calculate the magnitude and direction of the resultant of the forces shown below



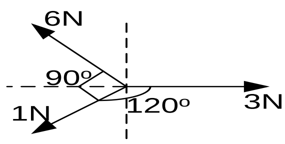
An(54.1N at 20° below the horizontal)

8. Find the resultant of the system of forces



An(40.6N at 61.0° to horizontal)

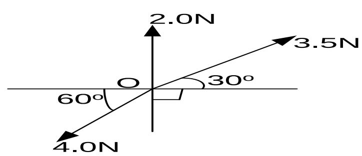
9. Three forces act on a body of mass 0.5kg as shown is the diagram. Find the position of the particle after 4 seconds.



An[3.44N, 6.88ms^{-2}, 55.2m]

UNEB 2008 No1

b

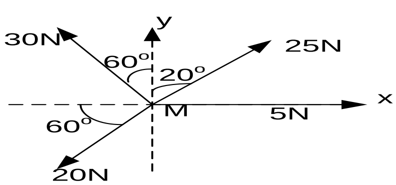


Three forces of 3.5N, 4.0N and 2.0N act at a point O as shown above. Find the resultant force. (4marks)

An[1.07N at 15.5° above the horizontal]

UNEB 2007 No 4

ii)

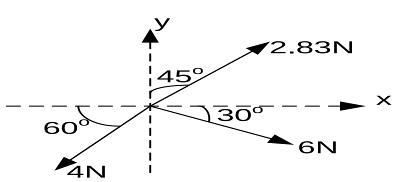


A body m of mass 6kg is acted on by forces of 5N, 20N, 25N and 30N as shown above. Find the acceleration of m [05 marks]

An[5.5ms^{-2}]

UNEB NOV/DEC 1998 No1

c)



Forces of 2.83N, 4.00N and 6.00N act on a particle O as shown above. Find the resultant force on the particle [06marks]

3.2.0: RELATIVE MOTION

It comprises of;
1-Relative velocity
2-Relative path

3.2.1: Relative velocity

This is the velocity a body would have as seen by an observer on another body. Suppose A and B are two moving bodies, the velocity of A relative to B is the velocity of A as it appears to an observer on B.

It's denoted by **${}_AV_B = V_A - V_B$**

Note that ${}_AV_B \neq {}_BV_A$ since [**${}_BV_A = V_B - V_A$**]

There are two methods used in calculations
- Geometric method
- Vectorial method

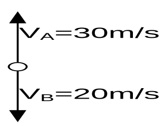
1. Vector method

Find component of velocity for each object separately
Therefore **${}_AV_B = V_A - V_B$**

Example

1. Particle A is moving due to north at $30ms^{-1}$ and particle B is moving due south at 20m/s. find the velocity of A relative to B.

Solution



${}_AV_B = V_A - V_B$

${}_AV_B = \begin{pmatrix} 0 \\ 30 \end{pmatrix} - \begin{pmatrix} 0 \\ -20 \end{pmatrix} = \begin{pmatrix} 0 \\ 50 \end{pmatrix}$

$/{}_AV_B/ = \sqrt{0^2 + 50^2}$ = 50m/s due north

2. A cruiser is moving at 30km/hr due north and a battleship is moving at 20km/hr due north, find the velocity of the cruiser relative to the battleship.

Solution

$V_C = \begin{pmatrix} 0 \\ 30 \end{pmatrix}$ $V_B = \begin{pmatrix} 0 \\ 20 \end{pmatrix}$

${}_CV_B = V_C - V_B$

${}_CV_B = \begin{pmatrix} 0 \\ 30 \end{pmatrix} - \begin{pmatrix} 0 \\ 20 \end{pmatrix} = \begin{pmatrix} 0 \\ 10 \end{pmatrix}$

due north

$/{}_CV_B/ = \sqrt{0^2 + 10^2}$

${}_CV_B$ =10km/h due north

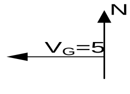
3. A particle A has a velocity of 4i+6j-5k (m/s) while particle B has a velocity of -10i-2j+6k (m/s). find the velocity of A relative to B

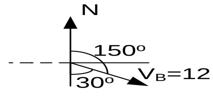
Solution

$${}_AV_B = V_A - V_B = \begin{pmatrix} 4 \\ 6 \\ -5 \end{pmatrix} - \begin{pmatrix} -10 \\ -2 \\ 6 \end{pmatrix} = \begin{pmatrix} 14 \\ 8 \\ -11 \end{pmatrix} ms^{-1}$$

4. A boy runs at 5km/h due west and a girl runs 12km/hr at a bearing of 150°. Find the velocity of the girl relative to the boy.

Solution

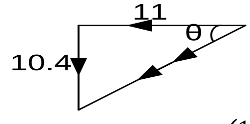




${}_GV_B = V_G - V_B$

${}_GV_B = \begin{pmatrix} -5 \\ 0 \end{pmatrix} - \begin{pmatrix} 12Sin30 \\ -12Cos30 \end{pmatrix} = \begin{pmatrix} -11 \\ -10.4 \end{pmatrix}$

$/{}_GV_B/ = \sqrt{(-11)^2 + (-10.4)^2}$ = 15.14km/hr

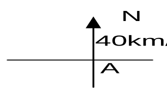


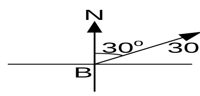
$\theta = tan^{-1}\left(\frac{10.4}{11}\right) = 43.4°$

Relative velocity is 15.14km/hr at 43.4° below the horizontal.

5. Plane A is flying due north at 40km/hr while plane B is flying in the direction N30°E at 30km/hr. Find the velocity of A relative to B.

Solution

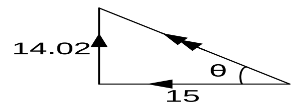




$_AV_B = V_A - V_B$

$_AV_B = \begin{pmatrix}0\\40\end{pmatrix} - \begin{pmatrix}30Sin30\\-30Cos30\end{pmatrix} = \begin{pmatrix}-15\\14.02\end{pmatrix}$

$/_AV_B/ = \sqrt{(-15)^2 + (14.02)^2}$ = 20.53km/hr

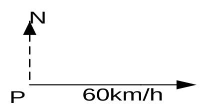


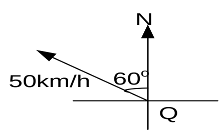
$\theta = tan^{-1}\left(\frac{14.02}{15}\right) = 43.07°$

The relative velocity is 20.53 at N46.93°W

6. Ship P is steaming at 60km/hr due east while ship Q is steaming in the direction N60°W at 50km/hr. Find the velocity of P relative to Q.

Solution



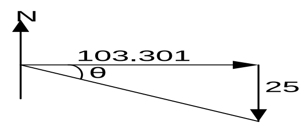


$V_p = \begin{pmatrix}60\\0\end{pmatrix}$ $\quad V_q = \begin{pmatrix}-50Sin60\\50Cos60\end{pmatrix}$

pVq=Vp-Vq

$pVq = \begin{pmatrix}60\\0\end{pmatrix} - \begin{pmatrix}-50Sin60\\50Cos60\end{pmatrix} = \begin{pmatrix}103.301\\-25\end{pmatrix}$

$/pVq/ = \sqrt{(103.301)^2 + (-25)^2}$ = 106.3km/hr



$\theta = tan^{-1}\left(\frac{25}{103.301}\right) = 13.6°$

Direction $S(90 - 13.6)°E$

Relative velocity is 106.3km/hr at S76.4°E

7. To a cyclist riding due north at 40km/hr, a steady wind appears to blow from west at 30km/hr. find the true velocity of the wind.

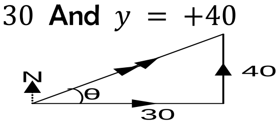
Solution

$Vc = \begin{pmatrix}0\\40\end{pmatrix}$ $\quad wVc = \begin{pmatrix}30\\0\end{pmatrix}$ $\quad Vw = \begin{pmatrix}x\\y\end{pmatrix}$

Vc =Vw−Vc

$\begin{pmatrix}30\\0\end{pmatrix} = \begin{pmatrix}x\\y\end{pmatrix} - \begin{pmatrix}0\\40\end{pmatrix}$

$x = 30$ And $y = +40$



$V\,w = \begin{pmatrix}30\\40\end{pmatrix}$

$Vw = \sqrt{30^2 + 40^2}$ = 50km/hr

$\theta = tan^{-1}\left(\frac{40}{30}\right) = 53.13°$

Direction $N(90 - 53.13)°E$

N36.87°E

Trial:3

1. Car A is moving East wards at 20m/s and car B is moving Northwards at 10m/s. find the
   i) Velocity of A relative to B **An [10$\sqrt{5}$ m/s]**
   ii) Velocity of B relative to A **An [10$\sqrt{5}$ m/s]**

2. In EPL football match, a ball is moving at 5m/s in the direction of N45°E and the player is running due north at 8m/s. Find the velocity of the ball relative to the player.
An[5.69m/s at S38.38°E].

3. A ship is sailing south East at 20km/hr and a second ship is sailing due west at 25km/hr. Find the magnitude and direction of the velocity of the first ship relative to the second. **An [41.62km/hr at S70.13°E]**

4. On a particular day wind is blowing N30°E at a velocity of 4m/s and a motorist is driving at 40m/s in the direction of S60°E
   a) Find the velocity of the wind relative to motorist **An [40.2m/s at N54.28°W]**
   b) If the motorist changes the direction maintaining his speed and the wind appears to blow due East. What is the new direction of the motorist? **An[N85.03°W]**

3.2.2: RELATIVE PATH

Consider two bodies A and B moving with V_A and V_B from points with position vectors R_A and R_B respectively.

Position of A after time t is

$R_{At}=OA + t \; x \; V_A$

Position of B after time t is

$R_{Bt}= OB + t \; x \; V_B$

Relative path

${}_AR_B=R_{At}-R_{Bt}$

${}_AR_B =(OA+tV_A)-(OB +tV_B)$

${}_AR_B =(OA-OB)+t(V_A-V_B)$

${}_AR_B = (OA-OB) + t({}_AV_B)$

EXAMPLE

1. A car A and B are moving with their respective velocities $2i - j$ and $i + 3j$, if their position vectors are $4i + j$ and $2i - 3j$ respectively. Find the path of A relative to B

   i) At any time t

   ii) At t=2s

Solution

i) $V_A=\begin{pmatrix}2\\-1\end{pmatrix}$ $V_B=\begin{pmatrix}1\\3\end{pmatrix}$

$OA = \begin{pmatrix}4\\1\end{pmatrix}$ $OB=\begin{pmatrix}2\\-3\end{pmatrix}$

${}_AR_B=$ (OA-OB) + t(${}_AV_B$)

${}_AR_B = \left[\begin{pmatrix}4\\1\end{pmatrix}-\begin{pmatrix}2\\-3\end{pmatrix}\right]+t\left[\begin{pmatrix}2\\-1\end{pmatrix}-\begin{pmatrix}1\\3\end{pmatrix}\right]$

${}_AR_B=\begin{pmatrix}2\\4\end{pmatrix}+t\begin{pmatrix}1\\-4\end{pmatrix}$

ii) When t=2 ${}_AR_B=\begin{pmatrix}2\\4\end{pmatrix}+2\begin{pmatrix}1\\-4\end{pmatrix}$

${}_AR_B=\begin{pmatrix}2\\4\end{pmatrix}+\begin{pmatrix}2\\-8\end{pmatrix}=\begin{pmatrix}4\\-4\end{pmatrix}$

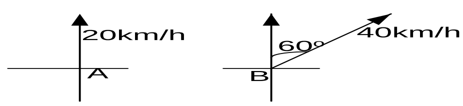
2. Two ships A and B move simultaneously with velocities 20km/hr and 40km/hr respectively. Ship A moves in the northern directions while ship B moves in N60°E. Initially ship B is 10km due west of A. determine

   a) The relative velocity of A to B

   b) The relative path of A to B

Solution

a)



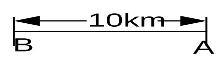
$V_A = \begin{pmatrix}0\\20\end{pmatrix}$ $V_B= \begin{pmatrix}40Sin60\\40Sin60\end{pmatrix}$

${}_AV_B = V_A-V_B$

${}_AV_B=\begin{pmatrix}0\\20\end{pmatrix}-\begin{pmatrix}40Sin60\\40Sin60\end{pmatrix} = \begin{pmatrix}-34.64\\0\end{pmatrix}$

${}_AV_B$=34.64km/hr

b)



${}_AR_B = (OA - OB) + t({}_AV_B)$

$OB=\begin{pmatrix}0\\0\end{pmatrix}$ $OA=\begin{pmatrix}10\\0\end{pmatrix}$

${}_AR_B=\left[\begin{pmatrix}10\\0\end{pmatrix}-\begin{pmatrix}0\\0\end{pmatrix}\right]+t\begin{pmatrix}-34.64\\0\end{pmatrix}$

${}_AR_B=\begin{pmatrix}10\\0\end{pmatrix}+t\begin{pmatrix}-34.64\\0\end{pmatrix}$

3.2.3: SHORTEST DISTANCE AND TIME TO SHORTEST DISTANCE [DISTANCE AND TIME OF CLOSEST APPROACH]

When two particles are moving simultaneously with specific velocities, time will come when they are closest to each other **without** colliding

Numerical calculations

There three methods used

- Geometrical
- Vector
- Differential

1. Vector

Consider particles A and B moving with velocities V_A and V_B from point with positions vectors OA and OB respectively.

Then **shortest distance** **d**= /$_A$**R**$_B$ /

For minimum distance to be attained then $_A$**V**$_B$ • $_A$**R**$_B$ = **0** This gives the time

Or time $= \frac{/AB\ .\ \mathrm{AVB}/}{/AVB/^2}$ Where **AB.** $_A$**V**$_B$ is a dot product

2. Differential

The minimum distance is reached when $\frac{d}{dt}$ /$_A$**R**$_B$ / ^{2} = **0** This gives the time

Minimum distance **d** = /$_A$**R**$_B$ /

EXAMPLE

1. A particle P starts from rest from a point with position vector $2j + 2k$ with a velocity $(j + k)$m/s. A second particle Q starts at the same time from a point whose position vector is $-11i - 2j - 7k$ with a velocity of $(2i + j + 2k)$m/s . Find;
   i) The shortest distance between the particles
   ii) The time when the particles are closest together
   iii) How far each has travelled by this time

Solution:

Method I vector

i) OP $= \begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix}$ Vp $= \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} m/s$

$OQ = \begin{pmatrix} -11 \\ -2 \\ -7 \end{pmatrix}$ $V_Q = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$m/s

$_P$V$_Q$ = V$_P$ –V$_Q$

$_P$V$_Q$ $= \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \\ -1 \end{pmatrix}$

$_P$R$_Q$ $= (OP - OQ)$ + ($_P$V$_Q$)t

$\mathrm{PRQ} = \left[\begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix} - \begin{pmatrix} -11 \\ -2 \\ -7 \end{pmatrix}\right] + \begin{pmatrix} -2 \\ 0 \\ -1 \end{pmatrix} t$

$_P$R$_Q$ $= \begin{pmatrix} 11 \\ 4 \\ 9 \end{pmatrix} + \begin{pmatrix} -2 \\ 0 \\ -1 \end{pmatrix} t$

For minimum distance

$_P$V$_Q$. $_P$R$_Q$ $= 0$

$\begin{pmatrix} -2 \\ 0 \\ -1 \end{pmatrix} . \begin{pmatrix} 11 - 2t \\ 4 \\ 9 - t \end{pmatrix} = 0$

$-22 + 4t + 0 - 9 + t = 0$

$t = \frac{31}{5}$ $\therefore\ t = 6.2s$

ii) Shortest distance d=/$_P$R$_Q$/

$_P$R$_Q$ $= \begin{pmatrix} 11 \\ 4 \\ 9 \end{pmatrix} + \begin{pmatrix} -2 \\ 0 \\ -1 \end{pmatrix} t$

t=6.2

$_P$R$_Q$ $= \begin{pmatrix} 11 \\ 4 \\ 9 \end{pmatrix} + \begin{pmatrix} -2 \\ 0 \\ -1 \end{pmatrix}$**6.2**$= \begin{pmatrix} -1.4 \\ 4 \\ 2.8 \end{pmatrix}$

/$_P$R$_Q$/= $\sqrt{(-1.4)^2 + 4^2 + 2.8^2}$

/$_P$R$_Q$/= **5.08m**

iii) How far each has travelled

R$_P$ = OP +V$_P$t

R$_P$ $= \begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} 6.2 = \begin{pmatrix} 0 \\ 8.2 \\ 8.2 \end{pmatrix}$

/R$_P$/= $\sqrt{0^2 + 8.2^2 + 8.2^2}$=**11.6m**

R$_Q$ = OQ +V$_Q$t

R$_Q$ $= \begin{pmatrix} -11 \\ -2 \\ -7 \end{pmatrix} + \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} 6.2 = \begin{pmatrix} 1.4 \\ 4.2 \\ 5.4 \end{pmatrix}$

/R$_Q$/ =$\sqrt{1.4^2 + 4.2^2 + 5.2^2}$= **6.8m**

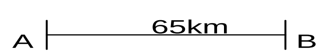
2. Initially two ships A and B are 65km apart with B due East of A. A is moving due East at 10km/hr and B due south at 24km/hr. the two ships continue moving with these velocities. Find the least distance between the ships in the subsequent motion and the time taken to the nearest minute for such a situation to occur.

Solution

least distance d = /$_A$R$_B$/

For least distance ($_A$V$_B$. $_A$R$_B$) = 0

But $_A$V$_B$ $= \begin{pmatrix} 10 \\ 24 \end{pmatrix}$



A $= \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ **B**$= \begin{pmatrix} 65 \\ 0 \end{pmatrix}$

${}_{A}R_{B} = (OA - OB) + {}_{A}V_{B}t$

$= \left[\begin{pmatrix}0\\0\end{pmatrix} - \begin{pmatrix}65\\0\end{pmatrix}\right] + t\begin{pmatrix}10\\24\end{pmatrix}$

${}_{A}R_{B} = \begin{pmatrix}-65+10t\\24t\end{pmatrix}$

${}_{A}V_{B} \cdot {}_{A}R_{B} = 0$

$\begin{pmatrix}10\\24\end{pmatrix} \cdot \begin{pmatrix}-65+10t\\24t\end{pmatrix} = 0$

$-650 + 100t + 576t = 0$

$t = \frac{650}{676} \quad \therefore \quad t = 0.96hrs$

Trial:4

1. A ship A is 8km due North of Ship B, ship A is moving at 150kmh^{-1} due west while B is moving at 200km/hr due N30^{o}W. After what time will they be nearest together and how far apart will they be. **An(2.22km, 0.043hrs)**
2. The point p is 50km west of q. Two air crafts A and B fly simultaneously from p and q velocities are 400km/hr N50^{o}E and 500km/hr N20^{o}W respectively. Find;
   (i) The closest distance between the air crafts
   (ii) The time of flight up to this point **An(20.35km, 5.24 minutes)**
3. Ship A steams North-west at 60km/hr whereas B steams southwards at 50km/hr, initially ship B was 80km due north of A. find;
   (i) The velocity of A relative to B
   (ii) The time taken for the shortest distance to be reached
   (iii) The shortest distance between A and B.
   An(101.675km/hr at N24.7^{o}W, 42.9minutes, 33.382km)

3.3.0: Motion of bodies with different frames of reference

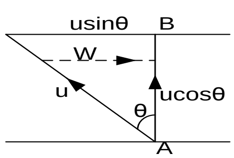
It involves crossing the river and flying space

3.3.1: Crossing the river

There are three cases to consider when crossing a river

a. Case I (shortest route)

If the water is not still and the boat man wishes to cross **directly opposite** to the starting point.
In order to cross point A to another point B directly opposite A (perpendicularly), then the course set by the boat must be upstream of the river.



u is the speed of the boat in still water,
w is the speed of the running water

At point B: $u\sin\theta = w$

$\sin\theta = \frac{w}{u}$

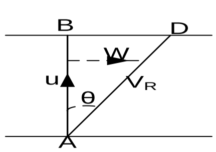
$\theta = \sin^{-1}\frac{w}{u}$

θ is the direction to the vertical but the direction to the bank is $(90-\theta)°$

Time taken $= \frac{AB}{u\cos\theta}$

b. Case II. The shortest time/as quickly as possible

If the boat man wishes to cross the river as quickly as possible, then he should steer his boat directly from A to B as shown. The river pushes him down stream.



Time to cross the river $t = \frac{AB}{u}$

Distance covered downstream is = wxt

Or distance downstream = $w\frac{AB}{u}$

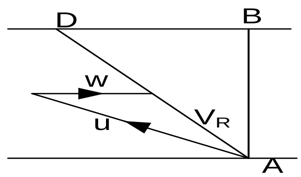
$\tan\theta = \frac{w}{u} \qquad \theta = \tan^{-1}\frac{w}{u}$

The resultant velocity downstream V_R

$V_R^2 = W^2 + u^2$

$V_R = \sqrt{w^2 + u^2}$

C. Case III



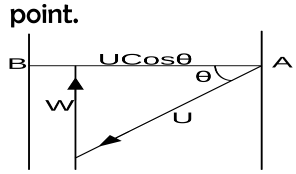
Resultant velocity $\overrightarrow{V_R} = \overrightarrow{W} + \overrightarrow{U}$

EXAMPLES

1. A river with straight parallel bank 400m apart flows due north at 4km/hr. Find the direction in which a boat travelling at 12km/hr must be steered in order to cross the river from East to West along the course perpendicular to the banks. Find also the time taken to cross the river.

Solution

Hint. Since the course is perpendicular to the bank, then it requires crossing directly to the opposite point.



W=4km/hr U=12km/hr

AB=400m=0.4km

Sinθ=$\frac{w}{u}$ θ=$Sin^{-1}\frac{4}{12}$ θ=19.47°

The direction is (90-19.47) to the bank.

Direction is 70.53° to the bank

Time taken =$\frac{AB}{uCos\theta}$

Time taken =$\frac{0.4}{12Cos19.47}$= 0.035hrs

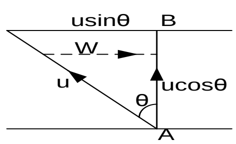
Time =2.1 minutes

2. A man who can swim at 6km/hr in still water would like to swim between two directly opposite points on the banks of the river 300m wide flowing at 3km/hr. Find the time he would take to do this.

Solution

U=6km/hr W= 3km/hr

AB=300m AB = 0.3km



$Sin\theta = \frac{w}{u}$

$$\theta = Sin^{-1}\left(\frac{3}{6}\right) = 30°$$

Time taken $= \frac{AB}{uCos\theta}$

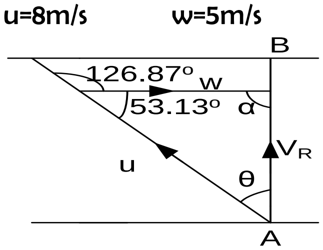
Time taken $= \frac{0.3}{6Cos30}$ = 0.058hrs=3.46minute

He must swim at 30° to AB in order to cross directly and it will take 3.46minutes

3. A man who can swim at 8m/s in still water crosses a river by steering at an angle of 126.87° to the water current. If the river is 75m wide and flows at 5m/s, find;
   (i) The velocity with which the person crosses the river
   (ii) The time he takes to do this

Solution

u=8m/s w=5m/s AB=75m



α is not 90°

Using cosine rule

$V_R^2 = 8^2 + 5^2 - 2\,u.wcos53.13$

$V_R = \sqrt{8^2 + 5^2 - 2\,x8x5cos53.13}$

V_R= $6.4m/s$

The person crosses with 6.4m/s.

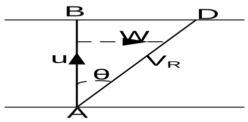
ii) Time taken =$\frac{AB}{uCos\theta}$

But V_R= $Ucos\theta$

Time $= \frac{75}{6.4}$ = 11.72 seconds

4. A man who can swim at 2m/s at in still water wishes to swim across a river 120m wide as quickly as possible. If the river flows at 0.5m/s, find the time the man takes to cross and how far down streams he travels.

Solution



$U = 2m/s \;\; w = 0.5m/s \quad AB = 120m$

$t = \frac{AB}{u} = \frac{120}{2} = 60s$

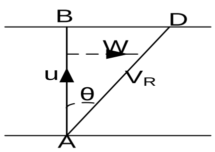
Distance downstream $= wt = 0.5x60$

Distance downstream $= 30m$

5. A boat can travel at 3.5m/s in still water. A river is 80m wide and the current flows at 2m/s, calculate
   a) The shortest time to cross the river and the distance downstream that the boat is carried.
   b) The course that must be set to apoint exactly opposite the starting point and the time taken for crossing

Solution

a) U=3.5m/s, w=2m/s AB=80m

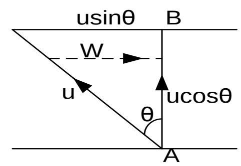


Shortest time $t = \frac{AB}{u} = \frac{80}{3.5}$ = **22.95**

Distance downstream BD=wt= $2x22.9$

Distance downstream BD=45.8m

b. U=3.5m/s, w=2m/s, AB=80



$Sin\theta = \frac{w}{u}$

$\theta = Sin^{-1}\left(\frac{2}{3.5}\right)$ =**34.8°**

The course must be 34.8° to AB.

Time taken $= \frac{AB}{uCos\theta} = \frac{80}{3.5Cos34.8} = 27.8s$

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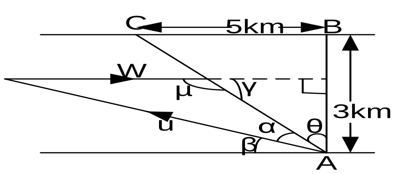
6. A boat crosses a river 3km wide flowing at 4m/s to reach a point on the opposite bank 5km upstream. The boat's speed in still water is 12m/s. Find the direction in which the boat must be headed. (04marks)

Solution

In order for a boat to cross to a point C upstream on the opposite bank then the course set must be such that the resultant velocity of the boat is along AC upstream.

U=12m/s, w=4m/s, AB=3km, AC=5km



$\tan\theta = \frac{5}{3}$ $\quad\theta$=59.04°

But $\gamma + \theta$=90°

$\gamma = 90 - 59.04$

γ =30.96°

But $\mu + \gamma$ =180°

μ +30.96°=180°

μ =180°-30.96°

μ=149.04°

Also using sin rule $\frac{w}{sin\alpha} = \frac{u}{sin\mu}$

$$\frac{4}{sin\alpha} = \frac{12}{sin149.04}$$

$\alpha = sin^{-1}\left(\frac{4sin149.04}{12}\right)$

α=9.87°

But β + α + θ=90°

β +9.87+59.04=90°

β =21.09°

The boat must be headed at 21.09° to the river bank upstream

Trial:4

1. A man who an row at 0.9m/s in still water wishes to cross the river of width 1000m as quickly as possible. If the current flows at a rate of 0.3m/s. Find the time taken for the journey. Determine the direction in which he should point the boat and position of the boat where he lands **An [1111.11s, 71.57° to the bank, 333.33 downstream]**

2. A man swims at $5kmh^{-1}$ in still water. Find the time it takes the man to swim across the river 250m wide, flowing at $3kmh^{-1}$, if he swims so as to cross the river;
   (i) By the shortest route **An [178.6s]**
   (ii) In the quickest time **An[217.4s]**
3. A boy can swim in still water at 1m/s, he swims across the river flowing at 0.6m/s which is 300m wide, find the time he takes;
   (i) If he travels the shortest possible distance
   (ii) If he travels as quickly as possible and the distance travelled downstream. [**375s,180m]**
4. Rain drops of mass $5x10^{-7}kg$ fall vertically in still air with a uniform speed of 3m/s. if such drops are falling when a wind is blowing with a speed 2m/s,
   (i) what is the angle which the paths of the drops make with the vertical
   (ii) what is the kinetic energy of a drop [**33.7^{o},$3.25x10^{-6}$J]**
5. A boy wishes to swim across a river 100m wide as quickly as possible. The river flows at 3km/hr and the boy can swim at 4km/h in still water. Find the time that the boy takes to cross the river and how far downstream he travels. **An [90s,75m].**

CHAPTER 4: NEWTON'S LAWS OF MOTION

LAW I : Everybody continues in its state of rest or uniform motion in **a straight line** unless acted upon by an external force.

This is sometimes called the law of **inertia**

Definition

Inertia is the reluctance of a body to start moving once its at rest or to stop moving if its already in motion.

Explain why a passenger jerks forward when a fast moving car is suddenly stopped.

Passengers jerk forward because of inertia. When the car is suddenly stopped, the passenger tends to continue in uniform motion in a straight line because the force that acts on the car does not act on the passenger

LAW II: The rate of change of momentum of a body is directly proportional to the applied force and takes place in the direction of the force.

Consider a mass m moving with velocity u. If the mass is acted on by a force F and its velocity changes to v;

By Newton's law of motion

F $\alpha\ \frac{mv-mu}{t} = \frac{k(mv-mu)}{t} = km\frac{(v-u)}{t} = kma$

Since $a = \frac{v-u}{t}$

When $F = 1N$, $m = 1kg$ and $a = 1ms^{-2}$

$1 = kx1x1$

$k = 1$

$\boxed{F = ma}$

Note: F must be the resultant force

LAW III: To every action there is an equal but opposite reactions.

$$\boldsymbol{F_1 = -F_2}$$

Example of 3rd law of motion

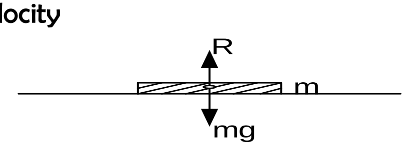
- A gun moves backwards on firing it.
- A ball bounces on hitting the ground.

Rocket engine propulsion

Fuel is burnt in the combustion chamber and exhaust gases are expelled at a high velocity. This leads to a large backward momentum. From conservation of momentum an equal forward momentum is gained by the rocket, due to continuous combustion of fuel there is a change in the forward momentum which leads to the thrust hence maintaining the motion of the rocket

4.1.0: IDENTIFICATION OF FORCES AND THE APPLICATION OF NEWTON'S LAWS

1. Consider a body of mass **m** placed on either a stationary platform or a platform moving at a constant velocity

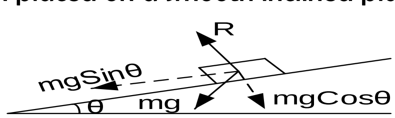


R *is* normal reaction

Mg is gravitational pull [weight]

$R = mg$ since (a=0) constant velocity

2. Mass m placed on a smooth inclined plane of angle of inclination θ



R = mgCosθ

NB:

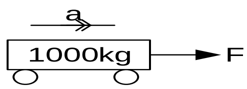
- All objects placed on, or moving on an inclined plane experience a force $mgsin\theta$ **down** the plane. [It doesn't matter what direction the body is moving]
- If the plane is **rough** the body experiences a frictional force whose direction is opposite to the direction of motion.

Examples:

1. A car of mass 1000kg is accelerating at 2ms^{-2}.
   (i) What resultant force acts on the car?.
   (ii) If the resistance to the motion is 1000N, what force is due to the engine?

Solution

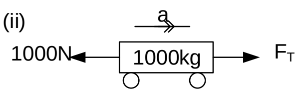
(i)



$F = ma = 1000x2 = 2000N$

Resultant force is 2000N

(ii)



The resistance force should act in opposite direction to the force due to the engine

$F_T - 1000 = ma$

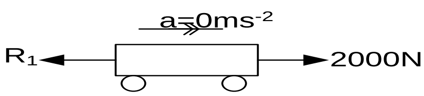
$F_T - 1000 = 1000x2$

$F_T = 3000N$

Force due to the engine is 3000N

2. A car moves along a level road at a constant velocity of 22m/s. If its engine is exerting a forward force of 2000N, what resistance is the car experiencing

Solution



Using $F = ma$

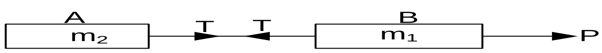
$2000 - R_1 = ma$

But a = 0 since it moves with constant velocity

$2000 - R_1 = 0$

$R_1 = 2000N$

3. Two blocks A and B connected as shown below on a horizontal friction less floor and pulled to the right with an acceleration of 2ms^{-2} by a force P, if m_1 = 50kg and m_2 = 10kg. what are the values of T and P



Solution

Using $F = ma$

For m₁: $P - T = 50x2 = 100$.......[1]

For m₂ $T = 10x2 = 20N$

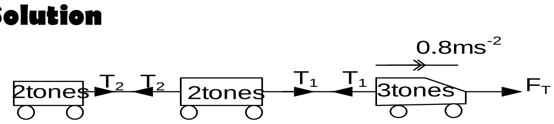
Put into equation (1) $P - T = 100$

$P - 20 = 100$

$P = 120N$

4. A Lorry of 3 tones pulls 2 trailers each of mass 2 tones along a horizontal road, if the lorry is accelerating at 0.8ms^{-2}, calculate
   a) Net force acting on the whole combination
   b) The tension in the coupling between the lorry and 1st trailer.
   c) The tension in the coupling between the 1st and 2nd trailer.

Solution



For the lorry: $F_T - T_1 = 3000x0.8 = 2400 \ldots..(1)$

For 1st trailer: $T_1 - T_2 = 2000x0.8 = 1600 \ldots..(2)$

For 2ND trailer: T$T_2 = 2000x0.8 = 1600N$

Put into [2]: $T_1 - T_2 = 1600$

$T_1 - 1600 = 1600$

$T_1 = 3200N$

Put into [1] F$-T_1 = 2400$

$F - 3200 = 2400$

$F = 5600N$

Exercise: 5

1. A large card board box of mass 0.75kg is pushed across a horizontal floor by a force of 4.5N. the motion of the box is opposed by a frictional force of 1.5N between the box and the floor , and an air resistance force given by kv^2 where $k = 6.0x10^{-2}kgm^{-1}$and v is the speed of the box in m/s. calculate;
   (a) The acceleration of the box
   (b) Its speed **An(4.0m/s², 7.1m/s)**
2. A stone of mass 500g is thrown with a velocity of $15ms^{-1}$ across the frozen surface of a lake and comes to rest in 40m. what is the average force of the friction between the stone and the ice
3. A 5000kg engine pulls a train of 5 trucks, each of 2000kg along a horizontal track. If the egnie exerts a force of 50000N and frictional resistance is 5000N calculate;
   (a) The net accelerating force
   (b) The acceleration of the train

(c) The force of truck 1 on truck 2

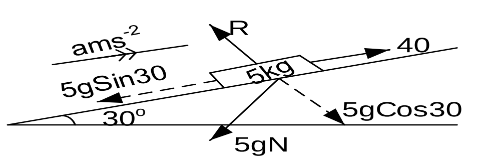
4. A dummy is used in a test crash to test the suitability of the seat belt. If the dummy had a mass of 65kg and it was brought to rest in a distance of 65cm form a velocity of 12m/s, calculate
   (a) the mean deceleration during the crash
   (b) The average force exerted on the dummy during the crash
5. A box of 50kg is pulled up from a ship with an acceleration of $1ms^{-2}$ by a vertical rope attached to it.
   i. Find the tension on the rope.
   ii. What is the tension in the rope when the box moves up with a uniform velocity of $1ms^{-1}$ ($g=9.8ms^{-2}$) **An [540N, 490N]**
6. A lift moves up and down with an acceleration of $2ms^{-2}$. In each case, calculate the reaction of the floor on a man of mass 50kg standing in the lift. (take g = $9.8ms^{-5}$) **An[590N, 390N]**

Motion on inclined planes

Example

1. A body of mass 5kg is pulled up a smooth plane inclined at 30° to the horizontal by a force of 40N acting parallel to the plane. Find
   a) Acceleration of the body
   b) Force exerted on the body by the plane

Solution



a) Resolving parallel to the plane: $F = ma$

$40 - 5gsin30 = ma$

$40 - 5x9.81sin30 = 5xa$

$a = 3.095ms^{-2}$

b) Force exerted on the body by the plane is the normal reaction

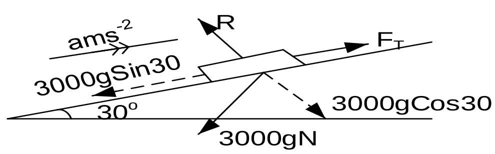
$R = 5gcos30 = 5x9.81cos30 = 42.4N$

2. A lorry of mass 3 tones travelling at 90km/hr starts to climb an incline of 1 in 5. Assuming the tractive pull between its tyres and the road remains constant and that its velocity reduces to 54km/h in a distance of 500m. Find the tractive pull

Solution

$u = 90km/h = \frac{90x1000}{3600} = 25ms^{-1}$

$v = 54km/h = \frac{54x1000}{3600} = 15ms^{-1}$



Resolving along the plane

$F_T - 3000gsin\theta = 3000a$

$F_T - 3000x9.81x\frac{1}{5} = 3000a$

$F - 5886 = 3000a$....................(i)

But $v^2 = u^2 + 2as$

$15^2 = 25^2 + 2ax500$

$a = -0.4ms^{-2}$

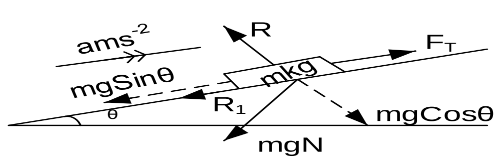
put into (i) $F - 5886 = 3000a$

$F = -3000x0.4 + 5886 = 4686N$

The tractive force is 4686N

3. A train travelling uniformly at 72km/h begins an ascent on 1 in 75. The tractive force which the engine exerts during the ascent is constant at 24.5kN, the resistance due to friction and air is also constant at 14.7kN, given the mass of the whole train is 225 tones. Find the distance a train moves up the plane before coming to rest.

Solution



1 in 75 means $sin\theta = \frac{1}{75}$ ∴ $\theta = 0.76°$

resistance force: $R_1 = 14.7kN$

tractive force : $F_T = 24.5kN$

$F_T - (mgsin\theta + R_1) = ma$

$$24500-(225000x9.81x\frac{1}{75}+14700) = 22500a$$

$$a = -0.087ms^{-2}$$

its deceleration $= 0.087ms^{-2}$

$v^2 = u^2 + 2as$ [$v = om/s$ comes to rest]

$$u = 72km/h = \frac{72x1000}{3600} = 20ms^{-1}$$

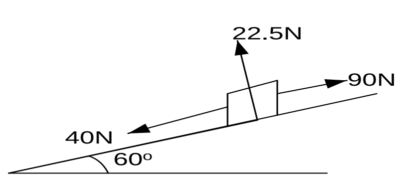
$$0^2 = 20^2 + 2(-0.087)s$$

$$-400 = -0.174s$$

$$S = 2298.85m$$

Exercise:6

1. The resistance to the motion of the train due to friction is equal to 1/160 of the weight of the train, if the train is travelling on a level road at $72kmh^{-1}$ and comes to the foot of an incline of 1 in 150 and steam is then turned off, how far will the train go up the incline before it comes to rest. **An(1579.99m)**
2. 12m length of the slope. If the truck starts from the bottom of the slope with a speed of 18km/h, how far up will it travel before coming to rest **An(71.43m).**
3. A car of 1 tonne accelerates from 36kmh to $72\ kmh^{-1}$ while moving $0.5kmh^{-1}$ up a road inclined at an angle of α to the horizontal, where $sin\alpha = \frac{1}{20}$. If the total resistive force to its motion is 0.3kN, find the driving force of the car engine **An(1009N).**
4. A railway truck of mass 6.0 tonnes moves with an acceleration of $0.050ms^{-2}$ down a track which is inclined to the horizontal at an angle α where $sin\alpha = \frac{1}{120}$. Find the resistance to motion **An($2.0x10^2$N).**
5. A body of mass 5.0kg is pulled along a smooth horizontal ground by means of force of 40N acting at 60^0 above the horizontal. Find
   (a) Acceleration of the body
   (b) Force the body exerts on the ground **An($4.0ms^{-2}$, 15.4N).**
6. A railway engine of mass 100 tones is attached to a line of truck of total mass 80 tones. Assuming there is no resistance to motion, find the tension in the coupling between the engine and the leading truck when the train
   (a) has an acceleration of $0.020ms^{-2}$
   (b) Is moving at constant velocity **An(25.6kN).**
7. A bullet of mass $8.00x10^{-3}$kg moving at $320ms^{-1}$ penetrates a target to a depth of 16.0mm before coming to rest. Find the resistance offered by the target, assuming it to be uniform. **An(1.6kN, 0N).**
8. A body of mass 3.0kg slides down a plane which is inclined at 30^0 to the horizontal. Find the acceleration no of the body , if:
   (a) The plane is smooth
   (b) There is a frictional resistance of 9.0N **An($5.0ms^{-2}$, $2.0ms^{-2}$).**
9. A car of mass 1000kg tows a caravan of mass 600kg up a road which rises 1m vertically for every 20m of its length. There are constant frictional resistance of 200N and 100N to the motion of the car and to the motion of the caravan respectively. The combination has an acceleration of $1.2ms^{-2}$ with the engine exerting a constant driving force. Find
   (a) Driving force
   (b) Tension in the tow- bar **An(3.02kN, 1.12kN).**
10. A 25kg block rests at the top of a smooth plane whose length is 2.0m and whose height at elevated end is 0.5m. how long will it take for the block to slide to the bottom of plane when released **An(1.258s).**
11.



Three forces act on a block as shown, the block is placed on a smooth plane inclined at 60^0 calculate;
   a) Acceleration of the block up the plane
   b) Gain in kinetic energy in 5s after moving from rest **An($1.5ms^{-2}$, 140.625J)**

4.1.1: MOTION OF CONNECTED PARTICLES

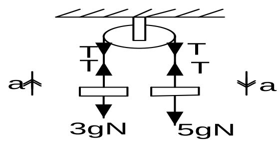
When two particles are connected by a light inextensible string passing over a smooth pully and allowed to move freely, then as long as the string is tight, the following must be observed.

- Acceleration of one body in general direction of motion is equal to the acceleration of the other
- The tension T in the string is constant.

Examples

1. Two particles of masses 5kg and 3kg are connected by a light inelastic string passing over a smooth fixed pulley. Find;
   (i) Acceleration of the particles
   (ii) The tension in the string
   (iii) The force on the pulley

Solution



Using $F = ma$

For 5kg mass: $5g - T = 5a$...............(i)

For 3kg mass: $T - 3g = 3a$ ....(ii)

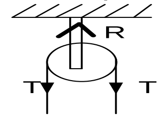
Adding (i) and (ii): $2g = 8a$

$a = \frac{2x9.81}{8} = 2.45ms^{-2}$

ii) $T - 3g = 3a$

$T = 3x2.45 + 3x9.81 = 36.78N$

iii) Force on the pulley

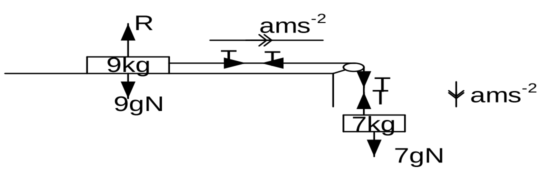


$R = 2T = 2x36.78 = 73.56N$

Force on the pulley is 73.56N

2. A mass of 9kg resting on a smooth horizontal table is connected by a light string passing over a smooth pulley at the edge of the table, to the pulley is a 7kg mass hanging freely; find
   (i) Common acceleration
   (ii) The tension in the string
   (iii) The force on the pulley in the system if its allowed to move freely.

Solution



Using $F = ma$

For 7kg mass: $7g - T = 7a$..............(i)

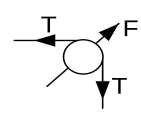
For 9kg mass: $T = 9a$............................(ii)

Put (ii) into (i): $7g - 9a = 7a$

$a = \frac{7g}{16} = \frac{7x9.81}{16} = 4.292ms^{-2}$

(ii) Tension : $T = 9a = 9\,x\,4.292 = 38.63N$

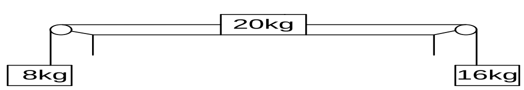
(iii) The force on the pulley



$F = \sqrt{T^2 + T^2} = T\sqrt{2} = 38.63\sqrt{2}$

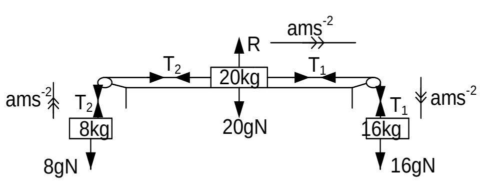
Force on the pulley $= 54.63N$

3.



The figure shows a block of mass 20 kg resting on a smooth horizontal table. Its connected by strings which pass over pulleys at the edges of the table to two loads of masses 8kg and 16kg which hang vertically. Calculate;
   (i) Acceleration of 16kg mass
   (ii) Tension in each string
   (iii) Reaction on each pulley

Solution



Using $F = ma$

For 16kg mass: $16g - T_1 = 16a$..........[1]

For 20kg mass: $T_1 - T_2 = 20a$............[2]

For 8kg mass: $T_2 - 8g = 8a$ ....[3]

Adding 1 and 2: $16g - T_2 = 36a$.............[x]

And (3) and (x): $8g = 44a$

$a = \frac{8x\,9.81}{44} = 1.784ms^{-2}$

ii) Tension in each string

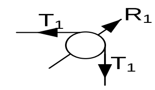
$16g - T_1 = 16a$

$T_1 = 16x9.81 - 16x1.784 = 128.416N$

$T_2 - 8g = 8a$

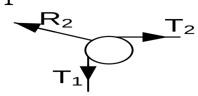
$T_2 = 8x1.784 + 8x9.81 = 92.752N$

iii) Reaction on each pulley



$R_1 = \sqrt{T_1^2 + T_1^2} = T_1\sqrt{2} = 128.416x\sqrt{2}$

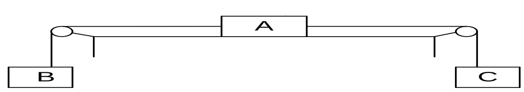
$R_1 = 181.61N$



$R_2 = T_2\sqrt{2} = 92.752\sqrt{2} = 131.171N$

Exercise:7

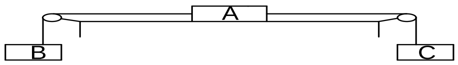
1. Two particles of masses 7kg and 3kg are connected by a light inelastic string passing over a smooth fixed pulley. Find;
   (i) Acceleration of the particles (ii) The tension in the string
   (iii) The force on the pulley **An(3.92ms^{-2} , 41.16N, 82.32N)**
2. Two particles of masses 6kg and 2kg are connected by a light inextensible string passing over a smooth fixed pulley. With the masses hanging vertically, the system is released from rest. Find;
   (i) Acceleration of the particles (ii) The tension in the string
   (iii) Distance moved by the 6kg mass in the first 2 seconds of motion
   An(4.9ms^{-2} , 3N, 9.8m)
3. A man of mass 70kg and a bucket of bricks of mass 100kg are tied to the opposite ends of a rope which passes over a frictionless pulley so that they hang vertically downwards
   (a) what is the tension in the section of the section of rope supporting the man
   (b) What is the acceleration of the bucket **An(807.06N, 1.73ms^{-2})**
4. Two particles of masses 20g and 30g are connected to a fine string passing over a smooth pulley, when released freely find;
   (i) Common acceleration (ii) The tension in the string
   (iii) The force on the pulley **An [1.962ms^{-2} , 0.235N ,0.471N]**
5. A mass of 5kg is placed on a smooth horizontal table and connected by a light string to a 3kg mass passing over a smooth pulley at the edge of the table and hanging freely. If the system is allowed to move, calculate;
   a) The common acceleration of the masses **An[3.68m/s^2, 18.4N, 26N]**
   b) The tension in the string **c)** The force acting on the pulley
6. Two objects of mass 3kg and 5kg are attached to the ends of a cord which passes over a fixed frictionless pulley placed at 4.5m above the floor. The objects are held at rest with 3 kg mass touching the floor and the 5kg mass at 4m above the ground and then released, what is
   (i) The acceleration of the system **An(2.45ms^{-2}).**
   (ii) The tension of the cord **An(36.75N).**
   (iii) Time will elapse before the 5kg object hits the floor **An(1.81s).**
7.



The diagram shows a particle A of mass M = 2kg resting on a horizontal table. It is attached to particles B of m =5kg and C of m= 3kg by light inextensible strings hanging over light smooth pulleys. If the system is allowed to move from rest, find the common acceleration of the particle and the tension in each string given that the surface of the table is rough and the coefficient of friction between the particle and the surface of the table is ½ **An[0.98ms^{-2}, 32.37N, 44.15N]**

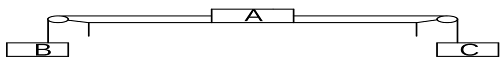
[Hint: friction force = coefficient of friction x normal reaction]

8.



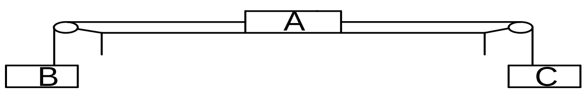
The diagram shows a particle A of mass $2kg$ resting on a rough horizontal table of coefficient of friction 0.5. It is attached to particles B of mass $5kg$ and C of mass $3kg$ by light inextensible strings hanging over light smooth pulleys. If the system is allowed to move from rest, find the common acceleration of the particle and the tension in each string. **An[0.98ms^{-2}, 32.37N, 44.15N]**

9.



The diagram shows a particle A of mass $5kg$ resting on a rough horizontal table. It is attached to particles B of mass $3kg$ and C of mass $2kg$ by light inextensible strings hanging over light smooth pulleys. If the system is released from rest, body B descends with an acceleration of 0.28ms^{-2}, find the c coefficient of friction between the body A and the surface of the table **An[$\frac{1}{7}$]**

10.



The diagram shows a particle A of mass $10kg$ resting on a smooth horizontal table. It is attached to particles B of mass $4kg$ and C of mass $7kg$ by light inextensible strings hanging over light smooth pulleys. If the system is allowed to move from rest, find the common acceleration of the particle and the tension in each string. **An [1.4ms^{-2}, 44.8N, 58.8N]**

4.1.2: LINEAR MOMENTUM AND IMPULSE

Momentum is the product of mass and velocity of the body moving in a straight line

Momentum (p) = mass x velocity

$$\boxed{\vec{p} = m\vec{v}}$$

Momentum is a vector quantity

Definition

Linear momentum (p) is the product of the mass and the velocity of the body moving in a straight line.

IMPULSE

This is the product of the force and time for which the force acts on a body

i.e. Impulse (I) = Force(F) x time (t)

$$\vec{I} = \vec{F}\,t$$

The unit of impulse is Ns.

An impulse produces a change in momentum of a body. If a body of mass(m) has it velocity changed from u to v by a force F acting on it in time t, then from Newton's 2nd law.

$$F = \frac{mv - mu}{t}$$

$$Ft = mv - mu$$

$$I = Ft$$

$$I = mv - mu$$

Impulse = change in momentum

Example

1. A body of mass 5kg is initially moving with a constant velocity of 2ms^{-1}, when it experiences a force of 10N is 2s, find
   (i) The impulse given to the body by the force
   (ii) The velocity of the body when the force stops acting

Solution

$$I = ft = 10x2 = 20Ns$$

$$I = mv - mu$$

$$20 = 5v - 5x2$$

$$v = 6m/s$$

2. A girl of mass 50kg jumps onto the ground from a height of 2m. Calculate the force which acts on her when she lands

(i) As she bends her knees and stops within 0.2 s
(ii) As she keeps her legs straight and stops in 0.05s

Solution

i) $v^2 = u^2 + 2gs$

$v^2 = 0^2 + 2x9.81x2$

$v = \sqrt{39.24} = 6.3ms^{-1}$

Using $F = \frac{\text{mv-mu}}{\text{t}}$

$F = \frac{50(6.3\text{-}0)}{0.2} = 1575N$

ii) ii) $F = \frac{mv-mu}{t}$

$F = \frac{50(6.3\text{-}0)}{0.05} = 6300N$

3. Water leaves horse pipe at a rate of 5.0kgs^{-1} with a speed of 20ms^{-1} and is directed horizontally on a wall which stops it. Calculate the force exerted by the water on the wall.

Solution

Force due to water$=$ $mass\ per\ second\ xvelocity\ change$

Force due to water$=$ $5\ x(20-0) = 100N$

4. A horse pipe has a hole of cross-sectional area 50cm^2 and ejects water horizontally at a speed of 0.$3ms^{-1}$. If the water is incident on a vertical wall and its horizontal velocity becomes zero. Find the force the water exerts on the wall.

Solution

Force due to water$=$ $mass\ per\ second\ xvelocity\ change$

Force due to water$=$ $(areaxvelocityx\ density)xvelocity\ change = \rho Av^2$

Force due to water$=$ $0.3x50x10^{-4}x1000\ x(0.3-0)$ =**0.45N**

5. A helicopter of mass 1.0x10^{3}kg hovers by imparting a downward velocity v to the air displaced by its rotating blades. The area swept pout by the blades is 80m^2. Calculate the value of v. (density of air=1.3kgm^{-3})

Solution

$$F = \rho Av^2$$
$$mg = \rho Av^2$$
$$1.0x10^3x9.81 = 80xvx1.3x(v-0)$$
$$1.0x10^3x9.81 = 104v^2$$
$$v = 9.8m/s$$

6. Sand falls onto a conveyor belt at a constant rate of 2kgs^{-1}. The belt is moving horizontally at 3ms^{-1}. Calculate

(a) The extra force required to maintain the speed of the belt
(b) Rate at which this force is dong work
(c) The rate at which the kinetic energy of the sand increases

Solution

Force =mass per second $x$velocity change

$= 2x3\ = 6N$

Rate of doing work =force x velocity change

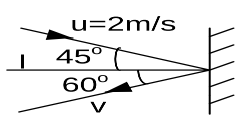
$= 6x3 = 18Js^{-1}$

Rate of k.e $= \frac{1}{2}m\ x\ (velocity\ change)^2$

$= \frac{1}{2}x2x3^2 = 9Js^{-1}$

7. A ball of mass 0.25kg moving in a straight line with a speed of 2ms^{-1} strikes a vertical wall at an angle of 45^o to the normal. The wall gives it an impulse in the direction of the normal and the ball rebounds at an angle of 60^o to the normal. Calculate the magnitude of the impulse and the speed with which the ball rebounds.

Solution



Impulse $I\ = mv - mu$

$$I = m\left[\begin{pmatrix}-vCos60\\-vSin60\end{pmatrix} - \begin{pmatrix}2cos45\\-2sin45\end{pmatrix}\right]$$

$$I = \frac{1}{4}\left[\begin{pmatrix}\frac{-1}{2}v\\\frac{-\sqrt{3}}{2}v\end{pmatrix} - \begin{pmatrix}\sqrt{2}\\-\sqrt{2}\end{pmatrix}\right] = \frac{1}{4}\begin{pmatrix}-\frac{v}{2}-\sqrt{2}\\-\frac{\sqrt{3}}{2}v+\sqrt{2}\end{pmatrix}$$

Since I is perpendicular to the wall then the vertical component is zero

$$-\frac{v}{2}-\sqrt{2} = 0$$
$$V = -2\sqrt{2}m/s$$

$$I = \frac{1}{4}\begin{pmatrix}-\frac{-2\sqrt{2}}{2}-\sqrt{2}\\-\frac{\sqrt{3}}{2}x-2\sqrt{2}+\sqrt{2}\end{pmatrix} = \frac{1}{4}\begin{pmatrix}0\\\sqrt{6}+\sqrt{2}\end{pmatrix} = \begin{pmatrix}0\\0.966\end{pmatrix}$$

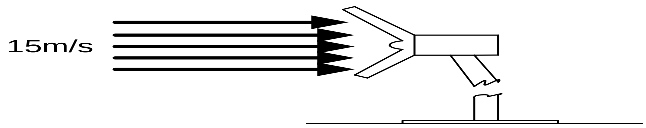
$$I = 0.966Ns$$

Exercise:8

1. A horizontal jet of water leaves the end of a hose pipe and strikes a wall horizontally with a velocity of 20m/s. if the end of the pipe has a diameter of 2cm, calculate the force that will be exerted on the wall. **An(125.7N)**
2. Water emerges at $2ms^{-1}$ from a hose pipe and hits a wall at right angles. The pipe has across-sectional area of $0.03m^2$. calculate the force on the wall assuming that the water does not rebound.(density of water $1000kgm^{-3}$) **An(120N)**
3. Water is squirting horizontally at $4.0ms^{-1}$ from a burst pipe at a rate of $3.0kgs^{-1}$. The water strikes a vertical wall at right angles and runs down it without rebounding. Calculate the force the water exerts on the wall **An(12N)**
4. A machine gun fires 300 bullets per minute horizontally with a velocity of $500ms^{-1}$. Find the force needed to prevent the gun moving back-ward if the mass of each bullet $8.0x10^{-3}kg$ **An(20N)**
5. Coal is falling onto a conveyor belt at a rate of 540 tones every hour. The belt is moving horizontally at $2.0ms^{-1}$. Find the extra force required to maintain the speed of the belt **An($3.0x10^2N$)**
6. A helicopter of total mass 1000kg is able to remain in a stationary position by imparting a uniform downward velocity to a cylinder of air below it of effective diameter 6m. assuming the density of air to be 1.2kgm , calculate the downward velocity given to air **An($17.2ms^{-1}$)**
7. (a) The rotating blades of a hovering helicopter seeps out an area of radius 4.0m imparting a down ward velocity of $12ms^{-1}$ to the air displaced. Find the mass of the helicopter.(density of air $1.3kgm^{-3}$) **An(940kg)**
   (b) the sped of rotation of the blades of the helicopter is now increased so that the air has a down ward velocity of $13ms^{-1}$. Find the upward acceleration of the helicopter **An($1.7ms^{-2}$)**
8. Find the force exerted on each square meter of a wall which is at right angles to a wind blowing at $20ms^{-1}$. Assume that the air does not rebound.(density of air $1.3kgm^{-3}$) **An(520N)**
9. Hail stones with an average mass of 4.0g fall vertically and strike a flat roof at $12ms^{-1}$. In a period of 5.0minutes, 6000 hailstones fall on each square meter of roof and rebound vertically at $3.0ms^{-1}$. Calculate the force on the roof if it has an area of $30m^2$ **An(36N)**
10. A hose with a nozzle 80mm in diameter ejects a horizontal stream of water at a rate of $0.044m^3s^{-1}$.
    (a) With what velocity will the water leave the nozzle
    (b) What will be the force exerted on a vertical wall situated close to the nozzle and at right-angle to the stream of water, if after hitting the wall;
        (i) The water falls vertically to the ground
        (ii) The water rebounds horizontally**An(8.75m/s, 385N, 770N)**
11. An astronaut is outside her space capsule in a region where the effect of gravity can be neglected. She uses a gas gun to move herself relative to the capsule. The gas gun fires gas from a muzzle of area $1.60mm^2$ at a speed of $150ms^{-1}$. The density of the gas is $0.800kgm^{-3}$ and the mass of the astronaut including her space suit is 130kg. calculate
    (a) The mass of gas leaving the gun per second
    (b) The acceleration of the astronaut due to gun, assuming that the change in mass is negligible **An($1.92x10^{-2}kgs^{-1}$, $2.22x10^{-2}ms^{-2}$)**
12. Sand is poured at a steady rate of $5.0gs^{-1}$ on to the pan of a direct reading balance calibrated in grams. If the sand falls from a height of 0.20m on to the pan and it does not bounce off the pan then, neglecting any motion of the pan, calculate the reading on the balance 10s after the sand first hits the pan. **An(0.051kg)**
13. A top class tennis player can serve the ball, of mass 57g at an initial horizontal speed of 50m/s. the ball remains in contact with the racket for 0.050s. calculate the average force exerted on the ball during the serve **An(57N)**
14. A motor car collides with a crash barrier when travelling at 100km/h and is brought to rest in 0.1s.
    (a) if the mass of the car and its occupants is 900kg calculate the average force on the car
    (b) Because of the seat belt, the movement of the driver whose mass is 80kg, is restricted to 0.20m relative tot the car. Calculate the average force exerted by the belt on the driver

An($2.5x10^5$N, $1.54x10^4$N)

15. A stone of mass 80kg is released at the top of a vertical cliff. After falling for by 3s, it reaches the foot of the cliff, and penetrates 9cm into the ground. What is;
    (a) The height of the cliff
    (b) The average force resisting penetration of the ground by the stone **An(45m, 400N)**
16. The blades of a large wind turbines, designed to generate electricity, sweeps pout an area of 1400m^2 and rotates about a horizontal axis which points directly into a wind of speed 15m/s



    (a) Calculate the mass of air passing per second through the area swept out by the blades (take the density of air to be 1.2 kg/m^3)
    (b) The mean speed of the on the far side of the blades is reduced to 13m/s. how much kinetic energy is lost by the air per second **An($2.5x10^4$kg/s, $7.1x10^5$J/s))**
17. A ball of mas $6.0x10^{-2}$kg moving at 15ms^{-1} hits a wall at right angles and bounces off along the same line at 10ms^{-1}
    (a) What is the magnitude of the impulse of the wall on the ball
    (b) The ball is estimated to be in contact with the wall for $3.0x10^{-2}$s, what is the average force on the ball **An(1.5Ns,50N)**
18. A body of mass 2.0kg and which is at rest is subjected to a force of 200N for 0.2s followed by a force of 400N for 0.30s acting in the same direction. Find
    (a) The total impulse on the body
    (b) The final speed of the body **An(160Ns,80ms^{-1})**

4.1.3: WHY LONG JUMPER BEND KNEES

By bending the knees, the time taken to come to rest is increased, which reduces the rate of change of momentum, therefore the force on the jumpers legs is reduced thus less pain on the legs.

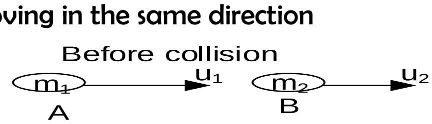
Questions

1. Explain why, when catching a fast moving ball, the hands are drawn backwards while ball is being brought to rest.
2. Explain why a long jumper must land on sand
3. Why is it much more painful to be hit by a hailstone of mass 0.005kg falling at 5m/s which bounces off your head than by a raindrop of the same mass and falling at the same velocity but which breaks up on hitting you and does not bounce? (numerical answered is required)

4.1.4: LAW OF CONSERVATION OF LINEAR MOMENTUM

It states that for a system of colliding bodies, their total linear momentum remains constant in a given direction provided no external forces acts on them.

Suppose a body A of mass m, and velocity U_1, collides with another body B of mass m_2 and velocity U_2 moving in the same direction



By principle of conservation of momentum

$$\boxed{\underbrace{m_1u_1 + m_2u_2}_{\text{Total momentum before collision}} = \underbrace{m_1v_1 + m_2v_2}_{\text{Total momentum after collision}}}$$

4.1.5: Proof of the law of conservation of momentum using Newton's law

Let two bodies A and B with masses m_1 and m_2 moving with initial velocities u_1 and u_2 and let their velocities after collision be v_1 and v_2 respectively for time t with (v_1<v_2)

By Newton's 2^{nd} law:

Force on m_1: $F_1 = \frac{m_1(v_1-u_1)}{t}$

Force on m_2: $F_2 = = \frac{m_2(v_2-u_2)}{t}$

By Newton's 3rd law: $\boldsymbol{F_1 = -F_2}$

$$\frac{m_1(v_1-u_1)}{t} = -\frac{m_2(v_2-u_2)}{t}$$

$$m_1v_1 - m_1u_1 = -m_2v_2 + m_2u_2$$

$$\therefore\ m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

Hence $m_1u_1 + m_2u_2 =$ **constant**

4.1.6: COLLISIONS

In an isolated system, momentum is always conserved but this is not always true of the kinetic energy of the colliding bodies.

In many collisions, some of the kinetic energy is converted into other forms of energy such as heat, light and sound.

Types of collisions

1. **Elastic collisions**

   It is also perfectly elastic collision. This is a type of collision in which all kinetic energy is conserved. Eg collision between molecules, electrons.
2. **Inelastic collision**

   This is a type of collision in which the kinetic energy is not conserved.
3. **Completely inelastic collision**

   This is a type of collision in which the bodies stick together after impact and move with a common velocity. Eg a bullet embedded in a target
4. **Explosive collision (super elastic)**

   This is one where there is an increase in K.E.

Summary

Elastic collision	Inelastic collision	Perfectly inelastic
❖ Linear momentum is conserved	❖ Linear momentum is conserved	❖ Linear momentum is conserved
❖ Kinetic energy is conserved	❖ K.e is not conserved	❖ K.e is not conserved
❖ Bodies separate after collision	❖ Bodies separate after collision	❖ Bodies stick together and move with a common velocity
❖ Coefficient of restitution (elasticity)=1 (e=1)	❖ Coefficient of restitution is less than 1 (e<1)	❖ e=0

4.1.7: Mathematic treatment of elastic collision

Consider an object of mass m, moving to the right with velocity u_1. If the object makes a head-on elastic collision with another body of mass m_2 moving with a velocity u_2 in the same direction. Let v_1 and v_2 be the velocities of the two bodies after collision.

By conservation of momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$m_1(u_1-v_1) = m_2(v_2-u_2)$-------------------[1]

For elastic collision k.e is conserved

$\frac{1}{2} m_1u_1^2 + \frac{1}{2} m_2u_2^2 = \frac{1}{2} m_1{v_1}^2 + \frac{1}{2} m_2{v_2}^2$

$m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2)$ ---------------[2]

from equation 1 and 2 then

$$\frac{m_1(u_1-v_1)}{m_1(u_1^2-v_1^2)} = \frac{m_2(v_2-u_2)}{m_2(v_2^2-u_2^2)}$$

$$\frac{(u_1-v_1)}{(u_1+v_1)(u_1-v_1)} = \frac{(v_2-u_2)}{(v_2+u_2)(v_2-u_2)}$$

$$\frac{1}{(u_1+v_1)} = \frac{1}{(v_2+u_2)}$$

$$u_1+v_1 = v_2+u_2$$

$$v_2-v_1 = -(u_2-u_1)$$

Example

1. A particle P of mass m_1, travelling with a speed u_1 makes a head-on collision with a stationary particle Q of mass m_2. If the collision is elastic and the speeds of P and Q after impact are v_1 and v_2 respectively. Show that for $\beta = \frac{m_1}{m_2}$

(i) $\frac{u_1}{v_1} = \frac{\beta+1}{\beta-1}$ (ii) $\frac{v_2}{v_1} = \frac{2\beta}{\beta-1}$

Solution

By law of conservation of momentum

$m_1u_1 = m_1v_1 + m_2v_2$--------------[x]

$(u_1 - v_1) = \frac{m_2}{m_1}v_2$

Therefore $u_1 - v_1 = \frac{v_2}{\beta}$

$\beta(u_1 - v_1) = v_2$----------------------- [1]

for elastic collision k.e is conserved

½ $m_1u_1^2$ =½ $m_1{v_1}^2$ +½ $m_2{v_2}^2$

$m_1(u_1^2 - v_1^2) = m_2({v_2}^2\ )$

$\frac{m_1}{m_2}({u_1}^2 - {v_1}^2) = {v_2}^2$

$\beta(u_1^2 - v_1^2) = v_2^2$----------------------[2]

equating [1] and [2]

$\beta(u_1^2 - v_1^2) = [\beta(u_1 - v_1)]^2$

$\beta(u_1^2 - v_1^2) = \beta^2(u_1 - v_1)(u_1 - v_1)$

$(u_1 - v_1)(u_1 + v_1) = \beta(u_1 - v_1)(u_1 - v_1)$

$(u_1 + v_1) = \beta(u_1 - v_1)$

$v_1 + \beta v_1 = \beta u_1 - u_1$

$v_1(1 + \beta) = u_1(\beta - 1)$

$\frac{u_1}{v_1} = \frac{\beta+1}{\beta-1}$

ii) From $\frac{u_1}{v_1} = \frac{\beta+1}{\beta-1}$ ------------------ [xx]

from equation[1] : $v_2 = \beta(u_1 - v_1)$

$v_2 = \beta u_1 - \beta v_1$

$u_1 = \frac{v_2+\beta v_1}{\beta}$ put into (xx)

$$\frac{\left(\frac{v_2 + \beta v_1}{\beta}\right)}{v_1} = \frac{(1+\beta)}{(\beta-1)}$$

$(v_2 + \beta v_1)(\beta - 1) = (1 + \beta)\beta v_1$

$\beta v_2 + \beta^2 v_1 - v_2 - \beta v_1 = \beta v_1 + \beta^2 v_1$

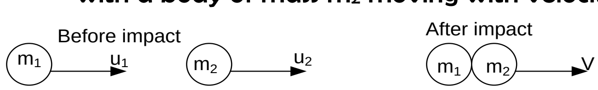
$\beta v_2 - v_2 = 2\beta v_1$

$v_2(\beta - 1) = 2\beta v_1$

$\frac{v_2}{v_1} = \frac{2\beta}{\beta-1}$

4.1.8: Mathematical treatment of perfectly inelastic collision

Suppose a body of mass m, moving with velocity u_1 to the right makes a perfectly inelastic collision with a body of mass m_2 moving with velocity u_2 in the same direction



By law of conservation

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v$$

$$v = \frac{m_1u_1 + m_2u_2}{m_1 + m_2}$$

Total kinetic energy before collision

k.e$_i$ $= \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2$

Total kinetic energy after collision

k.e$_f$ $= \frac{1}{2}(m_1 + m_2)v^2$

Loss in k.e= k.e$_i$ – k.e$_f$

$$= \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 - \frac{1}{2}(m_1 + m_2)v^2$$

Numerical examples

1. Ball P, Q and R of masses m_1, m_2 and m_3 lie on a smooth horizontal surface in a straight line. The balls are initially at rest. Ball P is projected with a velocity u_1 towards Q and makes an elastic collision with Q. if Q makes a perfectly in elastic collision with R, show that R moves with a velocity.

$$v_2 = \frac{2\,m_1m_2u_1}{(m_1 + m_2)(m_2 + m_3)}$$

Solution

Elastic collision of P and Q:

Conservation of momentum:

$m_1u_1 = m_1v_P + m_2v_Q$

$v_P = u_1 - \frac{m_2v_Q}{m_1}$.......(1)

Conservation of kinetic energy:

$\frac{1}{2}m_1u_1^2 = \frac{1}{2}m_1v_P^2 + \frac{1}{2}m_2v_Q^2$.....(2)

Putting [1] into [2

$m_1u_1^2 = m_1\left(u_1 - \frac{m_2v_Q}{m_1}\right)^2 + m_2v_Q^2$

$v_Q = \frac{2m_1u_1}{m_1+m_2}$......(3)

In elastic collision of Q and R:

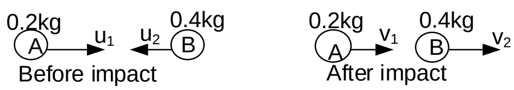
$m_2v_Q + m_3 0 = (m_2+m_3)v_2$

$m_2\frac{2m_1u_1}{m_1+m_2} = (m_2+m_3)v_2$

$v_2 = \frac{2m_1m_2u_1}{(m_1+m_2)(m_2+m_3)}$

2. A 0.2kg block moves to the right at a speed of $1ms^{-1}$ and meets a 0.4kg block moving to the left with a speed of $0.8ms^{-1}$. Find the find velocity of each block if the collision is elastic.

Solution



By law of conservation

$M_1U_1 + M_2U_2 = M_1V_1 + M_2V_2$

$(0.2x1) + (0.4x - 0.8) = 0.2v_1 + 0.4v_2$

$0.2 - 0.32 = 0.2v_1 + 0.4v_2$

$v_1 + 2v_2 = -0.6$---------------------- [1]

for elastic collision K.E is conserved

$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

$0.2x1^2 + 0.4x(-0.8)^2 = 0.2v_1^2 + 0.4v_2^2$

$0.2 + 0.256 = 0.2v_1^2 + 0.4v_2^2$

$v_1^2 + 2v_2^2 =$2.28 -----------------------[2]

But from [1] $v_1 = -0.6 - 2v_2$ put into (2)

$v_1^2 + 2v_2^2 =$2.28

$2v_2^2 + (0.6 - 2v_2)^2 = 2.28$

$6v_2^2 + 2.4v_2 - 1.92 = 0$

$v_2 =$0.4m/s , $v_2 = -$0.8m/s

$v_2 =$0.4m/s is correct since m_2 is in front it supposed to move faster

Therefore from (1)

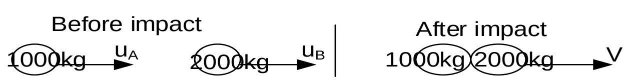
$v_1 + 2v_2 = -0.6$

$v_1 + 2x0.4 = -0.6$

$v_1 = -1.4m/s$

3. A truck of mass 1 tonne travelling at 4m/s collides with a truck of mass 2 tonnes moving at 3m/s in the same direction. If the collision is perfectly inelastic, calculate;
   (i) Common velocity
   (ii) Kinetic energy converted to other forms during collision

Solution



By law of conservation of momentum

$M_AU_A + M_BU_B = (M_A + M_B)V$

$(1000x4) + (2000x3) = (1000 + 2000)v$

$V = 3.3333ms^{-1}$

ii) Initial K.e $= \frac{1}{2}M_AU_A^2 + \frac{1}{2}M_BU_B^2$

$= ½\,x\,1000x\,4^2 + ½x2000x\,3^2 = 17000J$

Final k.e$_f = \frac{1}{2}(M_A + M_B)V^2$

$= ½(1000 + 2000)(3.3333)^2$

$= 16666.67J$

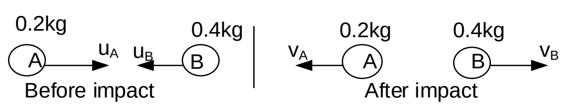
Kinetic energy converted = k.e$_i$ – k.e$_f$

$= 17000 - 16666.67$

$= 333.33Joules$

4. Two particles of masses 0.2kg and 0.4kg are approaching each other with velocities $4ms^{-1}$ and $3ms^{-1}$ respectively. On collision, the first particle reverses, its direction and moves with a velocity of $2.5ms^{-1}$. find the;
   (i) velocity of the second particle after collision
   (ii) percentage loss in kinetic energy

Solution



By law of conservation of momentum

$M_AU_A + M_BU_B = M_AV_A + M_BV_B$

$0.2x4 + 0.4x - 3 = 0.2x2.5 + 0.4V_B$

$V_B = 0.25m/s$

The velocity of the second particle is 0.25m/s in opposite direction

ii) Initial k.e $_i = \frac{1}{2}M_AU_A^2 + \frac{1}{2}M_BU_B^2$

$= ½\,(0.2x4^2 + 0.4x[-3]^2) = 3.4J$

Final K.e$_f = \frac{1}{2}M_AV_A^2 + \frac{1}{2}M_BV_B^2$

$= \frac{1}{2}x\,0.2x2.5^2 + \frac{1}{2}x0.4x0.25^2 = 0.6475J$

Loss in kinetic energy = k.e$_i$ – k.e$_f$

$= 3.4 - 0.6375 = 2.7625J$

% loss in k.e. $= \frac{loss\ of\ k.e}{\text{k.e}_\text{i}} x100\%$

$= \frac{2.7625}{3.4} x100\% = 81.25\%$

5. A bullet of mass $1.5\ x\ 10^{-2}kg$ is fired from a rifle of mass $2.7x10^{2}kg$ with a muzzle velocity of 100km/h. Find the recoil velocity of the rifle.

Solution

$V_b = \frac{100\ x\ 1000}{60x60} = 27.78m/s$

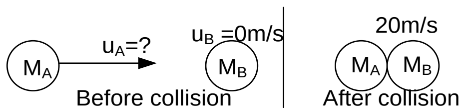
$M_g V_g = \text{M}_\text{b} V_b$

$3V_g = 1.5\ x\ 10^{-2}x27.78$

$V_g = 0.14m/s$

6. A bullet of mass 20g is fired into a block of wood of mass 400g lying on a smooth horizontal surface. If the bullet and the wood move together with the speed of 20m/s. Calculate
   (i) The speed with which the bullet hits the wood
   (ii) The kinetic energy lost

Solution



By the principle of conservation of momentum

$M_A U_A + M_B U_B = (M_A + M_B)V$

$(0.02xU_A) + (0.4x0) = (0.02 + 0.4)x20$

$u_A = 420m/s$

The original velocity of the bullet was *420m/s*

Initial K.e $= \frac{1}{2} M_A U_A^2 + \frac{1}{2} M_B U_B^2$

$= \frac{1}{2}\ x\ 0.02x\ 420^2 + \frac{1}{2} x0.4x0^2 = 1764J$

Final K.e$_f$ $= \frac{1}{2}(M_A + M_B)V^2$

$= {}^1/_2\, x(0.02 + 0.04)x(20)^2 = 84J$

Loss in kinetic energy = k.e$_i$ – k.e.$_f$

$= 1764 - 84 = 1680J$

7. A particle **P** of mass $m_1$moving at a speed u_1 collides head on with a stationery particle Q of mass m_2. the collision is perfectly elastic and the speeds of P and Q after impact are v_1 and v_2 respectively. Given that $\alpha = \frac{m_2}{m_1}$
   (i) Determine the value of α if $u_1 = 20v_2$
   (ii) Show that the fraction of energy lost by P is $\frac{4\alpha}{(1+\alpha)^2}$

Solution

(i) $m_1 u_1 = m_1 v_1 + m_2 v_2$

$m_1(u_1 - v_1) = m_2 v_2$

$(u_1 - v_1) = \alpha v_2$..........................(1)

$v_1 = u_1 - \alpha v_2$..........................(2)

½ $m_1 u_1^2$ =½ $m_1 {v_1}^2$ +½ $m_2 {v_2}^2$

$m_1(u_1^2 - v_1^2) = m_2({v_2}^2\ )$

$(u_1^2 - v_1^2) = \alpha v_2^2$----------------------[3]

equating [3] ÷ [1] : $\frac{\alpha(u_1^2 - v_1^2)}{\alpha(u_1 - v_1)} = \frac{\alpha v_2^2}{\alpha v_2}$

$\frac{(u_1 - v_1)\ (u_1 + v_1)}{(u_1 - v_1)} = \frac{v_2^2}{v_2}$

$(u_1 + v_1) = v_2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots ((4)$

Put (2) into (4)

$(u_1 + u_1 - \alpha v_2) = v_2$

$2u_1 =(1 + \alpha)\ v_2 \ldots\ldots\ldots\ldots\ldots\ldots ((5)$

but $u_1 = 20v_2$

$40v_2 = (1 + \alpha)\ v_2$

$\alpha = 39$

(iii) k.e of p before collision= ½ $m_1 u_1^2$

k.e of p after collision= ½ $m_1 v_1^2$

energy lost = ½$m_1 u_1^2$ – ½ $m_1 v_1^2$

fraction of energy lost $= \frac{½ m_1 u_1^2 - ½\, m_1 v_1^2}{½\, m_1 u_1^2}$

fraction of energy lost $= \frac{(u_1^2 - v_1^2)}{u_1^2} = \frac{(u_1 - v_1)\ (u_1 + v_1)}{u_1^2}$

from (i) above $(u_1 + v_1) = v_2$, $(u_1 - v_1) = \alpha v_2$

$u_1 = \frac{(1+\alpha)}{2}\ v_2$

fraction of energy lost $= \frac{(\alpha v_2)\ (v_2)}{\left[\frac{(1+\alpha)}{2} v_2\right]^2} = \frac{4\alpha}{(1+\alpha)^2}$

8. A body explodes and produces two fragments of masses $\boldsymbol{m}$ and $\boldsymbol{M}$. If the velocities of the fragments are $\boldsymbol{u}$ and $\boldsymbol{v}$ respectively, show that the ratio of kinetic energies of the fragments is

$$\frac{\boldsymbol{E_1}}{\boldsymbol{E_2}} = \frac{\boldsymbol{M}}{\boldsymbol{m}}$$

Where E_1 is the kinectic energy of m and E_2 is the kinectic energy of M

Solution

$$E_1 = \frac{1}{2}mu^2 \qquad and \qquad E_2 = \frac{1}{2}Mv^2$$

By law of conservation of linear momentum :

$$mu = -Mv$$

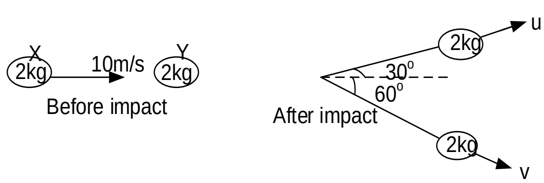
$$\therefore v = \frac{-mu}{M}$$

$$E_2 = \frac{1}{2}M\left(\frac{-mu}{M}\right)^2 = \frac{1}{2}\frac{m^2u^2}{M}$$

$$\frac{E_1}{E_2} = \frac{\left(\frac{1}{2}mu^2\right)}{\left(\frac{1}{2}\frac{m^2u^2}{M}\right)} = \frac{M}{m}$$

9. An object X of mass 2kg, moving with a velocity 10ms^{-1} collides with a stationary object Y of equal mass. After collision X moves with speed U at an angle of 30^o to its initial direction while Y moves with a speed of Y at an angle of 90^o to the new direction.
   (i) Calculate the speeds U and Y (05marks)
   (ii) Determine whether the collision is elastic or not. (03marks)

Solution



$$(\rightarrow): 2x10 = 2ucos30 + 2vcos60$$

$$20 = 2u\frac{\sqrt{3}}{2} + 2v\frac{1}{2}$$

$$v = 20 - u\sqrt{3} \ldots\ldots\ldots\ldots.. [1]$$

$$(\uparrow): 0 = 2usin30 - 2vsin60$$

$$2usin30 = 2vsin60$$

$$\frac{u}{2} = v\frac{\sqrt{3}}{2}$$

$u = v\sqrt{3}$.............[2]

Put into [1]: $v = 20 - \sqrt{3}\, v\sqrt{3}$

$$4v = 20$$

v =5ms^{-1}

$u = v\sqrt{3}. = 5\sqrt{3} = 8.66ms^{-1}$

i. Total K.E before collision

K.e= $\frac{1}{2}x2x10^2 = 100J$

Total K.e after collision

$= {}^1\!/_2\, x2x(5)^2 + 1/2x2x(5\sqrt{3})^2 = 100J$

Since kinetic energy is conserved then the collision is elastic

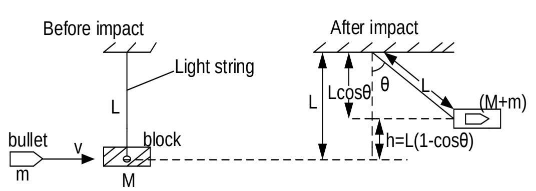
Exercise:9

1. A 4kg ball moving at 8m/s collides with a stationery ball of mass 12kg, and they stick together. Calculate the final velocity and the kinetic energy lost in impact **An [2m/s, 96J]**
2. A body of mass 6kg moving at 8ms^{-1} collides with a stationary body of mass 10kg and sticks to it. Find the sped of the composite body immediately after impact **An(3m/s)**
3. A bullet of mass 6g is fired from a gun of mass 0.50kg. if the muzzle velocity of the bullet is 300ms^{-1}, calculate the recoil velocity of the gun **An(3.6m/s)**
4. A body A of mass 4kg moves with a velocity of 2ms^{-1} and collides head on with another body, B of mass 3kg moving in the opposite direction at 5ms^{-1}. After the collision the bodies move off together with v. Calculate v **An(-1m/s)**
5. A mass A of 6kg moving a velocity of 5m/s collides with a mass B of mass 8kg moving in the opposite direction at 3m/s.
   (a) calculate the final velocity if the masses stick together on impact
   (b) If the masses do not stick together but mass A continues along the same direction with a velocity of 0.5m/s after impact. Calculate the velocity of B. **An (0.43m/s, 0.38m/s)**
6. A sphere of mass 3kg moving with velocity 4m/s collides head–on with a stationary sphere of mass 2kg and imparts to it a velocity of 4.5m/s. calculate the;
   (i) velocity of the 3kg sphere after the collision.
   (ii) amount of energy lost by the moving bodies in the collision **An (1m/s, 2.25J)**
7. A 2kg object moving with a velocity of 8m/s collides with a 3kg object moving with a velocity 6ms^{-1} along the same direction. If the collision is completely inelastic, calculate the decrease in kinetic energy collision. **An [2.4J]**
8. Two bodies A and B of mass 2kg and 4kg moving with velocities of 8m/s and 5m/s respectively collide and move on in the same direction. Object A's new velocity is 6m/s.

(i) Find the velocity of B after collision
(ii) Calculate the percentage loss in kinetic energy. **An(6m/s, 5.26%)**

9. A railway truck of mass 4×10^4 kg moving at a velocity of 3m/s collides with another truck of mass 2×10^4kg which is at rest. The coupling join and the trucks move off together
   (i) What fraction of the first trucks initial kinetic energy remains as kinetic energy of two trucks after collision **An [$^2/_3$]**
   (ii) Is energy conserved in a collision such as this, explain your answer
10. A particle of mass 2kg moving with speed $10ms^{-1}$ collides with a stationary particle of mass 7kg. Immediately after impact the particles move with the same speeds but in opposite directions. Find the loss in kinetic energy during collision. **An(28J)**
11. A 2kg object moving with a velocity of $6ms^{-1}$ collides with a stationary object of mass 1kg. If the collision is perfectly elastic, calculate the velocity of each object after collision. **An[$2ms^{-1}$, $8ms^{-1}$]**
12. A body of mass **m** makes a head on , perfectly elastic collision with a body of mass **M** initially at rest. Show that $\frac{\Delta E}{E_0} = \frac{4\left(\frac{M}{m}\right)}{\left(1+\frac{M}{m}\right)^2}$ where $E_0$is original kinetic energy of the mass **m** and ΔE the energy it loses in the collision
13. A metal sphere of mass m_1, moving at velocity u_1 collides with another sphere of mass $m_2$moving at velocity $u_2$in the same direction. After collision the spheres stick together and move off as one body. Show that the loss in kinetic energy E during collision is given by
$$\boldsymbol{E = \frac{\beta(u_1+u_2)^2}{2(m_1+m_2)}\ \ where\ \beta = m_1m_2}$$
14. A stationary radioactive nucleus disintegrates into an α −particle of relative atomic mass 4, and a residual nucleus of relative atomic mass 144. If the kinetic energy of the α −particle is 3.24×10^{-13}J, what is the kinetic energy of the residual nucleus **An(9×10^{-15}J)**
15. On a linear air-track the gliders float on a cushion of air and move with negligible friction. One such glider of mass 0.50kg is at rest on a level track. A student fires an air rifle pellet of mass 1.5×10^{-3}kg at the glider along the line of the track. The pellet embeds it's in the glider which recoil with a velocity of 0.33m/s. calculate the velocity ta which the pellet struck
An(1.1×10^2m/s)

4.1.9: BALLISTIC PENDULUM



Resolving along the vertical gives $Lcos\theta$
But $L = Lcos\theta + h$
$h = L - Lcos\theta = L(1-cos\theta)$
The device illustrates the laws of conservation of momentum and mechanical energy

a) **During impact**
- ❖ Mechanical energy is not conserved because of friction and other non conservative forces
- ❖ Linear momentum is conserved in the horizontal direction along which there is no external force

If V_c is the velocity of combined mass just after collision
$Mv + mx0 = (M + m)V_c$
$mv = (m+M)V_c$…………………………………(i)
The block was initially at rest.

b) **Swing after impact**
- ❖ Mechanical energy is conserved. The conserved gravitational force causes conversion of $k.e$ to $p.e$.
- ❖ Momentum is not conserved because an external resultant force (pull of the earth / weight) acts on the bullet-block system.

From (i) $k.e. = p.e.$

$$\frac{1}{2}(M+m)V_c^2 = (M+m)gh$$

$V_c^2 = 2gh$.........................(x)

But $h = L(1-cos\theta)$

$$V_c^2 = 2gL(1-cos\theta)$$

$\boldsymbol{V_C} = \sqrt{2gL(1-cos\theta)}$ ....(2)

θ is the angle of swing

Factors on which angle of swing depends

- The speed of the bullet
- The length of the string

NB; the angle can be obtained from

$h = L(1-cos\theta)$

$\frac{h}{L} = (1-cos\theta)$

$Cos\theta = \frac{L-h}{L}$

$\theta = Cos^{-1}\left(\frac{L-h}{L}\right)$

OR: $\boldsymbol{V_C} = \sqrt{2gL(1-cos\theta)}$

$\frac{v_c^2}{2gL} = (1-cos\theta)$

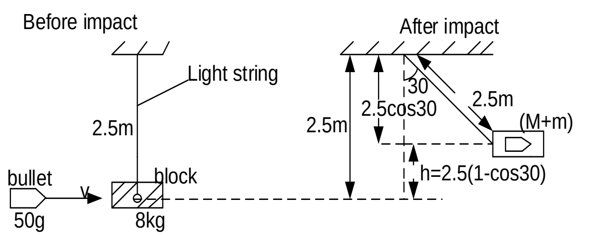
$Cos\theta = \left(\frac{2gL-v_c^2}{2gL}\right)$

$\theta = cos^{-1}\left(\frac{2gL-v_c^2}{2gL}\right)$

Examples

1. A bullet of mass 50g is fired horizontally into a block of wood of mass 8kg which is suspended by a string of length 2.5m. after collision the block swing upwards through an angle 30°. Calculate the velocity of the bullet assuming that it gets embedded in the block just after collision.

Solution



$h = L(1-cos\theta) = 2.5(1-\cos 30) = 0.335m$

Before impact (law of conservation of momentum)

$$mv + Mx0 = (M+m)V_C$$

$$\frac{50}{1000}v = \left(\frac{50}{1000}+8\right)V_C$$

$$0.05v = 8.05Vc$$

$$V_C = \frac{v}{161}$$

After impact (By conservation of mechanical energy)

$\frac{1}{2}(m+M)V_C^2 = (m+M)gh$

$\frac{1}{2}(8+0.05)V_C^2 = (0.05+8)\,x9.81x0.335$

$$V_C^2 = 6.5727$$

$$V_C = 2.564m/s$$

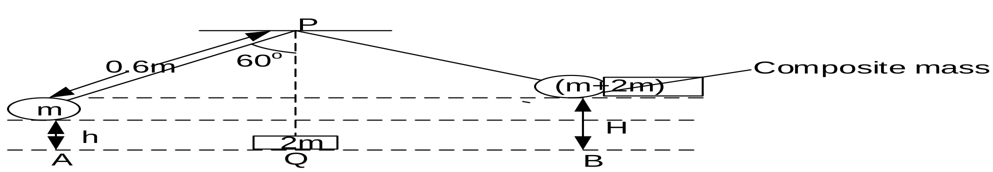
V_C is the velocity of bullet block system

But $V_C = \frac{v}{161}$

$V = 161V_C = 161x2.564 = 412.804m/s$

The velocity of the bullet is 412.804ms⁻¹

2. A steel ball of mass m is attached to an inelastic string of length 0.6m. The string is fixed to a point P so that the steel ball and the string can move in a vertical plane through P. The string is held out at an angle of 60° to the vertical and then released. At Q vertically below P, the wall makes a perfectly inelastic collision with the lump of plasticine of mass 2m so that the two bodies move together after collision



Calculate

(i) The velocity of the composite just after collision
(ii) The position of the composite mass with respect to point Q when the mass first comes to rest.
(iii) The composite mass now oscillates about the point Q, state two possible reasons why the composite mass finally comes to rest.

Solution

$h = L(1 - cos\theta) = 0.6(1 - cos60) = 0.3m$

Applying the law of conservation of energy at A

$$P.E = K.E$$

$$mgh = \frac{1}{2}mv^2$$

$$V = \sqrt{2gh} = \sqrt{2x9.81x0.3} = 2.43m/s$$

The velocity of mass m just before collision is 2.43m/s

Applying law of conservation of momentum at Q where collision occurs

i) $mv + 2mx0 = (m + 2m)Vc$

$2.43m = 3mV_C$

$V_c = 0.81ms^{-1}$

The velocity of the composite just after collision is $0.81ms^{-1}$

ii) Principle of mechanical energy at B

$K.E = P.E$

$\frac{1}{2}M_cV_c^2 = M_cgH$ but $M_c = (m + 2m)$

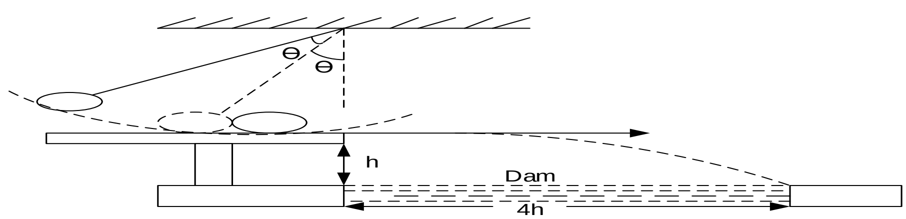
$H = \frac{1}{2}\frac{V_c^2}{g} = \frac{1}{2}x\frac{0.81^2}{9.81} = 0.33m$

iii) -Frictional force

-Air resistance

Exercise 10

1. A bullet of mass 40g is fired horizontally into freely suspended block of wood of mass 1.96kg attached at the end of an inelastic string of length 1.8m. given that the bullet gets embedded in the block and the string is deflected through an angle of 60° to the vertical . Find:
   (i) The initial velocity of the bullet **An[210m/s]**
   (ii) The maximum velocity of the block **An[42m/s]**
2. A bullet of mass 20g travelling horizontally at $100ms^{-1}$ embedded itself in the centre of a block of wood of mass 1kg which is suspended by a light vertical string 1m in length. Calculate the maximum inclination of the string to the vertical . **An(36.1°)**
3. A bullet of mass 50g travelling horizontally at $600ms^{-1}$ strikes a block of wood of mass 2kg which is suspended by a light vertical string so that its free to swing. The penetrates the block completely and emerges on the other side travelling at $400ms^{-1}$ in the same direction. As a result the block swings such that the string makes an angle of 25° with the horizontal. Calculate the lenght of the string. **An(1.719m)**
4. A block of wood of mass 1.00kg is suspended freely by a thread. A bullet of mass 10g is fired horizontally at the block and becomes embedded in it. The block swings to one suede rising a vertical distance of 50cm. with what speed did the bullet hit the block **An[319.4m/s]**
5. A circular ring is tied to a roof using a string of length, l and displaced such that it makes an angle of 2θ with the vertical, where $\theta = 30°$. It is then released to throw a speherical ball horizontally across the dam at a hegth h. It collides in elastically with the ball when at angle θ and move together until the ball leaves the bench horizontally to cross the dam of width 4h.



if the bench is frictionless and the masses are equal, show that $h = \frac{l(\sqrt{3}-1)}{32}$. Hence if $l = 128cm$ find the velociy with whoich the ball hits the ground

UNEB 2017 NO.1

(a) (i) State Newton's laws of motion (03marks)

(ii) A molecule of gas contained in a cube of side l strikes the wall of the cube repeatedly with a velocity u. Show that the average force F on the wall is given by $F = \frac{mu^2}{l}$ where m is the mass of the molecule (04marks)

(b) (i) Define the **linear momentum** and state the **law of conservation of linear momentum**. (02marks)

(ii) A body of mass m_1 moving with a velocity u, collides with another body of mass m_2 at rest. If they stick together after collision, find the common velocity with which they will move (04marks)

UNEB 2016 No 2

(a) (i) What is meant by **efficiency** of **a machine**. (01mark)

(ii) A car of mass $1.2x10^3kg.$ moves up an incline at a steady velocity of 15ms^{-1} agaisnt a frictional force of $6.0x10^3N$. The incline is such that the car rises 1.0m for every 10m along the incline. Calculate the out put power of the car engine. **An**$(1.077x10^5W)$ (04marks)

(b) (i) Define the **impulse** and **momentum**. (02marks)

(ii) An engine pumps water such that the velocity of the water leaving the nozzle is 15ms^{-1}. If the water jet is directed perpendicularly onto a wall and comes to a stop at the wall, calculate the pressure exerted on the wall **An**$(2.25x10^5Nm^{-2})$ (04marks)

(c) (i) Define **inertia** (01mark)

(ii) Explain why a body placed on a rough plane will slide when the angle of inclination is increase.

(d) (i) State the conditions for a body to be in equilibrium under action of coplanar forces. (02marks)

(ii) Briefly explain the three states of equilibrium. (03marks)

UNEB 2013 No 3(a)

(I) State the law of conservation of linear momentum (01mark)

(II) A body explodes and produces two fragments of masses m and M. If the velocities of the fragments are u and v respectively, show that the ratio of kinetic energies of the fragments is

$$\frac{E_1}{E_2} = \frac{M}{m}$$

Where E_1 is the kinectic energy of m and E_2 is the kinectic energy of M (04marks)

UNEB 2011 NO.2

(a) State Newton's laws of motion (03marks)

(b) Use Newton's laws of motion to show that when two bodies collide their momentum is conserved (04marks)

(c) Two balls P and Q travelling in the same line in opposite directions with speeds of 6ms^{-1} and 15ms^{-1} respectively make a perfect inelastic collision. If the masses of P and Q are 8kg and 5kg respectively, find the

(i) The velocity of P (04marks)

(ii) Change in kinetic energy **An[v=2.08ms^{-1},278.38J]** (04marks)

(d) (i) what is an impulse of a force (01marks)

(ii)Explain why a long jumper should normally land on sand. (04marks)

UNEB 2010 NO.1

(a) i) State the law of conservation of linear momentum (01mark)

ii)Use Newton's laws to derive the a(i) (04marks)

(b) Distinguish between elastic and inelastic collision (01mark)

(c) An object X of mass M, moving with a velocity 10ms^{-1} collides with a stationary object Y of equal mass. After collision X moves with speed U at an angle of 30^o to its initial direction while Y moves with a speed of Y at an angle of 90^o to the new direction.

(iii) Calculate the speeds U and Y **An**$(v = 5\text{ms}^{-1}\ u = 8.66\text{ms}^{-1})$ (05marks)

(iv) Determine whether the collision is elastic or not. $An(50mJ)$ (03marks)

UNEB 2009 NO.1

a) i) Define the term impulse (01mark)

ii) State Newton's laws of motion (03marks)

b) A bullet of mass 10g travelling horizontally at a speed of 100ms^{-1} strikes a block of wood of mass 900g suspended by a light vertical string and is embedded in the block which subsequently swings freely. Find the;

(i) Vertical height through which the block rises (04marks)

(ii) Kinetic energy lost by the bullet (03marks)

[Hint k.e. lost $=\frac{1}{2}m_b u_b^2 - \frac{1}{2}m_b V_C^2$]

Where V_C is velocity of combined system.

m_b -is mass of the bullet

u_b is initial velocity of the bullet **An(6.2×10^{-2}m , 49.99J)**

UNEB 2008 NO 4

a) State
   (i) Newton's laws of motion (03 marks)
   (ii) The principle of conservation of momentum (01 mark)

b) A body A of mass M_1 moves with velocity U_1 and collides head on elasticity with another body B of mass M_2 which is at rest. If the velocities of A and B are V_1 and V_2 respectively and given that $x=\frac{m_1}{m_2}$ Show that;
   i) $\frac{u_1}{v_1} = \frac{x+1}{x-1}$ (04 marks)
   ii) $\frac{v_2}{v_1} = \frac{2x}{x-1}$ (03 marks)

c) Distinguish between conservative and non conservative forces (02 marks)

d) A bullet of mass 40g is fired from a gun at $200ms^{-1}$ and hits a block of wood of mass 2kg which is suspended by a light vertical string 2m long. If the bullet gets embedded in the wooden block
   (i) Calculate the maximum angle the string makes with the vertical (06 marks)
   (ii) State factors on which the angle of swing depends **An (53.4^o)** (01 mark)

UNEB 2006 No 2(c)

(i) State the work - energy theorem (01 mark)

(ii) A bullet of mass 0.1kg moving horizontally with a speed of $420ms^{-1}$ strikes a block of mass 2.0kg at rest on a smooth table becomes embedded in it. Find the kinetic energy lost if they move together.
An[8400J] (04 marks)

UNEB 2005

C i) Define linear momentum (01 mark)

i) State the law of conservation of linear momentum (01 mark)

ii) Show that the law in c(ii) above follows Newton's law of motion (03 marks)

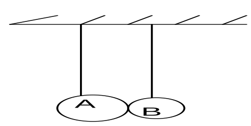
iii) Explain why, when catching a fast moving ball, hands are drawn back while the ball is being brought to rest. (02 marks)

d). A car of mass 1000kg travelling at uniform velocity of $20ms^{-1}$, collides perfectly inelastically with a stationary car of mass 1500kg, calculate the loss in kinetic energy of the car as a result of collision
An[1.68×10^5J) (04 marks)

UNEB 2001 No 1

c)State the conditions under which the following will be conserved in a collision between two bodies.
   i) Linear momentum [01mark]
   ii) Kinetic energy [01mark]

d] Two pendula of equal length L have bobs A and B of masses 3m and m respectively the pendulum are lung with bobs in contact as shown.

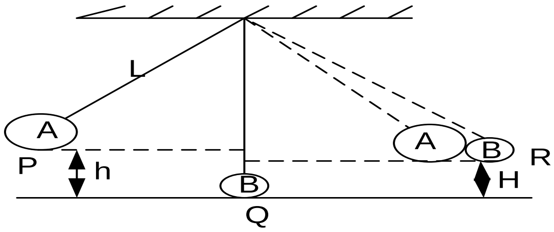


The bob A is displaced such hat the string makes an angle θ with the vertical and released. If A makes a perfectly inelastic collision with B, find the height to which B rises [08marks]

Solution

i) Linear momentum is conserved if there is no external resultant acting on the colliding bodies.

ii) Total kinetic energy is conserved if the collision is perfectly elastic i.e the bodies separate after collision
   d]



At P: $h = L(1 - cos\theta)$

$P.e = K.e$ by conservation of energy

$3mgh = \frac{1}{2}3mv^2$

Where v is the velocity with which A is released

$3mgh = \frac{1}{2}3mv^2$

$gh = \frac{v^2}{2}$

$$gL(1 - cos\theta) = \frac{v^2}{2}$$

$v^2 = 2gL(1 - cos\theta)$

$v = \sqrt{2gL(1 - Cos\theta)}$ ----------------- [1]

At Q: Momentum is conserved

$3mv + mx0 = (3m + m)Vc$

Where Vc is the velocity of the combination

$3mv = 4mVc$

$3v = 4Vc$

$3\sqrt{2gL(1 - Cos\theta)} = 4Vc$

$Vc = \frac{3}{4}\sqrt{2gL(1 - Cos\theta)}$----------------[2]

At R : mechanical energy is conserved

$½\ (3m + m)V_C^2 = (3m + m)gH$

$H = \frac{V_C^2}{2g} = \frac{\left(\frac{3}{4}\sqrt{2gL(1-Cos\theta)}\right)^2}{2g} = \frac{9gL(1-Cos\theta)}{16}$

B rises $\frac{9gL(1-Cos\theta)}{16}$

UNEB 2000 No 1

a) i)State Newton's laws of motion [03marks]

ii) Define impulse and derive its relation to linear momentum of the body on which it acts. [03marks]

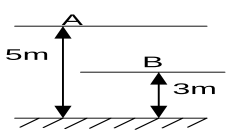
c) A ball of mass 0.5kg is allowed to drop from rest from a point at a distance of 5.0m above the horizontal concrete floor. When the ball first hits the floor, it rebounds to a height of 3.0m.

i) What is the speed of the ball just after the first collision with the floor [04marks]

ii) if the collision last 0.01s, find the average force which the floor exerts on the ball [05marks]

Solution

c)



i) By law of conservation of energy

k.e after collision =p.e at height of 3m

$½\, mv^2 = mgh$

$v^2 = 2gh$

$v = \sqrt{2x9.81x3} = 7.67m/s$

Where v is the velocity with which it rebounds from the floor .

ii) Force $= \frac{change\ in\ momentun}{time}$

k.e on hitting floor =p.e at height of 5m

$mgh =$ ½ mu^2

$u = \sqrt{2gh} = \sqrt{2x9.81x5}$ =**9.9ms^{-1}**

Since velocity is a vector quantity

$v = -$**7.67** since it rebounds (moves in opposite direction)

$F = \frac{mv-mu}{t} = \frac{(0.5x9.9)-(0.5x-7.67)}{0.01} = 878.5N$

UNEB 1997 No 2

a) Define the terms momentum [01marks]

b) A bullet of mass 300g travelling at a speed of 8ms^{-1} hits a body of mass 450g moving in the same direction as the bullet at 15ms^{-1}. The bullet and body move together after collision. Find the loss in kinetic energy [06marks]

c) i) State the work energy theorem [01mark]

ii)A ball of mass 500g travelling at a speed of 10ms^{-1} at 60^0 to the horizontal strikes a vertical wall and rebounds with the same speed at 120^0 from the original direction. If the ball is in contact with the wall for 8x10^{-3}, calculate the average force exerted by the ball.

Ans[625N] [06marks]

FORCE

Force is anything which changes a body's state of rest or uniform motion in a straight line
The unit of force is **a newton**
Definition: A Newton is a force which gives a body of mass 1kg an acceleration of $1ms^{-2}$

CONSERVATIVE AND NON CONSERVATIVE FORCES

1. **A conservative force** is a force for which the work done in moving a body around a closed path is zero.

Examples of conservative forces

- Gravitational force
- Elastic force
- Electric force
- Magnetic force

2. **A non-conservative force** is a force for which the work done in moving a body around a closed is not zero.

Examples of non- conservative force

- Frictional force
- Air resistance
- Viscous drag

Differences between conservative forces and non- conservative forces

Conservative forces	Non-conservative forces
Work done around a closed path is zero	Work done around a closed path is not zero
Work done to move a body from one point to another is independent on the path taken	Work done to move a body from one point to another is dependent on the path taken
Mechanical energy is conserved	Mechanical energy is not conserved

4.2.0: SOLID FRICTION

Friction is the force that opposes relative motion of two surfaces in contact.

4.2.1: Types of friction

1. Static friction

It's a force that opposes the tendency of a body to slide over another.
Limiting friction is the maximum frictional force between two surfaces in contact when relative motion is just starting.

2. Kinetic/sliding/dynamic friction

It's the force that opposes relative motion before two surfaces which are already in motion.

4.2.2: Laws of friction

1st law : Frictional forces between two surfaces in contact oppose their relative motion.
2nd law : Frictional forces are independent of the area of contact of the surfaces provided that normal reaction is constant.
3rd law : The limiting frictional force is directly proportional to the normal reaction but independent of relative velocity of surfaces.

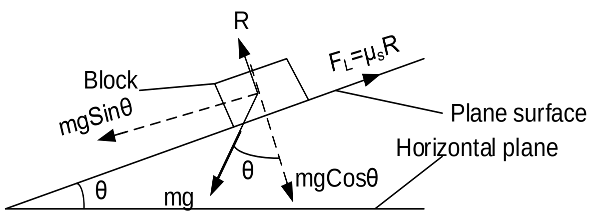
4.2.3: Molecular explanation for occurrence of friction

- Surfaces have very small projections and when placed together the actual area of contact of two surfaces is very small, hence the pressure at points of contact is very high. Projections merge to produce welding and the weldings have to be broken for relative motion to occur. This explains the fact that friction opposes relative motion between surfaces in contact

- When the area between the surfaces is changed, the actual area of contact remains constant. Therefore no change in friction. This explains the fact that friction is independent of the area of contact provided normal reaction is constant
- Increasing normal reaction, increases the pressure at the welds. This increases the actual area of contact to support the bigger load, and hence a greater limiting frictional force . Therefore friction is proportional to normal reaction.

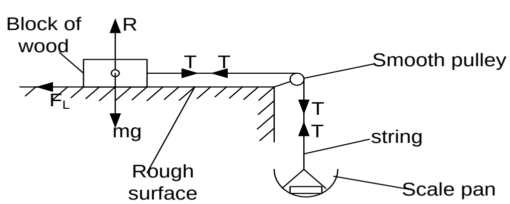
4.2.4: MEASUREMENT OF COEFFICIENT OF STATIC FRICTION

Method I



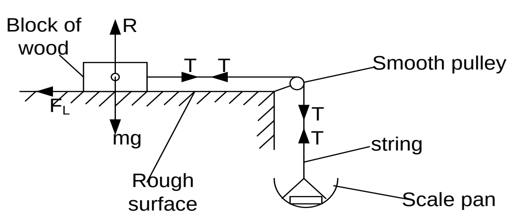
- Place a block on a horizontal plane.
- tilt the plane gently, until it **just begins** to slide.
- Measure and record the angle of tilt θ
- $\mu_s = tan\theta$
- Repeat the experiment with blocks of different masses
- Find the average value of μ_s

Method 2



- The mass m of the wooden block is determined and placed on a horizontal plane surface.
- A string is attached to the block and passed over a smooth pulley carrying a scale pan at the other end.
- Small masses are added to the scale pan one at a time, till the block just slides
- The total mass M of the scale pan and the masses added is obtained.
- Coefficient of static friction $\boldsymbol{\mu} = \frac{\boldsymbol{m}}{\boldsymbol{M}}$

4.2.5: Measurement of coefficient of kinetic friction

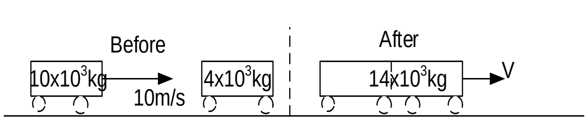


- The mass m of the wooden block is determined and placed on a horizontal plane surface.
- A string is attached to the block and passed over a smooth pulley carrying a scale pan at the other end.
- Small masses are added to the scale pan one at a time, till the block moves with a uniform speed
- The total mass M of the scale pan and the masses added is obtained.
- Coefficient of kinetic friction $\boldsymbol{\mu} = \frac{\boldsymbol{m}}{\boldsymbol{M}}$

EXAMPLES

1. A truck of mass 10 tones moving at 10ms^{-1} draws into a stationary truck of mass 4 tones. They stick together and skid to a stop a long a horizontal surface. Calculate the distance through which the trucks skid, if the coefficient of kinetic friction is 0.25.

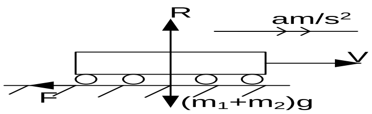
Solution



By law of conservation of momentum

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v$$
$$10^4x10 + (\ 4x10^3x0) \ = [10^4 + 4x10^3]v$$
$$v = 7.143ms^{-1}$$

When they skid to a s top, they experience a friction force



$F = \boldsymbol{\mu}R$ but $R = (m_1 + m_2)g$

$F = \boldsymbol{\mu}(m_1 + m_2) = 0.25(104 + 4x103)x9.81$

$Frictional\ force\ = 34335N$

Frictional force is the only resultant forces, therefore from Newton's 2nd law of motion

$34335 = \ (m_1 + m_2)A$

$34335 = (10^4 + 4x10^3)a$

$a\ \approx\ 2.453ms^{-2}$

The trucks come to a stop then

$a = -2.453ms^{-2}$(a deceleration)

To get the distance the trucks come to rest

u=7.143ms^{-1} v = 0m/s, a =2.453ms^{-2}

$v^2 = u^2 + 2as$

$0^2 = 7.143^2x2x - 2.453s$

$S\ = 10.4m$

Alternatively : Work done against friction=loss in k.e

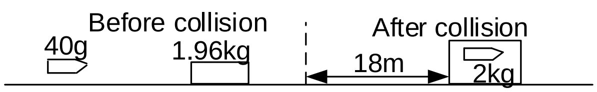
$$\boldsymbol{\mu}(m_1 + m_2)gxs = \frac{1}{2}(m_1 + m_2)v^2$$

$$0.25x9.81xs = \frac{1}{2}x(7.143)^2$$

$s = 10.4m$

2. A 40g bullet strikes a 1.96 kg block of wooden placed on a horizontal surface just in front of the gun. The coefficient of kinetic friction between the block and the surface is 0.28. If the impact drives the block a distance of 18.0m before it comes to rest, what was the muzzle speed of the bullet

Solution



During impact: $m_1u_1 + m_2u_2 = (m_1 + m_2)v$

$0.04u\ = 2v \ldots\ldots\ldots\ldots\ldots i$

$F = \boldsymbol{\mu}R$ but $R = (m_1 + m_2)g$

$F = \boldsymbol{\mu}(m_1 + m_2)g = 0.28(2)x9.81$

$Frictional\ force\ = 5.4936N$

Frictional force is the only resultant forces, therefore from $F = ma$

$5.4936 = \ (m_1 + m_2)a$

$5.4936 = (2)a$

$a\ \approx\ 2.7468ms^{-2}$

The trucks come to a stop then

$a = -2.7468ms^{-2}$ (a deceleration)

u=7.143ms^{-1} v = 0m/s, a =2.7468ms^{-2}

$v^2 = u^2 + 2as$

$0^2 = v^2 - 2.7468x18$

$v = 9.94ms^{-1}$

$0.04u\ = 2v$

$0.04u\ = 2x9.94$

$u = 497ms^{-1}$

Alternatively

Work done against friction=loss in k.e

$$\boldsymbol{\mu}(m_1 + m_2)gxs = \frac{1}{2}(m_1 + m_2)v^2$$

$$0.28x9.81x18 = \frac{1}{2}x(v)^2$$

$v = 9.94ms^{-1}$

Put into $0.04u\ = 2v$

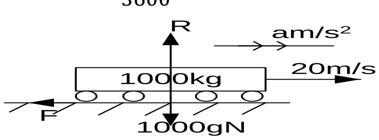
$0.04u\ = 2x9.94$

$u = 497ms^{-1}$

3. A car of mass 1000kg moving along a straight road with a speed of 72kmh^{-1} is brought to rest by a speedy application of brakes in a distance of 50m. Find the coefficient of kinetic friction between the tyres and the road.

Solution

$u = \frac{72x1000}{3600} = 20m/s$



$F = \mu R$ But $R = mg = 1000x9.81 =\ 9810N$

$F =\ 9810\ \mu$ --------- [1]

$F = ma$ --------------- [2]

To get the distance the car comes to rest

u=20m/s , v=0m/s, s=50m

$v^2 = u^2 + 2as$

$0 = 20^2\ + 2ax50$

$a = -4ms^{-2}$

$F = 1000x - 4 = -4000N$

Frictional force =4000N

$F = 9810\ \mu$

$4000 = 9810\ \mu$

$\mu = 0.41$

Coefficient of friction $= 0.41$

Alternatively

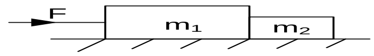
Work done against friction=loss in k.e

$$\boldsymbol{\mu}(m)gxs = \frac{1}{2}(m)v^2$$

$$\boldsymbol{\mu}x9.81x50 = \frac{1}{2}x(20)^2$$

$\mu = 0.408$

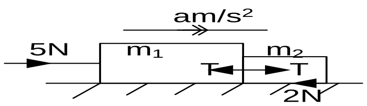
4. Two blocks of masses m_1=3kg and m_2=2kg are in contact on a horizontal table. A constant horizontal force F=5N is applied to the block of mass m_1 in the direction shown



There is a constant frictional force of 2N between the table and the block of mass m_2 but no frictional force between the table and the block of mass m_1. Find:

i) The acceleration of the two blocks
ii) The force of contact between the blocks

Solution



By Newton's 2nd law

For block m_1, $5 - T = 3a$ ---------- [1]

For block m_2 : $T - 2 = 2a$----------- [2]

Adding 1 and 2: $3 = 5a$

$a = 0.6ms^{-2}$

but from 2: $T - 2 = 2a$

$T = 2x0.6 + 2 = 3.2N$

Acceleration of two blocks =$0.6ms^{-2}$

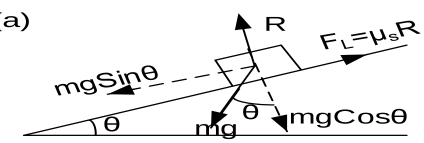
Force of contact= $3.2N$

5. A block of wood of mass 150g rests on an inclined plane. If the coefficient of static friction between the surface of contact is 0.3. find;

a) The greatest angle to which the plane may be tilted without the block slipping
b) The force parallel to the plane necessary to prevent slipping when the angle of the plane to the horizontal is 30°.

Solution

(a)

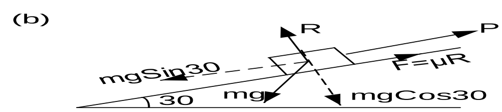


For the block not to slip then it experiences limiting friction

For limiting friction μ =tanθ

$\theta = tan^{-1}(\mu) = tan^{-1}(0.3) \ = 16.7°$

(b)



Using $F = ma$

$P \ + \mu\, R - mgsin30 = ma$

($a = 0$) no motion but $R = mgcos30$

$P + 0.3x\dfrac{150}{1000}x9.81cos30 = \dfrac{150}{1000}x9.81sin30$

P= $(\frac{150}{1000}x9.81sin30 - 0.3x\frac{150}{1000}x9.81cos30)$

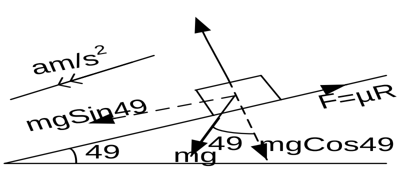
$P = 0.353N$

6. A car of mass 500kg moves from rest with the engine switched off down a road which is inclined at an angle 49° to the horizontal

a)Calculate the normal reaction

b)If the coefficient of friction between the tyres and surface of the road is 0.32. Find the acceleration of the car

Solution



a) $R = mgcos49$

R= $500x9.81cos49 = 3217.97N$

b) Using $F = ma$

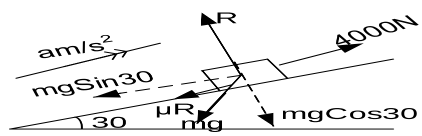
$mgsin49 - \mu R \ = ma$

$500x9.81sin49 - 0.32x3217.97 = 500a$

$a = 5.34ms^{-2}$

7. A car of mass 1000kg climbs a truck which is inclined at 30° to the horizontal. The speed of the car at the bottom of the incline is 36kmh⁻¹. If the coefficient of kinetic friction is 0.3 and engine exerts a force of 4000N how far up the incline does the car move in 5s?

Solution



u =36kmh^{-1} $= \frac{36x\ 1000}{3600} = 10ms^{-1}$

Using $F = ma$

$4000 - (mgsin30 + \mu R) = ma$

$4000 - (1000x9.81sin30 + 0.3mgcos30)$=$1000a$

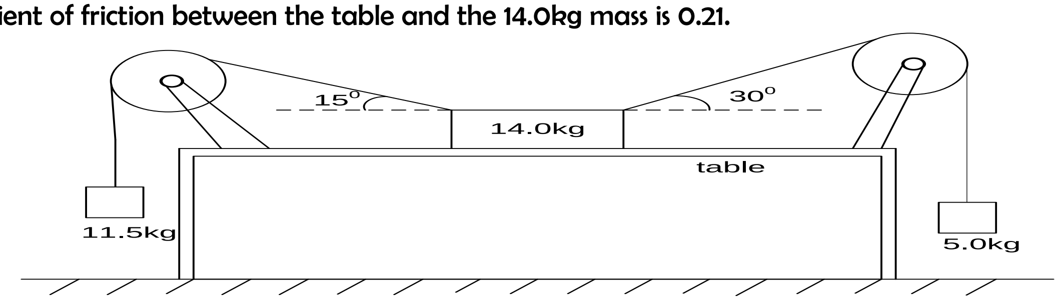
$a = -3.45\ ms^{-2}$

$S = ut + ½\ at^2$

$S = 10x5 + \frac{1}{2}x - 3.45x5^2$

$S = 6.9m$

8. The below shows three masses connected by inextensible strings which pass over smooth pulleys. The coefficient of friction between the table and the 14.0kg mass is 0.21.



If the system is released from rest, determine the

(i) Acceleration of the 14.0kg mass **An**(1.67ms^{-1})

(ii) Tension in each string **An(93.6N, 57.4N)**

Exercise:11

1. A particle of weight 4.9N resting on a rough inclined plane of angle equal to tan^{-1}(5/12) is acted upon by a horizontal force of 8N. If the particle is on the point of moving up the plane, find coefficient of friction between the particle and the plane. **An (**μ **= 0.72)**
2. A box of mass 2kg rests on a rough inclined plane of angle 25°. The coefficient of friction between the box and the plane is 0.4.Find the least force applied parallel to the plane which would move the box up the plane. **An[15.39N]**
3. A particle of mass 0.5kg is released from rest and slides down a rough plane inclined at 30° to the horizontal. It takes 6 seconds to go 3 meter.
   i. Find the coefficient of friction between the particle and the plane
   ii. What minimum horizontal force is needed to prevent the particle from moving?

   An[0.56, 0.086N]
4. A parcel of mass 2kg is placed on a rough plane inclined at 45° to the horizontal, the coefficient of friction between the parcel and the plane is 0.25. Find the force that must be applied to the plane so that the parcel is just.
   i. Prevented from sliding down the plane
   ii. On the point of moving up the plane. **An[10.39N, 17.32N]**

CHAPTER 5: WORK, ENERGY AND POWER

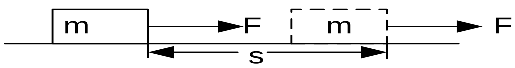
5.1.0: Work

5.1.1: Work done by a constant force

Work is said to be done when energy is transferred from one system to another

Case 1

When a block of mass m rests on a smooth horizontal



When a constant force F acts on the block and displaces it by x, then the work done by F is given by

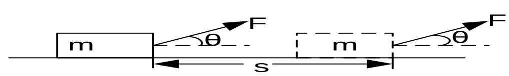
$$\boldsymbol{W = Fs}$$

Definition

Work is defined as the product of force and distance moved in the direction of the force

Case II

If the force does not act in the direction in which motion occurs but at an angle to the it as shown below



$$\boldsymbol{W = (F cos\theta)s}$$

Definition

Work done is also defined as the product of the component of the force in the direction of motion and displacement in that direction

Note

1. Work done either can be positive or negative. If it is positive, then the force acts in the same direction of the displacement but negative if it acts oppositely.
   The work done by friction when it opposes one body sliding over it is negative.
2. Work and energy are scalar quantities and their S.I unit is Joules

Definition

A joule is the work done when a force of 1N causes a displacement of 1m in the direction of motion

Dimension of work

$W = Fs$

$[W] = [F]\,[s]$

$= MLT^{-2}L$

[W] = ML^2T^{-2}

Explain why it is easier to walk on a straight road than an inclined road up hill.

When walking on a level ground, work is done only against the frictional force. While when walking up hill, work is done against both frictional force and the component of the weight of the person along the plane of the hill.

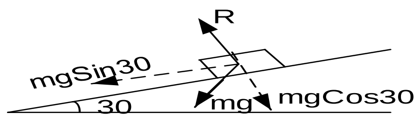
Explain whether a person carrying a bucket of water does any work on the bucket while walking on a level road

There is no net force on the bucket in the horizontal direction. The only force he exerts on the bucket is against the weight of the bucket and this force is perpendicular to the direction of motion. Therefore work done is $W = Fcos\theta = FCO90 = 0$. Hence the man does no work on the bucket

Examples

1. A block of mass 5kg is released from rest on a smooth plane inclined at an angle of 30° to the horizontal and slides through 10m. Find the work done by the gravitation force.

Solution



Work done by gravitational force

$W = mgsin30xd$

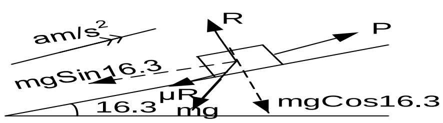
$W = 10x5x9.81sin30 = 245.25J$

2. A rough surface is inclined at $\tan^{-1}\left(\frac{7}{24}\right)$ to the horizontal. A body of mass 5kg lies on the surface and is pulled at a uniform speed a distance of 75cm up the surface by a force acting along a line of greatest slope. The coefficient of friction between the body and the surface is $\frac{5}{12}$. Find;

   a) Work done against gravity
   b) Work done against friction

Solution

$\theta = \tan^{-1}\left(\frac{7}{24}\right) = 16.3°$



a) Work done against friction

$W = \mu Rd \quad But \;\; R = mgcos\theta$

$W = \mu mgcos\theta d$

$$W = \frac{5}{12}x5x\frac{75}{100}x9.81cos16.3 = 14.71J$$

b) Work done against gravity

$W = mgsin\,\theta xd$

$$W = 5x9.81sin16.3x\frac{75}{100} = 10.35J$$

5.2.0 : ENERGY

This is the ability to do work.

When an interchange of energy occurs between two bodies, we can take the work done as measuring the quantity of energy transferred between them.

THE PRINCIPLE OF CONSERVATION OF ENERGY

It states that energy is neither created nor destroyed but changes from one form to another

5.2.1: KINETIC ENERGY

Kinetic energy is the energy possessed by a body due to its motion.

Formulae of kinetic energy

Consider a body of mass m accelerated from rest by a constant force, F so that in a distance, s it gains velocity, v

Then $v^2 = u^2 + 2as$ but $(u = 0)$

$a = \frac{v^2}{2s}$

$F = ma = \frac{mv^2}{2s}$

work done $= FxS = \frac{mv^2}{2s}s$

$w = \frac{mv^2}{2}$

by law of conservation of energy

$work\ done\ = k.e\ gained$

$$\boxed{k.e = \frac{1}{2}mv^2}$$

5.2.2: WORK-ENERGY THEOREM

It states that the work done by the net force acting on a body is equal to the change in its kinetic energy.

WORK-ENERGY THEOREM FORMULAR

Consider a body of mass m accelerated from u by a constant force F so that in a distance s it gains velocity v

$v^2 = u^2 + 2as$

$a = \frac{v^2-u^2}{2s}$ ----------------------- [1]

resultant $force\ F = ma = \frac{m(v^2-u^2)}{2s}$

But work done $= Fxs = \frac{m(v^2-u^2)}{2s}s$

$W = \frac{m(v^2-u^2)}{2}$

$$\boxed{W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2}$$

This is the work-energy theorem.

Examples

1. A car mass 1000kg moving at 50ms^{-1} skid to rest in 4s under a constant retardation. Calculate the magnitude of the work done by the force of friction

Solution

a) Using $v = u + at$

$0 = 50 + 4a$

$a = -12.5m/s^2$

Frictional force $= ma$

$= 1000x - 12. = 12500N$

$S = ut + ½\, at^2$

$S = 50x4 + ½\, x - 12.5x4^2$

$S = 100m$

W$= FxS = 12500x100$

Work done $= 1.25x10^6J$

Alternatively

$W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

W$= \frac{1}{2}x1000x50^2 - \frac{1}{2}\,x1000x0^2$

Work done $= 1.25x10^6J$

2. A bullet travelling at 150ms^{-1} will penetrate 8cm into a fixed block of wood before coming to rest. Find the velocity of the bullet when it has penetrated 4cm of the block.

Solution

Loss in k.e energy=work done against resistance

$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = w$

$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = FxS$

$\frac{1}{2}mx0^2 - \frac{1}{2}mx150^2 = maxs$

$-\frac{1}{2}mx150^2 = max\frac{8}{100}$

$a = -140625$ms^{-2}

Using $v^2 = u^2 + 2as$

$v^2 = 150^2 + 2x(-140625)x\frac{4}{100}$

$v = 106.06ms^{-1}$

3. A constant force pushes a mass of 4kg in a straight line across a smooth horizontal surface. The body passes through a point A with a speed of 5m/s and then through a point B with a speed of 8m/s. B is 6m form A. Find the magnitude of the force acting on the mass.

Solution

$v^2 = u^2 + 2as$

$a = \frac{8^2 - 5^2}{2x6} = 3.25ms^{-2}$

$F = ma = 4x3.25 = 13N$

OR $W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

$Fx6 = \frac{1}{2}x4x(8^2 - 5^2)$

$F = 13N$

4. A body of mass 5kg slides over a rough horizontal surface. In sliding 5m, the speed of the body decrease from 8m/s to 6m/s, find

(i) Frictional force

(j) Coefficient of friction

Solution

$W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

$Fx5 = \frac{1}{2}x5x(8^2 - 6^2)$

$F = 14N$

$F = \mu R$

$\mu = \frac{14}{5x9.81} = 0.286$

Alternatively $v^2 = u^2 + 2as$

$a = \frac{6^2 - 8^2}{2x5} = -2.8ms^{-2}$

$F = ma = 5x2.8 = 14N$

5. A bullet of mass 15g is fired towards a fixed wooden block and enters the block when travelling horizontally at 400m/s. It comes to rest after penetrating a distance of 25cm. find the

(i) work done against resistance of the wood

(ii) Magnitude of the resistance

Solution

(i) $W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

$W = \frac{1}{2}x0.015x(400^2 - 0^2) = 1200J$

(ii) $W = FxS$

$1200 = Fx0.25$

$F = 4800N$

6. A particle of mass 2kg is released from rest and falls freely under gravity. Find its speed when it has fallen a distance of 10m

Solution

$W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

$2x9.8x10 = \frac{1}{2}x2x(v^2 - 0^2)$

$v = 14m/s$

7. A particle of mass 5kg falls vertically against a constant resistance. The particle passes through two points A and B 2.5m apart with A above B. Its speed is 2m/s when passing through A and 6m/s when passing. through B. Find the magnitude of the resistance

Solution

$W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

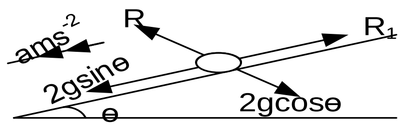
$(5g - R)x2.5 = \frac{1}{2}x5x(6^2 - 2^2)$

$R = 17N$

Incline planes

1. A rough slope of length 5m is inclined at angle of 30° to the horizontal. A body of mas 2kg is released from rest at the top of the slope and travels down the slope against a constant resistance. The body reaches the bottom of the slope with speed of 2m/s, find the magnitude of the resistance

Solution



$W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

$(2gsin\theta - R)x5 = \frac{1}{2}x2x(2^2 - 0^2)$

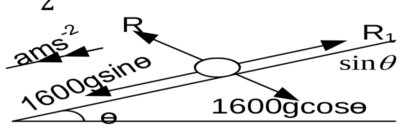
$R = 9N$

2. A car of mass 1600kg slides down a hill of slope 1 in 25. When the car descends 200m along at the hill, its speed increases from $3ms^{-1}$ to $10ms^{-1}$. Calculate
   (i) The change in the total kinetic energy
   (ii) Average value of resistance to motion

Solution

(i) $\Delta k.e = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

$= \frac{1}{2}x1600(10^2 - 3^2) = 72{,}800J$



$\sin\theta = \frac{1}{25}$

$v^2 = u^2 + 2as$

$a = \frac{10^2 - 3^2}{2x200} = 0.228ms^{-2}$

using $F = ma$

$1600g\ sin\theta - R_1 = 1600a$

$R_1 = 1600x9.8x\frac{1}{25} - 1600x0.228 = 262.4N$

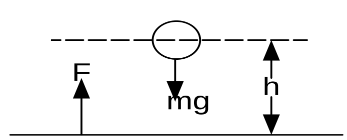
OR $W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

$(1600gsin\theta - R)x200 = \frac{1}{2}x1600(10^2 - 3^2)$

$R = 263.2N$

5.2.3: GRAVITATIONAL POTENTIAL ENERGY

Potential energy is the energy that a body has due to its position in a gravitational field. Consider a body of mass m on the surface of the earth moved up a height h by a greater Force F.



Work done = $Fx\ S$

work done $= mgxh$

But work done =P.E gained at maximum height

$\boxed{P.E = mgh}$

Note

When a body is moving vertically upwards, it losses K.E but gains P.E and when moving downwards, it losses P.E and gains K.E

Definition

Elastic potential energy is energy possessed by a stretched or compressed elastic material eg spring.

P.E (elastic)= ½ ke^2

Where k is the spring constant and e is the compression /extension

THE PRINCIPLE OF CONSERVATION OF MECHANICAL ENERGY

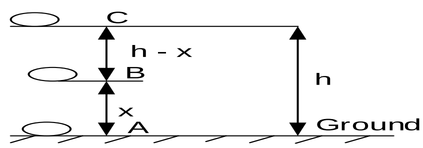
States that in a mechanical system the total mechanical energy is a constant provided that no dissipative forces act on the system.

Examples of dissipative forces are;
Frictional force, air resistance, viscous drag

Examples of principle of conservation of M.E

i) A body thrown vertically upwards

Consider a body of mass m projected vertically upwards with speed u from a point on the ground. Suppose that it has a velocity v at a point B at a height h above the ground provided no dissipative forces act.



At point A

$P.E = 0$ and K.E $= ½\, \mathrm{m}u^2$

Total energy $= K.E + P.E = ½\, \mathrm{m}u^2$

At point B

K.E = ½ mv^2 and $P.E = mgx$

But $v^2 = u^2 - 2gx$

Total energy $= ½\, m(u^2 - 2gh) + mgx = ½\, mu^2$

At point C

K.E = ½ mv^2 and P.E $= mgh$

$but\ v^2 = u^2 - 2gh$

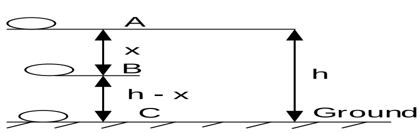
Total energy $= mgh + ½\,\mathrm{m}\,(u^2 - 2gh)$

$= ½\, mu^2$

Since the total mechanical energy at all points is constant then the mechanical energy of a an object projected vertically upwards is conserved provided there is no dissipative force.

ii) A body falling freely from a height above the ground

Consider a body of mass 'm' at a height 'h' from the ground surface and at rest



At point A

$K.E = 0$ (at rest) and P.E $= mgh$

Total energy $= K.E + P.E = mgh$

At point B

K.E = ½ mv^2 and $P.E = mg(h - x)$

$but\ v^2 = 2gx$

Total energy $= ½\,\mathrm{m}\, 2gx + mg\,(h - x) = mgh$

At point C (just before impact)

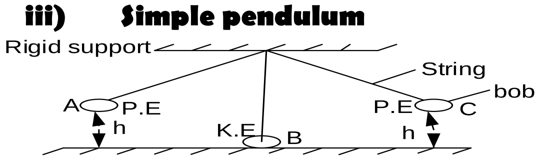
K.E = ½ m$v^2$and $P.E = 0$ (ground level)

$v^2 = 2gh$

Total energy $= mgh$

Since the total mechanical energy at all points is constant then the mechanical energy of a freely falling object is conserved provided there is no dissipative force.

iii) Simple pendulum

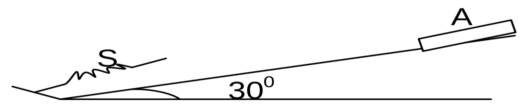


It consists of a bob that oscillates about equilibrium position B.

- At extreme ends A and C of the swing, the energy is potential energy and maximum since h is maximum.
- When passing through rest position B, the energy is kinetic energy and maximum; since the velocity at B is maximum and h = 0.
- At intermediate positions (i.e. between AB and BC) the energy is partly kinetic and partly potential.

Example

1. A block of mass 1kg is released from rest and travels down a rough incline of 30° to the horizontal a distance of 2m before striking a spring of force constant 100Nm^{-1}. The coefficient of friction between the block and the plane is 0.1

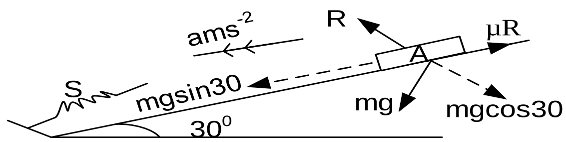


Calculate the:

(i) velocity of B just before it strikes the spring

(ii) maximum compression of the spring

solution



$$\boldsymbol{F = ma}$$

$ma = mgsin30 - \mu R$ **but** $R = mgcos30$

$$ma = mgsin30 - 0.1mgcos30$$

$$a = 4.055ms^{-2}$$

$$v^2 = u^2 + 2as$$

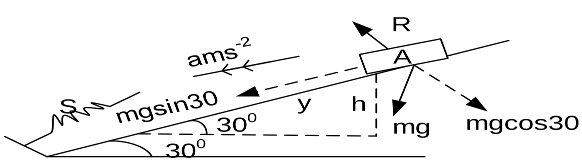
$$v = \sqrt{0^2 + 2x4.055x2} = 4.027ms^{-1}$$

$$(ii) \quad \frac{1}{2}ke^2 = \frac{1}{2}mv^2$$

$$e = \sqrt{\frac{1x(4.027)^2}{100}} = 0.4027m$$

2. A block of mass 0.2kg is released from rest and travels down a rough incline of 30° to the horizontal. The block compresses a spring of force constant $20Nm^{-1}$placed at the bottom of the plane by 10cm before its brought to rest. Find the distance the block travels down the incline before it comes to rest and its speed just before it reaches the spring

solution



by conservation of energy

$$\frac{1}{2}ke^2 = mgh$$

but h=ysin30

$$\frac{1}{2}x20x0.1^2 = 0.2x9.81ysin30$$

$$y = 0.1m$$

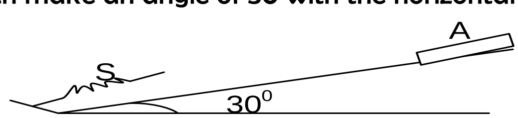
$$\frac{1}{2}ke^2 = \frac{1}{2}mv^2$$

$$\frac{1}{2}x20x0.1^2 = \frac{1}{2}mv^2$$

$$0.1 = \frac{1}{2}x0.1v^2$$

$$v = 1ms^{-1}$$

3. An ideal mass less spring is compressed 3.0cm by a force of 100N. the same spring is placed at the bottom of a frictionless inclined plane which make an angle of 30 with the horizontal as shown below



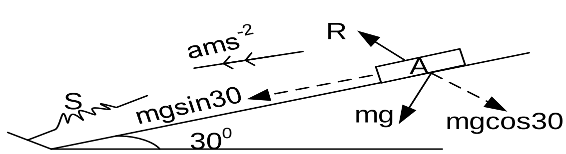
A 4.0kg mas released from rest at top of the incline and is brought to rest after compressing the spring 5.0cm. Find:

(I) The speed of the mass just before it reaches the spring

(II) The distance through which the mass slides before it reaches the spring

(III) The time taken by the mass to reach the spring

Solution



$$\frac{1}{2}ke^2 = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{3330x(5x10^{-2})^2}{4}} = 1.44ms^{-1}$$

(ii) $\boldsymbol{F = ma}$

$$ma = mgsin30$$

$$a = 9.81sin30 = 4.9ms^{-2}$$

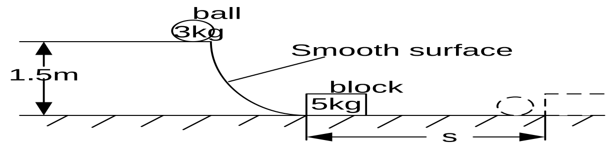
$$v^2 = u^2 + 2as$$

$$s = \frac{1.44^2 - 0^2}{2x4.9} = 0.21m$$

(iii) $v = u + at$

$$t = \frac{1.44 - 0}{4.9} = 0.294m$$

4. A ball of mas 3kg slides down a friction less surface and then strikes a stationary 5kg block on a horizontal surface as shown below



The coefficient of kinetic friction between the block and the table is 0.1. if the ball and the block stick together, how far do they slide before coming to rest

Solution

Before collision By conservation of energy:

$\frac{1}{2}mu^2 = mgh$

$$u^2 = 2x9.81x1.5$$

$$u = 5.42m/s$$

During collision: $m_1 u_1 + m_2 u_2 = (m_1 + m_2)v$

$$3x5.42 = 8v$$

$$v = 2.03ms^{-1}$$

After collision:

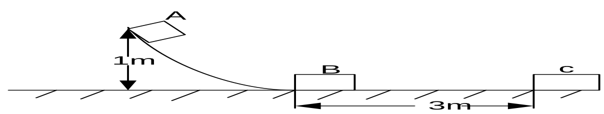
Work done against friction=loss in k.e

$$\boldsymbol{\mu}(m_1 + m_2)gxs = \frac{1}{2}(m_1 + m_2)v^2$$

$$0.1x9.81xs = \frac{1}{2}x(2.03)^2$$

$$s = 2.1m$$

5. A ball of mas 2kg is released from rest at point A on a friction less track which is one quadrant of circles of radius 1m as shown below.



The block reaches point B with a velocity of 4m/s. From point B, it then slides on a level road to point C where it comes to rest.

(i) find the coefficient of sliding friction on the horizontal surface

(ii) how much work was done against friction as the body slides down from A to B

Solution

From B to C:

Work done against friction=loss in k.e

$$\boldsymbol{\mu}(m)gxs = \frac{1}{2}(m)v^2$$

$$\boldsymbol{\mu}x9.81x3 = \frac{1}{2}x(4)^2$$

$$\mu = 0.272$$

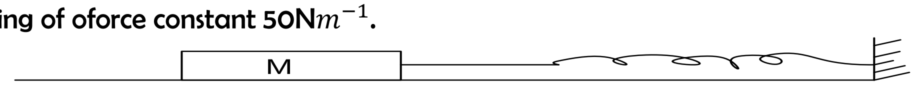
From A to B:

Work done against friction=change in M.E

$$= mgh - \frac{1}{2}(m)v^2$$

$$= 2x9.81x1 - \frac{1}{2}x2x(4)^2 = 3.62J$$

6. The figure below hows a wooden block M of mass 990g resting on a rough horizontal surface and attached to a spring of oforce constant 50Nm^{-1}.



When a sharp nail of mass 10g shot at close range ito the block, th spring is compressed by a distance of 20cm. If th work done against friction is $9x10^{-2}J$, Find the intial speed of the nail just before collision with the block.

Solution

After collision By conservation of energy:

K.e of the naill and block=incresae in P.E+Work again friction

$$\frac{1}{2}(m + M)\,v^2 = \frac{1}{2}kx^2 + 9x10^{-2}J$$

$$\frac{1}{2}(0.01 + 0.99)\,v^2 = \left(\frac{1}{2}x50x0.02^2 + 9x10^{-2}J\right)$$

$$v = 0.0141m/s$$

Befefore collision: $m_1 u_1 + m_2 u_2 = (m_1 + m_2)v$

$$(0.01u) + 0.99x0 = (0.01 + 0.99)x0.0141$$
$$u = 1.41m/s$$

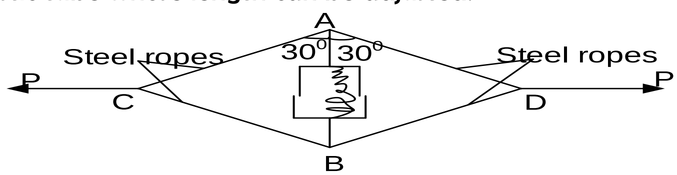
7. A car of mas 1000kg increases its speed from 10ms^{-1} to 20ms^{-1} while moving 500m up a road inclined at an angle θ to the horizontal where $\sin\theta = \frac{1}{20}$. There is a constant resistance to motion of 300N. Find the driving force exerted by the engine, assuming that it is constant

   Solution

   Work done by engine=incresae in P.E+incresae in K.E+Work against resistance

$$F_D\boldsymbol{x500} = mghsin\,\theta + \left(\frac{1}{2}mv^2 - \frac{1}{2}mu^2\right) + Fxs$$
$$F_D\boldsymbol{x500} = 1000x9.81x500x\frac{1}{20} + \frac{1}{2}x1000(20^2 - 10^2) + 300x500$$
$$F_D = 1100N$$

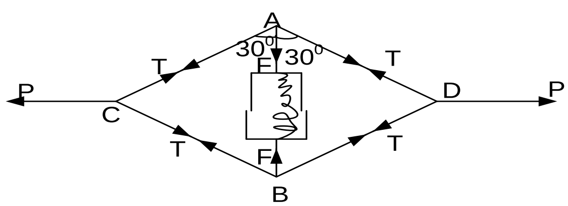
8. A muscle exerciser consists of two steel ropes attached to the ends of a strong spring of force constant 500N/m contained in a plastic tube whose length can be adjusted.



The spring has an uncompressed length of 0.80m. The ropes are pulled with equal and opposite forces, P so that the string is compressed to a length of 0.60m and the ropes make an angle of 30° with the length of the springs. Calculate;

(i) Tension in each rope (ii) Force p

Solution



$e = 0.8 - 0.6 = 0.2m$

$F = Ke = 2Tcos30°$

$T = \frac{ke}{2cos30°} = \frac{500x0.2}{2cos30°} = 57.7N$

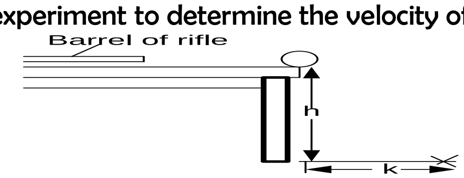
$(\rightarrow): P = 2Tsin30°$

$P = 2x57.7x0.5 = 57.7N$

Exercise:12

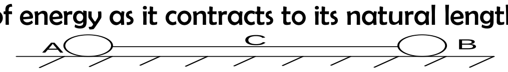
1. A car of mass 800kg and moving at 30ms^{-1} along a horizontal road is brought to rest by a constant retarding force of 5000N. calculate the distance the car moves while coming to rest. **An(72m)**
2. A car of mass 1200kg moves 300m up a road which is inclined at an angle θ to the horizontal where $\sin\theta = \frac{1}{15}$. By how much does the gravitational potential energy of the car increase **An(2.4x10^{5}J)**
3. A car of mass 800kg moving at 20ms^{-1} is brought to rest by the application of brakes in a distance of 100m. Calculate the work done by the brakes and the force they exert assuming that it is constant an d that there is no other resistance to motion **An(1.6x10^{5}J, 1.6x10^{3}N)**
4. The speed of a dog-sleigh of mass 80kg and moving along horizontal ground is increased form 3.0ms^{-1} to 9.0ms^{-1} over a distance of 90m. find;
   (a) The increase in the k.e of the sleigh
   (b) The force exerted on the sleigh by the dogs assuming that it is constant and there is no resistance to motion **An(2.9x10^{3}J, 32N)**
5. A simple pendulum consisting of a small heavy bob attached to a light string of length of length 40cm is released from rest with the string at 60° to the downward vertical. Find the speed of the pendulum bob as it passes through its lowest point **An(2.0ms^{-1})**

6. A car of mass 900kg accelerates from rest to a speed of $20ms^{-1}$ while moving 80m along a horizontal road. Find the tractive force exerted by the engine, assuming that it is constant and that there is a constant resistance to motion of 250N **An($2.5x10^3$N)**
7. A child of mass 20kg starts from rest at the top of a playground slide and reaches the bottom with a speed of $5.0ms^{-1}$. The slide is 5.0m long and there is a difference in height of 1.6m between the top and the bottom. Find
   (i) The work done against friction
   (ii) The average frictional force **An(70J, 14N)**
9. Two particles of masses 6.0kg and 2.0kg are connected by a light inextensible string passing over a smooth pulley. The system is released from rest with the string taut. Find the speed of the particles when the heavier one has descended 2.0m **An($4.5ms^{-1}$)**
10. A ball of mass 50g falls from a height of 2.0m and rebounds to a height of 1.2m. How much kinetic energy is lost on impact **An(0.4J)**
11. A student devises the following experiment to determine the velocity of a pellet from an air rifle



A piece of plasticine of mass **M** is balanced on the edge of a table such that it just fails to fall off. A pellet of mass, **m** is fired horizontally into the plasticine and remains embedded in it. As a result the plasticine reaches the floor a horizontal distance k away. The height of the table is **h**

   (i) show that the horizontal velocity of the plasticine with pellet embedded is $k\left(\frac{g}{2h}\right)^{1/2}$
   (ii) obtain an expression for the velocity of the pellet before impact with the plasticine
12. A model railway truck P, of mass 0.20kg and a second truck, Q o of mass 0.10kg are at rest on two horizontal straight rails, along which they can move with negligible friction. P is acted on by a horizontal force of 0.10N which makes an angle of 30^0 with the track. After P has travelled 0.50m, the force is removed and P then collides and sticks to Q. calculate;
   (a) The work done by the force
   (b) The speed of P before the collision
   (c) The speed of the combined trucks after collision **An**($4.3x10^{-3}$J, 0.66m/s, 0.44m/s)
13. A particle A of mass 2kg and a particle B of mass 1kg are connected by a light elastic string C and initially held at rest 0.9m apart on a smooth horizontal table with the string in tension. They are simultaneously released. The string releases 12J of energy as it contracts to its natural length.



Calculate the velocity acquired by each of the particles and find where the particles collide
An(2m/s, 4m/s, 0.3m from A)
14. A particle of mass 3kg and a particle Q of mass 1kg are connected by a light elastic string and initially held at rest on a smooth horizontal table with the string in tension. They are simultaneously released. The string releases 24J of energy as it contracts to its natural length. Calculate the velocity acquired by each of the particles . **An(2m/s, 4m/s, 0.3m from A)**
15. A bullet of mass $2.0x10^{-3}$kg is fired horizontally into a free- standing block of wood of mass $4.98x10^{-1}$kg, which it knocks forward with an initial speed of 1.2m/s
   (a) Estimate the speed of the bullet
   (b) How much kinetic energy is lost in the impact **An(300m/s, 89.64J)**
   (c) What becomes of the lost kinetic energy
16.

As shown in the diagram, two trolleys P and Q of mass 0.50kg and 0.30kg respectively are held together on a horizontal track against a spring which is in a state of compression.

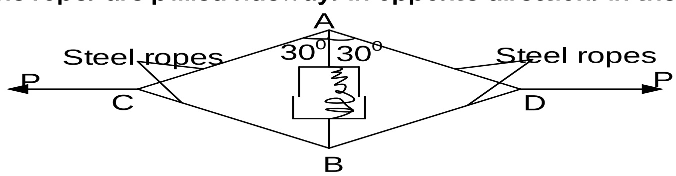
(a) When the spring is released the trolley separate freely and P moves to the left with an initial velocity of 6m/s. calculate

(i) Initial velocity of Q

(ii) The initial total kinetic energy of the system

(b) Calculate the initial velocity of Q if trolley P is held still when the spring under the same compression as before us released **An(10m/s, 24J, 12.5m/s)**

17. A muscle exerciser consists of two steel ropes attached to the ends of a strong spring contained in a telescopic tube. When the ropes are pulled sideways in opposite directions in the diagram below



The spring has an uncompressed length of 0.8m. the force F in newton required to compress the spring to a length x in meters is given by $F = 500(0.80 - x)$

The ropes are pulled with equal and opposite forces, P so that the string is compressed to a length of 0.60m and the ropes make an angle of 30° with the length of the springs

(a) Calculate the force F

(b) the work done in compressing the spring

(i) by considering forces at A or B, calculate the tension in each rope

(ii) by considering forces at C or D, calculate the force P **An(100N, 10J, 57.7N, 57.7N)**

5.3.0: POWER

It's the rate of doing work.

Its units are watts(W) or joule per second [Js^{-1}]

Power $= \frac{\text{Work done}}{\text{time}}$

P$= \frac{\text{F x d}}{\text{t}}$

P$= Fx\ \frac{d}{t}$

$P = Fxv$

Dimensions of power

$[P] = [F]x[v]$

[P] $= MLT^{-2}LT^{-1}$

[P] $= ML^2T^{-3}$

Numerical examples

1. A ball of mass of 0.1kg is thrown vertically up wards with an initial speed of $20m^{-1}$. Calculate

i) the time taken to return to the thrower

ii) the maximum height

iii) the kinetic and potential energy of the ball half way up.

Solution

i) Using $v = u + gt$

$0 = 20 - 9.81t$

$t = 2.04s$

Time to return to the thrower$= 2x2.04$

T$= 4.08s$

ii) max height (v=om/s)

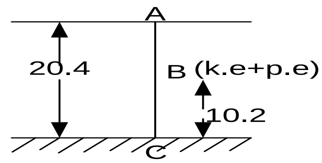
$v^2 = u^2 - 2gs_{max}$

$0 = 20^2 - 2x9.81s_{max}$

$s_{max} = \frac{400}{2x9.81}$

$s_{max} = 20.39m$

iii)



$k.e\ = \frac{1}{2}mv^2$ --------- (1)

But $v^2 = u^2 + 2gs$

$v^2 = 20^2 + 2x9.81x10.2$

$v = 14.14m/s$

$k.e = \frac{1}{2}x0.1x14.14^2$

$k.e\ = 9.96J$

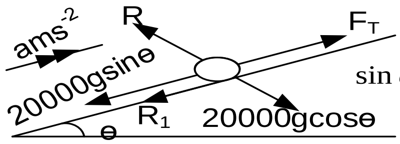
$p.e = mgh$

$p.e = 0.1x9.81x10.2$

$p.e = 10.01\,J$

2. A train of mass 20000kg moves at a constant speed of $72kmh^{-1}$ up a straight incline against a frictional force of 128. The incline is such that the train rises vertically one meter for every 100m travelled along the incline. Calculates the necessary power developed by the train.

Solution



$\sin\theta = \frac{1}{100}$

Using $F = ma$

$$F_T - (mgSin\theta + R_1) = ma$$

$a = 0ms^{-2}$ constant speed

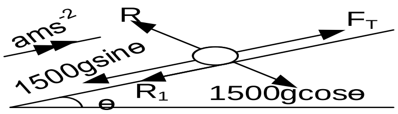
$$\frac{P}{v} - (20000x9.81\frac{1}{100} + 128) = 0$$

$$\frac{P}{20} = 2088N$$

Power$= 41760\text{W}$

3. A car of mass 1.5 metric tones moves with a constant speed of 6m/s up a slope of inclination $sin^{-1}\left(\frac{1}{7}\right)$. Given that the engine of the car is working at a constant rate of 18kW. Find the resistance to the motion

Solution



$\sin\theta = \frac{1}{7}$

Using $F = ma$

$$F_T - (mgSin\theta + R_1) = ma$$

$a = 0ms^{-2}$constant speed

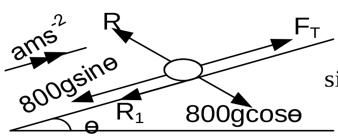
$$\frac{18000}{6} - (1500x9.81x\frac{1}{7} + R_1) = 0$$

$$R_1 = 900N$$

4. A car of mass 800kg with the engine working at a constant rate of 15kW climbs a hill of inclination 1 in 98 against a constant resistance to motion of 420N. Find the
   (i) Acceleration of a car up a hill when travelling with a speed of 10m/s
   (ii) Maximum speed of the car up the hill

Solution

(i)



$\sin\theta = \frac{1}{98}$

Using $F = ma$

$$F_T - (mgSin\theta + R_1) = ma$$

$$\frac{15000}{10} - (800x9.81x\frac{1}{98} + 420) = 800a$$

$$a = 1.25ms^{-2}$$

(i) $F_T - (mgSin\theta + R_1) = ma$

$a = 0ms^{-2}maximum\ sp$eed

$$\frac{15000}{v} - (800x9.81x\frac{1}{98} + 420) = 0$$

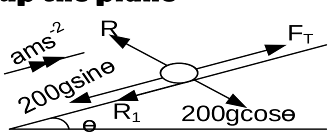
$$v = 30m/s$$

5. The maximum power developed by the engine of a car of mass 200kg is 44kW. When the car is travelling at $20kmh^{-1}$ up an incline of 1 in 8 it will accelerate at $2ms^{-2}$. At what rate will it accelerate when travelling down an incline of 1 in 16 at $60kmh^{-1}$. If in both cases the engine is developing the maximum power and the resistance to motor is the same.

Solution

Case I : up the plane

(i)



$\sin\theta = \frac{1}{8}$

$$v = 20kmh^{-1} = \frac{20x1000}{3600} = 5.5556ms^{-1}$$

Using $F = ma$

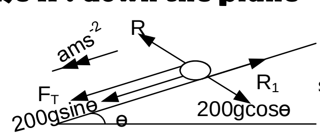
$$F_T - (mgSin\theta + R_1) = ma$$

$$\frac{44000}{5.5556} - (200x9.81x\frac{1}{8} + R_1) = 200a$$

$$R_1 = 7274.75N$$

Retarding force$= 7275N$

Case II : down the plane



$\sin\theta = \frac{1}{16}$

$$v = \frac{60x1000}{3600} = 16.6667ms^{-1}$$

Using $F = ma$

$$F_T + (mgSin\theta - R_1) = ma$$

$$\frac{44000}{16.6667} + (200x9.81x\frac{1}{16} - 7275) = 200a$$

$a = -22.56$ms^{-2}

PUMP RAISING AND EJECTING WATER.

Consider a pump which is used to raise water from a source and then eject it at a given speed. The total work done is sum of potential energy in raising the water and kinetic energy given to the water. The work done per second gives the rate (power) at which the pump is working.

$$\boxed{\boldsymbol{work\ done\ per\ second = P.E\ given\ to\ water\ per\ second + K.E\ given\ to\ water\ per\ second}}$$

Example

1. A pump raises water through a height of 3.0m at a rate of 300kg per minute and delivers it with a velocity of 8.0ms-1. Calculate the power output of the pump

Solution

$$\boldsymbol{work\ done\ per\ second = P.E\ given\ to\ water\ per\ second + K.E\ given\ to\ water\ per\ second}$$

$$\boldsymbol{work\ done\ per\ second} = (mass\ per\ second x g x h) + \left(\frac{1}{2} x mass\ per\ second x v^2\right)$$

$$\boldsymbol{work\ done\ per\ second} = \left(\frac{300}{60} x 9.81 x 3\right) + \left(\frac{1}{2} x \frac{300}{60} x (8)^2\right) = \boldsymbol{310J}$$

2. A pump draws 3.6$m^3$of water of density 1000kgm^{-3} from a well 5m below the ground in every minute, and issues it at ground level r a pipe of cross-sectional area 40cm^2. Find
   (i) The speed with which water leaves the pipe
   (ii) The rate at which the pump is working
   (iii) If the pump is only 80% efficient, find the rate at which it must work
   (iv) Find the power wasted

Solution

ii) $volume\ per\ second = area\ x\ velocity$

$$\frac{3.6}{60} = 40x10^{-4}\ v$$

$$v = 15\ ms^{-1}$$

iii) Mass per second =volume per second $x\rho = \frac{3.6}{60} x1000 = 60kgs^{-1}$

$$work\ done\ per\ second = P.E\ given\ to\ water\ per\ second + K.E\ given\ to\ water\ per\ second$$

$$work\ done\ per\ second = (mass\ per\ second x g x h) + \left(\frac{1}{2} x mass\ per\ second x v^2\right)$$

$$work\ done\ per\ second = (60x9.81x5) + \left(\frac{1}{2} x60x15^2\right) = 9693W$$

iv) Efficiency $= \frac{power\ output}{power\ input} x100\%$

$$80\% = \frac{9693}{power\ input} x100\%$$

$$power\ input = 12116.25W$$

v) Power wasted= power output – power input

Power wasted= $12116.25 - 9693 = 2423.25W$

EXERCISE:13

1. A man of mass 75kg climbs 300m in 30 minutes. At what rate is he working **An[125W]**
2. A pump with a power output of 600W raises water from a lake a height of 3.0m and delivers it with a velocity of 6.0ms^{-1}. What mass of water is removed from the lake in one minute **An[7500kg]**
3. What is the power output of a cyclist moving at a steady speed of 5.0ms^{-1} along a level road against a resistance of 20N **An[100W]**
4. What is the maximum speed which a car can travel along road when its engine is developing 24kW and there is a resistance to motion of 800N **An[30ms^{-1}]**
5. A crane lifts an iron girder of mass 400kg at a steady speed of 2.0ms^{-1}. At what rate is the carne working **An[8000W]**
6. A man of mass 70kg rides a bicycle of mass 15kg at a steady speed of 4.0ms^{-1} up a road which rises 1.0m fro every 20m of its length. What power is the cyclist developing if there is a constant resistance to motion of 20N **An[250W]**

7. A lorry of mass 2000kg moving at 10m/s on a horizontal surface is brought to rest in a distance of 12.5m by the brakes being applied.
   (a) Calculate the average retarding force
   (b) What power must the engine produce if the lorry is to travel up a hill of 1 in 10 at a constant speed of 10m/s, frictional resistance being 200N. **An[8000N, 22000W]**
8. A car of mass 900kg travelling at 30m/s along a level road is brought of rest in a distance of 35m by its brakes.
   (a) Calculate the average exerted by the brakes
   (b) If the same car travels up a slope of 1 in 15 at a constant speed of 25m/s, what power does the engine develop if the total frictional resistance is 120N
9. A bullet of mass 50g travelling horizontally at 500ms^{-1} strikes as stationary block of wood and after travelling 10cm, it emerges from the block travelling at 100ms^{-1}. Calculate the average resistance of the block to the motion of the bullet. **An[60000N]**
10. A horizontal force of 2000N is applied to a vehicle of mass 400kg which is initially at rest on a horizontal surface. If the total force opposing the motion is common at 800N, calculate;
    (a) The acceleration of the vehicle
    (b) The kinetic energy of the vehicle 5s after the force is first applied
    (c) The total power developed 5s after the force is first applied **An[3.0m/^{2}, 45kJ, 30kW]**
11. A lorry of mass 3.5x10^{4}kg attains a steady speed v while climbing an incline of 1 in 10, with the engine operating at 175kW. Find v (neglect friction) **An[5.0m/s]**
12. A point A is vertically below the point B. A particle of mass 0.1kg is protected from A vertically up wards with a speed 21ms^{-1} and passes through point B with speed 7ms^{-1}. Find the distance from A to B **An[20m]**
13. The friction resistance to the motion of a car of mass 100kg is 30VN where V is the speed in ms^1. Find the steady speed at which the car ascends a hill of inclination sin^{-1}($\frac{1}{10}$). If the power exerted by the engine is 12.8kW. **An[V=10m/s]**
14. A load of 3Mg is being hauled by a rope up a slope which rises 1 in 140. There is a retardation force due to friction of 20gN per Mg at a certain instant when the speed is 16kmh^{-1} and the acceleration is 0.6ms$^{-2.}$ Find the pull in the rope and the power exerted at the instant. **An[2598N, 11.55kW]**
15. A car of mass 2 tones moves from rest down. A road of inclination sin^{-1} ($\frac{1}{20}$) to the horizontal. Given that the engine develops a power of 64.8kW when it is travelling at a speed of 54kmh^{-1} and the resistance to motion is 500N, find the acceleration. **An[2.4m/s^2]**
16. A car is driven at a uniform speed of 48kmh^{-1} up a smooth incline of 1 in 8. If the total mass of the car is 800kg and the resistance are neglected calculate the power at which the car is working. **An[1.31x10^{4}W]**
17. A train whose mass is 250Mg run up an incline of 1 in 200 at a uniform rate of 32km/h. The resistance due to friction is equal to the weight of 3Mg. at what power is the engine working?. **An[370.2kW]**
18. A train of mass 1x10^{5}kg acquires a uniform speed of 48kmh^{-1}from rest in 400m. Assuming that the frictional resistance of 300gN. Find the tension in the coupling between the engine and the train. And the maximum power at which the engine is working during 400m run, the mass of the engine may be neglected. **An[25162N, 335.5kW]**
19. A car of mass 2000kg travelling at 10ms^{-1} on a horizontal surface is brought to rest in a distance of 12.5m by the action of its brakes. Calculate the average retarding force. What power must the engine develop in order to take the vehicle up an incline of 1 in 10 at a constant speed of 10ms^{-1} if the frictional resistance is equal to 2000N. **An[8000N, 21600N]**
20. A water pump must work at a constant rate of 900W and draws 0.3m^3 of water from a deep well and issue it through a nozzle situated 10m above the level from which the water was drawn after every minute. If thee pump is 75% efficient, find;
    i) Velocity with which the water is ejected
    ii) The cross-sectional area of the nozzle **An (8.6ms^{-1}, $5.81cm^2$)**

UNEB 2017 No1c

A bullet of mass 10g moving horizontally with a velocity of 300m/s into a block of wood of mass 290g which rests on a rough horizontal floor. After impact, the block and bullet move together and come to rest when the block has trvaelled a distance of 15m. calculate the coefficient of slidding friction between the block and the floor. **An(**0.34 **)** (07marks)

UNEB 2015 No1

(a) (i) What is meant by **a conservative force** (01mark)

(ii) Give **two** examples of a conservative force (01mark)

(b) (i) State the law of conservation of **mechanical energy** (01mark)

(ii) A body of mass m, is projected vertically upwards with speed, u. Show that the law of conservation of mechanical energy is obeyed through its motion (05marks)

(iii) Sketch a graph showing variation of kinetic energy of the body with time (01mark)

(c) (i) Describe an experiment to measure the coefficient of static friction (04marks)

(ii) State two disadvantages of friction (01marks)

(d) A bullet of mass 20g moving horizontally strikes and gets embedded in a wooden block of mass 500g resting a horizontal table. The block slides through a distance of 2.3m before coming to rest. If the coefficient of kinetic friction between the block and the table is 0.3, calculate the

(i) Friction force between the block and the table (02marks)

(ii) Velocity of the bullet just before it strikes the block (04marks)

An($1.53N$**,** $95.68m/s$ **)**

UNEB 2014 No3

(a) Define **work and energy** (02marks)

(b) Explain whether a person carrying a bucket of water does any work on the bucket while walking on a level road (03marks)

(c) A pump discharges water through a nozzle of diameter 4.5 cm with a speed of 62ms^{-1} into a tank 16 m above the intake.

(i) Calculate the work done per second by the pump in raising the water if the pump is ideal

(ii) Find the power wasted if the efficiency of the pump is 73% (02marks)

(iii) Account for the power lost in (c) (ii) (02marks)

An($2.05x10^5\ J\ s^{-1}$**,** $7.6x10^4 W$ **)**

(d) (i) State the **work-energy theorem** (01mark)

(ii) Prove the work-energy theorem for a body moving with constant acceleration.

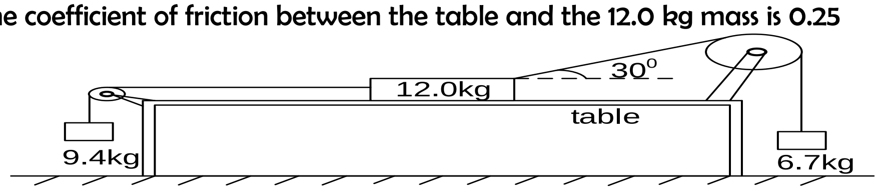
(e) Explain briefly what is meant by internal energy of a substance (03marks)

UNEB 2013 No1

(a) Using the molecular theory, explain the laws of friction between solid surface (06marks)

(b) With the aid of a labeled diagram, describe how the coefficient of static friction for an interface between a rectangular block of wood and a plane surface can be determined. (06marks)

(c) The diagram below shows three masses connected by inextensible strings which pass over smooth pulleys. The coefficient of friction between the table and the 12.0 kg mass is 0.25

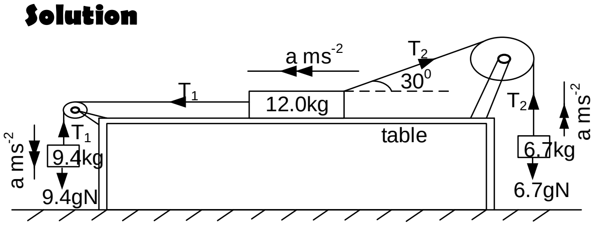


If the system is released from rest, determine the

(i) Acceleration of the 12.0kg mass (05marks)

(ii) Tension in each string (03marks)

Solution



$F = ma$

9.4kg mass: $9.4gN\ - T_1 =$ 9.4a

$T_1 = 9.4g - 9.4a$..............(1)

For 6.7kg mass: $T_2 - 6.7gN =$ 6.7a

$T_2 = 6.7a + 6.7g$...................(2)

For 12kg mass:

$T_1 - (T_2\ Cos30° + 0.25R) =\ 12a$...........(3)

But $R + T_2\ Sin30° =\ 12gN$

$\therefore R = 12g - T_2\,Sin30°$
put into(3)
$T_1 - (T_2\,Cos30° + 0.25[12g - T_2\,Sin30])= 12a$
Put equation(1)
9.4g-9.4a-$T_2\,Cos30°$-0.25x12g+$0.25xT_2\,Sin30 = 12a$
$a = 0.53\,m\,s^{-2}$
Acceleration of 12kg mass is $0.53\,m\,s^{-2}$

(i) Tension in each string
$T_1 = 9.4g - 9.4a$
$T_1 = 9.4x9.81 - 9.4x0.53 = 87.2N$
Also $T_2 = 6.7a + 6.7g$
$T_2 = 6.7x0.573 + 6.7x9.81 = 69.3N$

UNEB 2013 No4d

A simple pendulum of length 1m has a bob of mass 100g. it is displaced from its mean position A and to a position B so that the string makes an angle of 45° with the e vertical. Calculate the;

(i) Maximum potential energy of the bob **An(0.287J)** [03marks]
(ii) Velocity of the bob when the string makes an angle of 300 with the vertical (neglect air resistance) **An(1.766m/s)** [04marks]

UNEB2010No3

(c) i) State the laws of solid friction [03marks]
ii) With the aid of a well labeled diagram describe an experiment to determine the co-efficient of kinetic friction between the two surfaces. [05marks]
d) A body slides down a rough plane inclined at 30° to the horizontal. If the co-efficient of kinetic friction between the body and the plane is 0.4. Find the velocity after it has travelled 6m along the plane.
An[4.25m/s] [05marks]

UNEB2008 No2

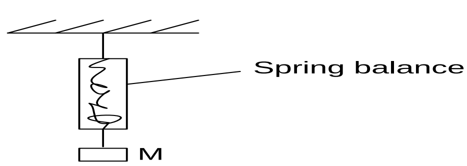
a) i) state the laws of friction between solid surfaces [03marks]
ii) Explain the origin of friction force between two solid surfaces it contact. [03marks]
(iii) Describe an experiment to measure the co-efficient of kinetic friction between two solid surfaces.
b) i) A car of mass 1000kg moves a long a straight surface with a speed of 20ms^{-1}. When brakes are applied steadily, the car comes to rest after travelling 50m. Calculate the co-efficient of friction between the surface and the types. **An[$\mu = 0.408$]** [04marks]
c) ii) State the energy changes which occur from the time the brakes are applied to the time the car comes to rest. **An[kinetic energy$\rightarrow$ $heat + sound\ energy$]** [02marks]
d) i)State two disadvantages of friction [01marks]
e) ii) Give one method of reducing friction between solid surfaces. [01mark]

UNEB2007No3

a) i)State the laws of solid friction [03marks]
ii) Using the molecular theory, explain the laws stated in a i). [03marks]
b) Describe an experiment to determine the co-efficient of static friction for an interface between a rectangular block of wood and plane surface. [04marks]
c) i)State the different between conservative and non conservative forces, giving one example of each.
ii) State the work-energy theory. [01marks]
iii) A block of mass 6.0 kg is protected with a velocity of 12ms^{-1} up a rough plane inclined at 45° to the horizontal if it travels 5.0m up the plane. Find the frictional force. **An[44.8N]** [04marks]

UNEB2006No2

a) i)Define force and power [02marks]
ii)Explain why more energy is required to push a wheelbarrow uphill than on a level ground.
b)



A mass M is suspended from a spring balance as shown above. Explain what happens to the reading on the spring balance when the set up is raised slowly to a very high height above the ground. [02marks]

c) i)State the work-energy theorem [01mark]

Solution

b)As the setup is raised to a high height, acceleration due to gravity reduces, the weight of M decreases and it reading of the spring balance reduces proportionately.

UNEB2005No1

a) i)What is meant by conservation of energy? [01mark]

ii)Explain how conservation of energy applies to an object falling from rest in a vacuum. [02marks]

UNEB2004 N01

a) State the laws of friction [04marks]

b) A block of mass 5.0kg resting on the floor is given horizontal velocity of $5ms^{-1}$ and comes to rest in a distance of 7.0m. Find the co-efficient of kinetic friction between the block and the floor.

An[0.182] [04marks**]**

c) i)State the laws of conservation of linear momentum [01mark]

ii)What is perfectly inelastic collision? [01mark]

d) A car of mass 1500kg rolls from rest down a round inclined to the horizontal at an angle of 35^0, through 50m. The car collides with another car of identical mass at the bottom of the incline. If the two vehicles interlock on collision and the co-efficient of kinetic friction is 0.20, final the common velocity of the vehicle.

An[20.05m/s] [08marks]

[Hint loss of p.e at the top=gain in k.e at the bottom + work done against friction]

e) Discuss briefly the energy transformation which occur in(d) above. [01mark]

An[Potential energy$\rightarrow kinetic\ energy + sound + heat$**]**

UNEB 2001 No1

a) i)State the principle of conservation of mechanical energy. [01mark]

ii)Show that a stone thrown vertically upwards obey, the principle in (c) throughout its upward motion. [04marks]

CHAPTER 6: STATICS

Is a subject which deals with equilibrium of forces $e.g$ the forces which act on a bridge.

Coplanar forces

Those are forces acting on the same point (plane).

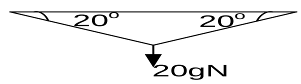
6.1.0: Conditions necessary for mechanical equilibrium

When forces act on a body then it will be in equilibrium when;

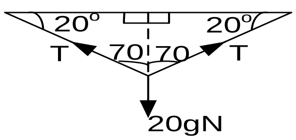
1. The algebraic sum of all forces on a body in any direction is zero
2. The algebraic sum of moments of all forces about any point is zero

Examples

1. A mass of 20kg is hang from the midpoint P of a wire as shown below. Calculate the tension in the wire take g=9.8ms^{-1}

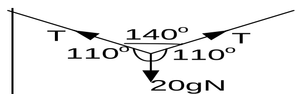


Solution



Method I : lami's theorem

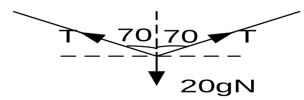
(Apply to only three forces in equilibrium)



$$\frac{20gN}{Sin140} = \frac{T}{Sin110}$$

$$T = \frac{20x9.81sin110}{Sin140} = 286.83N$$

METHOD II: Resolving



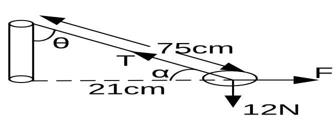
Resolving vertically

$Tsin70 + Tcos70 = 20gN$

T = $\frac{20x9.81}{2\ cos70} = 286.83N$

2. One end of a light in extensible string of length 75cm is fixed to a point on a vertical pole. A particle of weight 12N is attached to the other end of the string. The particle is held 21cm away from the pole by a horizontal force. Find the magnitude of the force and the tenion in the string

Solution

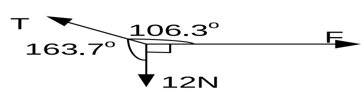


$Sin\theta = \frac{21}{75}$ $\therefore \theta = 16.3°$

Also $cos\ \alpha = \frac{21}{75}$

$\alpha = 73.7°$

Using Lami's theorem



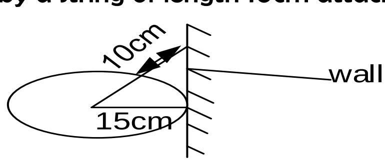
$$\frac{F}{Sin163.7} = \frac{12}{Sin106.3}$$

$F = 3.51N$

Also $\frac{T}{Sin90} = \frac{12}{Sin106.3}$

$T = 12.5N$

3. A sphere of weight 20N and radius 15cm rests against a smooth vertical wall. A sphere is supported in its position by a string of length 10cm attached to a point on the sphere and to a point on the wall as shown.

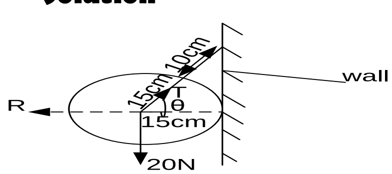


i) copy the diagram and show the forces acting on the sphere

ii) Calculate the reaction on the sphere due to the wall.

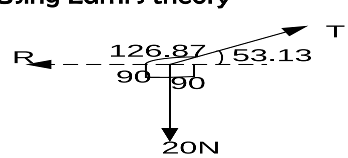
iii) Find the tension in the string

Solution



$Cos\theta = \frac{15}{28}$ $\therefore \theta = 53.13°$

Using Lami's theory



$$\frac{20}{Sin126.87} = \frac{T}{Sin90}$$

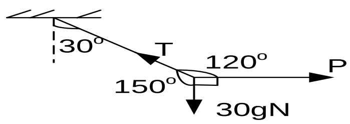
$T = 25N$

$$\frac{R}{Sin143.13} = \frac{20}{Sin126.87}$$

$R = 15N$

4. A mass of 30kg hangs vertically at the end of a light string. If the mass is pulled aside by a horizontal force P so that the string makes an angle 30° with the vertical. Find the magnitude of the force P and the tension in the string.

Solution

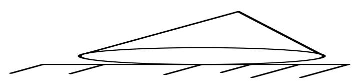


$$\frac{30x9.81}{Sin120} = \frac{20}{Sin150}$$
$$P = 169.91N$$

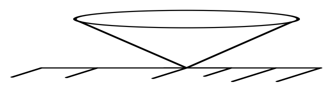
$$\frac{T}{Sin90} = \frac{30x9.81}{Sin120}$$
$$T = 339.83N$$

6.1.1: Types of equilibrium

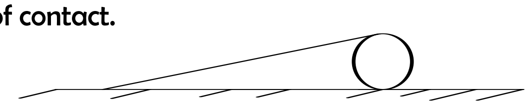
1. **Stable equilibrium**.
   Stable equilibrium is when a body returns to its original position after being displaced slightly and its center of gravity rises. A body under stable equilibrium has Large base area, the center of gravity is in the lowest position.



2. **Unstable equilibrium**.
   Un Stable equilibrium is when a body does not return to its original position after being displaced slightly and its center of gravity is lowered. A body under un stable equilibrium has Low base area, the center of gravity is in the highest position.



3. **Neutral equilibrium**.
   The body is said to be in a neutral equilibrium if the center of gravity is neither raised nor lowered during displacement and the body remains in the displaced position.
   A body under neutral equilibrium has a small area of contact The center of gravity is always at the same height directly above the point of contact.



6.3.4: CENTER OF GRAVITY

This point where the resultant force on the body due to gravity acts.

DETERMINATION OF CENTRE OF GRAVITY OF AN IRREGULAR LAMINA

- Make three holes near the edge of the card board
- Suspend the sheet form one hole and allow it to swing freely
- Hung a pendulum bob form the same point of suspension
- Trace the outline of the pendulum on the sheet
- Repeat the procedure above using the other holes.
- The point of intersection of the three outlines is the centre of gravity of the board

Definition: A uniform body is one whose center of gravity is the same point as its geometrical centre

6.2.1: Moment of a force

This is the product of a force and the perpendicular distance of its line of action from the pivot.
The unit of a moment is Nm and it's a vector quantity.
Moment of a force = Force x perpendicular distance of its line of action from pivot.

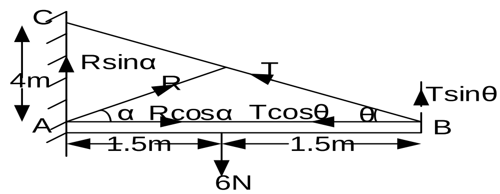
6.2.2: Principle of moments

It states that when a body is in mechanical equilibrium, the sum of clockwise moments about a point is equal to the sum of anticlockwise moments about the same point.

6.2.3: Beams hinged against the wall

1. A Uniform beam AB, 3.0m long and of weight 6N is hinged at a wall at A and is held stationary in a horizontal position by a rope attached to B and joined to a point C on the wall, 4.0m vertically above A. Find
   (i) the tension T in the rope
   (ii) the magnitude and direction of the Reaction R at the hinge.

Solution



tan$\theta = \frac{4}{3}$ θ= 53.13°

Taking moments about A at equilibrium

$(Tsin53.13)\ x\ 3\ =\ 9$

$T\ =\ 3.75N$

$(\uparrow)\ Rsin\alpha\ +\ Tsin\theta\ =\ 6$

$Rsin\alpha =\ 6 - 3.75sin53.13$

$Rsin\alpha\ =\ 3$-------------------------------i

$(\rightarrow)\ Rcos\alpha\ =\ Tcos\theta$

$Rcos\alpha\ =\ 3.75cos53.13$

$Rcos\alpha = 2.238$-----------------------------ii

i/ii tan$\alpha = \frac{3}{2.238}$ $\alpha\ =\ 53.3°$

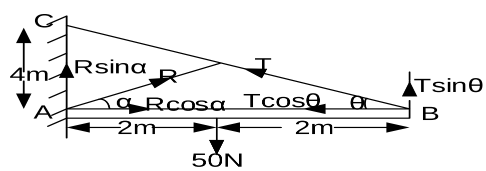
Put into 1; $Rsin53.3\ = 3$

$R\ =\ 3.74N$

The reaction at A is 3.74 at 53.28° to the beam

2. A uniform beam AB of length 4m and weight 50N is freely hinged at A to a vertical wall and is held horizontal in equilibrium by a string which has one end attached at B and the other end attached to a point C on the wall, 4m above A. find
   (i) the tension T in the rope
   (ii) the magnitude and direction of the Reaction R at the hinge.

Solution



tan$\theta = \frac{4}{4}$ $\therefore\ \theta\ =\ 45°$

Taking moments about A

$Tsin45x4\ =\ 50x2$

$T\ =\ 35.36N$

$(\uparrow)$: $Rsin\ \alpha\ +\ Tsin\ \theta\ =\ 50$

$Rsin\ \alpha +\ 35.36sin45\ =\ 50$

$Rsin\ \alpha\ =\ 24.997$............................(i)

$(\rightarrow)$:$Rcos\ \alpha\ =\ Tcos\ \theta$

$Rcos\ \alpha\ =\ 35.36cos45$

$Rcos\ \alpha\ =\ 25$............................(ii)

(i)/(ii) $tan\ \alpha = \frac{24.997}{25} \therefore\ \alpha\ =\ 45°$

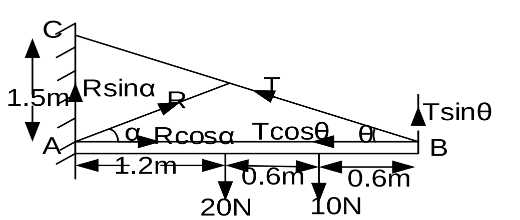
Put into (ii); $Rcos\ \alpha\ =\ 25$

$Rcos45\ =\ 25$

$R\ =\ 35.36N$ at $45°$ to the beam

3. A uniform beam AB of mass 20kg and length 2.4m is hinged at a point A in a vertical wall and is maintained in a horizontal position by means of a chain attached to B and to point C in a wall 1.5m above. If the bar carries a load of 10kg at a point 1.8m from A. calculate.
   i) The tension in the chain
   ii) The magnitude and direction of the reaction between the bar and the wall

Solution



tan$\theta = \frac{1.5}{2.4}$ $\therefore\ \theta$ = 32.01°

Taking moments about A

$Tsin\ \theta\ x\ 2.4\ =\ 20gNx1.2\ +\ 10gN\ x\ 1.8$

$Tx2.4sin32.01 = 20x9.8x1.2\ +\ 10x9.8x1.8$

$T\ =\ 323.87N$

Tension in the chain = $323.87N$

(ii) Reaction at the wall

$(\uparrow)\ R\ sin\ \alpha\ +\ Tsin\theta\ =\ 20gN\ +\ 10gN$

$R\ sin\ \alpha\ +\ 323.87sin32.01\ =\ 30gN$

$R\ sin\ \alpha\ =\ 122.63$........................(i)

$(\rightarrow)$; $R\ cos\ \alpha$ = Tcosθ

$R\ cos\ \alpha\ =\ 32.87cos32.01$

$R\ cos\ \alpha\ =\ 274.63$ ....(ii) (i)/(ii) $tan\ \alpha\ =\ 0.446528055$

$\alpha\ =\ 24.1°$ Put α in eqn (ii)

$$R\ cos24.1\ =\ 274.63$$

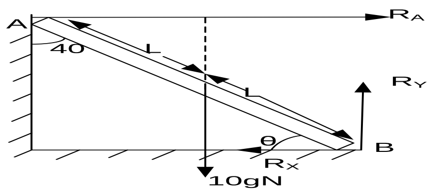
$$R\ =\ 300.85N$$

Reaction at A is 300.85 at 24.1° to the horizontal

6.2.4: Ladder problems

1. A uniform rod AB of mass 10kg is smoothly hinged at B and rests in a vertical plane with the end A against a smooth vertical wall. If the rod makes an angle of 40° with the wall, find the reaction on the wall and the magnitude of the reaction at B

Solution



let length of the ladder be 2L

$$\theta = 90° - 40° = 50°$$

Taking moments about B

$$R_A\ x\ 2Lsin50\ =\ 10x9.81lcos50$$

$$R_A\ =\ 41.16N$$

$(\uparrow)$; $R_Y\ =\ 10gN = 10x9.81 = 98.1N$

$(\rightarrow)$: $R_X\ =\ R_A$

: $R_X\ =\ 41.16$

$$R = \sqrt{(R_X)^2 + (R_Y)^2} = \sqrt{(41.16)^2 + (98.1)^2}$$

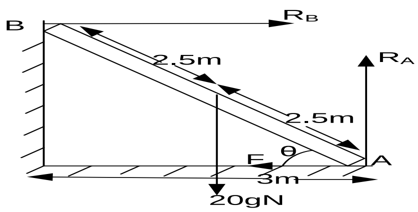
$$R\ = 106.38N$$

$$\alpha = tan^{-1}\left(\frac{R_Y}{R_X}\right) = tan^{-1}\left(\frac{98.1}{41.16}\right) = 67.24°$$

Reaction at B is $106.38N$ at $67.24°$ to the beam.

2. uniform ladder which is 5m long and has a mass of 20kg leans with its upper end against a smooth vertical wall and its lower end on a rough ground. The bottom of the ladder is 3m from the wall. Calculate the functional force between the ladder and the ground and the coefficient of friction

Solution



$Cos\theta = \frac{3}{5}$ $\therefore$ θ = 53.13°

Resolving vertically: $R_A = 20gN$

$R_A = 20x9.81 = 196.2N$

Taking moments about A

$R_B\ x\ 5sin\theta = 20x9.81x2.5cos\theta$

$R_B\ 5sin53.13 = 20x9.81x2.5cos53.13$

$R_B = 73.56N$

Resolving horizontally: R_B= F

$$F\ =\ 73.56N$$

But $F\ =\ \mu x R_A$

$73.56\ = \mu x196.2$

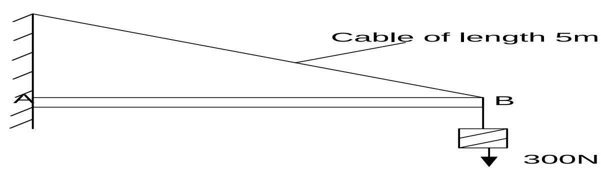
$\mu\ =\ 0.37$

Exercise: 14

1. A particle whose weight is 50N is Suspended by a light string which is 35°° to the vertical under the action of horizontal force F. Find
   (a) The tension in the string
   (b) Force F **An(61.0N, 35.0N)**
2. A particle of weight W rests on a smooth plane which is inclined at 40° to horizontal. The particle is prevented from slipping by a force of 50.0N acting parallel to the plane and up a line of greatest slope. Calculate
   (a) W
   (b) Reaction due to the plane **An(77.8N, 59.6N)**
3. Two light strings are perpendicular to each other and support a particle of weight 100N. the tension in one of the strings is 40.0N. Calculate the angle this string makes with the vertical and the tension in the other string **An(66.4°, 91.7N)**
4. A uniform pole AB of weight 5W and length 8a is suspended horizontally by two vertical strings attached to it at C and D where AC=DB=a. A body of weight 9W hangs from the pole at E where ED=2a. calculate the tension in each string **An(5.5W, 8.5W)**
5. AB is a uniform rod of length 1.4m. It is pivoted at C, where AC=0.5m, and rests in horizontal equilibrium when weighs of 16N and 8N are applied at A and B respectively. Calculate
   (a) the weight of the rod

(c) (b) the magnitude of the reaction at the pivot **An(4N, 28N)**

6. A uniform rod AB of length 4a and weight W is smoothly hinged at its upper end, A. the rod is held at 30° to the horizontal by a string which is at 90° to the rod and attached to it at C where AC=3a, find
   (d) the tension in the string
   (e) reaction at A **An(0.58W, 0.578W)**
7. A sphere of weight 40N and radius 30cm rests against a smooth vertical wall. The sphere is supported in this position by a string of length 20cm attached to a point on the sphere and to the a point on the wall. Find
   (a) tension in the string
   (b) reaction due to the wall **An(50N, 30N at 90° to the wall)**
8. A uniform ladder which is 5m long and has a mass of 20kg leans with its upper end against a smooth vertical wall and i6ts lower end on rough ground. The bottom of the ladder is 3m from the wall. Calculate the frictional forces between the ladder and ground **An(75N)**
9. One end of a uniform plank of length 4m and weight 100N is hinged to the vertical wall. An inelastic rope, tied to the other end of the plank is fixed at a point 4m above the hinge. Find
   i. The tension in the rope'
   ii. The reaction of the wall on the plank **An(388.9N, 302.1N at 24.4° to horizontal)**
10.



The figure shows a uniform rod AB of weight 200N and length 4m, the beam is hinged to the wall at A.
   i. Find the tension in the cable
   ii. The horizontal and vertical components of the force exerted on the beam by the wall
   iii. The reaction of the wall on the beam at point A

   An(666.7N, 533.3N, 99.98N, 542.59 at 10.6° to the horizontal)
11. A uniform beam AB of length 2L rests with end A in contact with a rough horizontal ground. A point C on the beam rests against a smooth support. AC is of length $\frac{3l}{2}$ with C higher than A and AC making an angle of 60° with the horizontal. If the beam is in limiting equilibrium, find the coefficient of friction between the beam and the ground.
12. A uniform ladder of mass 25kg rests in equilibrium with its base on a rough horizontal floor and its top against a smooth vertical wall. If the ladder makes an angle of 75° with the horizontal, find the magnitude of the normal reaction and of the frictional force at the floor and state the minimum possible value of the coefficient of friction μ between the ladder and the floor.
13. A ladder 12m long and weighing 200N is placed 60° to the horizontal with one end B leaning against the smooth wall and the other end A on the ground. Find;
   a) reaction at the wall **An(57.7N)**
   b) reaction at the ground **An(208.2N at 73.9° to the horizontal).**

6.3.0: Couples

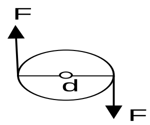
A couple is a pair of **equal**, **parallel** and **opposite** forces with different lines of action acting on a body.

Examples

- Forces in the driver's hands applied to a steering wheel
- Forces in the handles of a bike
- Forces in the peddles of a bike
- Forces experienced by two sides of a suspended rectangular coil carrying current in a magnetic field.

6.3.1: Moment of a couple (torque of a couple)

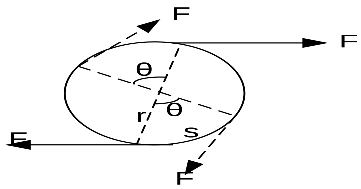
It is defined as the product of one of the forces and the far distance between the lines of action of the forces



Moment of a couple or torque of couple = F x d

6.3.2: Work done by a couple

Consider two opposite and equal forces acting tangentially on a wheel of radius r, suppose the wheel rotates through an angle θ radians as shown below.



Work done by each force $= FxS$

But $s = \frac{\theta}{360} x2\pi r$

360° = 2π $rads$

Work done by each force $= F\ x\ r\theta$

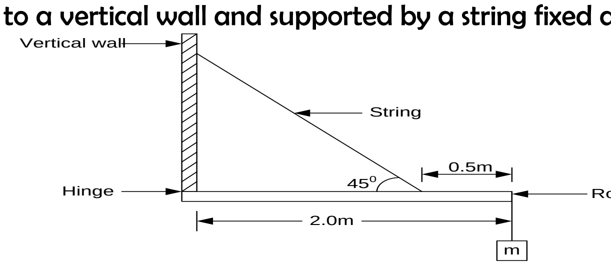
Total work done by the couple $= 2Fr\theta$

UNEB 2015 No 2

(a) (i) State the **principle of moments** (1marks)

(ii) Define the terms **center of gravity** and **uniform body** (2marks)

(b) The figure below shows a body, m of mass 20kg supported by a rod of negligible mas horizontally hinged to a vertical wall and supported by a string fixed at 0.5m from the other end of the rod



Calculate the

(i) Tension in the string (3marks)

(ii) Reaction at the hinge (3marks)

(iii) Maximum additional mass which can be added to the mas of 20 kg before the string can break given that the string cannot support a tension of more than 500N (2marks)

An(370N,270N,7.03kg)

UNEB 2009 No 2

a) Define the following terms
   i) Velocity (2marks)
   ii) Moment of a force

c)(i) State the condition necessary for mechanical equilibrium t be attained.(2 marks)

ii)A uniform ladder of mass 40kg and length 5m, rest with its upper end against a smooth vertical wall and with its lower end at 3m from the wall on a rough ground. Find the magnitude and direction of the force exerted at the bottom of the ladder **An**[418.7N at an angle of 69.4° to the horizontal**].** (06 marks)

UNEB 2006 No 2

c) State the condition for equilibrium of a rigid body under the action of coplanar forces. (2mk)

d) A 3m long ladder at an angle 60° to the horizontal against a smooth vertical wall on a rough ground. The ladder weighs 5kg and its centre of gravity is one third from the bottom of the ladder.
   i) Draw a sketch diagram to show the forces acting on the ladder. (2mk)
   ii) Find the reaction of the ground on the ladder. (4mk)

(Hint Reaction on the ladder =$\sqrt{R^2 + F^2}$) An(49.95N at 79.11° to the horizontal)

UNEB 2006 No1

e) Describe an experiment to determine the centre of gravity of a plane sheet of material having an irregular shape. (4 marks)

UNEB 2005 No2

f) (i) Define centre of gravity (1 mark)

(ii) Describe an experiment to find the centre of gravity of a flat irregular card board. (3 marks)

UNEB 2002 No2

d) (i)Define moment of a force (1 mark)

(ii) A wheel of radius 0.6m is pivoted at its centre. A tangential force of 4.0N acts on the wheel so that the wheel rotates with uniform velocity find the work done by the force to turn the wheel through 10 resolutions.

Solution

Work done $= force\ x\ distances$

But distance= circumference x number of revolutions

$= 2\pi\ rx10$

$W = F\ x\ d = 4x2\ \pi\ rx0.6x10$

$W\ = 150.79J$

UNEB 2000 No3

b) State the conditions for equilibrium of a rigid body under the action of coplanar forces.(2mk)

d) A mass of 5.0kg is suspended from the end A of a uniform beam of mass 1.0kg and length 1.0m. The end B of the beam is hinged in a wall. The beam is kept horizontal by a rope attached to A and to a point C in the wall at a height 0.75m above B

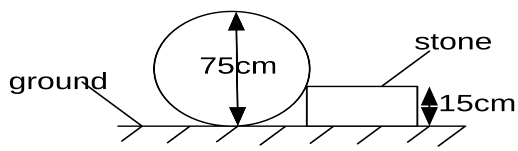
i. Draw a diagram to show the forces on the beam (2 marks)
ii. Calculate the tension in the rope (4 marks)
iii. What is the reaction exerted by the hinge on the beam (5 marks)

An (89.8N, 72.01N, at 3.95° to the beam)

UNEB 1998 No1

d) (i)Explain the term unstable equilibrium (3mk)

(ii) An oil drum of diameter 75cm and mass 90kg rests against a stone as shown



Find the least horizontal force applied through the centre of the drum, which will cause the drum to roll up the stone of height 15cm.

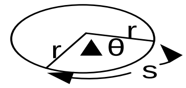
An(1177.2N) (5 marks)

CHAPTER 7: CIRCULAR MOTION

This is the motion of the body with a uniform speed around a circular path of fixed radius about a center.

Terms used in circular motion

Consider a body of mass m initially at point A moving with a constant speed in a circle of radius r to point B in a time Δt, the radius sweeps out an angle $\Delta\theta$ at the centre



1. Angular velocity (ω)

This is the is the rate of change of the angle for a body moving in a circular path.

Or rate of change of angular displacement i.e $\boldsymbol{\omega} = \frac{\Delta\boldsymbol{\theta}}{\Delta\boldsymbol{t}}$

For large angles and big time intervals. $\boldsymbol{\omega} = \frac{\theta}{t}$

Angular velocity is measured in radians per second (rads^{-1})

2. Linear speed (v)

If the distance of the arc AB is, s and the speed is constant then velocity.

$v = \frac{Arc\ length}{time} = \frac{s}{\Delta t}$

But $s = \frac{\Delta\theta}{360}\ x 2\pi r = \Delta\theta r$

Since $360° = 2\pi$ rads

$\therefore\ \ v = \frac{\Delta\theta r}{\Delta t} = r\ \omega$

Where $\frac{\Delta\theta}{\Delta t} = \omega$

$\boxed{v = r\ \omega}$–units are ms^{-1}

Definition

Velocity is the rate of change of displacement for a body moving a round a circular path about a fixed point or centre.

3. **Period T**

This is the time taken for the body to describe one complete are revolution

T$= \frac{Circumference[distance\ around\ acircle]}{velocity} = \frac{2\pi r}{v}$

But $v = r\ \omega$

T$= \frac{2\pi r}{\omega r}$

$\boxed{T = \frac{2\pi}{\omega}}$ units seconds.

1. Acceleration

Centripetal acceleration is defined as the rate of change of velocity of a body moving in a circular path and is always directed towards the centre.

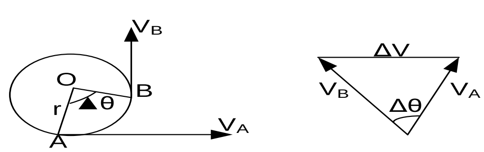
7.1.0: Derivation of $a = \frac{v^2}{r}$

Question:

Show that the acceleration of a body moving round a circular path with speed v is given by $\frac{\boldsymbol{V^2}}{\boldsymbol{r}}$ where r is the radius of the path.

Solution

Consider a body of mass m moving around a circular path of radius r with uniform angular velocity ω and speed V. If initially the body is at point A moving with velocity V_A and after a small time internal Δt, the body is at point B where its velocity is V_B with the radius having moved an angle $\Delta\theta$



Acceleration, a$= \frac{change\ in\ velocity}{time} = \frac{V_B - V_A}{\Delta\ t}$

but $V_B - V_A = V\Delta\theta$

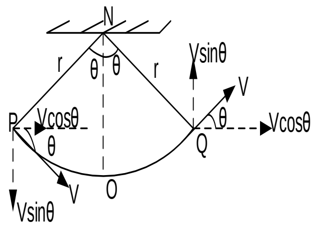
$a = \frac{V\Delta\theta}{\Delta t}$

$\frac{\Delta\theta}{\Delta t} = \omega = \frac{\mathrm{v}}{\mathrm{r}}$

$\boxed{a = \frac{\mathrm{v}^2}{\mathrm{r}}}$

Question

A volume of mass m is oscillated from a fixed point by a string of length r with a constant speed V. Shows that the acceleration of the body is $\frac{v^2}{r}$ and directed towards the centre.



Acceleration a= $\frac{change\ in\ velocity}{time}$

Horizontal component

$$a_X = \frac{vCos\theta - vCos\theta}{t}$$

$a_x = \frac{0}{t}$

a_x =0 ,the body does not accelerate horizontally.

Vertical component

$a_y = \frac{vSin\theta - -vSin\theta}{t}$

$a_y = \frac{2vSin\theta}{t}$ ------------(1)

but time (t) $= \frac{Arc\ length\ PQ}{Velocity}$

$t = \frac{2\theta}{360} x \frac{2\,\pi\, r}{v} = \frac{2\,\theta x\, 2\,\pi\, r}{2\,\pi\, v}$

where $360° = 2\pi rads$

$$t = \frac{2\,\theta\, r}{V}$$

put into equation (1)

$a_y = \frac{2\ v\ sin\theta}{\left(\frac{2\,\theta\, r}{v}\right)} = \frac{v^2 Sin\,\theta}{r\,\theta}$

for small angle sinθ $\approx \theta$

$a_y = \frac{v^2\theta}{r\,\theta}$

$$\boxed{a_y = \frac{v^2}{r}}$$

EXAMPLE

1. A particle moves along a circular path of radius 3.0m with an angular velocity of 20 rad s^{-1} calculate;
   a) The linear speed of the particle
   b) Angular velocity in revolutions per second
   c) Time for one revolution
   d) The centripetal acceleration

Solution

$r = 3m$ $\omega = 20$ rads^{-1}

a) Linear speed $v =$ r$\boldsymbol{\omega}$

$v = 20x3$ =60ms^{-1}

b) Angular velocity in rev per second gives the frequency

$\omega = 2\pi f$ $\therefore$ $f = \frac{\omega}{2\pi}$

$f = \frac{20}{2\pi} = 3.18$ rev per second

c) Time for one revolution (T)

$T = \frac{1}{f} = \frac{1}{3.18} = 0.31\ s$

d) Acceleration $a = \frac{v^2}{r}$

$a = \frac{60^2}{3} = 1200 ms^{-2}$

2. A body is fixed on the string and whirled in a circle of radius 10cm. If the period is 5s. find
   i) The angular velocity
   ii) The speed of the body in the circle
   iii) The acceleration of the body
   iv) The frequency

Solution

i) $\omega = \frac{\theta}{t}$

its whirled in a circle

$(\theta = 360° = 2\pi)$

$\omega = \frac{2\pi}{t} = \frac{2x\frac{22}{7}}{5} = 1.26$rads^{-1}

ii) $v = \boldsymbol{\omega}$ r

$v = 1.26x\frac{10}{100} = 0.13$ms^{-1}

iii) $a = \omega^2 r$

$a = (1.26)^2 x$0.1$= 0.169 ms^{-2}$

iv) $f = \frac{2\,\pi}{\omega} = \frac{2x\frac{22}{7}}{1.26} = 0.2Hz$

EXERCISE:15

1. A particle of mass 0.2kg moves in a circular path with an angular velocity of 5 rads^{-1} under the action of a centripetal force of 4N. What is the radius of the particle. **An(0.8m).**
2. Calculate the tension in the wire of hammer throwers when a hammer of mass 7kg is being swung round at 1 rev per second in a circle of radius 1.5m **An(414N)**
3. What force is required to cause a body of mass 3g to move in a circle of radius 2m at a constant rate of 4 revolutions per second. **An(3.8N)**
4. A particle moves along a circular path of radius 3m with an angular velocity of 20. Calculate the
   (i) Linear speed of particle
   (ii) Angular velocity in revolutions per second
   (iii) Time taken for one revolution. **An(60ms^{-1}, $3.2 rev\ s^{-1}$, $0.31s$)**
5. An astronaut is trained in a centrifuge that has an arm of length 6m. if the astronaut can stand an acceleration of 9g ms^{-2}, what is the maximum number revolutions per second that the centrifuge may make?

7.1.1: CENTRIPETAL AND CENTRIFUGAL FORCES

If a body is moving in a circle, it will experience an initial outward force called **centrifugal force**. These forces always act away from the center and are perpendicular to the direction of motion.
In order for the body to continue moving in a circle without falling off, there must be an equal and opposite force to the centrifugal force. This force which counter balances the centrifugal force is called the **centripetal force** and always acts towards the center of the motion.

Definition

Centripetal force is an inward force towards the center of the circle required to keep a body moving in a circular path
If the mass of the body is m then the centripetal force
$F =$ma

But a = $\frac{v^2}{r}$

$\boxed{F = \frac{m\,v^2}{r}}$ This is the expression for the centripetal force Or $\boxed{F = mr\,\omega^2}$

Question

Explain why there must be a force acting on a particle which is moving with uniform speed in a circular path. Write down an expression for its magnitude.

Solution

- If a body is moving along circular path, there must be a force acting on it, for if there were not, it would move in a straight line in accordance with Newton's first law.
- Since the body is moving with a constant speed, this force cannot at any stage have a component in direction of motion of a body. For it did, it would increase or decrease the speed of the body. The force on the body must therefore be perpendicular to direction of motion and directed towards the center.

7.1.2: Examples of centripetal forces

1. **A car moving around a circular truck**: For a car negotiating a corner or moving on a circular path, the frictional force between the wheels and the surface provides the necessary centripetal force required to keep it on the track.

2. **A car moving on banked track**:
   For a banked track, the centripetal force is provided by the frictional force and the horizontal components of the normal reaction.

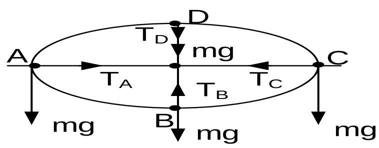
3. a) **Tension on the string keeping a whirling body in a vertical circle**.
   The tension force in the string provides the necessary centripetal force
   b) **For the conical pendulum, the horizontal component of the tension in the string** provides the necessary centripetal force

4. **Gravitational force on planets**
   For a planet orbiting round the sun or satellite revolving about the earth, the gravitational force between the two bodies provides the necessary centripetal force required to keep the satellite in the orbit.

5. **Electrostatics force on the electrons**
   For electrons moving round the nucleus, the electrostatics force provides the necessary centripetal force.

7.1.3: Motion in a vertical cycle

Consider a body of mass m attached to a string of length r and whirled in a vertical circle with a constant speed V. If there is no air resistance to the motion, then the net force towards the centre is the centripetal force.



At point A: $T_A = \frac{m\,v^2}{r}$ ------------------------(2)

At point B: $T_B = \frac{m\,v^2}{r} + mg$ - ----------------(3)

At point C: $T_C = \frac{m\,v^2}{r}$--------------------------(4)

At point D: $T_D = \frac{m\,v^2}{r} - mg$ ------------------(5)

The maximum tension in the vertical circle is experienced at B

$T_{max} = \frac{m\,v^2}{r} + mg$

The minimum tension is experienced on the top of the circle at point D

$T_{min} = \frac{m\,v^2}{r} - mg$

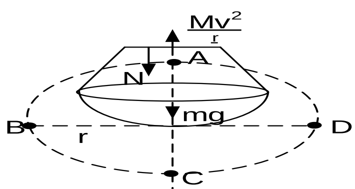
Note

If the speed of whirling is increased the string will most likely break at the bottom of the circle. Motion is tangential to the circle and when string breaks the mass will fly in a parabolic path.

Question

Explain why a bucket full of water can be swung round a vertical circle without spilling.

Solution

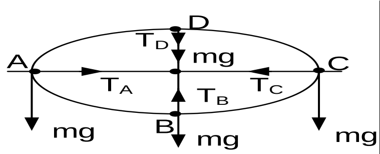


When the bucket is inverted vertically above the point of support, the weight of the water is less than the required centripetal force, the reaction at bucket base on the water proiveds the rest of the centripetal force so the water stays in the bucket

Examples

1. An object of mass 3kg is whirled in a vertical circle of radius 2m with a constant sped of $12ms^{-1}$, calculate the maximum and minimum tension in the string

Solution



Maximum tension is at B

$T - mg = \frac{m\,v^2}{r}$

T $= \frac{3x12^2}{2} + 3x9.81 = 245.43N$

Minimum tension is at D

T$= \frac{m\,v^2}{r} - mg$

$T = \frac{3x12^2}{2} - 3x9.81$

T $= 186.57N$

2. A stone of mass 800g is attached to string of length 60cm which has a breaking tension of 20N. The string is whirled in a vertical circle the axis of rotation at a height of 100cm from the ground.
   i) What is the angular velocity where the string is most likely to break?
   ii) How long will it take before the stone hits the ground?
   iii) Where the stone hit the ground

Solution

i) The string breaks when $T_{max} = \frac{m\,v^2}{r} + mg$

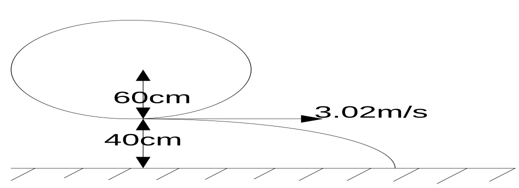
$20 = 0.8(9.81 + \frac{v^2}{0.6})$

$v = 3.02ms^{-1}$

But $v = r\,\omega$

$\omega = \frac{3.02}{0.6} = 5.03rads^{-1}$

ii)



$y = -40cm$ (below the point of projection)

$-0.4 = 3.02tsin0 - ½\,x9.81t^2$

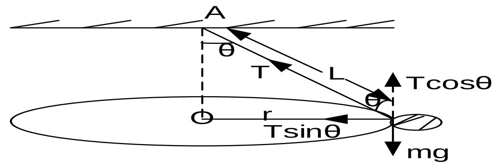
$t = 0.286s$

iii) Horizontal range

$x = utcos\theta = 3.02x0.285cos0 = 0.86m$

7.1.4: MOTION IN A HORIZONTAL CIRCLE [CONICAL PENDULUM]

Consider a body of mass , m tied to a string of length L whirled in a horizontal circle of radius r at a constant speed V



If the string is fixed at A and the centre O of the circle is directly below O, the horizontal components of the tension provides the necessary centripetal force.

$(\rightarrow)\ T\sin\theta = \frac{mv^2}{r}$ ----------- (1)

$(\uparrow)\ T\cos\theta = mg$ ----------- (2)

(1) ÷ (2): $\tan\theta = \frac{v^2}{rg}$

$\boxed{v^2 = rg\tan\theta}$ -------- (3)

but also $\sin\theta = \frac{r}{L}$

r$= L\sin\theta$

and $v = r\,\omega$

put into equation (3)

$(r\,\omega)^2 = rg\tan\theta$

$\omega^2 = \frac{g\tan\theta}{r}$

But $r = L\sin\theta$

$\omega^2 = \frac{g\tan\theta}{L\sin\theta} = \frac{g}{L\sin\theta}\,{\sin\theta}/{\cos\theta}$

$\boxed{\omega = \sqrt{\frac{g}{L\cos\theta}}}$ ---------- (4)

Also $T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{g}{L\cos\theta}}}$

$\boxed{T = 2\pi\sqrt{\frac{L\cos\theta}{g}}}$

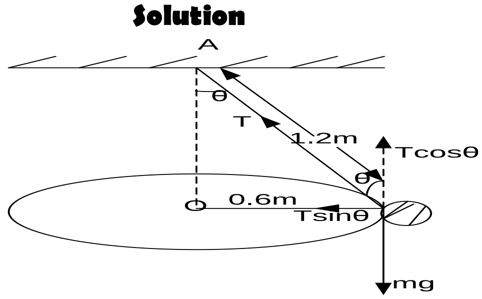
Explain why a mass attached to a string rotating at a constant speed in a horizontal circle will fly off at a tangent if the string breaks

- ❖ When a mass is whirled in a horizontal circle, the horizontal component of the tension ($T\sin\theta$) provides the necessary centripetal force which keeps the body moving in a circle without falling off.
- ❖ When the string breaks, the mass will not have any centripetal force and will continue in a straight line along the tangent.

Example

1. A stone 0.5kg is tied to one end of a string 1.2m long and whirled in a horizontal circle of diameter 1.2m. Calculate;
   i) The length in the string ii) The angular velocity iii) The period of motion

Solution



i) $(\uparrow) T\cos\theta = 0.5gN$ ---(1)

But $\sin\theta = \frac{0.6}{1.2}$ ∴ θ= 30°

put into: (1) $T\cos 30 = 0.5x9.81$

$T = 5.60N$

ii) Angular velocity

$\omega = \sqrt{\frac{g}{L\cos\theta}}$

$\omega = \sqrt{\frac{9.81}{1.2\cos 30}}$

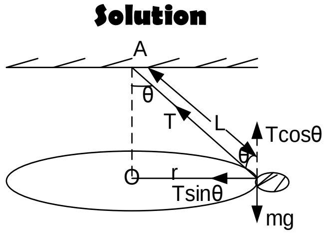
ω =3.07rads^{-1}

iii)Period , $T = \frac{2\pi}{\omega}$

$T = \frac{2x\frac{22}{7}}{3.07} = 2.05s$

2. A body of mass 4kg is moving with a uniform speed 5ms^{-1} in a horizontal circle of radius 0.3m, find:
   i) The angle the string makes with the vertical
   ii) The tension on the string

Solution



$(\rightarrow)\ T\sin\theta = \frac{mv^2}{r}$ ....[1]

$(\uparrow) T\cos\theta = mg$ ………..[2]

[1]÷[2] $\tan\theta = \frac{v^2}{rg}$

$\theta = \tan^{-1}\frac{v^2}{rg}$

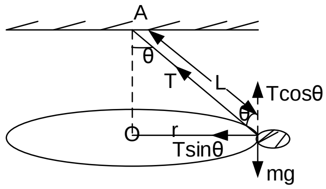
$\theta = \tan^{-1}\left(\frac{5^2}{0.3x9.81}\right) = 83.3°$

ii) $T\cos\theta = mg$

$T = \frac{4x9.81}{\cos 83.3} = 336.33N$

3. The period of oscillation of a conical pendulum is 2s. If the string makes an angle of 60° with the vertical at the point of suspension, Calculate;
   i) The length of the string ii) The velocity of the mass

Solution



$\theta = 60°$

$sin60° = \frac{r}{l}$

r= $l$sin60°------------- (1)

(↑) $Tcos\theta = mg$

(→) $Tsin\theta = \frac{m\,v^2}{r}$

$Tan\theta = \frac{v^2}{rg}$

$v^2 = rgtan60$ ------- (2)

Also $T = \frac{2\,\pi}{\omega} = \frac{2\,\pi}{2} = 3.14$rads^{-1}

But $v = r\,\omega = 3.14r$

Put into equation (2)

$v^2 = rgtan60$

$(3.14r)^2 = rgtan60$

$r = \frac{g\,tan60}{314^2}$

put into equation (1)

$r = Lsin60$

$\frac{g\,tan60}{314^2} = lsin60$

$L = \frac{g\,tan60}{3.14^2 sin60} = 1.986m$

OR

$T = 2\pi\sqrt{\frac{Lcos\theta}{g}}$

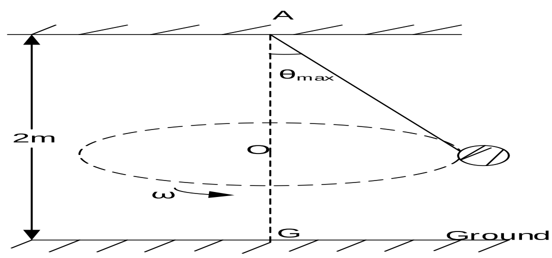
$L = \frac{T^2\,g}{4\pi^2\,cos\theta}$

$L = \frac{2^2\,x9.81}{4\left(\frac{22}{7}\right)^2\,cos60} = 1.986m$

$v = r\,\omega$

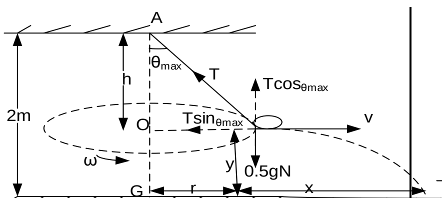
$v = \frac{2\pi}{T}$r= $\frac{2\pi}{2}$ $l$sin60°=5.4ms^{-1}

4. Stone of mass 0.5kg is tied to one end of the string 1m long. The point of suspension of the string is 2m above the ground. The stone is whirled in the horizontal circle with increasing angular velocity. The string will break when the tension in it is 12.5N and the angle θ is to the maximum (θ_{max})as shown in the figure below;



i) Calculate the angle θ_{max}
ii) Calculate the angular velocity of the stone when the string breaks
iii) How far from the point G on the ground will the stone hit the ground
iv) What will be the speed of the stone when it hits the ground

Solution



(↑) $Tcos\theta_{max} = 0.5gN$

$cos\theta_{max} = \frac{0.5x9.81}{12.5}$

θ_{max}= 66.9°

(→) Tsinθ $= m\omega^2$r

Also sinθ $= \frac{r}{l}$

T $\frac{r}{1} = m\omega^2$r

T=m ω^2

$\omega^2 = \frac{12.5}{0.5}$ rads^{-1}

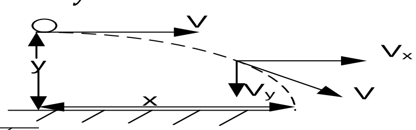
ω = 5rads^{-1}

$Cos\theta_{max} = \frac{h}{1}$

$h = cos66.9 = 0.39m$

$y + h = 2$

$y = 2 - 0.39 = 1.61m$



Using $y = utsin\theta - ½\,gt^2$

$y = -1.61$ below the point of projection

$-1.61 = ut\,sin0 - ½\,x9.81t^2$

$-1.61 = -½\,x9.81t^2$

Horizontal distance

$x = vcos\theta t$

$x = vcos0x0.57 = 0.57v$

but $v = \omega r$

$x =$0.57 ωr

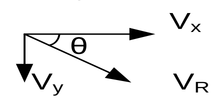
$x = 0.57x5xsin66.9$

where $sin66.9 = \frac{r}{l}$

$x = 2.63m$

$\therefore G = r + x$

$G = 2.62 + sin66.9 = 3.54m$



v_x is constant

v_x =ucosθt

$v_x = vcos0x0.57 = 0.57v$

$v_x = 0.57\omega r = 0.57x5xsin66.9$

v_x =4.599ms^{-1}

$v_y = usin\theta + gt$

$v_y = usin0 + 9.91x0.57$

$v_y = 5.592ms^{-1}$

$v_R = \sqrt{V_X^2 + V_y^2}$

$v_R = \sqrt{4.599^2 + 5.592^2}$ =7.24ms^{-1}

θ =tan$^{-1}\frac{5.592}{4.599}$

The speed as it hits the ground is 7.24ms^{-1}.

EXERCISE:16

1. A stone of mass 500g is attached to string of length 50cm which will break when the tension in it exceeds 20N. The string is whirled in a vertical circle the axis of rotation at a height of 100cm from the ground.
   i) What is the angular velocity where the string is most likely to break?

ii) Where will the stone hit the ground **An(7.8rad**s^{-1}, $1.25m$)

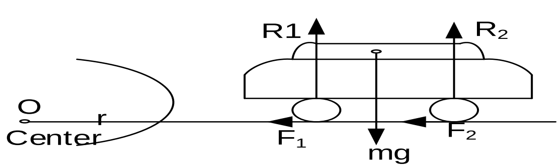
3. A bucket of water is swung in a vertical circle of radius 64.0m in such a way that the bucket is upside down when it is at the top of the circle. What is the minimum speed that the bucket ,may at this point if the water is to remain in it. **An[25.06ms^{-1}]**
4. An aero plane loop a path in a vertical circle of radius 200m, with a speed of $40ms^{-1}$ at the top of the path. The pilot has a mass of 80kg. what is the tension in the strap holding the pilot into his seat when he is at the top of the path **An[60N]**
5. An astronaut loop a path in a horizontal circle of radius 5m. if he can withstand a maximum acceleration of 78.5ms^{-2} .What is the maximum angular velocity at which the astronaut can remain conscious **An[$3.96rads^{-1}$]**
6. A body of mass 20kg is whirled in a horizontal circle using an inelastic string which has a breaking force of 400N. If the breaking speed is at 9ms^{-1}. Calculate the angle which the string makes with the horizontal at the point of breaking. **An(θ=29.3°).**
7. A particle of mass 0.2kg is attached to one end of a light inextensible string of length 50cm. The particle moves in a horizontal circle with an angular velocity of 5.0rads^{-1} with the string inclined at θ to the vertical. Find the value of θ. **An(37°)**
8. A particle of mass 0.25kg is attached to one end of a light in extensible string of length 3.0m. The particle moves in a horizontal circle and the string sweeps out the surface of a cone. The maximum tension that the string can sustain is 12N. Find the maximum angular velocity of the particle. **An[4rads^{-1}].**
9. A particle of mass 0.30kg moves with an angular velocity 0f 10rads^{-1} in a horizontal circle of radius 20cm inside a smooth hemispherical bowl. Find the reaction of the bowl on the particle and the radius of the bowl. **An[6.7N, 22cm]**
10. A child of mass 20kg sits on a stool tied to the end of an inextensible string 5m long, the other end of the string being tied to a fixed point. The child is whirled in a horizontal circle of radius 3m with a child not touching ground.
    i) Calculate the tension on the string
    ii) Calculate the speed of the child as it moves around the circle. **An[245.25N, 4.695ms^{-1}]**

7.1.5: MOTION OF A CAR ROUND A FLAT HORIZONTAL TRACK [NEGOTIATING A BEND]

Consider a car of mass m moving round a circular horizontal arc of radius r with a speed v

A) Skidding of the car

Skidding is the failure of a vehicle to negotiate a curve as a result of having a centripetal force less than the centrifugal force and the car goes off the track or moves away from the centre of the circle.
Consider a car of mass m taking a flat curve of radius r at a speed v. F_1 and F_2 are the frictional forces due to the inner tyre and outer tyre respectively. R_1 and R_2 are the normal reactions due to inner and outer tyres respectively.



$(\uparrow): R_1 + R_2 = mg$--------- (1)

$(\rightarrow): F_1 + F_2 = \frac{m v^2}{r}$ -----(2)

The frictional forces F_1 and F_2 provide the necessary centripetal force

But $F_1 = \mu R_1$, $F_2 = \mu R_2$

$\mu R_1 + \mu R_2 = \frac{m v^2}{r}$

$\mu (R_1 + R_2) = \frac{m v^2}{r}$ --------------------- (3)

Put equation (1) into equation (3)

$\mu mg = \frac{mv^2}{r}$

$v^2 = rg\mu$

The maximum speed with which no skidding occurs is given by

$\boxed{v_{max} = \sqrt{\mu rg}}$

For no skidding

$\mu \geq \frac{v^2}{rg}$ Or $v^2 \leq \mu rg$

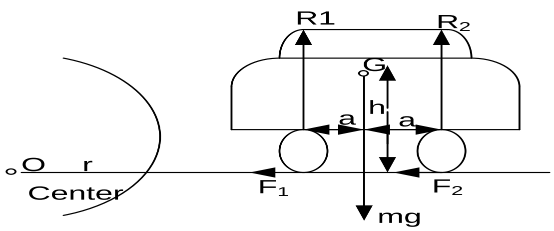
Conditions for no skidding/side slips

For a car to go round a bend successfully without skidding then:

1- The speed should not exceed $(\mu rg)^{\frac{1}{2}}$ or $[v \leq \sqrt{\mu rg}]$
2- The radius of the bend should be made big
3- Coefficient of friction should be increased
4- Centre of gravity should be low

B) Overturning/toppling of a car

Consider a car of mass m moving around a horizontal (flat)circular bend of radius r at speed v. let the height of the centre of gravity above the track be "h" and the distance between the wheels be "2a".



$(\uparrow)$: $R_1 + R_2$ =mg----- (1)

$(\rightarrow)$: $F_1+F_2 = \frac{m\,v^2}{r}$ -----(2)

Taking moments about G

Clockwise moments =anticlockwise moments

$F_1.h + F_2.h + R_1.a = R_2.a$

$(F_1 + F_2)h + R_1 a = R_2.a$ -------------- (3)

Put equation 2 into equation 3

$\frac{m\,v^2}{r}.h + R_1 a = R_2.a$

$\frac{m\,v^2}{r}.\frac{h}{a} = (R_2 - R_1)$----------------------[4]

Equation 1+Equation 4

$R_1 + R_2 + \frac{m\,v^2}{r}.\frac{h}{a} = (R_2 - R_1) + mg$

$2R_1 = mg - \frac{mv^2h}{ra}$

$R_1 = \frac{m}{2}\left(g - \frac{v^2h}{ra}\right)$----------------------- (5)

A car just topples or upsets when $R_1 = 0$

$\frac{m}{2}\left(g - \frac{v^2h}{ra}\right)$ =0

$g = \frac{v^2h}{ra}$

$v_{max} = \sqrt{\frac{rag}{h}}$

Note

R_1 is the reaction of the inner tyre

- When R_1>0: The wheels in the inner side of the curve is in contact with the ground
- When R_1=0: The wheels in the inner side of the curve are at the point of loosing contact with the ground
- When R_1<0: The inner wheels have lost contact with the ground and the vehicle has over turned

Ways prevent toppling/overturning

i) Reduce the speed when negotiating a corner ($v^2 \leq \frac{rag}{h}$)

ii) Increase radius of a corner (r $>\frac{v^2h}{ra}$)

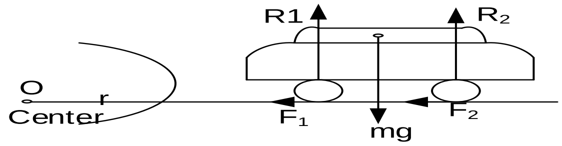
iii) The distance between the tyres should be made big (a $>\frac{v^2h}{ra}$)

iv) Reduce distance from the ground to the centre of gravity (h) or C.O.G of the car should be low (h< $\frac{rag}{v^2}$)

EXAMPLE

1. A car of mass 1000kg goes round a bend of radius 100m at a speed of 50km/hr without skidding. Determine the coefficient of friction between the tyres and the road surface

Solution



$(\uparrow)$: $R_1 + R_2 = mg$----- (1)

$(\rightarrow)$: $F_1 + F_2 = \frac{m\,v^2}{r}$

$\mu(R_1 + R_2) = \frac{m\,v^2}{r}$-------------------------[2]

Put equation (1) and equation 2: μ mg= $\frac{m\,v^2}{r}$

$\mu = \frac{v^2}{rg} = \frac{\left(\frac{50x1000}{3600}\right)^2}{100x9.81} = 0.1965$

MOTION OF A CAR ON A BANKED TRACK

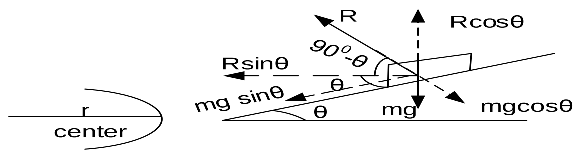
Definition :Banking a track is the building of a track round a corner with the outer edge raised above the inner one.

Banking ensures that only the horizontal component of normal reaction contributes towards the centripetal force.

Banking also enables the car to go round a bend at a higher speed for the same radius compared to a flat track.

A) NO SIDE SLIPP [No frictional force]

Consider a car of mass m negotiating a banked track at a speed v and radius of the bend is r.



$(\uparrow)$:$Rcos\ \theta = mg$ ------- (1)

$(\rightarrow)$:$Rsin\ \theta = \frac{m\,v^2}{r}$----(2)

(2) ÷ (1): $\frac{RSin\theta}{RCos\theta} = \frac{m\,v^2}{rmg}$

$\tan\theta = \frac{v^2}{rg}$

$v^2 = rgtan\ \theta$

θ is the angle of banking and v is the designed speed of the banked track.

Examples

1. A racing car of mass 1000kg moves around a banked track at a constant speed of 108km/hr, the radius of the track is 100m. Calculate the angle of banking and the total reaction at the tyres.

Solution

$\theta = tan^{-1}\left(\frac{v^2}{rg}\right) = tan^{-1}\left[\frac{\left(\frac{108x1000}{3600}\right)^2}{100x9.81}\right]$ =42.5°

Resolving vertically:$Rcos\theta = mg$

$R\ = \frac{1000x9.81}{cos42.5} = 13305N$

Exercise :17

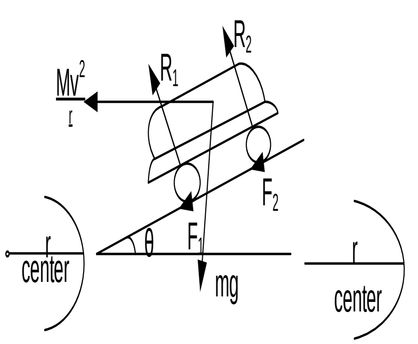
1. A road banked at 10° goes round a bend of radius 70m. At what speed can a car travel round the bend without tending to side slip. **An[11ms⁻¹]**
2. A car travels round a bend of radius 400m on a road which is banked at an angleθ to the horizontal. If the car has no tendency to skid when traveling at 35ms^{-1}, find the value of θ **An[17.34°]**
3. A driver has to drive a car in a horizontal circular path of radius 105m around a bend that is banked at 45° to the horizontal. The driver finds that he must drive with a speed of at least 21ms^{-1} if he is to avoid slipping sideways. Find the coefficient of friction between the tyres of the car and road **An[0.4]**

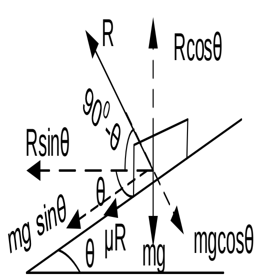
B) SKIDDING/SLIDE SLIPP

The frictional force must be there whose direction depends on the speed of the car.

(i) MAXIMUM SPEED/GREATEST SPEED

If the car is moving at speed v, greater than the designed speed v, the force Rsinθ is enough to provide the necessary centripetal force. The car will tend to slid outwards from the circular path, the frictional force would therefore oppose their tendency up to the maximum value .





$(\uparrow)$: $Rcos\ \theta = mg + \mu\ Rsin\ \theta$

$R(cos\ \theta\ - \mu\ sin\ \theta) = mg$ ------- (1)

$(\rightarrow)$: $Rsin\ \theta + \mu R\ cos\theta = \frac{m\,v^2}{r}$

$R\ (sin\theta + \mu\ \ cos\ \theta) = \frac{m\,v^2}{r}$ ------ (2)

(2)÷ (1): $\frac{R(\sin\theta + \mu\ \cos\theta)}{R(\cos\theta - \mu\sin\theta)} = \frac{m\,v^2}{rmg}$

$\frac{(Sin\theta + \mu Cos\theta)}{(Cos\theta - \mu Sin\theta)} = \frac{v^2}{r}g$

$v_{max}^2\ = \ rg\frac{(Sin\theta + \mu Cos\theta)}{(Cos\theta - \mu Sin\theta)}$

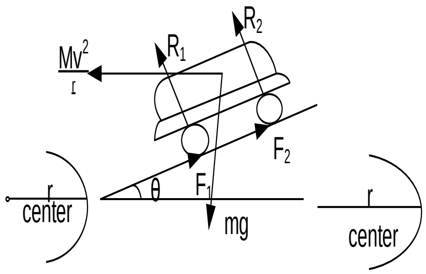
Or divide the right hand side by cosθ

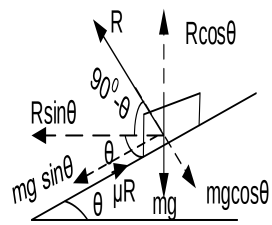
$v_{max}^2\ = \ rg\left[\frac{(tan\theta + \mu)}{(1 - \mu tan\theta)}\right]$

(ii) MINIMUM SPEED/LEAST SPEED

If the speed v, is less than the designed speed v the component of the reaction $Rsin\ \theta$ produces an acceleration greater than the centripetal acceleration ($\frac{v^2}{r}$) which is required to keep the car on circular path.

The car tends to slip down the banked track and this tendency is opposed by the frictional force acting upwards.





$(\uparrow): Rcos\ \theta + \mu\ Rsin\ \theta = mg$

$R(cos\ \theta + \mu sin\ \theta) = mg$ ------------- (1)

$(\rightarrow): Rsin\ \theta\ -\ \mu\ Rsin\ \theta = \frac{\mathrm{m\ v^2}}{\mathrm{r}}$

$R(sin\ \theta -\ \mu cos\ \theta\ ) = \frac{\mathrm{m\ v^2}}{\mathrm{r}}$------------- (2)

(2)÷ (1): $\frac{(Sin\theta - \mu Cos\theta)}{(Cos\theta + \mu Sin\theta)} = \frac{v_{min}^2}{rg}$

Divide the right hand side by cosθ

$v_{min}^2\ =\ rg\left[\frac{(tan\theta - \mu)}{(1+\mu tan\theta)}\right]$

Example

1. A car travels round a bend which is banked at 22°. If the radius of the curve is 62.5m and the coefficient of friction between the road surface and tyres of the car is 0.3, calculate the maximum and minimum speed at which the car can negotiate the bend without skidding.

Solution

$v_{max}^2\ =\ rg\left[\frac{(tan\theta + \mu)}{(1-\mu tan\theta)}\right]$

$v_{max} = \left[62.5x9.81\left(\frac{tan22+0.3}{1-0.3tan22}\right)\right]^{\frac{1}{2}}$ =22.15ms⁻¹

$v_{min}^2\ =\ rg\left[\frac{(tan\theta - \mu)}{(1+\mu tan\theta)}\right]$

$v_{min} = \left[62.5x9.81\left(\frac{tan22-0.3}{1+0.3tan22}\right)\right]^{\frac{1}{2}}$ =7.54ms⁻¹

2. On a level race track, a car just goes round a bend of radius 80m at a speed of 20ms⁻¹ without skidding. At what angle must the track be banked so that a speed of 30ms⁻¹ can just be reached without skidding, the coefficient of friction being the same in both cases.

Solution

Case I: of a level track

For no skidding $V_{max} = \sqrt{\mu rg}$

$20^2 =\ \mu x 80 x 9.81$

$\mu =\ 0.51$

Case II: on a banked track

$v_{max}^2 = \mathrm{rg}\left[\frac{(tan\theta + \mu)}{(1-\mu tan\theta)}\right]$

$30^2 = 80x9.81\left[\frac{(tan\theta + \mu)}{(1-\mu tan\theta)}\right]$

$\frac{900}{80x9.81} = \frac{(tan\theta+0.51)}{(1-0.51tan\theta)}$

$tan\theta = \frac{0.6368}{1.58468}$

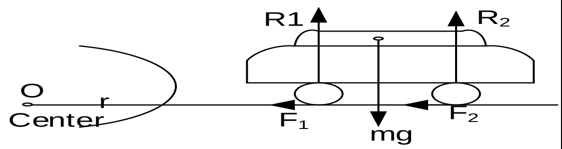
$\theta = 21.89°$

EXERCISE:18

1. A racing car of mass 2 tonnes is moving at a speed of 5ms⁻¹ round a circular path. If the radius of the track is 100m. calculate;
   i) Angle of inclination of the track to the horizontal if the car does not tend to side slip
   ii) The reaction to the wheel if it's assumed to be normal to the track. **An[1.5°, 19606.7N]**
2. A car travels round a bend banked at an angle of 22.6°. if the radius of curvature of the bend is 62.5m and the coefficient of friction between the tyres of the car and the road surface is 0.3. Calculate the maximum and minimum speed at which the car negotiates the bend without skidding. **An [22.38ms⁻¹, 7.96ms⁻¹]**
3. A car moves in a horizontal circle of radius 140cm around a banked corner of a track. The maximum speed with which the car can be driven around the corner without slipping occurring is $42ms^{-1}$. If the coefficient of friction between the tyres of the car and the surface of the track is 0.3. find the angle of banking **An[71.1°]**

Question: Explain why a car travels at a higher speed round a banked track without skidding unlike the flat tracks of the same radius.

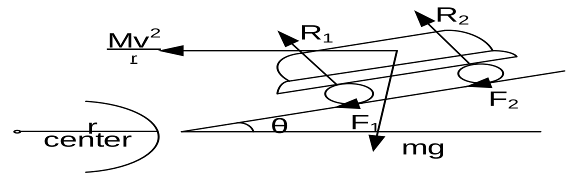
Solution



Along a circular arc on a horizontal road the frictional force provides the centripetal force

$F_{max} = \frac{\mathrm{m\,v^2}}{\mathrm{r}} = \mu$ R

At a higher speed, the frictional force is not sufficient enough to provide the necessary centripetal force and skidding would occur.



On a banked track the centripetal force is provided by both the horizontal component of normal reaction R and component of the frictional force. $F_c = F cos\,\theta + R sin\,\theta = \frac{\mathrm{m\,v^2}}{\mathrm{r}}$

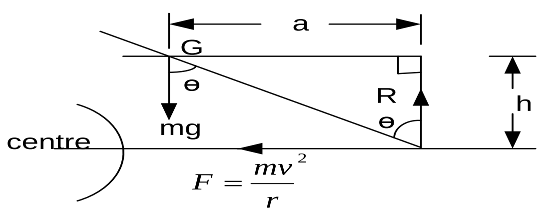
For $0° < \theta < 90°$, $\mu cos\,\theta + sin\,\theta > \mu$ therefore $V_1 < V_2$

This is enough to keep the car on the track even at high speed.

7.1.7: MOTION OF A CYCLIST ROUND A BEND

A cyclist must bend towards the centre while travelling round the bend to avoid toppling. When the cyclist bends, the weight creates a couple which opposes the turning effect of the centrifugal forces. Consider the total mass of the cyclist and his bike to be m round the circle of radius r at a speed v.

A) No skidding



$F = \frac{mv^2}{r}$

(↑): $R = mg$ ---------- (1)

(→): $\mu R = \frac{\mathrm{m\,v^2}}{\mathrm{r}}$-------------------- (2)

Put 1 into 2: $\mu mg = \frac{\mathrm{m\,v^2}}{\mathrm{r}}$

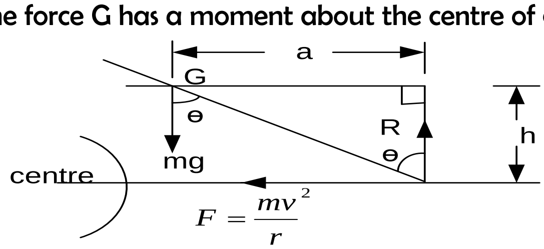
$\boldsymbol{v^2 = \mu r g}$

V is the max speed at which a cyclist negotiates a bend of radius r without skidding

For no skidding : $\mathrm{v}^2 \leq \mu r g$

B) No toppling/over turning

The force G has a moment about the centre of gravity G(F.h) which tends to turn the rider out.



$F = \frac{mv^2}{r}$

(↑) $R = mg$

Taking moment about G: $\frac{m\,v^2}{r}.h = R.a$

$\frac{a}{h} = \frac{\frac{m\,v^2}{r}}{R}$

But $\tan\theta = \frac{a}{h}$

$\tan\theta = \frac{\frac{m\,v^2}{r}}{mg}$

$\boxed{\mathrm{v}^2 = \mathrm{rgtan}\theta}$

v is the speed at which a cyclist can negotiate a corner without toppling

For no toppling $\mathrm{v}^2 \leq \mathrm{rgtan}\theta$

Why it is necessary for a bicycle rider moving round a circular path to lean towards a center of the path

When a rider moves round a circular path, the frictional force provides the centripetal force. The frictional force has a moment about the centre of gravity of the rider, the rider therefore tends to fall off from the centre of the path if this moment is not counter balanced. The rider therefore leans toward the center of the path so that his reaction provides a moment about the center of gravity , which counter balances the moment due to friction.

UNEB 2014No1

(b) (i) Define angular velocity. (01mark)

(ii) satellite is revolving around the earth in a circular orbit at an altitude of $6\ x10^5 m$ where the acceleration due to gravity is $9.4ms^{-2}$. Assuming that the earth is spherical, calculate the period of the satellite. **An[$5.42x10^3 s$]** (03marks)

UNEB2013No3

(b) Show that the centripetal acceleration of an object moving with constant speed, v, in a circle of radius, r, is $\frac{v^2}{r}$ (04marks)

(c) A car of mass 1000kg moves round a banked track at a constant speed of 108kmh^{-1}. Assuming the total reaction at the wheels is normal to the track, and the radius of curvature of the track is 100m, calculate the;

(i) Angle of inclination of the track to the horizontal. **An[42.5°]** (04marks)

(ii) Reaction at the wheels **An[13305N]** (02marks)

UNEB 2012 No3

a) Explain what is meant by centripetal force (2mks)

b) i) Derive an expression for the centripetal force acting on a body of mass m moving in a circular path of radius r (6mrk)

ii) A body moving in a circular path of radius 0.5m makes 40 revolutions per second. Find the centripetal force if the mass is 1kg (3mk)

c) Explain the following;

i) a mass attached to a string rotating at a constant speed in a horizontal circle will fly off at a tangent if the string break (02mk)

ii) a cosmonaut in a satellite which is in a free circular orbit around the earth experiences the sensation of weightlessness even though there is influence of gravitation field of the earth.

Solution

b) ii f=40revs^{-1} r=50m m=1kg

ω =$2\pi f$ = 2x$\frac{22}{7}$x40 =251.43rads^{-1}

F=m ω^2r=1(251.43)^{2}x50= 3.161x$10^2 N$

UNEB 2011 No1

a) Define the following terms

i) Uniform acceleration (1mk)

ii) Angular velocity (1mk)

b) i) what is meant by banking of a track

(ii) Derive an expression for the angle of banking θ for a car of mass, m moving at a speed, v around banked track of radius r. (4mk)

c) A bob of mass, m tied to an inelastic thread of length L and whirled with a constant speed in a vertical circle

i) With the aid of a sketch diagram, explain the variation of tension in the string along the circle (5mk)

ii) If the string breaks at one point along the circle state the most likely position and explain the subsequent motion of the bob. [2mk]

UNEB 2007 No1

d) Explain why the maximum speed of a car on a banked road is higher than that on an unbanked road.

e) A small bob of mass 0.20kg is suspended by an inextensible string of length 0.8m. The bob is then rotated in a horizontal circle of radius 0.4m. find the

i) linear speed of the bob (3mk)

ii) tension in the string (2mk)

UNEB 2005 No4

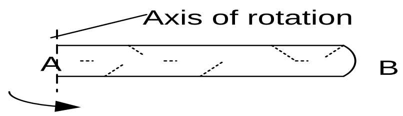
a) i) Define angular velocity (1mk)

ii) Derive an expression for the force F on a particle of mass m, moving with angular velocity ω in a circle of radius r.

b) A stone of mass 0.5kg is attached to a string of length 0.5m which will break if the tension in it exceeds 20N. The stone is whirled in a vertical circle, the axis of rotation being at a vertical height of 1m above he ground. The angular speed is gradually increased until the string breaks.
   i) In what position is the string most likely to break? Explain.
   ii) At what angular speed will the string break **An [7.78rads^{-1}, 1.24m]**
   iii) Find the position where the stone hits the ground when the string breaks
c) Explain briefly the action of a centrifuge

Solution

Action of a centrifuge



A centrifuge is used to separate substances of different densities e.g. milk and fat by whirling in a horizontal circle at a high speed.
The mixture placed in a tube and the tube is rotated in a horizontal circle. The liquid pressure at the closed end B is more than that at the open end A. this sets up a pressure gradient along the tube. This pressure gradient creates a large centripetal force that causes matter of small density to move inwards while that of higher density to move away from the centre when rotation stops, the tube is placed in a vertical position and the less dense substance comes to the top which are then separated from the mixture.

UNEB 2004 No2

a) Define the term angular velocity (1mk)
b) A car of mass m, travels round a circular track of radius, r with a velocity v.
   i) Sketch a diagram to show the forces acting on the car (2mks)
   ii) Show that the car does not overturn if $v^2 < \frac{arg}{2h}$, where a is the distance between the wheel, h is the height of the C.O.G above the ground and g is the acceleration due to gravity
c) A pendulum of mass 0.2kg is attached to one end of an inelastic string of length 1.2m. the bob moves in a horizontal circle with the string inclined at 30^0 to the vertical . Calculate;
   i) The tension in the string (2mk)
   ii) The period of the motion **An[2.27N, 2.04s]** (4mk)

UNEB 2003 No2

a) Define the following terms
   i) Angular velocity (1mk)
   ii) Centripetal acceleration (1mk)
b) i) Explain why a racing car travels faster on a banked track than one which is flat of the same radius of curvature. (4mk)
   ii) Derive an expression for the speed with which a car can negotiate a bend on a banked track without skidding (3mk)

UNEB 2002 No1

d) The period of oscillation of a conical pendulum is 2.0s. if the string makes an angel 60^0 to the vertical at the point of suspension, calculate the
   i) Vertical height of the point of suspension above the circle (3mk)
   ii) Length of the string (1mk)
   iii) Velocity of the mass attached to the string (3mk)

An[0.995m, 1.99m, 5.41ms^{-1}]

UNEB 2002 No2

b i) Derive an expression for the speed of a body moving uniformly in a circular path (3mk)
   ii) Explain why a force is necessary to maintain a body moving with a constant speed in a circular path.
c) A small mass attached to a string suspended from a fixed point moves in a circular path at a constant speed in a horizontal plane.
   i) Draw a diagram showing the forces acting on the mass (1mk)
   ii) Derive an equation showing how the angle of inclination of the string depend on the speed of the mass and the radius of the circular path (3mk).

CHAPTER 8: GRAVITATION

Gravitation deals with motion of planets in a gravitational field.

8.1.0: KEPLER'S LAWS OF GRAVITATION

Law I: Planets describe elliptical orbits with the sun at one focus

Law II: The imaginary line joining the sun and a planet sweeps out equal areas in equal time intervals

Law III: The squares of the periods of revolution of a planet about the sun is directly proportional to the cube of the mean distance from the sun to the planet. ie$T^2 \alpha r^3$

8.1.1: NEWTON'S LAWS OF GRAVITATION

It states that: the force of attraction between two bodies in the universe is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

i.e. $F \propto m_1m_2$ --------------------- (1)

$F \propto \frac{1}{r^2}$ --------------------------- (2)

Combining 1 and 2

$F \propto \frac{m_1m_2}{r^2}$

$F = \frac{Gm_1m_2}{r^2}$

G is the gravitational constant

$G = 6.67x10^{-11} Nm^2kg^{-2}$

This law is sometimes called the inverse square law of gravitation

8.1.2: DIMENSION OF G AND ITS UNITS

From $F = \frac{Gm_1m_2}{r^2}$

$G = \frac{F\,r^2}{m_1m_2}$

$[G] = \frac{[F\,][r^2]}{[m_1][m_2]} = \frac{M\,L\,T^{-2}L^2}{M^2}$

$[G] = M^{-1}L^3T^{-2}$

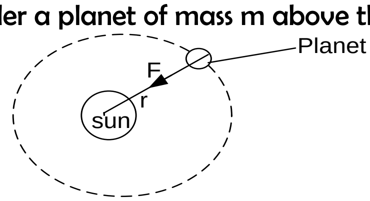
Units of G $= kg^{-1}m^3s^{-2}$ or Nm^2kg^{-2}

Exercise

1. Calculate the gravitational attraction of two cars 5m part if the masses of the cars are 1000kg and 1200kg. **An**$(3.2x10^{-6}N)$
2. Calculate the force between the sun and Jupiter if the mass of the sun is $2.0x10^{30}kg$, mass of Jupiter is $1.89x10^{27}kg$ and radius of Jupiter's orbit is $7.73x10^{11}m$. **An**$(4.22x10^{23}N)$

8.1.3: VERIFICATION OF KEPLER'S 3RD LAW

Consider a planet of mass m above the sun of m_s. If the distance separating the planet and the sun is r.



centripetal force should be provided by the gravitational force of attraction

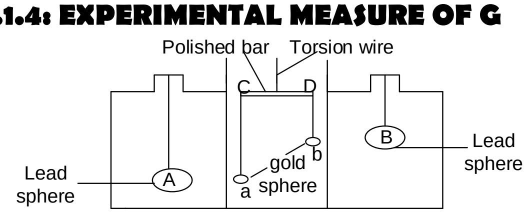
m r $\omega^2 = \frac{G\,mm_s}{r^2}$ but $\omega = \frac{2\pi}{T}$

$$m\,r\left(\frac{2\pi}{T}\right)^2 = \frac{G\,mm_s}{r^2}$$

$$T^2 = \left(\frac{4\pi^2}{Gm_s}\right)r^3$$

Since $\frac{4\pi^2}{Gm_s}$ is a constant $T^2\alpha\, r^3$

8.1.4: EXPERIMENTAL MEASURE OF G



- ❖ Two identical gold sphere a and b of mass m are suspended from the ends of a highly polished bar CD of length l
- ❖ Two large spheres A and B each of mass M are brought in position near a and b respectively.
- ❖ The distance d between a and A or b and B is measured and recorded
- ❖ The deflection θ, of bar CD is measured by lamp and scale method.

Torque of couple on CD = $\frac{G\,m\,M}{d^2}$x l

$$\frac{G\,m\,Ml}{d^2} = k\theta$$

Where k is torsional of wire per unit radian of twist

$$G = \frac{k\theta d^2}{G\,m\,Ml}$$

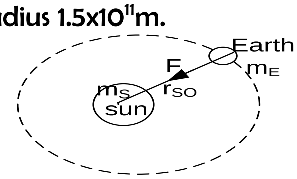
Note

- The high sensitivity of the quarts fibres enables the small deflection to be big enough to be measured accurately. The small size of the apparatus allowed it to be screened considerably from air convention currents.
- The constant k can be determined by allowing CD to oscillate through small angle and then observing its period of oscillation 'T' which was of the order of 3 minutes. If I is the known moment of inertia of the system about the torsion wire

$$T = 2\pi\sqrt{\frac{I}{k}}$$

8.1.5: MASS OF THE SUN

The mass of the sun can be estimated by considering the motion of the earth round the sun in an orbit of radius $1.5x10^{11}$m.



Force of attraction= Centripetal force

$$\frac{G\,M_E M_S}{r_{so}^2} = m_E\,\omega^2 r_{so}$$

$$m_s = \frac{\omega^2 r_{so}^3}{G}$$

But $\omega = \frac{2\pi}{T}$

$$m_s = \frac{4\pi^2 r_{so}^3}{GT^2}$$

r_{so} is radius of the orbit of the earth around the sun

r_{so} =$1.5x10^{11}$m

G =$6.67x10^{-11}Nm^{-2}kg^{-2}$

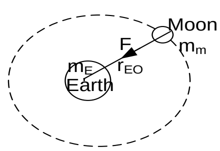
T =1yr≈365days =365x24x60x60s

r_{so} = $1.5x10^{11}$m

$$m_s = \frac{4x\left(\frac{22}{7}\right)^2 x\left(1.5x10^{11}\right)^3}{6.67x10^{11}x(365x24x60x60)^2} = 2.0x10^{30}kg$$

8.1.6: MASS OF THE EARTH

The mass of the earth can be estimated by considerably the motion of the moon round the earth in an orbit of radius $4x10^8$m



Force of attraction= Centripetal force

$$\frac{G\,M_E M_m}{r_{EO}^2} = m_m\,\omega^2 r_{EO}$$

$$m_E = \frac{\omega^2 r_{Eo}^3}{G}$$ But $\omega = \frac{2\pi}{T}$

$$m_E = \frac{4\pi^2 r_{Eo}^3}{GT^2}$$

r_{Eo} is the radius of the orbit of the moon about the earth.

r_{Eo} = $4x10^8$m

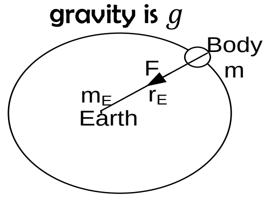
T=1 month =30days =30x24x60x60

G=$6.67x10^{-11}$

$$m_E = \frac{4x\left(\frac{22}{7}\right)^2 x\left(4x10^8\right)^3}{6.67x10^{11}x(30x24x60x60)^2} = 5.6x10^{24}kg$$

8.1.8: RELATION BETWEEN G AND g

Consider a body of mass m placed on the earth's surface of radius r_E where the acceleration due to gravity is g



Force of attraction F = $\frac{G\,M_E m}{{r_E}^2}$-------- (1)

If the body is on the earth's surface then it experiences a gravitational pull

$F = mg$ ------------------------------ (2)

Equating equation 1 and 2

$$\frac{G\,M_E m}{{r_E}^2} = mg$$

$$\boxed{G\,m_E = g\,{r_E}^2}$$

Where r_E is the radius of earth where

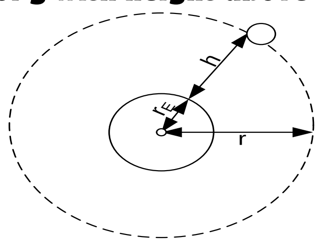
r_E =$6.4x10^6$m

Differences between G and g

G	g
Units are $m^3kg^{-1}s^{-2}$ or Nm^2kg^{-2}	Units are ms^{-2}
Occurs due to forces of attraction between two bodies	Acts on only one body
Does not vary with attitude	Varies with attitude

8.1.9: VARIATION OF g OF A BODY DURING FREE FALL

(i) Variation of g with height above the earth's surface



An object of mass m placed at a height h, above the surface of the earth where acceleration due to gravity at that height is g^1.
At a height h the gravitational force of attraction between the object and the earth is equal to the weight of the object.

$mg^1 = \frac{Gm_E\, m}{r^2}$---------------------- (1)

but $gr_E{}^2 = Gm_E$---------------------- (2)

(1)÷ (2) $\frac{mg^1}{gr_E{}^2} = \frac{Gm_E m}{r^2 Gm_E}$

$g^1 = \frac{gr_E{}^2}{r^2}$

Since $r_E{}^2$ and g are constant, $g^1 \propto \frac{1}{r^2}$

- Therefore for a point above the earth surface g varies inversely as the square of the distance r from the centre of the earth.

Examples

1. A body has a weight of 10N on the earth. What will its weight be on the moon if the ratio of the moon's mass to the earth's mass is $1.2x10^{-2}$ and the ratio of the moon's radius to that of the earth is 0.27?

Solution

Consider the body on the earth's surface

$mg = \frac{Gm_E m}{r_E{}^2}$

$g = \frac{Gm_E}{r_E{}^2}$--------------- (1)

Also on the moon's surface

$mg^1 = \frac{Gm_m\, m}{r_m{}^2}$

$g^1 = \frac{Gm_m}{r_m{}^2}$------------- (2)

eqn2 ÷ eqn 1

$\frac{g^1}{g} = \frac{m_m}{m_E} x \frac{r_E{}^2}{r_m{}^2}$

But $\frac{\mathrm{m_m}}{\mathrm{m_e}} = 1.2x10^{-2}$ and

$\frac{\mathrm{r_m}}{\mathrm{r_e}} = 0.27$

$\frac{g^1}{g} = 1.2x10^{-2} x \left(\frac{1}{0.27}\right)^2$

$g^1 = \frac{1.2x10^{-2}}{0.27^2} x9.81$

$g^1 = 1.6148ms^{-2}$

but weight 10N

$w = mg$

$10 = mx9.81$

$m = \frac{10}{9.81}$

m=1.0194kg

$w^1 = mg^1 = 1.0194x1.614$

$w^1 = 1.646N$

Alternatively

$W_m = \frac{Gm_m\, m}{r_m{}^2}$---------- (1)

$W_E = \frac{Gm_E m}{r_E{}^2}$

$10 = \frac{Gm_E m}{r_E{}^2}$---------- (2)

$\frac{W_E}{10} = \frac{\frac{Gm_E m}{r_E{}^2}}{\frac{Gm_m\, m}{r_m{}^2}}$

$W_E = \frac{m_m}{m_E} x \frac{r_E{}^2}{r_m{}^2} x10$

But $\frac{\mathrm{m_m}}{\mathrm{m_e}} = 1.2x10^{-2}$ and

$\frac{\mathrm{r_m}}{\mathrm{r_e}} = 0.27$

$W_m = 1.2x10^{-2} x \left(\frac{1}{0.27}\right)^2 x10$

$W_m = 1.65N$

2. The acceleration due to gravity on the surface of mars is about 0.4times the acceleration due to gravity on the surface of the earth. How much wound a body weigh on the surface of mars if it weighs 800N on the earth's surface .

Solution

$W_m = mg^1$

But $g^1 = 0.4g$

$W_m = mx0.4g$

$W_m = 0.4mg$

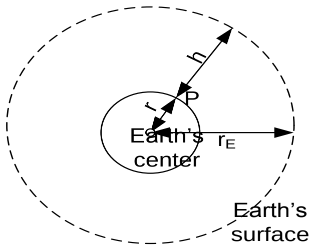
$W_m = 0.4x800$ since $mg = 800N$

$W_m = 320N$

EXERCISE:19

1. At what distance from the earth surface will the acceleration be $\frac{1}{8}$ of its value at the earth surface **An**$(1.18x10^7m)$
2. A body weighs 63N on earth surface. How much will it weigh at the height above the earth surface equal to half the radius of the earth **An**$(28N)$

(ii) Variation of g with depth below the earth surface



Consider the earth to be a uniform sphere of uniform density. Suppose a body at a point h meters from the surface of the earth measured towards the centre of the earth.

When the object is on the surface of the earth .

$$mg = \frac{Gm_Em}{r_E^2}$$

$$M_E = \frac{r_E^2 g}{G} \quad \text{------------------ (1)}$$

at p $\quad m_E{}^1 g^1 = \frac{G\, m_E{}^1\, m}{r^2}$

$$m = \frac{r^2 g^1}{G} \quad \text{---------------------------- (2)}$$

Where $m_E{}^1$ is the effective mass of that part of the earth which has a radius of r

Equation 2 divided by 1

$$\frac{m}{M_E} = \frac{r^2 g^1}{G} \Big/ \frac{r_E^2 g}{G}$$

$$\frac{m}{M_E} = \frac{r^2 g^1}{r_E^2 g} \quad \text{------------------------- (3)}$$

For masses of uniform spheres are proportional to the cube of their radii

i.e. $m \propto r^3$ and $M_E \propto r_E^3$

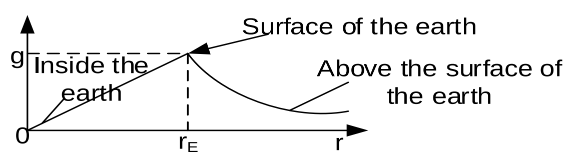
$$\frac{r^3}{r_E^3} = \frac{r^2 g^1}{r_E^2 g}$$

$$\frac{g^1}{g} = \frac{r}{r_E}$$

$$g^1 = g\frac{r}{r_E}$$

$\therefore g^1 \propto r$ for a point inside the earth

(iii) Graph of variation of acceleration of free fall from the centre of the earth



For points above the earth, the gravitational force obeys the inverse square law while for points inside the earth, g is proportional to the distance from the centre.

(iv) Variation of acceleration due to gravity with location on the surface of the earth

a) The earth is elliptical with the equatorial radius slightly greater than the polar radius. At the equator, the body is less attracted towards the earth than at the poles, acceleration due to gravity is greater at the poles than the equator
b) The earth rotates about its polar axis, weight of the body at the equator has to provide some centripetal force $m\omega^2 r$ where r is the equatorial radius, acceleration due to gravity is greater at the poles than the equator

8.2.0: MOTION OF SATELLITE

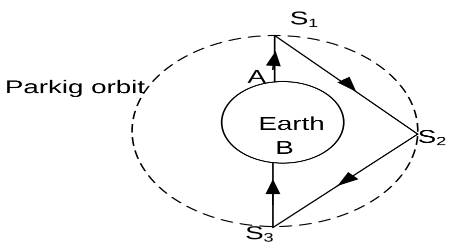
A satellite is a small body that moves in space round a planet

- Artificial satellites are grouped into
    i) Passive satellites, for these satellites, they simply reflect signals of the same strength from one point to another.
    ii) Active satellites, these satellites are able to amplify and retransmit signal that they pick from one point on the earth to another.

8.2.1: GEOSTATIONARY/SYNCHRONOUS ORBIT

These are communication satellites with orbital period of 24hrs and stays at the same point above the earth surface provided it is above the equator and its moving in the same direction as the earth is rotating.

8.2.2: HOW COMMUNICATION IS DONE USING A SATELLITE



- A set of three satellites are launched into geostationary or parking orbit
- Radio signals from A are transmitted to a geosynchronous satellite 1.
- These are re-transmitted from 1 to geosynchronous satellite 2.
- Then to geosynchronous satellite 3 which transmits to B

8.2.3: PARKING ORBIT

It's a path in space followed by a satellite which appears stationary when viewed from the earth surface.

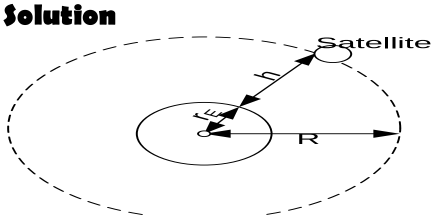
Note:

For an object (satellite) in parking orbit;
- It has a period of 24hrs
- Angular velocity relative to that of the earth is zero
- Direction of the object in the orbit is the same as the direction of rotation of the earth orbit about its axis.

Example

A communication satellite orbits the earth in synchronous orbits. Calculate the height of communication satellite above the earth.

Solution



Attractive force=Centripetal force:

$m\omega^2 R = \frac{Gm_E m}{R^2}$ but $\omega = \frac{2\pi}{T}$

$$\frac{m\,4\pi^2 R}{T^2} = \frac{Gm_E m}{R^2}$$

$$R = \left(\frac{T^2 Gm_E}{4\pi^2}\right)^{\frac{1}{3}}$$

$G = 6.67x10^{-11}$ Nm^{2}kg^{-2}

T =24hrs for synchronous orbits M_E=5.97x10^{24}kg

$$R = \left(\frac{[24x60x60]^2 x6.67x10^{-11}x5.97x10^{24}}{4x\left[\frac{22}{7}\right]^2}\right)^{\frac{1}{3}}$$

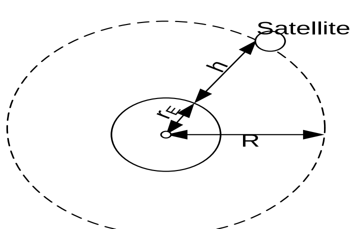
R =4.22x10^{7}m

But $R = r_E + h$ $\therefore$ r_E =6.4x10^6+h

h =4.22x10^7−6.4x10^6= 3.58 x10^{7}m

8.2.4: PERIOD OF A SATELLITE

Consider a satellite of mass m moving in a circular orbit of radius h above the earth surface.



Attractive force=Centripetal force:

$m\omega^2 R = \frac{Gm_E m}{R^2}$

$$\omega = \frac{2\pi}{T}$$

$$\frac{m\,4\pi^2 R}{T^2} = \frac{Gm_E m}{R^2}$$

$$\boxed{T^2 = \frac{4\pi^2 R^3}{Gm_E}}$$

OR Where $R = r_E + h$

But also $Gm_E = gr_E{}^2$

$$\boxed{T^2 = \frac{4\pi^2 R^3}{gr_E{}^2}}$$

Examples

1. If the moon moves round the earth in a circular orbit of radius $=4.0\times10^{8}$m and takes exactly 27.3 days to go round once, calculate the value of acceleration due to gravity g at the earth's surface. (04marks)

$m\omega^2R = \frac{Gm_Em}{R^2}$ but $\omega = \frac{2\pi}{T}$

$$\frac{m\,4\pi^2R}{T^2} = \frac{Gm_Em}{R^2}$$

But $G\,m_E = g\,{r_E}^2$

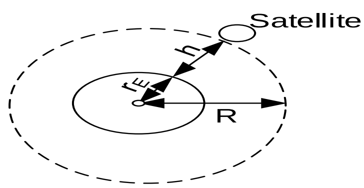
$$g = \frac{4\pi R^3}{T^2{r_E}^2}$$

$$g = \frac{4x\left(\frac{22}{7}\right)^2x\left(4.0x10^8\right)^3}{(27.3x24x60x60)^2x(6.4x10^6)^2} = 11.09\text{ms}^{-2}$$

2. The period of the moon round the earth is 27.3days. If the distance of the moon from the earth is 3.83×10^{5}km. Calculate the acceleration due to gravity at the face of the earth. **An [g=9.72ms^{-2}]**

3. Find the period of revolution of a satellite moving in a circular orbit round the earth at a height of 3.6×10^{6}m above the earth's surface.

Solution



Attractive force=Centripetal force:

$m\omega^2R = \frac{Gm_Em}{R^2}$ but $\omega = \frac{2\pi}{T}$

$$\frac{m\,4\pi^2R}{T^2} = \frac{Gm_Em}{R^2}$$

$$T = \left(\frac{4\pi^2R^3}{Gm_E}\right)^{\frac{1}{2}}$$

Where $R = \mathrm{r_E} + \mathrm{h}$ But also $Gm_E = g{r_E}^2$

$$T = \left(\frac{4\pi^2(\mathrm{r_E}+\mathrm{h})^3}{Gm_E}\right)^{\frac{1}{2}}$$

r_E is radius of earth $=6.4\times10^{6}$m

m_E is mass of earth $=6\times10^{24}$kg

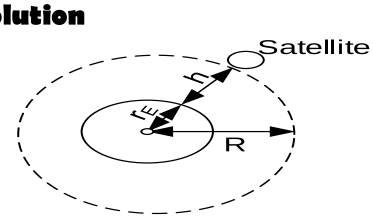
$$T = \left(\frac{4\left(\frac{22}{7}\right)^2(6.4\text{x}10^6+3.6\text{x}10^6)^3}{6.67x10^{-11}x6x10^{24}}\right)^{\frac{1}{2}}$$

T =9932.10555s

$T = 2.759Hrs$

- An artificial satellite move round the earth in a circular orbit in the plane of the equator at height 30,000km above the earth's surface (mass of earth $=6.0\times10^{24}$kg, radius of the earth= 6.4×10^{6}m,)
  - i) Calculate its speed
  - ii) What is the time between successive appearances over a point on the equator
  - iii) What will be the additional distance of the satellite if it was to appear stationery

Solution



h=30,000km $=3\times10^{7}$m, $r_E=6.4\text{x}10^6$m

G $=6.67x10^{-11}$Nm^{2}kg^{-2} $M_E=6.0\times10^{24}$kg

Attractive force=Centripetal force:

$$m\omega^2R = \frac{Gm_Em}{R^2}$$

$$\omega^2 = \frac{Gm_E}{R^3}$$

$$\omega = \sqrt{\frac{6.67x10^{-11}\text{x}6.0\text{x}10^{24}}{(6.4\text{x}10^6+3\text{x}10^7)^3}} = 9.1025\times10^{-5}\text{rads}^{-1}$$

$$v = r\omega = (6.4x10^6+3x10^7)x9.1025x10^{-5}$$

$v = 3.313\times10^{3}\text{ms}^{-1}$

(i) Its speed is 3.313×10^{3}ms^{-1}

(ii) Time required is the period

$$T = \frac{2\pi}{\omega} = \frac{2x\frac{22}{7}}{9.1025x10^{-5}} = 6.903\times10^{4}\text{s}$$

(iii) From Kepler's third law

$T^2 \propto R^3$

$T^2 = kR^3$-------------------------------- (x)

$R = \mathrm{r_E} + \mathrm{h}$

R $=(6.4\times10^{6}+3\times10^{7})$m

R $=36.4\times10^{6}$m

$19.2^2 = k(36.4\times10^{6})^3$----------------------(1)

If the satellite is stationery, then the geostationary T^1=24hrs

$(T^1)^2 \propto (R^1)^3$

$(T^1)^2 = k(R^1)^3$--------------------------(xx)

$(24)^2 = k(R^1)^3$---------------------------(2)

(2) ÷ (1): $\frac{(24)^2}{(19.2)^2} = \frac{\mathrm{k}\,(\mathrm{R}^1)^3}{\mathrm{k}(36.4\,\mathrm{x}106)3}$

$R^1=42.2\times10^{6}$m

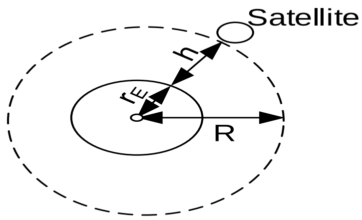
R^1=R+ extra distance

Extra distance $= 42.2x10^6 - 36.4\,x10^6 = 5.8\times10^{6}$m

8.2.5: ENERGY OF A SATELLITE

1. Kinetic energy

Consider a satellite of mass m moving in an orbit of radius R around the earth at a constant speed v



Centripetal force $F = \frac{mv^2}{R}$ ---------------- (1)

Force of attraction $F = \frac{Gm_E m}{R^2}$ ---------- (2)

(1) = (2): $\frac{mv^2}{R} = \frac{Gm_E m}{R^2}$

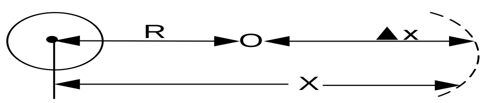
Introducing ½ on both sides

$$\frac{1}{2}mv^2 = \frac{Gm_E m}{2R}$$

$$\boxed{\boldsymbol{K.E} = \frac{Gm_E\, m}{2R}}$$

2. Potential energy

Consider a satellite of mass m brought from infinity into the region of earth's gravitational force.



From Newton's law of gravitation

$F = \frac{Gm_E m}{x^2}$

Work done $\Delta w = F\Delta x$

Total work done

$$\int_0^W dw = \int_\infty^R F dx$$

$$[w]_0^W = \int_\infty^R \frac{Gm_E m}{x^2} dx$$

$$W = Gm_E m \int_\infty^R x^{-2} dx$$

$$W = Gm_E m \left[\frac{-1}{x}\right]_\infty^R$$

$$W = Gm_E m \left[\frac{-1}{R} - \frac{-1}{\infty}\right]$$

$$W = Gm_E m \left[\frac{-1}{R} - 0\right] = \frac{-Gm_E m}{R}$$

But work done = $p.e$

$$\boxed{P.E = \frac{-Gm_E m}{R}}$$

Definition

Gravitational potential energy P.E is the work done in bringing a unit mass from infinity to that point.

3. Mechanical energy/total energy

$$M.E = K.E + P.E$$

$$M.E = \frac{Gm_E\, m}{2R} + \frac{-Gm_E\, m}{R}$$

$$\boxed{M.E = \frac{-Gm_E\, m}{2R}}$$

Note:

i) Mechanical energy and kinetic energy only differ by the sign therefore their magnitude is the same

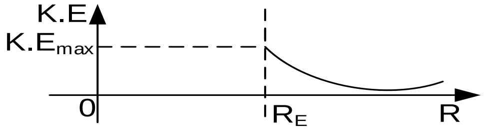
ii) If the radius of the orbit of the satellite decreases, the gravitational potential energy of the satellite becomes more negative implying that it has decreased.
   - Decrease in radius however causes an increase in the kinetic energy, resulting in an increase in the speed of the satellite in its new orbit.
   - Decrease in orbital radius also results into the mechanical energy becoming more negative hence it has decreased.

8.2.6: EFFECT OF FRICTION ON A SATELLITE

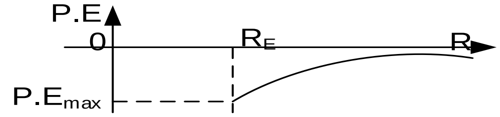
- If a satellite is located within the earth atmosphere as it moves in its orbit, the atmospheric gasses offer frictional resistance to its motion. The satellite thus would be expected to do work to overcome this resistance and is so doing, it falls to an orbit of lower radius.
- The decrease in the radius causes the total energy $\left(\frac{-Gm_E\, m}{2R}\right)$ to decrease while the kinetic energy of the satellite $\left(\frac{Gm_E\, m}{2R}\right)$ increases resulting into an increase in the speed of the satellite in its new orbit. Because of the increase of the speed the satellite becomes hotter and it may burnout.

Question Explain why any opposition to the forward motion of a satellite may cause it to burn.

A graph of K.E with variation of distance from centre of the earth



A graph of P.E with variation of distance from centre of the earth



Examples

1. A satellite of mass 100kg is in a circular orbit at a height $3.59 x 10^7$m above the earth surface
   i) Calculate the kinetic energy, potential energy and the mechanical energy of the satellite in this orbit
   ii) State what happens when the mechanical energy of the satellite is reduced

Solution

i) $K.E = \frac{Gm_E\, m}{2R}$

R =r_e+h

K.E. = $\frac{6.67x10^{-11}x6x10^{24}x100}{2x(6.4x10^6+3.59x10^7)}$

K.E. = $4.7.5x10^8$J

$P.E. = -\frac{Gm_E\, m}{R}$

R =r_e+h

$P.E. = -\frac{6.67x10^{-11}x6x10^{24}x100}{(6.4x10^6+3.59x10^7)}$

$P.E. = -$ 9.4992 x10^8J

$M.E = P.E + K.E$

$= -9.4992\, x10^8 + 4.75x10^8$

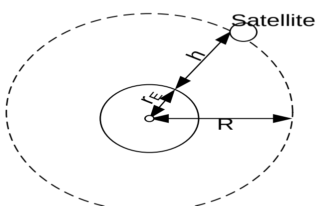
M.E= $-4.75x10^8$J

(ii)

- ✓ Frictional force increases
- ✓ Satellite falls to orbit of small radius
- ✓ PE reduces
- ✓ K.E increases
- ✓ Satellite becomes hot and may burn

2. A 10^3 kg mass satellite is launched in a parking orbit about the earth
   i) Calculate the height of the satellite above the surface of the earth
   ii) Calculate the mechanical energy of the satellite [$R_E = 6.4x10^6 m, g = 9.81ms^{-2}, G = 6.67x10^{-11}Nm^2kg^{-2}$]

Solution



Attractive force=Centripetal force:

$m\omega^2 R = \frac{Gm_E m}{R^2}$ but $\omega = \frac{2\pi}{T}$

$R = \left(\frac{T^2 Gm_E}{4\pi^2}\right)^{\frac{1}{3}}$ But $G\, m_E = g\, {r_E}^2$

$$R = \left(\frac{T^2 g\, {r_E}^2}{4\pi^2}\right)^{\frac{1}{3}}$$

T=24hrs for parking orbits M_E=5.97x10^{24}kg

$$R = \left(\frac{[24x60x60]^2 x9.81x(6.4x10^6)^2}{4x\left[\frac{22}{7}\right]^2}\right)^{\frac{1}{3}}$$

$R = 4.22x10^7 m$

But $R = R_E + h$

$h = 4.24x10^7 - 6.4x10^6 = 3.6x10^7 m$

$M.E = -\frac{Gm_E\, m}{2R}$

But $G\, m_E = g\, {r_E}^2$

$$M.E = \frac{9.81x(6.4x10^6)^2 x1000}{2x4.22x10^7}$$

M.E= $-4.74x10^9$ J

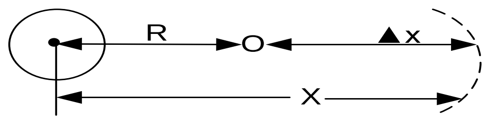
EXERCISE 20

1. A satellite of mass 1000kg is launched on a circular orbit of radius 7.2x10^6m about the earth. Calculate the mechanical energy of the satellite [ME=6x10^{24}kg, , $G = 6.67x10^{-11}Nm^2kg^{-2}$] **An [-2.78x$10^{10}$J**]
2. An artificial satellite is launched at a height of 3.6x10^7mabove the earth's surface
   i) Determine the speed with which the satellite must be launched to maintain in the orbit. **An**[**3.08x10^3**]ms^{-1}
   ii) Determine the period of the satellite. **An[24hrs]**

8.2.7: GRAVITATIONAL POTENTIAL [U]

Gravitational potential at a point in the gravitational field is defined as the work done to move a one kilogram mass (1kg) from infinity to that part. Ie $U = \frac{w}{m}$

Consider a body of mass 1kg moved from infinity to appoint O where the distance from the centre of the earth to O is R



From Newton's law of gravitation

F = $\frac{GMm}{x^2}$

$m = 1kg$

Work done $\Delta w \ = F\Delta x$

Total work done

$$\int_0^W dw = \int_\infty^R F dx$$

$$[w]_0^W = \int_\infty^{R_E} \frac{GM}{x^2} dx$$

$$W \ = GM \int_\infty^R x^{-2} dx$$

$$W \ = GM \left[\frac{-1}{x}\right]_\infty^R \ = \ GM \left[\frac{-1}{R} - \frac{-1}{\infty}\right]$$

$$W \ = \ GM \left[\frac{-1}{R} - 0\right] = \frac{-GM}{R}$$

But work done = $potential\ U$

$$\boxed{U = \frac{-GM}{R}}$$

Generally On the earth surfaces $U \ = \frac{-G\ m}{R_E}$

Note:

The amount of work done against the gravitational force of mas M to move the mass a distance r_1 to position r_2 is given by

$$W \ = \ GMm \left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

Example

A body of mass 15kg is moved from the earth's surface to a point 2.8x10^{6}m above the earth. If the radius of the earth is 6.4x10^{6}m and its mass is 6.0x10^{24} kg calculate the work done in taking the body to that point

Solution

$$W = Gm_E m \left(\frac{1}{R} - \frac{1}{R_E}\right)$$

R =R$_e$+h

$R = (6.4x10^6 + 2.8x10^6)$

R =9.2x10^6

$$w = 6.67x10^{-11}x6.4x10^{24}x15 \left(\frac{1}{9.2x10^6} - \frac{1}{6.4x10^6}\right)$$

work done in taking the body to that point

W = 2.85x10^{8}J

8.2.8: VELOCITY OF ESCAPE

This is the minimum velocity with which a body is projected from the surface of the earth so that it escapes from the earth's gravitational pull.

Derivation of formulae

Suppose a rocket of mass is fired from the earth's surface so that it just escapes from the gravitational influence of the earth

K.E lost = P.E lost

½ mV$_{esc}$^{2}= $0 - \frac{-Gm_E m}{R_E}$

V$_{esc}$ = $\sqrt{\frac{2\ Gm_E}{R_E}}$

$G\ m_E = g\ {R_E}^2$

$V_{esc} = \sqrt{2gR_E} = \sqrt{2x9.81x6.4x10^6}$

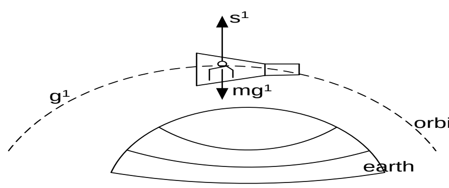
V$_{esc}$ = $11.2km/h$

Note

- ❖ The sun and the earth have an atmosphere while the moon doesn't have an atmoshphere because, they have very high masses compared to the moon, molecules of air move with average velocities less than their escape velocities and gravitational acceleration keeps the atmosphere around the sun and the earth. While molecules of air around the moon move at average velocities much greater than the escape velocity of the moon, they escape from the moon leaving it with no atmosphere

- ❖ Light gases like Neon, Argon, helium have mean thermal speed more than 3 time of air. This means that their speeds are higher than the mean speeds of air and this explain why they are rare in the earth atmosphere.
- ❖ For other planets escape velocity is given by $V_{esc} = \sqrt{\frac{2\ Gm}{R}}$

8.2.9: WEIGHTLESSNESS



The sensation of weight is caused by the reaction of the floor on the person. In orbit an astronaut and the floor have the same acceleration as acceleration due to gravity. The floor therefore exerts no supporting force on the astronaut (zero reaction)

The astronaut therefore experiences a sensation of **weightless.**

Definition

Weightlessness is the condition of a zero reaction and a body moves with the same acceleration as acceleration due to the gravity.

UNEB 2017No2

(a) State **Kepler's laws** of planetary motion (03marks)

(b) Use Newton's law of gravitation to derive the dimension of the universal gravitational constant.(03marks)

(c) A statellite is revolving at a height h above the surface of the earth with a period , T

(i) Show that the accerelatio due to gravity g on the earht's surface is given by $g = \frac{4\pi^2(r_e+h)^3}{T^2 r_e^2}$ where r_e is the radius of the earth (06marks)

(ii) What is meant by **parking orbit** (02mark)

(d) A satellite revolves in a circular orbit at a height of $600km$ above the earth's surface. Find the

(i) Speed of the satellite **An** $7.5764x10^3ms^{-1}$ **or An** $7.542x10^3ms^{-1}$ (03marks)

(ii) Periodic time of the satellite **An** $5805.2s$ or **An** $5802.2s$ (03marks)

UNEB 2016No3

(a) What is meant by **conservative forces** and give **two** examples of conservative forces. (02marks)

(b) Explain the following

(i) Damped oscillations (02mark)

(ii) Forced oscillations (02marks)

(c) (i) State **Newtons law of gravitation** (01marks)

(ii) Show that Newton's law of gravitation is consistent with Kepler's third law (05marks)

(d) If the earth takes 365 days to make one complete revolution around the sun, calculate the mass of the sun (04marks)

(e) Explain briefly how satellites are used in world wide radio or television communication. (04marks)

UNEB 2015No3

(a) State **Kepler's laws** of planetary motion (03marks)

(b) (i) What is a parking orbit (01mark)

(ii) Derive an expression for the period, T of a satellite in a circular orbit of radius r, above the earth in terms of mass of the earth m, gravitational constant G and r (03marks)

(c) (i) A satellite of mas 200kg is launched in a circular orbit at a height of $3.59x10^7m$ above the earth's surface. Find the mechanical energy of the satellite **An** $-9.41x10^8 J$ (03marks)

(ii) Explain what will happen to the satellite if the mechanical energy was reduced

(d) Describe a laboratory ,method of determining the universal gravitational constant, G (06marks)

UNEB 2013No4(a)

(i) State Kepler's laws of planetary motion (03marks)

(ii) Estimate the mass of the sun, if the orbit of the earth around the sun is circular (04mks)

UNEB 2012 No3(c)

(ii) A cosmonaut in a satellite which is in a free circular orbit around the earth experience the sensation of weightlessness even though there is influence of the gravitational field of the earth Explain (03marks)

UNEB 2011 No1(d)

A body of mass 15kg is moved from the earth's surface to a point $2.8x10^6$m above the earth. If the radius of the earth is $6.4x10^6$m and its mass is $6.0x10^{24}$ kg calculate the work done in taking the body to that point
An **$2.85x10^8$J** (06marks)

UNEB 2008 No3(c)

(i) with the aid of a diagram, describe an experiment to determine the universal gravitational constant G. (06marks)

(ii) If the moon moves round the earth in a circular orbit of radius =$4.0x10^8$m and takes exactly 27.3 days to go round once calculate the value of acceleration due to gravity g at the earth's surface. (04marks)

UNEB 2007 No2

a) State Kepler's laws of planetary motion (3mk)

b) i) A satellite moves in a circular orbit of radius R about a planet of mass m, with period T. show that $R^3 = \frac{G\,m\,T^2}{4\,\pi^2}$ where G is the universal gravitational constant (04marks)

ii) The period of the moon round the earth is 27.3days. If the distance of the moon from the earth is $3.83x10^5$km. Calculate the acceleration due to gravity at the face of the earth.
An [g=9.72ms^{-2}] (04marks)

iii) Explain why any resistance to forward motion of an artificial satellite results into an increase in its speed. (04marks)

c) i) what is meant by weightlessness (02marks)

ii) Why does acceleration due to gravity vary with location on the surface of the earth (03marks)

UNEB 2004 No2

d) Explain and sketch the variation of acceleration due to gravity with distance from the centre of the earth. (06marks)

UNEB 2003 No2

c) Show how to estimate the mass of the sun if the period and orbital radius of one of its planets are known.

d)The gravitational potential U at the surface of a planet of mass m and radius R is given by $U = -\frac{G\,m}{R}$ where G is the gravitational constant. Derive an expression for the lowest velocity, v which an object of mass m must have at the surface of the planet if it is the escape from the planet (04marks)

e) Communication satellite orbits the earth in synchronous orbits. Calculate the height of a communication satellite above the earth **An [$3.6x10^7$m]** (04marks)

UNEB 2000 No 4

a) State Keplers law's of gravitation (03marks)

b) I)Show that the period of a satellite in a circular orbit of radius r about the earth is given by

$$T=\left(\frac{4\,\pi^2}{G\,M_E}\right)^{\frac{1}{2}} r^{\frac{3}{2}}$$

Where the symbols have usual meanings (05marks)

ii)Explain briefly how world wide, radius or television communication can be achieved with the help of satellites (04marks)

c) A satellite of mass 100kg in a circular orbit at a height of $3.59x10^7$m above the earth's surface

(i) Find the mechanical energy (04marks)

(ii) Explain what would happened if the mechanical energy was decreased (04marks)

CHAPTER 9: SIMPLE HARMONIC MOTION (S.H.M)

Definition

This is the periodic motion of a body whose acceleration is directly proportional to the displacement from a fixed point and is directed towards the fixed point.

$$a \,\alpha - x$$
$$a = -\omega^2 x$$

The negative signs means the acceleration and the displacement are always in opposite direction.

9.1.0: Characteristics of SHM

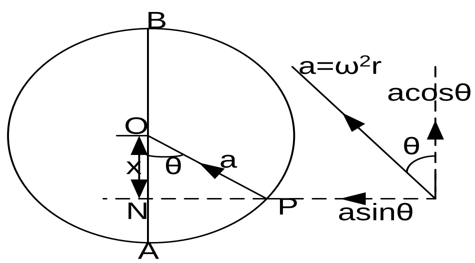
(1) It's a periodic motion (to and fro motion)
(2) Mechanical energy is always conserved
(3) The acceleration is directed towards a fixed point
(4) Acceleration is directly proportional to its displacement

9.1.1: PRACTICAL EXAMPLES OF S.H.M

- Pendulum clocks
- Pistons in a petrol engine
- Strings in music instruments
- Motor vehicle suspension springs
- Balance wheels of a watch

a) Acceleration $\ddot{x}$ or a

The acceleration of N is as a result of the acceleration of p. This is equal to the vertical component



$a_N = acos\theta$

but $a = \omega^2 r$ and $\theta = \omega t$

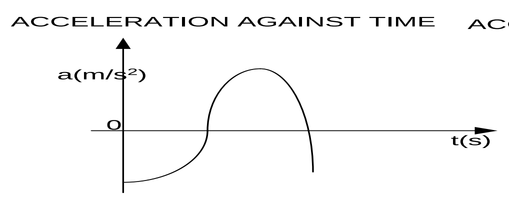
$a_N = \omega^2 rcos\omega t$

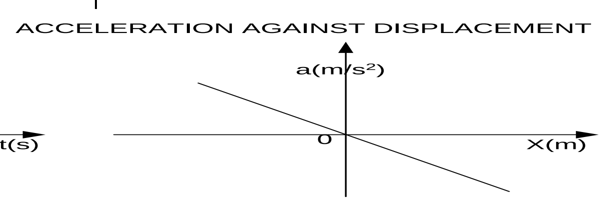
but fom equation 1 $x = rcos\omega t$

$a_N = \omega^2 x$

$\boxed{a_N = -\omega^2 x}$

$a_{max} = -\omega^2 r$





b) Period T

This is the time taken for one complete oscillation. .i.e. N moving from A to B and back to A.

$T = \frac{distance}{speed}$

$T = \frac{2\pi}{v}$ but $v = r\omega$

$T = \frac{2\,\pi\, r}{\omega\, r}$

$T = \frac{2\,\pi}{\omega}$

c) Frequency f

This is the number of complete oscillation made in one second

$f = \frac{1}{T}$ | $f = \frac{\omega}{2\pi}$

d) Velocity in terms of displacement

Velocity of a body executing S.H.M can be expressed as a function of displacement x. this is obtained from the acceleration

$a = -\,\omega^2 x$

$a = \frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt}$

but $\frac{dx}{dt} = v$

$a = v \cdot \frac{dv}{dx}$

$v \cdot \frac{dv}{dx} = - \omega^2 x$

$vdv = - \omega^2 xdx$

integrating both sides

$\int v\, dv = - \omega^2 \int x\, dx$

$\frac{v^2}{2} = -\frac{\omega^2 x^2}{2}$ + C ....[1]

Where C is a constant of integration

When t = 0 v=0 and $x = r$(amplitude)

$\frac{0^2}{2} = -\frac{\omega^2 r^2}{2} + C$

$C = \frac{\omega^2 r^2}{2}$

Put into [1]: $\frac{v^2}{2} = -\frac{\omega^2 x^2}{2} + \frac{\omega^2 r^2}{2}$

$v^2 = \omega^2 r^2 - \omega^2 x^2$

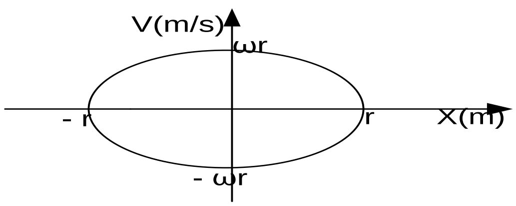
$\boxed{v^2 = \omega^2(r^2 - x^2)}$

Velocity is maximum when x =0

$v^2 = \omega^2 r^2$

$\boxed{\boldsymbol{v_{max} = \omega r}}$

AGRAPH OF VELOCITY AGAINST DISPLACEMENT



From $\boldsymbol{v^2} = \omega^2 r^2 - \omega^2 x^2$

$\boldsymbol{v^2} + \omega^2 x^2 = \omega^2 r^2$

$\frac{\boldsymbol{v^2}}{\omega^2 r^2} + \frac{x^2}{r^2} = 1$

This an ellipse

Example

1. A particles moves in a straight line with S.H.M. Find the time of one complete oscillation when
   i) The acceleration at a distance of 1.2m is 2.4ms^{-2}
   ii) The acceleration at a distance of 20cm is 3.2ms^{-2}

Solution

i) From $a = -\omega^2 x$

Negative is ignored

$2.4 = \omega^2(1.2)$

$\omega^2 = \frac{2.4}{1.2}$

ω =1.4rads^{-1}

But $T = \frac{2\pi}{\omega}$

$T = \frac{2x\frac{22}{7}}{1.4} = 4.46s$

ii) $a = -\omega^2 x$

$3.2 = \omega^2(0.2)$

ω =4rads^{-1}

$T = \frac{2\pi}{\omega} = \frac{2x\frac{22}{7}}{4}$ =1.57second

2. A Particle moving with S.H.M has velocities of 4ms^{-1} and 3ms^{-1} at distances of 3m and 4m respectively from equilibrium position. Find
   i) amplitude , ii) period , iii) frequency
   iv) velocity of the particle as it passes through equilibrium position

Solution

(i) $v = 4ms^{-1}, x = 3m$

Using $v^2 = \omega^2(r^2 - x^2)$

$4^2 = \omega^2(r^2 - 3^2)$

$16 = \omega^2(r^2 - 9)$------- (1)

Also $v = 3ms^{-1}, x = 4m$

$3^2 = \omega^2(r^2 - 4^2)$

$9 = \omega^2(r^2 - 16)$------- (2)

(1)$\div$ (2): $\frac{16}{9} = \frac{\omega^2(r^2 - 9)}{\omega^2(r^2 - 16)}$

$16(r^2 - 16) = 9(r^2 - 9)$

$r^2 = 25$

$r = 5m$; Amplitude =5m

(ii) period put $r = 5m$ into (1)

$4^2 = \omega^2(r^2 - 3^2)$

$16 = \omega^2(5^2 - 9)$

$\omega^2 = 1$

$\omega = 1$

But $T = \frac{2\pi}{\omega} = \frac{2x\frac{22}{7}}{1}$ =6.28second

(iii) frequency $= \frac{1}{T} = \frac{1}{6.28}$ =0.16Hz

(iv) velocity as it passes equilibrium position at equilibrium $x = 0$

$v^2 = \omega^2(r^2 - x^2)$

$v^2 = 1^2(5^2 - 0^2)$

$v = 5m/$s

3. A body of mass 200g s executing S.H.M with amplitude of 20mm. The maximum force which acts upon it is 0.064N. calculate
   a) its maximum velocity b) its period of oscillation

Solution

$F = 0.064N$

Mass m=200g = 0.2kg

Amplitude r =20mm = 0.02m

a)$v_{max} = \omega r$

But $F = ma_{max}$

0.064 =0.2a_{max}

$a_{max} = 0.32m/s^2$

$a_{max} = -\omega^2 r$

0.32= $\omega^2 x0.02$

$\omega^2 = 16$

ω =4rads^{-1}

$v_{max} = \omega r = 4x0.02$

v_{max} = 0.08ms^{-1}

b) T= $\frac{2\pi}{\omega} = \frac{2\pi}{4} = \frac{2x22}{4x7}$

$T = 1.57$ seconds

4. A body of mass 0.30kg executes S.H.M with a period of 2.5s and amplitude of 4x10^{-2}m. determine
   i) Maximum velocity of the body
   ii) The maximum acceleration of the body

Solution

M=0.3kg, T=2.5s, r= $4x10^{-2}$m

i) $v_{max} = \omega r$

$\omega = \frac{2\pi}{T}$ $\quad \therefore v_{max} = \frac{2\pi}{T} r$

$$v_{max} = \frac{2x\frac{22}{7}x4x10^{-2}}{2.5} = 0.101m/s$$

ii) $a_{max} = \omega^2 r$

$$a_{max} = \left(\frac{2\pi}{T}\right)^2 r = \left(\frac{2x\frac{22}{7}}{2.5}\right)^2 x\,4x10^{-2}$$

$a_{max} = 0.25ms^{-2}$

5. A particle moves with S.H.M in a straight line with amplitude 0.05m and period 12s. Find
   a) speed as it passes equilibrium position
   b) maximum acceleration of the particle

Solution

a)speed at equilibrium

$v_{max} = \omega r$

$v_{max} = \frac{2\pi}{T} r$

$$v_{max} = \frac{2x\frac{22}{7}x0.05}{12} = 0.026ms^{-1}$$

b)$a_{max} = \omega^2 r$

$$a_{max} = \left(\frac{2\pi}{T}\right)^2 r$$

$$a_{max} = \left(\frac{2x\frac{22}{7}}{12}\right)^2 x\ 0.05$$

$a_{max} = 0.014ms^{-2}$

ENERGY CHANGES IN S.H.M

- In S.H.M there's always an energy exchange. At maximum displacement , all the energy is elastic potential energy while at equilibrium point all the energy is kinetic energy

a) Kinetic energy

It's the energy possessed by a body due to its motion

K.E = ½ mv²= ½ m$\omega^2(r^2 - x^2)$

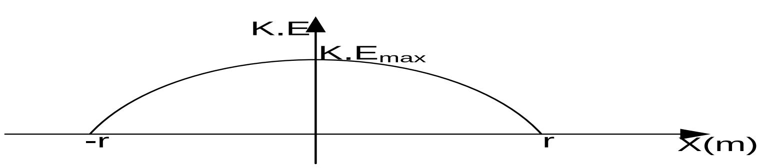
Note

i) The K.E is zero when the displacement x is equals to the amplitude

ii) K.E is maximum when the displacement x is zero

K.E_{max} = ½ m$\omega^2 r^2$

9.3.1: A graph of K.E against displacement



b) Elastic potential energy

This is the energy possessed by a body due to the nature of its particle i.e. compressed or stretched.

Force is applied to make particles stretch or compress and therefore the force does work, which work is stored in the body.

$\Delta w = F\Delta x$

But $F = kx$

$\Delta w = k\Delta x$

Total work done $\int_0^w dw = \int_0^x kx\,dx$

$w = \left[\frac{kx^2}{2}\right]_0^x = \frac{kx^2}{2}$

Elastic potential energy = $\frac{1}{2}kx^2$

Or $\Delta w = F\Delta x$

But $F = m\omega^2 x$

$\Delta w = m\omega^2 x\Delta x$

$$\int_0^w dw = \int_0^x m\omega^2 x\, dx$$

$W = ½\, m\omega^2 x^2$

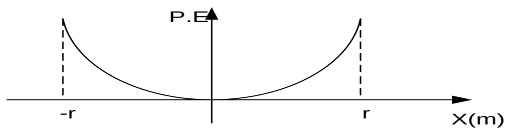
Elastic potential energy $= ½\, m\omega^2 x^2$

note :

i) Elastic potential energy is maximum when x is a maximum

ii) Elastic potential energy is zero when x=0 (equilibrium)

9.3.2: Graph of P.E against displacement



iii) Mechanical energy

This is the total energy possessed by a body due its motion and nature of its particles

$M.E = K.E + P.E$

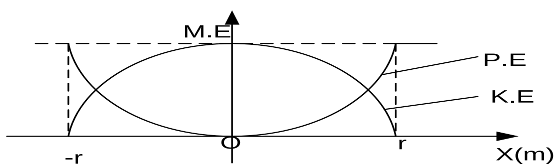
$= ½\, m\omega^2(r^2 - x^2) + ½\, m\omega^2 x^2$

M.E $= ½\, m\omega^2 r^2$

Note

Mechanical energy is constant

9.3.3: A graph of M.E against displacement



9.4.0: MECHANICAL OSCILLATION

There are three types of oscillation i.e.

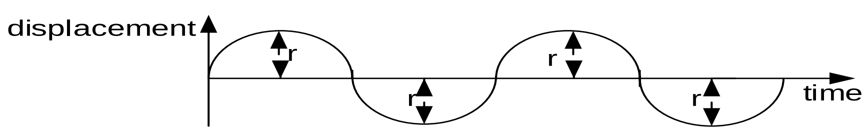
a) Free oscillation
b) Damped oscillation
c) Forced oscillation

a) Free oscillations

These are oscillations in which the oscillating systems does not do work against dissipative force such as air friction, and viscous drag and amplitude remains constant with time.

Eg a pendulum bob in a vacuum

Displacement- time graph



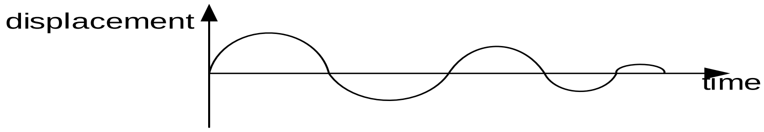
(b) Damped oscillations

These are oscillations in which the oscillating system loses energy to the surrounding due to dissipative forces and amplitude of these oscillations reduce with time

Types of damped oscillations

i) Under damped/slightly damped/lightly damped oscillations

Is when energy is lost and amplitude gradually decreases until oscillation dies away.

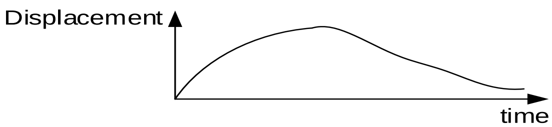


Examples

- ❖ Mass oscillating at the end of the spring oscillating in air
- ❖ Simple pendulum oscillating in air

ii)Over damped/highly damped/heavily damped

Is when a system does not oscillate when displaced but takes a very long time to return to equilibrium position.

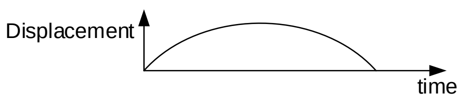


Example

- ❖ A horizontal spring with a mass on a rough surface

iii) Critically damped oscillations

Is when a system does not oscillate when displaced and returns to equilibrium position in a short time.



Example

- ❖ Shock absorber in a car

C) FORCED OSCILLATIONS

These are oscillations where the system is subjected to an external force and the system oscillates at the same frequency as the oscillating force.

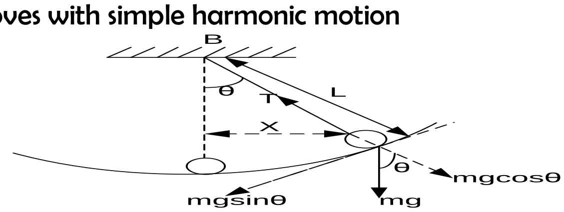
Example

- ❖ Oscillation of a guitar string
- ❖ Oscillation of a building during an earthquake
- ❖ Oscillation of air column in a musical pipe

Examples of S.H.M

9.2.1: SIMPLE PENDULUM

Consider a mass m suspended by a light inelastic string of length L from a fixed point B.
If the bob is given a small vertical displacement through an angle θ and released, we show that a bob moves with simple harmonic motion



Resolving tangentially: Restoring force $= -mgsin\theta$

By Newton's 2nd law: $ma = -mgsin\theta$

$ma = -mgsin\theta$……………………………….1

If the displacement is small, then θ is very small.

$$Sin\theta \approx \theta \approx \frac{x}{l}$$

$$ma = -mg\theta$$

$$a = -g\frac{x}{l} = -\left(\frac{g}{l}\right)x$$

it is in the form $a = -\omega^2 x$ and hence performs S.H.M with period

$$\omega^2 = \frac{g}{l}$$

$$\boxed{\omega = \sqrt{\frac{g}{l}}}$$

But $\omega = \frac{2\pi}{T}$

$$\boxed{T = 2\pi\sqrt{\frac{l}{g}}}$$

$$\boxed{f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}}$$

9.2.1: Determination of acceleration due to gravity (g) using a simple pendulum

- Starting with a measured length L of the pendulum, the pendulum is clamped between 2 wood pieces from a retort stand.
- A bob is then given a small angular displacement from the vertical position and released.
- The time t for 20 oscillation is obtained, find period T and henceT^2
- Repeat the procedure for different values of length of the string.
- A graph of T^2 against L is then drawn and its slope S calculated.

Hence acceleration due to gravity is obtained from $g = \frac{4\pi^2}{s}$

Factors which affect the accuracy of g when using a simple pendulum.

1. The nature of the string. The string should be inelastic
2. Air resistance (dissipative force). In present of air the motion of a simple pendulum is highly damped such that the oscillation dies out quickly that affecting the period.
3. The displacement of the bob from the equilibrium position should be small such that the oscillation remain uniform.
4. The mass of the bob should be small to minimize the effect of dimension of the object.
5. In accuracies in the timing and measuring extensions.

Examples ;

A bob of a simple pendulum moves simple harmonically with amplitude 8.0cm and period 2.00s. its mass is 0.50kg, the motion of the bob is un damped. Calculate maximum values of;

a) The speed of the bob, and
b) The kinetic energy of the bob.

Solution

a) m=0.5kg , r=8cm=0.08m, T=2s

$v_{max} = \omega r$

$v_{max} = \frac{2\pi}{T} r = \frac{2x\frac{22}{7}}{2} x0.08$

$v_{max} = 0.25 ms^{-1}$

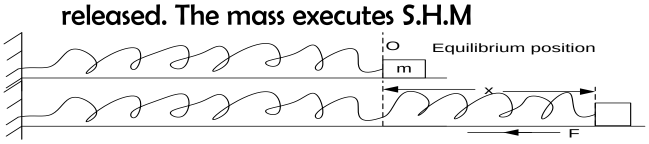
b) $K.E_{max} = ½\, m\, v_{max}^2$

$K.E_{max} = ½\, x0.5x(0.25)^2 = 1.563x10^{-2}J$

MASS ON A SRING

a) A horizontal spring attached to a mass

Consider a spring lying on a smooth horizontal surface in which one end of the spring is fixed and the other end attached to a particle of mass m. When the mass is slightly pulled a small distance x and the released. The mass executes S.H.M



By Hooke's law :$F = -kx$--------------------- (1)

By Newton's 2nd law: $ma = -kx ---- (2)$

$a = -\left(\frac{k}{m}\right) x$----------------------------- (3)

Where k is the spring constant

Equation (3) is in the form$[a = -\omega^2 x]$,

therefore the body performs S.H.M

$\therefore \omega^2 = \frac{k}{m}$

=> $\boxed{\omega = \sqrt{\frac{k}{m}}}$

But $\omega = \frac{2\pi}{T}$

$\frac{4\,\pi^2}{T^2} = \frac{k}{m}$

$T^2 = \frac{4\,\pi^2 m}{k}$

$\boxed{T = 2\pi\sqrt{\frac{m}{k}}}$

Also $f = \frac{1}{T}$

$\boxed{f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}}$

Example : UNEB 2011 No 4C

A horizontal spring of force constant 200 Nm^{-1} is fixed at one end and a mass of 2kg attached to the free end and resting on a smooth horizontal surface. The mass is pulled through a distance of 4.0cm and released. Calculate the;

i) Angular speed

ii) Maximum velocity attained by the vibrating body, acceleration when the body is half way towards the centre from its initial position.

Solution

i) From $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{200}{2}} = 10rads^{-1}$

ii) $v_{max} = \omega r$

$v_{max} = 10x\frac{4}{100}$ =0.4ms^{-1}

Note: the small distance pulled and released becomes the amplitude

$a = -\omega^2 x$

where its half towards the centre

$x = \frac{r}{2}$

$x = \frac{4x10^{-2}}{2}$

$a = -\omega^2 x = 10^2 x\frac{4x10^{-2}}{2} = 2ms^{-2}$

Alternatively

$F = ma$

$F = kx$

$k\frac{r}{2} = ma$

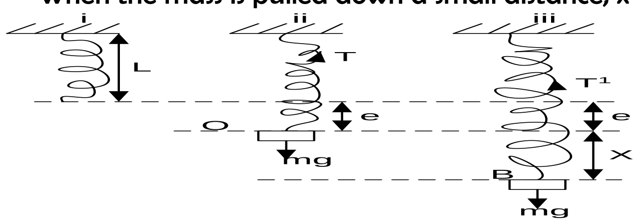
$a = \frac{200x4x10^{-2}}{2x2} = 2ms^{-2}$

b) Oscillation of mass suspended on a helical spring

Consider a helical spring or elastic string suspended from a fixed point.

When a mass is attached to the spring, it stretches by length, e and comes to equilibrium positions O.

When the mass is pulled down a small distance, x and released the motion will be simple harmonic.



In position (ii) the mass is in equilibrium position

$T = mg$

And by hooke's law $T = ke$

$mg = ke$ -------------------------(1)

In position (iii) after displacement through x

But by Hooke's law $T^1 = k(e + x)$

By Newton's 2nd law: $mg - k(e + x) = ma$

But from equation 1 $mg = ke$

$ke - k(e + x) = ma$

$ke - ke - kx = ma$

$-kx = ma$

$a = -\frac{k}{m}x$ -------------------------------- [2]

Equation 3 is in the form $a = -\omega^2 x$ and therefore performs S.H.M

$\omega^2 = \frac{k}{m}$

$\boxed{\omega = \sqrt{\frac{k}{m}}}$------------------------------- [3]

But $\omega = \frac{2\pi}{T}$

$\boxed{T = 2\pi\sqrt{\frac{m}{k}}}$

$\boxed{f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}}$

Note:

From [1] $mg = ke$

$\frac{k}{m} = \frac{g}{e}$

$\omega = \sqrt{\frac{g}{e}}$

$T = 2\pi\sqrt{\frac{e}{g}}$

$f = \frac{1}{2\pi}\sqrt{\frac{g}{e}}$

9.2.2: Determination of acceleration due to gravity using a vertically loaded spring

- Clamp a spring on a retort stand using pieces of wood
- Fix a horizontal pin to the free end of the spring to act as a pointer
- Place a vertical meter rule next to the pin and note its initial position
- Suspend a known mass, m at the free end of the spring, note and record the new position of the pointer
- Calculate the extension e produced
- Repeat the procedure above for several masses suspended in turns and tabulate.
- Plot a graph of **e** against **m**
- Find the slope, s of the graph

Hence acceleration due to gravity is determined from $g = ks$ where k is known spring constant

Alternatively

- Clamp a spring on a retort stand using pieces of wood
- Fix a horizontal pin to the free end of the spring to act as a pointer
- Place a vertical meter rule next to the pin and note its initial position
- Suspend a known mass, m at the free end of the spring, note and record the new position of the pointer
- Calculate the extension e produced
- Displace the mass through a small vertical displacement and released to oscillate. Note the time t for 20 oscillations.
- Find the period T and calculate T^2
- Repeat the procedure for several masses suspended in turns .
- Plot a graph of $\mathbf{T^2}$ against **e** and find the slope, s of the graph
- Hence acceleration due to gravity is determined from $g = \frac{4\pi^2}{s}$

Examples

1. A 100g mass is suspended vertically from a light helical spring and the extension in equilibrium is found to be 10cm. The mass is now pulled down a further 0.5cm and it is released from rest.
   i) Show that the subsequent motion is simple harmonic
   ii) Find the period of oscillation
   iii) What is the maximum kinetic energy of the mass

Solution

$m = 100g = 0.1kg,$
$e = 10cm = 0.1m\,,$
$r = 0.5cm = 0.005m$

From $\omega = \sqrt{\frac{k}{m}}$

$T = 2\pi\sqrt{\frac{m}{k}}$

But also $mg = ke$

Therefore $\frac{k}{m} = \frac{g}{e}$

$T = 2\pi\sqrt{\frac{e}{g}} = 2\pi\sqrt{\frac{0.1}{9.81}} = 0.63s$

$v_{max} = \omega r$

$v_{max} = \frac{2\pi}{T}r = \frac{2x\frac{22}{7}}{0.63}x0.05$

$v_{max} = 0.0499ms^{-1}$

$K.E_{max} = ½\, m\, v_{max}^2$

$= ½\, x0.1x(0.0499)^2$

$K.E_{max} = 1.245x10^{-4}J$

2. A mass hangs from a light spring. The mss is pulled down 30mm from its equilibrium position and then released from rest. The frequency of oscillation is 0.5Hz. calculate
   a) The angular frequency, ω of the osculation
   b) The magnitude of the acceleration at the instant it is released from rest

Solution

Distance pulled down ward and released becomes the amplitude

$\therefore r = 30mm = 30x10^{-3}m$

$f = 0.5Hz$

a) Angular frequency ω

$\omega = 2\pi f = 2\pi x0.5 == 3.14rad\ s^{-1}$

b) When it is released from rest the displacement is equals to amplitude and the acceleration is maximum.

$a_{max} = \omega^2 r$

$a_{max} = (3.14)^2 x30x10^{-3}$

$a_{max} = 0.296ms^{-2}$

Exercise:21

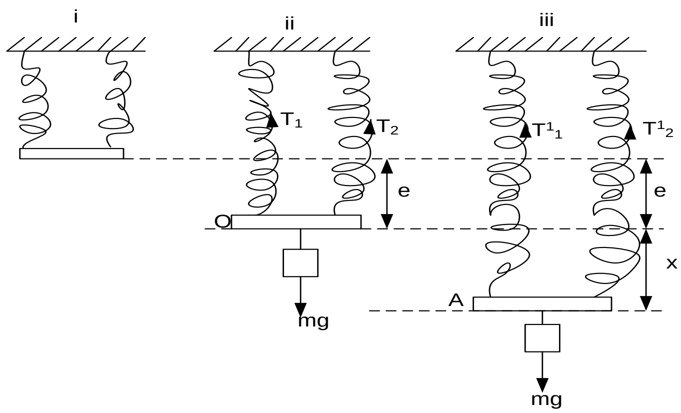
1. When a metal cylinder of mass 0.2kg is attached to the lower end of a light helical spring, the upper end of which is fixed, the spring extends by 0.16m. the metal cylinder is then pulled down a further 0.08m.
   i) Find the force that must be exerted to keep it there. **An [1.0N]**
   ii) The cylinder is then released. Find the period of vertical oscillation and the kinetic energy the cylinder posses when it passes through its mean position. **An[0.79s, 0.04J]**
2. A mass of 0.2kg is attached to the lower end of a helical spring and produces extension of 5.0cm. The mass is now pulled down at a further distance of 2cm and released. Calculate
   a) the force constant of the spring
   b) The period of the subsequent motion
   c) The maximum value of the acceleration during the motion **An[39.24Nm^{-1}, 0.45s, 3.924ms^{-2}]**

COMBINED SPRINGS

a) Vertical springs

9.2.3: Vertically loaded springs in parallel

Consider two springs of force constants k_1 and k_2 suspended from the same rigid support side by side. When a mass is attached to the mid point of a rod connected to the lower ends of the springs.
The system rests in equilibrium
When the mass is displaced a small distance vertically downwards and then released the system execute S.H.M



By hookes law: $T_1 = k_1x \quad and\ T_2 = k_2x$

Restoring force $= \ k_1x + k_2x$

By Newton's second law Restoring force $= ma$

$$-(\ k_1x + k_2x) \ = ma$$

$$a = \ -\left(\frac{k_1 + k_2}{m}\right)x \text{ ------------------------ (1)}$$

Equation 3 is in the form $a = -\omega^2x$ and therefore performs S.H.M

$$\omega^2 = \left(\frac{k_1 + \ k_2}{m}\right)$$

$$\boxed{\omega = \sqrt{\left(\frac{k_1 + k_2}{m}\right)}} \text{ ------------------------ (2)}$$

$$T = \frac{2\pi}{\omega}$$

Period $T = 2\pi\sqrt{\left(\frac{m}{k_1+k_2}\right)}$

$$f = \frac{1}{2\pi}\sqrt{\left(\frac{k_1 + k_2}{m}\right)}$$

Note: at equilibrium

$$mg \ = (\ k_1 + k_2)e$$

$$\frac{m}{(\ k_1+k_2)} = \frac{e}{g}$$

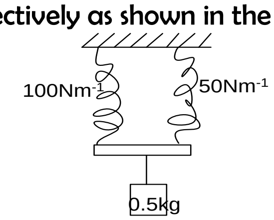
$$\omega = \sqrt{\frac{g}{e}}$$

$$T = 2\pi\sqrt{\left(\frac{e}{g}\right)}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{e}}$$

Examples

1. A mass of 0.5kg is suspended from the free ends of two springs of force constant 100Nm^{-1} and 50Nm^{-1} respectively as shown in the figure below.



Calculate ;
i) The extension produced
ii) Tension in each string
iii) Energy stored in the string
iv) Frequency of small oscillations when he mass is given a small vertical displacement

Solution

i) At equilibrium $mg = (\ k_1 + k_2)$ e

$$e = \frac{mg}{k_1 + k_2} = \frac{0.5x9.81}{100 + 50} = 0.0327m$$

ii) Tension in each string

From Hooke's law $T_1 = k_1e$

$$T_1 = 100x0.0327 = 3.27N$$

Also $T_2 = k_2e = 50x0.0327 = 1.635N$

iii) Energy stored is always stored as elastic potential energy of the spring

$$P.E_{Elatic} = \frac{1}{2}ke^2$$

$E_1 = \frac{1}{2}k_1e^2 = \frac{1}{2}x\ 100\ x\ (0.0327)^2$ =**0.0535J**

$E_2 = \frac{1}{2}k_2e^2 = \frac{1}{2}x\ 50\ x\ (0.0327)^2$ =**0.0267J**

$$P.E_{Elatic} = E_1 + E_2$$

$P.E_{Elatic} =$ **0.0535+ 0.026**

$P.E_{Elatic} = 0.0802J$

iv) Frequency

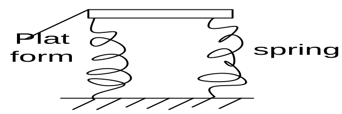
$$f = \frac{1}{2\pi}\sqrt{\frac{g}{e}} = \left(\frac{1}{2x\frac{22}{7}}\right)\sqrt{\frac{9.81}{0.0327}} = 2.757Hz$$

Alternatively

$$f = \frac{1}{2\pi}\sqrt{\left(\frac{k_1 + k_2}{m}\right)}$$

$$f = \left(\frac{1}{2x\frac{22}{7}}\right)\sqrt{\left(\frac{100 + 50}{0.5}\right)} = 2.757Hz$$

2. A light platform is supported by tow identical springs each having spring constants 20Nm^{-1} as shown below.



a) Calculate the weight which must be placed on the centre of the platform in order to produce a displacement of 3.0cm.
b) The weight remains on the platform and the platform is depressed a further 1.0cm and then released
   i) What is the frequency of the oscillation
   ii) What is the maximum acceleration of the platform

Solution

a) Compression $e = 3.0cm = 0.03m$

At equilibrium

$mg = T_1 + T_2$

$mg = (k_1 + k_2)$ e

$mg = (20 + 20)x0.03$

$mg = 1.2N$

$weight = 1.2N$

b) Amplitude r=1.0cm=0.01m

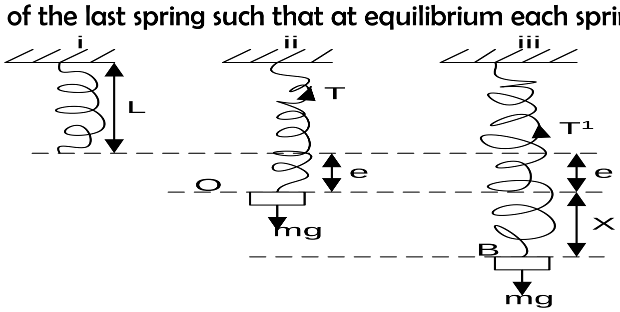
$$f = \frac{1}{2\pi}\sqrt{\frac{g}{e}} = \left(\frac{1}{2x\frac{22}{7}}\right)\sqrt{\frac{9.81}{0.03}} = 2.89Hz$$

$$a_{max} = \omega^2 r = (2\pi f)^2 r$$

$$a_{max} = \left(2x\frac{22}{7}x2.89\right)^2 x0.01 = 3.297ms^{-2}$$

9.2.4: Vertically loaded spring in series

Consider two springs of constants k₁ and k₂ suspended in series, mass m is then attached to the lower end of the last spring such that at equilibrium each spring extends by x_1 and x_2 respectively.



After a small displacement,

$x = x_1 + x_2$ ---------------------------- (1)

by hooks law $T = k_1 x_1$ and $T = k_2 x_1$

$\therefore x_1 = \frac{T}{k_1}$ and $x_2 = \frac{T}{k_2}$

$$x = \frac{T}{k_1} + \frac{T}{k_2}$$

$$x = T\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$$

$$x = T\left(\frac{k_1 + k_2}{k_1 k_2}\right)$$

by newton's 2nd law

$$T = \left(\frac{k_1 k_2}{k_1 + k_2}\right)x$$

by newton's 2nd law $T = ma$

$a = -\frac{1}{m}\left(\frac{k_1 k_2}{k_1 + k_2}\right)x$-----------------------[2]

it is in the form $a = -\omega^2 x$

$$\therefore \omega^2 = \frac{k}{m}$$

But $k = \frac{k_1 k_2}{k_1 + k_2}$

$$\omega^2 = \left(\frac{k_1 k_2}{k_1 + k_2}\right)\Big/ m$$

$$\boxed{\omega = \sqrt{\left(\frac{k_1 k_2}{k_1 + k_2}\right)\Big/_{m}}} \text{ ---------------- [3]}$$

$$T = 2\pi\sqrt{\frac{(k_1 + k_2)m}{k_1 k_2}}$$

$$f = \frac{1}{2\pi}\sqrt{\left(\frac{k_1 k_2}{k_1 + k_2}\right)\Big/_{m}}$$

Note

$$mg = ke$$

$$\therefore k = \frac{k_1 k_2}{k_1 + k_2}$$

Also

$$\omega = \sqrt{\frac{g}{e}}$$

$$T = 2\pi\sqrt{\left(\frac{e}{g}\right)}$$

$$f = \frac{\mathbf{1}}{2\pi}\sqrt{\frac{g}{e}}$$

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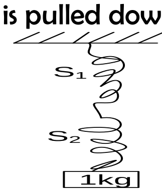
A mass of 1.0kg is hung from two springs S_1 and S_2 connected in series as shown
The force constant of the springs are 100Nm^{-1} and 200 Nm^{-1} respectively. Find

i) The extension produced in the combination
ii) The frequency of oscillation of the mass if it is pulled downwards and released



Solution

m=1kg , k_1=100Nm^{-1}, k_2=200 Nm^{-1}

At equilibrium $mg = ke$

$e = \frac{mg}{k}$ but k= $\frac{k_1 k_2}{k_1 + k_2}$

$$e = {}^{mg}\Big/_{\frac{k_1 k_2}{k_1 + k_2}} = 1x9.81\Big/\left(\frac{100x200}{100+200}\right) = 0.1472m$$

ii)

$$f = \frac{1}{2\pi}\sqrt{\frac{k_1 k_2}{k_1 + k_2}\Big/_{m}}$$

$$f = \frac{1}{2\pi}\sqrt{\left(\frac{100x200}{100 + 200}\right)\Big/_{1}}$$

$$f = 1.299Hz$$

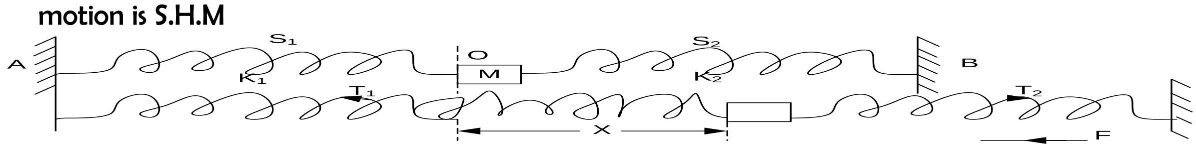
NB: For all S.H.M, the following assumptions hold

1- displacement from equilibrium position is small such that Hooke's law is obeyed throughout the motion
2- no dissipative forces act

b) horizontal springs

9.2.5: TWO HORIZONTAL SPRINGS WITH A MASS BETWEEN THEM

Consider two springs with spring constants K_1, and K_2 attached to fixed points and mass attached between them. Show that when the mass is displaced horizontally towards one side the resultant motion is S.H.M



Extension of $S_1 = x$
Compression of $S_2 = x$
Restoring force $F = -(T_1 + T_2)$
But by Hooke's law ; $T_1 = k_1 x$ and $T_2 = k_2 x$
$F = -(k_1 + k_2)x$ ------------------------ (1)
By Newton's 2nd law ; $ma = -(k_1 + k_2)x$

$a = -\left(\frac{k_1 + k_2}{m}\right)x$ ------------------------ (3)

Equation 3 in the form $a = -\omega^2 x$ and therefore it performs S.H.M

$$\omega^2 = \left(\frac{k_1 + k_2}{m}\right)$$

$$\boxed{\omega = \sqrt{\left(\frac{k_1 + k_2}{m}\right)}}$$ ------------------------ (4)

But $\omega = \frac{2\pi}{T}$

$$\frac{2\pi}{T} = \sqrt{\left(\frac{k_1 + k_2}{m}\right)}$$

$$T = \frac{2\pi}{\sqrt{\left(\frac{k_1 + k_2}{m}\right)}}$$

$$\boxed{T = 2\pi\sqrt{\left(\frac{m}{k_1 + k_2}\right)}}$$

$$f = \frac{1}{T}$$

$$\boxed{f = \frac{1}{2\pi}\sqrt{\left(\frac{k_1 + k_2}{m}\right)}}$$

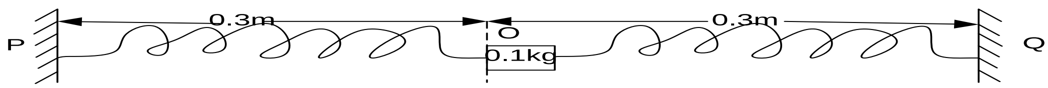
Note: when the springs are identical $k_1 = k_2 = k$

$$T = 2\pi\sqrt{\left(\frac{m}{2k}\right)}$$

$$f = \frac{1}{2\pi}\sqrt{\left(\frac{2k}{m}\right)}$$

Example

1. A mass of 0.1kg is placed on a frictionless horizontal surface and connected to two identical springs of negligible mass and a spring constant of 33.5Nm^{-1}. The springs are then attacked to fixed point p and Q on the surface as shown below.



The mass is given a small displacement along the line of the spring and released

i) Show that the system will execute S.H.M
ii) Calculate the period of oscillation
iii) If the amplitude of oscillation is 0.05m, calculate the maximum kinetic of the system.

Solution

ii) From $T = 2\pi\sqrt{\left(\frac{m}{k_1 + k_2}\right)}$

$$T = 2x\frac{22}{7}\sqrt{\left(\frac{0.1}{33.5 + 33.5}\right)} = 0.243s$$

iii) $r = 0.05m$

$v_{max} = \omega r$

$$v_{max} = \frac{2\pi}{T}r = \frac{2x\frac{22}{7}}{0.243}x0.05$$

$$v_{max} = 1.293ms^{-1}$$

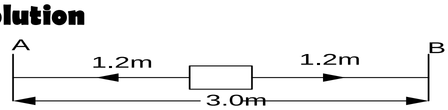
$$K.E_{max} = ½\, m\, v_{max}^2$$

$$= ½\, x0.1x(1.293)^2 = 0.084J$$

2. A body of mass 4kg rests on a smooth horizontal surface. Attached to the body are two pieces of light elastic strings each of length of 1.2m and force constant 6.25Nm^{-1}. The ends are fixed to two points A and B 3.0m apart as shown in the figure below. The body is then pulled through 0.1m towards B and then released.

i) Show that the body executes S.H.M
ii) Find the period of oscillation of the body
iii) Calculate the speed of the body when it is 0.03m from the equilibrium position

Solution



From $T = 2\pi\sqrt{\left(\frac{m}{k_1 + k_2}\right)}$

$$T = 2x\frac{22}{7}\sqrt{\left(\frac{4}{6.25 + 6.25}\right)} = 3.55s$$

iii) $v^2 = \omega^2(r^2 - x^2)$

Amplitude $r = 0.1\, m$

$$x = 0.03m$$

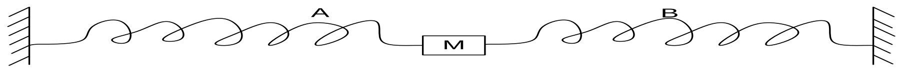
$$\omega = \frac{2\pi}{T}$$

$$v^2 = \left(\frac{2\pi}{T}\right)^2(r^2 - x^2)$$

$$v^2 = \frac{4x3.14^2}{3.55^2}(0.1^2 - 0.03^2)$$

$$v = 0.169m/s$$

3. The figure below shows a mass of 200g resting on a smooth horizontal table, attached to two springs A and B of force constants k_1 and k_2 respectively



The block is pulled through a distance of 8cm to the right and then released.

(i) Show that the mass oscillates with simple harmonic motion and find the frequency of oscillation if $k_1 = 120Nm^{-1}$ and $k_2 = 200Nm^{-1}$

(ii) Find the new amplitude of oscillation when a mass of 120g is dropped vertically onto the block as the block passes the equilibrium position. Assume that the mass sticks to the block

Solution

i) From $f = \frac{1}{2\pi}\sqrt{\left(\frac{k_1+k_2}{m}\right)}$

$$f = {}^{1}\Big/\left(2x\frac{22}{7}\right)\sqrt{\left(\frac{120+200}{0.2}\right)}$$

$$f = 6.37Hz$$

ii) By conservation of momentum:

$$m_1u = (m_1+m_2)v_{max}$$

$$v_{max} = \frac{m_1u}{(m_1+m_2)}$$

$v_{max} = \omega^1 r^1$ and $u_{wax} = \omega r$

$$\omega^1 = \sqrt{\left(\frac{k_1+k_2}{m}\right)} = \sqrt{\left(\frac{120+200}{0.32}\right)} = 10\sqrt{10}rads^{-1}$$

$$\omega = \sqrt{\left(\frac{k_1+k_2}{m}\right)} = \sqrt{\left(\frac{120+200}{0.2}\right)} = 40rads^{-1}$$

$$u_{wax} = \omega r = 40x0.08 = 3.2m/s$$

$$\therefore v_{max} = \frac{m_1u}{(m_1+m_2)}$$

$$10\sqrt{10}\,r^1 = \frac{0.2x3.2}{(0.2+0.12)}$$

$$r^1 = 0.632m$$

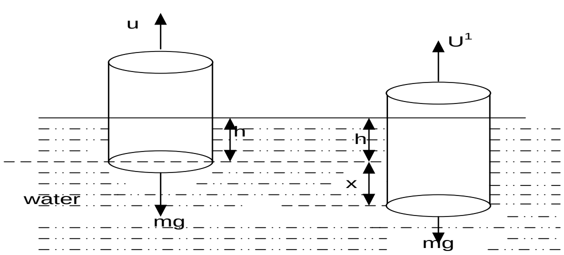
EXERCISE:22

A block of mass 0.1kg resting on a smooth horizontal surface and attached to two springs s_1 and s_2 of force constant 60Nm^{-1} and 100Nm^{-1}respectively. The block is pulled a distance of 4x10^{-2}m to the right and the released.

i) Show that the mass executes S.H.M and fixed the frequency of oscillation
ii) Find the new amplitude of oscillation when the block is added a mass of 0.06kg on top as the block passes the equilibrium position. **An (6.4 Hz, 0.032m)**

9.2.6: S.H.M OF A FLOATING CYLINDER

Consider a uniform cylindrical rod of length L and cross sectional area A and density, ρ floating vertically in a liquid of density, σ. When the rod is given a small downward displacement x and released, the rod executes S.H.M.



At equilibrium, $U = mg = Ah\delta g$ -------------- [1]

After a downward, restoring force $F = U^1 - mg$

$F = A(h+x)\delta g - Ah\delta g$ ---------- [2]

But $F = ma\ hence$

$$Ah\delta g - A(h+x)\delta g = ma$$

$$-A\delta g\,x = ma$$

$$a = -\left(\frac{A\delta g}{m}\right)x$$

But $m = A\,l\,\rho$

$$a = -\left(\frac{A\delta g}{A\,l\,\rho}\right)x$$

$a = -\left(\frac{\delta g}{l\,\rho}\right)x$-------------------------- [3]

it is the form $a = -\omega^2 x$

$$\omega^2 = \frac{\delta g}{l\,\rho}$$

$\boxed{\omega = \sqrt{\frac{\delta g}{l\,\rho}}}$---------------------------- [4]

T=$2\pi\sqrt{\left(\frac{\rho l}{\delta g}\right)}$

$$f = \frac{1}{2\pi}\sqrt{\frac{\delta g}{l\,\rho}}$$

Examples : UNEB 2000 No2b

1. A Uniform cylindrical rod of length 8cm, cross sectional area $0.02m^2$ and density $900kgm^{-3}$ floats vertically in a liquid of density $1000kgm^{-3}$. The rod is depressed through a distance of 0.005m and then released.
   i) Show that the rod performs S.H.M (5mk)
   ii) Find the frequency of the resultant oscillation (4mk)
   iii) Find the velocity of the rod when it is at a distance of 0.004m above the equilibrium position

Solution

ii) $f = \frac{1}{2\pi}\sqrt{\frac{\delta g}{l\,\rho}}$

$$f = \left(\frac{1}{2x\frac{22}{7}}\right)\sqrt{\frac{1000x9.81}{8x10^{-2}x900}} = 1.858Hz$$

iii) $v^2 = \omega^2(r^2 - x^2)$

r=0.005m , $x = 0.004m$, $\omega = 2\pi f$

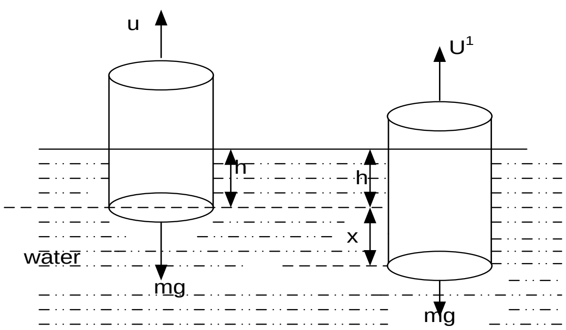
$v^2 = (2\pi f)^2(r^2 - x^2)$

$$v^2 = \left(2x\frac{22}{7}x1.858\right)^2 (0.005^2 - 0.004^2)$$

$v = 3.5x10^{-2}ms^{-1}$

2. A wooden rod of uniform cross sectional area A floats with a height h immersed in a liquid of density δ. The rod is given a slight downward displacement and released. Show that the resulting motion is S.H.M with a time period of $2\pi\sqrt{\frac{h}{g}}$

Solution



At equilibrium, $U = mg = Ah\delta g$ -------------- [1]

After a downward, restoring force $F = U^1 - mg$

$F = A(h + x)\delta g - Ah\delta g$ ---------- [2]

But $F = ma\ hence$

$$Ah\delta g - A(h + x)\delta g = ma$$

$$-A\delta g\ x = ma$$

$$a = -\left(\frac{A\delta g}{m}\right)x$$

But $m = Ah\delta$

$$a = -\left(\frac{A\delta g}{A\,h\,\delta}\right)x$$

$a = -\left(\frac{g}{h}\right)x$-------------------------- [3]

it is the form $a = -\omega^2 x$

$\omega^2 = \frac{g}{h}$

$\boxed{\omega = \sqrt{\frac{g}{h}}}$---------------------------- [4]

$$T = \frac{2\pi}{\omega}$$

$$T = 2\pi\sqrt{\left(\frac{h}{g}\right)}$$

Example

A cylindrical test tube of thin wall and mass 1kg with a piece of lead of mass 1kg fixed at its inside bottom floats vertically in the liquid.

When the test tube is slightly depressed and released it oscillates vertically with a period of one second $(T = 1s)$.

If some extra copper beads are put in the test tube, it floats vertically with a period of 1.5 seconds. Find the mass of the copper beads in the test tube.

Solution

$$T = 2\pi\sqrt{\left(\frac{h}{g}\right)}$$

$$1 = 2\pi\sqrt{\left(\frac{h_1}{9.81}\right)}$$

$h_1 = \frac{1^2x9.81}{4\pi^2}$ =0.2485m

Also $1.5 = 2\pi\sqrt{\left(\frac{h_2}{9.81}\right)}$

$h_2 = \frac{1.5^2 x 9.81}{4\pi^2}$ =0.5591m

at equilibrium U =weight of liquid displaced

$2g \ = \mathrm{A}\, h_1 \delta\, g$

$2 \ = \mathrm{A}\, h_1 \delta$ ------------------------------ (1)

Also when a mass m is added

$(2 + m)g = \mathrm{A}\, h_2 \delta\, g$

$(2 + m) = \mathrm{A}\, h_2 \delta$ -------------------- (2)

Equation 2 divided by equation 1

$$\frac{(2+m)}{2} = \frac{\mathrm{A}\, h_2\delta}{\mathrm{A}\, h_1\delta}$$

$$\frac{(2+m)}{2} = \frac{h_2}{h_1}$$

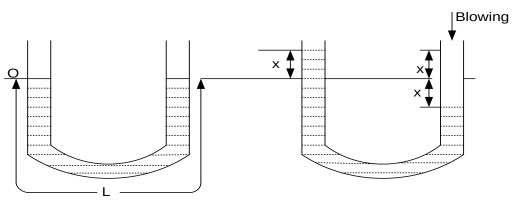
$$m = \frac{2\, x\, h_2}{h_1} - 2$$

$$m = \frac{2\, x\, 0.5591}{0.2485} - 2$$

$$m = 2.5kg$$

9.2.7: A LIQUID OSCILLATING IN A U-TUBE

Consider a column of liquid of density σ and total length l in a U-tube of uniform cross sectional area A. Suppose the level of the liquid on the right side is depressed by blowing gently down that side, the levels of liquid will oscillate for a short time about their respective or equilibrium positions O.



When the meniscus is at a distance, x , from equilibrium position, a differential height of liquid of, $2x$, is produced

Excess pressure on liquid $= 2x\delta g$ from $[p = h\delta g]$

Force on liquid ,$F = 2x\delta gA$

Restoring force $F = -ma$ ----------------[1]

Newton's 2nd law : $ma \ = -2x\delta gA$

$a = -\left(\frac{2\delta gA}{m}\right)$-----------------------------[2]

But mass of liquid in the tube =volume of liquid x $\delta = 2Al\delta$

$$a = -\left(\frac{2\delta gA}{2A\, l\, \delta}\right)x$$

$a = -\left(\frac{g}{l}\right)x$----------------------------- [3]

it is in the form $a = -\omega^2 x$

$$\omega^2 = \frac{g}{l}$$

$$\boxed{\omega = \sqrt{\frac{g}{l}}}$$

$$T \ = \frac{2\pi}{\omega}$$

$$\boxed{T \ = \ 2\pi\sqrt{\frac{l}{g}}}$$

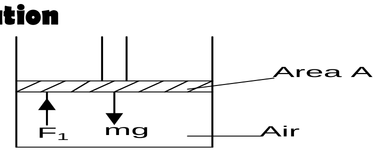
9.2.8:S.H.M IN A FRICTIONLESS AIR TIGHT PISTON

A volume v of air and pressure p is contained in a cylindrical vessel of cross section area A by frictionless air tight piston of mass m.

Show that on slight forcing down the piston and then releasing it, the piston will exert S.H.M given by

$$T = \frac{2\,\pi}{A}\sqrt{\frac{m\, v}{P}}$$

Solution



At Equilibrium

$F_1 = PA$

$PA = mg$ -------------------------------- [1]

When the piston is given a slight downward displaced x,

the restoring force $F_2 = P_2A - mg$

But by Newton's 2nd law

$ma = -[\,P_2A - mg]$

from Equation 1 $PA = mg$

$ma = -\,(P_2A - PA)$--------------------- [2]

Boyle's law. $[P_1V_1 = P_2V_2]$

$P_2(v - Ax) = Pv$

$$P_2 = \frac{Pv}{(v - Ax}$$

$$ma = -\left(\frac{Pv}{(v - Ax}A - PA\right)$$

$$ma = -PA\left(\frac{Ax}{v - Ax}\right)$$

For small displacement, $x \quad v - Ax \approx v$

$$ma = -PA\left(\frac{Ax}{v}\right)$$

$$ma = -A\left(\frac{PAx}{v}\right)$$

$$a = -\left(\frac{PA^2}{m\,v}\right)x$$

it is in the form $a = -\omega^2 x$

$$\omega^2 = \frac{PA^2}{m\,v}.$$

$$\omega = \sqrt{\frac{PA^2}{m\,v}}.$$

$$\omega = A\sqrt{\frac{P}{m\,v}}.$$

But $T = \frac{2\pi}{\omega}$

T=$\frac{2\pi}{A}\sqrt{\left(\frac{mv}{p}\right)}$

$$f = \frac{1}{T}$$

$$f = \frac{\boldsymbol{A}}{\boldsymbol{2\pi}}\sqrt{\frac{\boldsymbol{P}}{\boldsymbol{m\,v}}}$$

Example

A piston in a car engine performs S.H.M. The piston has a mass of 0.50kg and its amplitude of vibration is 45mm. the revolution counter in the car reads 750 revolutions per minute. Calculate the maximum force on the piston.

Solution

$r = 45mm = 45x10^{-3}m,\ m = \ 0.5kg$

$$f = 750\,rev/min$$

$$f = \frac{750}{60} = 12.5rev/s$$

But $a_{max} = \omega^2 r$

$\omega \ = 2\pi f$

$$a_{max} = \ (2\pi f)^2 r$$

$$a_{max} = \left(2x\frac{22}{7}x12.5\right)^2 x12.5$$

a_{max} =277.583ms^{-2}

$$F_{max} \ = \ ma_{max}$$

$$F_{max} \ = 0.5x277.583$$

$$F_{max} \ = 138.792N$$

UNEB 2017 No 3

a) (i) Define **simple harmonic motion** [1mk]

(ii) Sketch a displacement-time graph for a body performing simple harmonic motion [1mk]

b) A uniform cylindrical rod of length 16cm and density $920kgm^{-3}$ float vertically in a liquid of density $1000kgm^{-3}$. The rod is drepressed through a distance of 7mm and then released.

i) Show that the rod executes simple harmonic motion [06mk]

ii) Find the frequency of the resultant oscillations **An(1.299Hz)** [04mk]

iii) Find the velocity of the rod when it is at a distance of 5mm above the equilibrium position **An($3.998x10^{-2}ms^{-1}$)** [03mk]

c) What is meant by potential energy [01mk]

d) Describe energy changes which occur when a

(i) Ball is thrown upwards in air [03mk]

(ii) Loud speaker is vibrating [01mk]

UNEB 2013 No4

(b) Explain **Brownian motion** (03marks)

(c) Explain the energy changes which occur when a pendulum is set into motion (03marks)

An[p.e to k.e to p.e]

(d) A simple pendulum of length 1 m has a bob of mass 100g. It is displaced from its mean Position A to a position B so that the string makes an angle of 45° with the vertical. Calculate the ;

(i) Maximum potential energy of the bob (03marks)

(ii) Velocity of the bob when the string makes angle of 30° with the vertical. [Neglect air resistance] (04marks)

Solution

i) $P.e = mgh = mg(l - lCos\theta)$
$P.e = 0.1x9.81(1 - 1cos45) = 0.287J$
ii) By law of conservation of energy
$K.e = loss\ in\ P.e$

$$\frac{1}{2}mv^2 = mgh$$
$$\frac{1}{2}mv^2 = mgl(lCos45 - Cos30)$$
$$\boldsymbol{v} = \sqrt{2x9.81(Cos45 - 1Cos30)} = 1.766ms^{-1}$$

UNEB 2012 No 2

(a) Define the following terms as applied to oscillating motion
i) Amplitude [1mk]
ii) Period [1mk]
(b) State four characteristics of simple harmonic motion [2mk]
(c) A mass m, is suspended from a rigid support by a string of length, l. the mass is pulled a side so that the string makes an angle, θ with the vertical and then released.
i) Show that the mass executes simple harmonic motion with a period, $T = 2\pi\sqrt{\frac{l}{g}}$ [05mk]
ii) Explain why this mass comes to a stop [02mk]
(d) A piston in a car engine performs simple harmonic motion of frequency 12.5Hz. If the mass of the piston is 0.50kg and its amplitude of vibration is 45mm, find the maximum force on the piston. **An[139N]** [03mk]
(e) Describe an experiment to determine the acceleration due to gravity, g using a spiral spring of known force constant [06mk]

UNEB 2011 No 2

a) i) what is meant by simple harmonic motion [1mk]
ii) State two practical examples of simple harmonic motion [1mk]
iii) Using graphical illustration distinguish between under damped and critically damped oscillation [4mk]
b) i)describe an experiment to measure acceleration due to gravity using a spiral spring [6mk]
ii) State two limitations to the accuracy of the value it b (i) [02mk]

UNEB 2010 No 2

b) i) What is meant by a simple harmonic motion [1mk]
ii) Distinguish between damped and forced oscillations [2mk]
c) a cylinder of length l, cross sectional area A and density,δ, floats in a liquid of density, ρ, the cylinder is pushed down slightly and released.
i) Show that a performs simple harmonic oscillation [5mk]
ii) Derive the expression for the period of oscillation [2mk]
An(T=$2\pi\sqrt{\left(\frac{\delta\ l}{\rho\ g}\right)}$ **)**
d) A spring of force constant 40Nm^{-1} is suspended vertically. A man of 0.1kg suspended from the spring is pulled down a distance of 5mm and released. Find the,
i) Period of oscillation **An[0.314s]** [2mk]
ii) Maximum oscillation of the mass **An[2ms-^{2}]** [2mk]
iii) Net force acting on the mass when it is 2mm below the centre of oscillation. **An[0.08N]** [2mk]

UNEB 2009 No 3

(a) What is meant by simple harmonic motion (01marks)
(b) A cylindrical vessel of cross-sectional area A, contains air of volume V, at a pressure P, trapped by frictionless air tight piston of mass M,. The piston is pushed down and released.
(i) If the piston oscillates with s.h.m, show that the frequency is given by $f = \frac{\boldsymbol{A}}{\boldsymbol{2\pi}}\sqrt{\frac{\boldsymbol{P}}{\boldsymbol{m\,v}}}$ (06marks)
(ii) Show that the expression for, f in b(i) is dimensionally correct (02marks)
(c) Particle executing s.h.m vibrates in a straight line, given that the speeds of the particle are 4m$s^{-1}$and 2ms^{-1} when the particle is 3cm and 6cm respectively from equilibrium. calculate the;
(i) amplitude of oscillation **An(6.7x10^{-2}m)** (03marks)

(ii) frequency of the particle **An(10.68Hz)** (03marks)

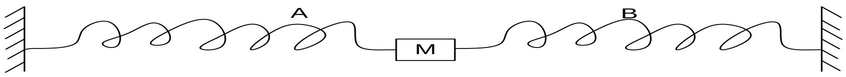
(d) Give two examples of oscillatory motions which execute s.h.m and state the assumptions made in each case

UNEB 2008 No3

a) (i) Define simple harmonic motion [01marks]

(ii) A particle of mass m executes simple harmonic between two point A and B about equilibrium position O. Sketch a graph of the restoring force acting on the particle as a function of distance r and moved by the particle [02marks]

b)



Two springs A and B of spring constants K_A and K_B respectively are connected to a mass m as shown. The surface on which the mass slides is frictionless.

(i) Show that when the mass is displaced slightly, it oscillates with simple harmonic motion of frequency given by

$$f = \frac{1}{2\pi}\sqrt{\left(\frac{k_A + k_B}{m}\right)}$$ [04marks]

(ii) If the two springs above are identical such that $k_A = k_B = 5Nm^{-1}$ and mass m=50g, calculate the period of oscillation **An[0.44s]** [03marks]

UNEB 2007 No 1

a) Define simple harmonic motion [01marks]

b) Sketch a graph of

i) velocity against displacement [03marks]

ii) acceleration against displacement for a body executing S.H.M

c) A glass U-tube containing a liquid is tilted slightly and then released

i) Show that the liquid oscillates with S.H.M [04marks]

ii) Explain why the oscillations ultimately come to rest [03marks]

UNEB 2007 No 4

b) i)What is meant by acceleration due to gravity [01mark]

ii)Describe how you would use a spiral string, a retort stand with a clamp, a pointer, seven 50g masses, meter rule and a stop clock to determine the acceleration due to gravity [6mk]

iii) State any two sources of errors in the experiment in bii) above. [01mark]

iv)A body of mass 1kg moving with simple harmonic motion has speed of $5ms^{-1}$ and $3ms^{-1}$ when it is at a distance of 0.1m and 0.2m respectively from the equilibrium point. Find the amplitude of motion [04marsk]

CHAPTER 10: ELASTICITY

If a force is applied to a material in such a way as to deform it (change its shape or size), then the material is said to be stressed and there will be change in relative positions of the molecules within the body and the material become strained. Stress which results in increase in length is called tensile stress and one which results in decrease in length is called compressive stress.

Terms used

1. **Elasticity:** This is the ability of the material to regain its original shape and size when the deforming load has been removed.
2. **Elastic material:** This is a material which regains its original shape and size when the deforming load has been removed. E.g. Rubber band, spring.
3. **Elastic deformation:** This is when a material can recover its original length and shape when the deforming load has been removed.
4. **Elastic limit:** This is the **maximum load** which a material can experience and still regain its original size and shape once the load has been removed.

The elastic limit sometimes coincides with the limit of proportionality.

5. **Proportional limit:** This is the **maximum load** a material can experience for which the extension created on it is directly proportional to the load applied.
6. **Hooke's law:** it staes that; the extension of a wire or spring is proportional to the applied load provided the proportional limit is not exceeded.
   The law shows that when the molecules of a material are slightly displaced from their mean positions, the restoring force is proportional to its displacement.

I.e. $F \alpha\ e$ $\boxed{F = ke}$ Where k is the constant of proportionality.

7. **Yield point:** this is a point at which there is a marked increase in extension when the stress or load is increased beyond the elastic limit.
   The internal structure of the material has changed and the crystal planes have effectively slid across each other. At yield poirt the material begin to show plastic behavior.
   Few materials exhibit yield point such as mild steel, brass and bronze.
8. **Plastic deformation:** this is when a material cannot recover its original shape and size when the deforming load has been removed.
9. **Breaking stress/ultimate tensile strength:** it is the maximum stress which can be applied to a material. Or it is the corresponding force per unit area of the narrowest cross section of the wire.
10. **Strength:** this is the ability of a material to with stand an applied force before breaking.

Or it is the maximum force which can be applied to a material without it breaking.

11. **Stiffness:** this is the ability of a material to resist changing its shape and size.
12. **Ductility:** it is the ability of the material to be permanently stretched. or it is the ability of the material to be stretched appreciably beyond elastic limit. It can be drawn into different shapes without breaking.
13. **Brittleness:** it is the ability of the material to break immediately it is stretched beyond to elastic limit.
14. **Toughness:** this is the ability of material to resist crack growth e.g. rubber
15. **Tensile stress:** it is force acting per unit area of cross-section of a material.

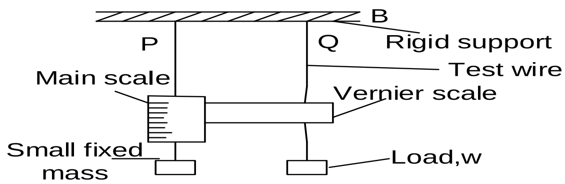
Stress $= \frac{F}{A}$

16. **Tensile strain**: it is the extension per unit original length of the material.

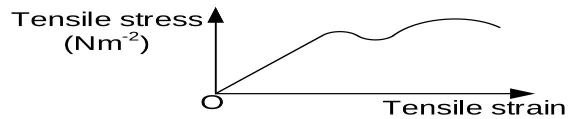
Strain $= \frac{e}{L}$

Strain has no units because it is a ratio of two similar units

10.1.0: Experiment to study elastic properties of steel



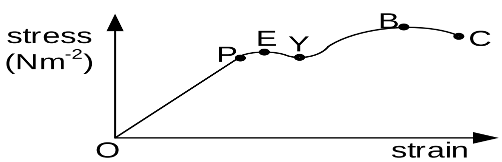
- Two long, thin identical steel wires are suspended besides each other from the same rigid support B
- The wire P is kept taut and free of kinks by weight A attached to its end
- The original length l of test wire Q is measured and recorded.
- The mean diameter d is determined and cross-sectional area $A = \frac{\pi d^2}{4}$ is found.
- Known weight, W is added to the free end of test wire Q and the corresponding extension e is read from the vernier scale.
- The procedure is repeated for different weights.
- Results are tabulated including values of tensile stress $\left(\frac{W}{A}\right)$ and tensile strain $\left(\frac{e}{L}\right)$
- The graph of tensile stress versus tensile strain is plotted as below.



10.1.1: Stress-strain graphs

1. **Ductile material** e.g. copper, steel, iron

A ductile material is one which can be permanently stretched



P-Proportionality limit
E-Elastic limit
Y-Yield ponit
B-Breaking stress
C-Breaking point

OP: stress ∝ starin, material regains all original length when the stress is removed and Hooke's law is obeyed

PE: material regains all original length when the stress is removed but Hooke's law is not obeyed

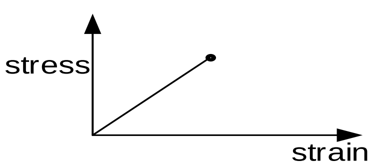
EY; material does not regains all original length when the load is removed

YB; No extension at all is regained when the load is removed

C: The wire breaks

2. **Brittle materials** e.g. glass, chalk, rocks and cast iron

These are materials that can not be permanently stretched. It breaks as soon as the elastic limit has been reached

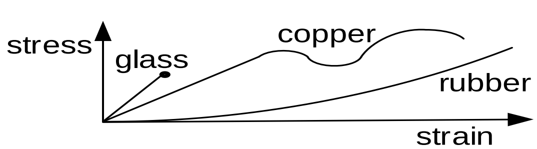


Brittle materials have only a small elastic region and do not under go plastic deformation. This behavior in glass is due to the existence of cracks in its surface. The high concentration of the stress at the crack makes the glass break.

3. **Rubber**

Rubber does not obey Hooke's law except for a smaller load. This is because rubber has coiled molecules which uncoil when stretched

10.1.2: Stress-strain graph for glass, copper and rubber



10.1.3: Energy changes/physical process

1. Elastic deformation

Atoms are slightly displaced from their equilibrium positions when the load is applied. The energy used to stretch the wire becomes elastic potential energy. When the stretching force is removed, the elastic potential energy of the atoms changes to kinetic energy and moves them back to their equilibrium position.

2. Plastic deformation

When the wire is stretched beyond the elastic limit, permanent displacement of atoms occurs. Crystals planes slide over each other. The movement of dislocations take place and on removing the stress, the original shape and size is not recovered due to energy loss in form of heat. At the breaking point the energy is used to break interatomic bonds

3. Work hardening

It is the strengthening of the material by repeatedly deforming it.
During repeated plastic deformation, atomic planes slide over each other and this increases plane dislocations which prevents further sliding of atomic planes
This explains why it is easier to break a copper wire by flexing it to and fro.

4. Annealing

It is a process by which a material restores its ductility.

Procedure

The metal is heated to high temperature above its melting point and maintained in this temperature for a period of time and relaxes the internal strains and hence the metal is re-crystallised and returns to the ductile state.

10.2.0: Young's modulus

It is also called the modulus of elasticity of a wire.
Young's modulus is the ratio of tensile stress to tensile strain of a material

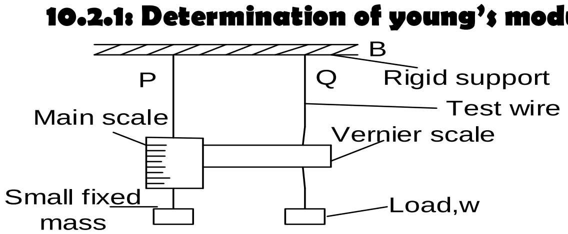
Young modulus $= \frac{stress}{strain}$ | $E = \frac{F}{A}\Big/\frac{e}{L}$ | $E = \frac{F\,L}{A\,e}$

A is area, L is original length, e is extension

Dimensions of young's modulus

$[E] = \frac{[F][L]}{[A][e]}$ | $[E] = \frac{(M\,L\,T^{-2})(L)}{L^2\,L}$ | $[E] = M\,L^{-1}T^{-2}$

10.2.1: Determination of young's modulus (Searle's apparatus)



- Two long, thin identical steel wires are suspended besides each other from the same rigid support B
- The wire P is kept taut and free of kinks by weight A attached to its end
- The original length l of test wire Q is measured and recorded.
- The mean diameter d is determined and cross-sectional area $A = \frac{\pi\,d^2}{4}$ is found.
- Known weights, W is added to the free end of test wire Q and the corresponding extension e is read from the vernier scale.
- The procedure is repeated for different weights.
- A graph of weight W against extension e is plotted and its slope (s) obtained.
- Young's modulus is obtained from $E = \frac{S\,L}{A}$

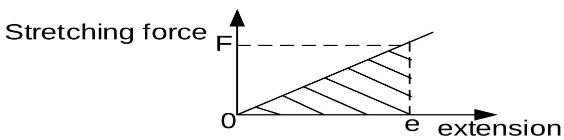
Precautions

✓ Two identical wires are used to eliminate errors due to thermal expansion as a result of temperature changes since they are affected equally.

- ✓ Both wires are suspended from the same support to eliminate errors in extension due to the yielding of the support
- ✓ Long wire are used to produce a measurable extension.
- ✓ thin wires are used to produce a measurable extension even with a small load. Otherwise if the wires were thick it requires a large load which would cause the support to yield.
- ✓ Micrometer/vernier readings are also taken when the load is removed to ensure that the elastic limit is not exceeded.
- ✓ Average diameter of wire is got to obtain accurate cross-sectional area
- ✓ Wires should be free from kinks to get accurate original length

10.2.2: Energy stored in a stretched material [strain energy]

Consider a material of an elastic constant k, stretched by a force, F to extend by e.
By Hooke's law, the extension is directly proportional to the applied force provided the elastic limit is not exceeded.



Work done = <u>area under the graph</u>

Work done $= \frac{1}{2}F\ e$

But $F = ke$

Work done $= \frac{1}{2}k\ e^2$

The work done to stretch the material is stored as elastic potential in the material

Energy stored $= \frac{1}{2}k\ e^2$

Or Energy stored $= \frac{1}{2}F\ e$

By calculus [integration]

If F is the force which gives an extension from O to e and $F\ =\ kx$ (from Hooke's law)

Work done $= \int_0^e F\ dx$

$= \int_0^e kx\ dx$

$= \left[\frac{k\ x^2}{2}\right]_0^e$

Work done $= \frac{1}{2}ke^2$

10.2.3: Energy stored per unit volume

Energy stored in the wire = $\frac{1}{2}F\ e$

If a wire is of cross sectional area A and natural length L, the volume = AL

Energy per unit volume = $\frac{Energy\ stored}{volume} = \frac{1}{2}F\ e \Big/ AL = \frac{Fe}{2AL}$

$$\boxed{\text{Energy per unit volume } = \frac{1}{2}\left(\frac{F}{A}\frac{e}{L}\right) \text{ or } \frac{1}{2}x\ stress\ x\ strain}$$

Numerical examples

1. A metal bar has a circular cross section of diameter 20mm. If the maximum permissible tensile stress is $8x10^7Nm^{-2}$, calculate the maximum force which the bar can withstand.

Solution

d = 20mm = $20x10^{-3}$m

stress= $\frac{Force}{Area}$

Force $= 8x10^7x\frac{\pi d^2}{4}$

$= 8x10^7x\frac{\left[\frac{22}{7}x\left(20x10^{-3}\right)^2\right]}{4}$

$Force\ = 2.513x10^4N$

2. Find the maximum load which may be placed on steel of diameter 1mm if the permitted strain must not exceed $\frac{1}{1000}$ and young's modulus for steel is $2x10^{11}Nm^{-2}$

Solution

Young modulus $= \frac{stress}{strain}$

Stress $= 2x10^{11}x\frac{1}{1000}$

Stress $= 2x10^8\ Nm^{-2}$

But stress $= \frac{F}{A}$

Force $= 2x10^8x\frac{\pi d^2}{4}$

$= 2x10^8x\frac{\left[\frac{22}{7}x(1x10^{-3})^2\right]}{4}$

Force $= 1.571x10^2N$

3. Calculate the energy stored in 2m long copper wire of cross-sectional area 0.55mm², if a force of 50N is applied to it

Solution

$$e = \frac{F\,L}{A\,E}$$

Energy stored $= \frac{1}{2}Fe$

$$= \frac{1}{2}x50x\frac{2.8x0.1}{1.2x10^{11}x0.5x10^{-6}}$$

$$= 0.04J$$

4. An elastic string of cross-sectional area 4mm² requires a force of 2.8N to increase its length by one tenth. Find young's modulus for the string if the original length of the string was 1m, find the energy stored in the string when it is extended.

Solution

A= 4mm²= $4x\ 10^{-6}$m², $F = 2.8N$,

$L = 1m$, e= $\frac{1}{10}m = 0.1m$

$$E = \frac{F\,L}{A\,e} = \frac{2.8\ x\ 1}{4x\ 10^{-6}\ x\ 0.1} = 7x10^6Nm^{-2}$$

Energy stored $= \frac{1}{2}Fe = \frac{1}{2}x2.8x0.1 = 0.14J$

5. A rubber cord of a catapult has a cross-sectional area of 1.2mm^2 and original length 0.72m, and is stretched to 0.84m to fire a small stone of mass 15g at a bird. Calculate the initial velocity of the stone when it just leaves the catapult. Assume that Young's modulus for rubber is $6.2x10^8\ Nm^{-2}$

Solution

$$e = 0.84 - 0.72 = 0.12m$$

$$F = \frac{EAeL}{l}$$

$$F = \frac{6.2x10^8\ x1.2x10^{-6}x0.12}{0.72} = 124N$$

Energy stored in rubber= ½ Fe

$$\frac{1}{2}x124x0.12 = 7.44J$$

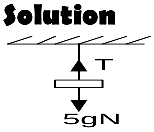
Kinetic energy of stone= $\frac{1}{2}mv^2$

$$\frac{1}{2}x0.015xv^2 = 7.44$$

$$v = 31.5ms^{-1}$$

6. A steel wire 10cm long and with a cross-sectional area of 0.01cm² is hung from a support and a mass of 5kg is suspended from its ends. Calculate the new length of the wire . the young modulus for steel=210GPa

Solution



$$T = 5gN = 5x9.81$$

$$T = 49.05N$$

But $F = T = 49.05N$

$$A = 0.01cm^2 = 0.01x10^{-4}m^2$$

$$e = \frac{F\,L}{A\,E} = \frac{49.05x\ 10x10^{-2}}{210x10^9x0.01x10^{-4}} = 2.38mm$$

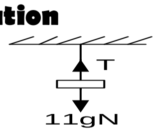
$$e = 0.0024m$$

New length $= 10.0024m$

7. A mass of 11kg is suspended from the ceiling by an aluminum wire of length 2m and diameter 2mm, what is;

   a) The extension produced
   b) The elastic energy stored in the wire (young's modulus of aluminum is 7x10¹⁰Pa)

Solution



$L = 2m$,$d = 2mm = 2x10^{-3}m$

$$T = 11gN = 11x9.81$$

$$T = 107.91N$$

But $F = T = 107.91N$

$$A = \frac{\pi d^2}{4} = \frac{\left[\frac{22}{7}x\left(2x10^{-2}\right)^2\right]}{4}$$

$$A = 3.14x10^{-6}m^2$$

$$e = \frac{F\,L}{A\,E}$$

$$e = \frac{107.91x\ 2}{3.14x10^{-6}\ x7x10^{10}}$$

$$e = 9.813x10^{-4}m$$

Energy stored = $\frac{1}{2}Te$

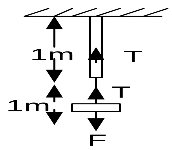
$= \frac{1}{2}\ x107.91x9.813x10^{-4}$

Energy stored = **5.29**$x10^{-2}$J

8. A cylindrical copper wire and a cylindrical steel wire, each of length 1m and having equal diameter are joined at one end to form a composite wire 2m long. This composite wire is subjected to a tensile stress until its length becomes 2.002m. calculate the tensile stress applied to the wire (young modulus of copper =$1.2x10^{11}$Pa and Steel =$2x10^{11}$Pa)

Solution

[Recall from S.H.M wire in series experience the same tension and weight]



Total extension, $e = 2.002 - 2$

$e = 0.002m$

$e = e_1 + e_2$-------------[1]

Note the two wires will experiences same stress

$0.002 = e_1 + e_2$

$e = \frac{F\,L}{A\,E}$

$0.002 = \frac{F\,L_1}{A\,E_1} + \frac{F\,L_2}{A\,E_2}$

$0.002 = \frac{F}{A}\left(\frac{1}{1.2x10^{11}} + \frac{1}{2x10^{11}}\right)$

$\frac{F}{A} = 1.5x10^8N$

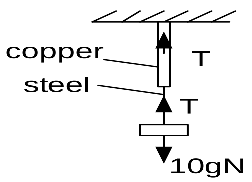
Stress $= 1.5x10^8N$

9. One end of a copper wire is welded to a steel wire of length 1.5m and diameter 1mm while the other end is fixed. The length of the copper wire is 0.8m while its diameter is 0.5mm. a bob 10kg is suspended from the free end of a steel wire. Find

   i) Extension which results ii) Energy stored in the compound wire

   (Young's modulus for copper= $1x10^{11}$Nm^{-2} and steel = $2x10^{11}$Nm^{-2})

Solution



$E_1 = 1x10^{11}, l_1 = 0.8m$

$d_1 = 0.5mm = 0.5x10^{-3}m$

$E_2 = 2x10^{11}, l_2 = 1.6m$

$d_1 = 1mm = 1x10^{-3}m$

Recall from S.H.M for series wires

$T = mg$

But $e = \frac{F\,L}{A\,E}$

$e_1 = \frac{10x9.81x0.8}{\frac{22}{7}x\frac{(0.5x10^{-3})^2}{4}x1x10^{11}}$

e_1 =3.997x10^{-3}m

$e_2 = \frac{10x9.81x1.6}{\frac{22}{7}x\frac{(1x10^{-3})^2}{4}x2x10^{11}}$

e_2 =9.9924x10^{-4} m

$e = e_1 + e_2$

e =9.9924x10^{-4} + 3.997x10^{-3}

$e = 1.039x10^{-3}m$

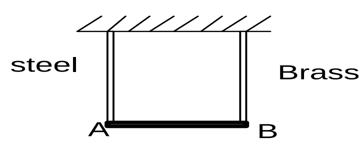
iii) Energy stored in composite

$= \frac{1}{2}$Fe

$= \frac{1}{2}x(10x9.81)x1.039x10^{-3}$

$= 5.10x10^{-2}J$

7.



A light rigid bar is suspended horizontally from two vertical wires, one of steel and one of brass as shown in the diagram. Each wire is 2.00m long. The diameter of the steel wire is 0.6mm and the length of the bar AB is 0.2m. when a mass of 10kg is suspended from the centre of AB the bar remains horizontal.

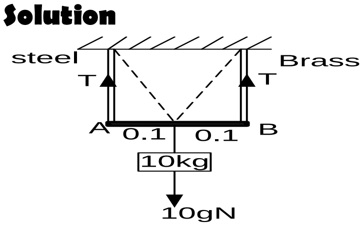
(i) What is the tension in each wire

(ii) Calculate the extension of the steel wire and the energy stored in it

(iii) Calculate the diameter of the brass wire

(iv) If the brass wires are replaced by another brass wire of diameter 1mm, where should the mass be suspended so that AB would remain horizontal.[young's modulus for steel = 2x10^{11}Pa and brass= 1x10^{11}Pa].

Solution



Assume that AB always remains horizontal

L_1 = 2m, d_1 =0.6x10^{-3}m, E_1 = 2x10^{11}pa, e_1 = ?,

L_2 = 2m, d_2 = ?, E_2 = 1x10^{11}pa, e_2 = ?

Taking moments about O: $0.1xT_1 = 0.1xT_2$

$T_1 = T_2 \dots\dots (i)$

Also: $10gN = T_1 + T_2 \dots .(\text{ii})$

$2T_1 = 10x9.81$

$T_1 = 49.05N$

Tension on each wire is $49.05N$

ii) for steel $e = \frac{F\,L}{A\,E}$

$e_1 = \frac{49.05x2}{\frac{22}{7}x\frac{(0.6x10^{-3})^2}{4}x2x10^{11}} = 1.735x10^{-3}m$

Energy stored in steel $= \frac{1}{2}T_1e_1$

$= \frac{1}{2}x49.05x1.735x10^{-3}$

Energy stored in steel is $4.26x10^{-2}J$

iii) For the bar AB to remain horizontal $e_1 = e_2$ and $L_1 = L_2$

For brass : $A_2 = \frac{T_2}{e_2}x\frac{L_2}{E_2}$

$A_2 = \frac{49.05}{1.735x10^{-3}} x \frac{2}{1x10^{11}} = 5.65x10^{-7}m^2$

$A_2 = \frac{\pi d^2}{4}$

$d^2 = \frac{4x\ 5.65x10^{-7}}{\frac{22}{7}}$

d = $8.485x10^{-4}m$

(v) Brass: $d = 1mm$

$A_2 = \frac{\pi\ (1x10^{-3})^2}{4} = 7.85x10^{-7}m^2$

$T_1 = \frac{e_1E_1A_1}{L_1}$ and $T_2 = \frac{e_2E_2A_2}{L_2}$

Taking moments about O

$yxT_1 = (0.2 - y)xT_2$

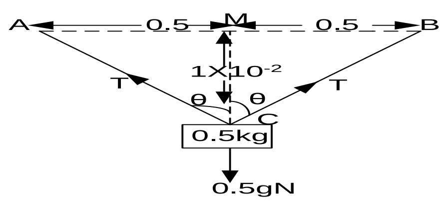
$y(2x10^{11}x2.825x10^{-7}) = (0.2-y)(1x10^{11}x7.85x10^{-7})$

$y = 0.116m$

Mass should be placed 0.116m from the steel wire

8. The ends of a uniform wire of cross-sectional area $10^{-6}m^2$ and negligible mass are attached to fixed points A and B which are 1m apart in the same horizontal plane. The wire is initially straight and outstretched. A mass of 0.5kg is attached to the mid point of the wire and hangs in equilibrium with the mid point at a distance 10mm below AB. Calculate the value of young's modulus for the wire

Solution



Using Pythagoras theorem

$CB^2 = 0.5^2 + (1x10^{-2})^2$

$CB^2 = 0.2501$

$CB = 0.5001m$

$AC = CB = 0.5001m$

Length ACB = $0.5001\ x\ 2 = 1.0002m$

Extension = $1.0002 - 1 = 2x10^{-4}m$

But $Tan\theta = \frac{0.5}{1x10^{-2}}$

$\theta = 88.9°$

(↑): $2Tcos\ \theta = 0.5g$

$2Tcos88.9 = 0.5x9.81$

$T = 127.75N$

$E = \frac{F\ L}{A\ e}$

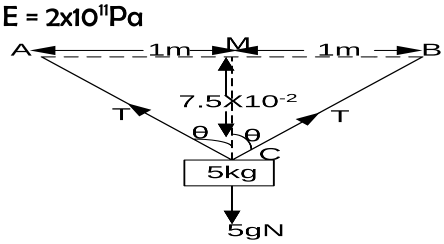
But $F = T$ (deforming force)

$E = \frac{127.75x1}{10^{-6}x2x10^{-4}} = 6.39x10^{11}Nm^{-2}$

9. The ends of a uniform wire of length 2m are fixed to two points which are 2m apart in the same horizontal line. When a 5kg mass is attached to the mid point of the wire, the equilibrium position is 7.5cm below the line AB. Given that the young's modulus of the material of the wire is $2x10^{11}$pa. find the;
   i. Strain in the wire
   ii. Stress in the wire
   iii. Energy stored in the wire.

Solution

M = 5kg, AB = 2m, L = 2m, $M_C = 7.5x10^{-2}m$,
E = $2x10^{11}$Pa



$CB^2 = 1^2 + (7.5x10^{-2})^2$

$CB = 1.003m$

$CB = AC = 1.003m$

Stretched length ACB = $2x1.003 = 2.006m$

Extension = $2.006 - 2 = 0.006m$

Strain = $\frac{e}{l} = \frac{0.006}{2} = 3x10^{-3}$

Stress = $E\ x$ strain= $2x10^{11}\ x\ 3\ x\ 10^{-3}$

Stress = $6x10^8Nm^{-2}$

Energy stored = $\frac{1}{2}Te$ ....(i)

(↑): $2Tcos\theta = 5g$ ....(ii)

Also $tan\theta = \frac{1}{7.5x10^{-2}}$

$\theta = 85.7°$

$2Tcos85.7 = 5x9.81$

$T = 327.92N$

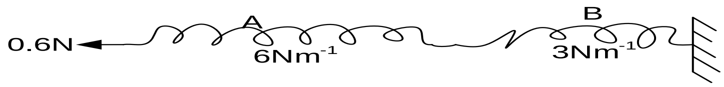
Energy stored = ½ $x327.92\ x\ 0.006$

$= 0.984J$

Exercise: 23 [use g = 10ms^{-2}]

1. A metal specimen has length of 0.5m. If the maximum permissible strain is not to exceed 10^{-3}, calculate its maximum extension **An ($5x10^{-4}$m)**
2. A metal bar of length 50mm and square cross-sectional side 20mm is extended by 0.015mm under a tensile load of 30kg, calculate
   a. Stress
   b. Strain in specimen
   c. Value of young's modulus for that metal. **An[$7.25x10^{-3}Nm^{-2}$, $3x10^{-4}$, $24.5Nm^{-2}$]**

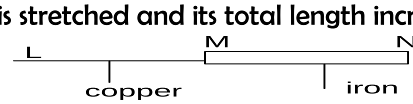
3.



A spring A of force constant 6Nm^{-1} is connected in series with a spring B of force constant 3Nm^{-1} as shown below. One end of the combination is securely anchored and a force of 0.6N is applied to the other end

a. By how much does each spring extend
b. What is the force constant of the combination **An[0.1 (A), 0.2m(B), 2Nm^{-1}]**

4. A copper wire and steel wire each of length 1.5 m and diameter 2mm are joined end to end to form a composite wire. The composite wire is loaded until its length becomes 3.003m. if young's modulus of steel is $2.0x10^{11}Pa$, and that of copper is $1.2x10^{11}Pa$
   (i) Find the strain in the copper and steel wires
   (ii) Calculate the force applied
   An[copper =0.0013, steel =$7.5x10^{-4}$, force=$4.7x10^{2}N$]
5. A thin steel wire initially 1.5m long and of diameter 0.50mm is suspended from a rigid support, calculate
   i. The final extension
   ii. Energy stored in a wire when a mass of 3kg is attached to the lower end. (young's modulus for steel = 2x10^{11}Nm^{-2}) **An [1.1mm, 1.7x10^{-2}J]**
6. Two wires of steel and phosphor bronze each of diameter 0.40cm and length 3.0m are joined end to end to form a composite wire of length 6.0m. calculate the tension in the wire needed to produce a total extension of 0.128cm in the composite wire.
   (Given that E of steel= $2.0x10^{11}Pa$ and E of bronze= $1.2x10^{11}Pa$) **An[100.5N]**
7. A copper wire LM is fused at one end M to an iron wire MN. The copper wire has length 0.9m and cross section 0.9x10^{-6}m^2. The iron wire has length 1.4m and cross-section 1.3x10^{-6}m^2. The compound wire is stretched and its total length increases by 0.01m



Calculate;
   a) The ratio of the extension of the two wires
   b) The extension of each wire
   c) The tension applied to the compound wire (young's modulus for copper = 1.3x10^{11}Nm^{-2}and 2.1x10^{11} pa) (Young's modulus for steel = 2x10^{11}Nm^{-2})
   An (Cu:Fe 3:2, 0.6mm, 4.0mm,780N)
8. a) Define stress, strain and the young's modulus
   b) i) Describe an experiment to determine the young's modulus for a material in the form of a wire
   ii) Which measurement require particular care, from the point of view of accuracy and why
   c) i) derive an expression for the potential energy stored in a stretched wire
   ii) A steel wire of diameter 1mm and length 1.m is stretched by a force of 50N, calculate the potential energy stored in the wire. ((young's modulus for steel = 2x10^{11}Nm^{-2}) **An 1.2x10^{-2}J**
   iii) The wire is further stretched to breaking where does the stored energy go
9. a) A heavy rigid bar is supported horizontally from a fixed support by two vertical wires A and B of the same initial length and which experience the same extension. If the ratio of the diameter of A and to that of B is 2 and the ratio of the young's modulus of A to that of B is 2, calculate the ratio of the tension in A to that in B. **An (8:1)**
   b) if the distance between the wires is D, calculate the distance of wire A from the centre of gravity of the bar**. (An**= $\frac{D}{9}$**)**
10. a) A rubber cord has a diameter of 5.0mm and on un stretched length of 1.0m. One end of the cord is attached to a fixed support A. When a mass of 1.0kg is attached to the other end of the cord so as to hang vertically below A, the cord is observed to elongate by 100mm, calculate the young's modulus of rubber.
   b) If the 1kg mass is now pulled down a further short distance and then released, what is the period of the resulting oscillations**?** **An [5.1x10^{6}Nm^{-2}, 0.63s]**
11. A uniform steel wire of density 7800kgm^{-3} weighs 26g and 250cm long, it lengthens by 1.2mm,

when stretched by a force of 80N, calculate;

a) The value of young's modulus for steel
b) The energy stored in the wire

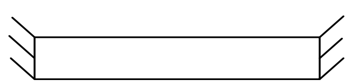
(Hint volume = Al = $\frac{mass}{density}$) Ans (2.03x10^{11}Nm^{-2}, 0.048J)

12. If the young modulus for steel is 2.0x10^{11}Nm^{-2}. Calculate the work done in stretching a steel wire 100cm is length and of cross-sectional area 0.030cm^2. When a load of 100N is slowly applied, the elastic limit not being exceeded
13. A gymnast of mass 70kg hangs by one arm from high bar. If the gymnasts whole weight is assumed to be taken by the humerous bore (in the upper arm), calculate the stress in the humerous if it has a radius of 1.5cm
14. Find the maximum load that can be support by a steel cable 1.5cm in diameter without its elastic limit being exceeded when the load is
    (a) In air (b) immerse in water
15. A hammer thrower swing a 7.25kg hammer in a horizontal circle at one revolution per second. If the hammer wire is 1.20m long, 1.5mm in diameter and made of steel. Calculate the extension produced in it. (mass of the wire its self may be neglect and young's modulus of steel 210GPas)
16. A copper wire 200cm long and 1.22mm in diameter is fixed horizontally between two supports 200cm apart. Find the mass of load which when suspended at the mid part of wire, produced a sag of 2cm at the point.(young's modulus for copper=1.2x10^{11}Nm^{-2})
17. A steel rod of mass 97.5g and of length 50cm is heated to 200°C and its end securely clamped. Calculate the tension in the rod when its temperature is reduced to 0°C.
18. A rubber cord a catapult has a cross-sectional area of 1.0mm^2 and un stretched length 10.0cm. It is stretched to 15cm and then released to project a missile of mass 5.0g. Calculate;
    (a) the energy stored in the rubber.
    (b) The velocity of projection
    (c) The maximum height that the missile could reach
    (young's modulus for rubber=5.0x10^{8}Pa)
19. A solid copper wire of cross-sectional area 8mm^2 and original length 1.10m is set up as a telephone line with a uniform tension 3.6x10^{3}N. Assuming that the wire stretches elastically. Calculate
    (a) The extension of the wire
    (b) The elastic energy store in the wire
    (c) Heat lost by the wire during cooling and find the change in elastic energy. If during cold weather the temperature falls by 15K

10.2.4: FORCE ON A BAR DUE TO THERMAL EXPANSION OR CONTRACTION

When a bar is heated and then prevented from contracting as it cools, a force is exerted at the ends of a bar.

Consider a metal of young's modulus E, cross sectional Area A at a temperature θ_2°c fixed between two rigid supports.



When the bar is cooled to a temperature θ_1°C, the bar can not contract hence there will be forces on the rigid support.

If α is the mean co-efficient of linear expansion then $L_\theta = L_0(1+\alpha\theta)$

L_θ is length of the bar at temperature θ°c

L_o is length of the bar at temperature 0°c

$L_2 = L_0(1+\alpha\theta_2)$ i

$L_1 = L_0(1+\alpha\theta_1)$ ii

Subtracting

$L_2 - L_1 = L_0\,\alpha\,(\theta_2 - \theta_1)$

$L_2 - L_1 = L_0\,\alpha\theta$

$\alpha\,\theta = \frac{L_2 - L_1}{L_0}$

But strain = $\frac{L_2 - L_1}{L_0}$

$\boxed{Strain = \alpha\,\theta}$ where θ = $\theta_2 - \theta_1$

Stress = E x strain

$\frac{F}{A} = Ex\alpha\,\theta$

$F = AE\,\alpha\,\theta$

$\boxed{F = AE\,\alpha\,\theta}$

Coefficient of linear expansion α is defined as the fractional increase in length at0°C for every degree rise in temperature.

Examples

1. A steel bar with cross-sectional area of 2cm^2 is heated, raising its temperature by 120°C and prevented from expanding. Calculate the resulting force in the bar young's modulus of steel = 1.0x10^{11}Nm^{-2} and linear expansitivity of steel = $1.2x10^{-5}K^{-1}$)

Solution

$Force = EA\,\alpha\,\theta = 2.1x10^{11}x2x10^{-4}x1.2x10^{-5}x120 = \mathbf{6.05x10^4N}$

2. Two identical steel bars A and B of radius 2.0mm are suspended from the ceiling. A mass of 2.0kg is attached to the free end of bar A, calculate the temperature to which B should be raised so that the bars are again of equal length. (young's modulus of steel = 1.0x10^{11}Nm^{-2} and linear expansitivity of steel = $1.2x10^{-5}K^{-1}$)

Solution

For steel bar A, $r = 2x10^{-3}m$, m = 2kg,

$E = 1x10^{11}Nm^{-2}$, $\alpha = 1.2x10^{-5}K^{-1}$

But $E = \frac{stress}{strain}$ Strain $= \frac{stress}{E}$

Stress $= \frac{F}{A}$

Strain $= \frac{F}{A\,E}$

Strain $= \frac{2x9.81}{\pi(2x10^{-3})^2x1x10^{11}} = 1.56x10^{-5}$

but strain $= \alpha\,\theta$

$\theta = \frac{1.56x10^{-5}}{1.2x10^{-5}} = 1.3K$

B should be raised by a temperature of 1.3K

b) A uniform metal bar of length 1m and diameter 2cm is fixed between two rigid supports at 25°C. if the temperature of the bar is raised to 75°C, find
   (i) The force exerted on the support.
   (ii) Energy stored in the bar at 75°C. (young's modulus of metal = 2x10^{11}Pa and coefficient of linear expansion = $1x10^{-5}$K^{-1})

Solution

i) Θ_1 = 25°C, Θ_2 = 75°C, E = 2x10^{11}Pa,

L = 1m, d = 2x10^{-2}m, α = $1x10^{-5}$K^{-1}

$$Force = EA\,\alpha\,\theta$$

$$F = 2x10^{11}\,x\,\frac{\frac{22}{7}x\left(2x10^{-2}\right)^2}{4}x1x10^{-5}(75-25)$$

$$F = 3.14x10^4\,N$$

ii) Energy stored $= \frac{1}{2}Fe$

but strain $= \alpha\,\theta$

and also strain $= \frac{e}{l}$

$\frac{e}{l} = \alpha\,\theta$

e $= l\,\alpha\,\theta$

Energy stored $= \frac{1}{2}$F $l\,\alpha\,\theta$

$= 3.14x10^4\,x1x10^{-5}x1x(75-25)$

$= 7.85J$

Exercise:24

1. A copper rod of length 0.8m and diameter 40mm is fixed between two rigid supports at a temperature of 20°C. the temperature of the rod is raised to 70°C, calculate;
   i. The force exerted on the rod at 70°C
   ii. Energy stored per unit volume at 70°C
   iii. Force exerted on the support if temperature was lowered to 45°C

   [E for copper = 1.2x10^{11}Nm^{-2}, α for copper between 20°C to 70°C is 1.7x10^{-5}K^{-1}]

   Ans (1.28x10^5 N, 43.52J, 4.33x10^{4}Jm^{-3}, 6.4x10^{4}N)

2. Two identical cylindrical steel bars each of radius 3.00m and length 7m rest in a vertical position with their lower end on a rigid horizontal surface. A mass of 4.0kg is placed on the top of one bar. The temperature of the other bar is to be altered so that the two bars are once again of equal length. Given that the coefficient of linear expansivity of steel is$= 1.2x10^{-5}K^{-1}$
   (i) By how much should the temperature be altered
   (ii) Find the energy store in the bar due to the temperature change. **An[0.58K, $0.96J$]**

UNEB 2017 No4

(a) (i) Define **elastic deformation** and **plastic deformation** (02mark)

(ii) Explain what is meant by work hardening (02marks)

(b) (i) Sketch using the same axes, stress-strain curves for a ductile material and rubber. (03marks)

(ii) Explain the features of the curve for rubber (03marks)

UNEB 2016 No1

(a) (i) Define **elastic limit** of a material (01mark)

(ii) Describe an experiment to determination the Young's modulus of a steel wire (06marks)

(b) Explain why tyres of a vehicle travelling on a hard surfaced road may burst. (04marks)

Solution

when a car moves on a hard surface, friction between the tyre and the surface causes heating. So the temperature of the air inside the tyre increases. The K.E of the air molecules in the tyre increase leading to increased pressure inside the tyre and the tyre may burst

UNEB 2015 No2

(c) (i) Define **young's modulus** (01mark)

(ii) Explain the precautions taken in the determination of Young's modulus of a wire (06marks)

(d) Explain why a piece of rubber stretches much more than a metal wire of the same length and cross-sectional area (02marks)

UNEB 2014 No2

(a) **(i)** What is meant by **Young's modulus** (01mark)

(ii) State **Hooke's law** (01mark)

(iii) Derive an expression for the energy released in a unit volume of a stretched wire in terms of stress and strain (04marks)

(b) A steel wire of length 0.6 m and cross- sectional area $1.5x10^{-6}m^2$ is attached at B to a copper wire BC of length 0.39 m and cross-sectional area $3.0x10^{-6}m^2$. The combination is suspended vertically from a fixed point at A and supports weight of 250 N at C. find the extension in each of the wires, given that Young's Modulus for steel is $2.0x10^{11}Pa$ and that of copper is $1.3x10^{11}Pa$.

An[$steel = 5.0x10^{-4}m, copper = 2.5x10^{-4}m$ **]** (05 marks)

(c) With the aid of a labeled diagram, describe an experiment to determine the Young's Modulus of steel wire (07marks)

(d) Explain the term plastic deformation in metals (02marks)

UNEB 2012 No1

a) State Hooke's law (1 mark)

b) A copper wire is stretched until it breaks

   i. Sketch a stress-strain graph for the wire and explain what happens to the energy used to stretch the wire at each stage. (4 marks)

   ii. Derive the expression for the work done by a distance e (3 marks)

c) (i) Define young's modulus (1 mark)

(II) Two identical steel bars A and B of radius 2.0mm are suspended from the ceiling. A mass of 2.0kg is attached to the free end of bar A, calculate the temperature to which B should be raised so that the bars are again of equal length. (young's modulus of steel = $1.0x10^{11}Nm^{-2}$ and linear expansitivity of steel = $1.2x10^{-5}K^{-1}$)

(d) why does an iron roof make cracking sound at night (2 marks)

Solution

(d) during the day, the roof is heated , it expands and buckle (bends) since it is fixed. At night, the roof contracts due to fall in temperature. As it straightens again sound is produced

UNEB 2010

a) i) describe the terms tensile stress and tensile strain as applied to a stretched wire. (2 marks)

b) ii) Distinguish between elastic limit and proportional limit (2 marks)

c) With the aid of a labeled diagram, describe an experiment to investigate the relationship between tensile stress and tensile strain of a steel wire (4 marks)

d) i) A load of 60N is applied to a steel wire of length 2.5m and cross sectional area of $0.22mm^2$. if young's modulus for steel is 210GPa, find the expansion produced. (3 marks)

ii) If the temperature rise of 1k causes a fractional increase of 0.001%, find the change in the length of a steel wire of length 2.5mm when the temperature increases by 4K. (3 marks)

Solution

F = 60N, L = 2.5m, A = 0.22mm^2 = 0.22x10^{-6}m^2, E = 210Gpa or E = 210x10^{9}Pa

Expansion required is the extension

$$E = \frac{F\,L}{A\,e}$$

$$e = \frac{F\,L}{AE} = \frac{60x2.5}{0.22x210x10^{9}x10^{-6}} = 3.247\ x\ 10^{-3}m$$

ii) 1K gives 0.001%

$$\%\text{extension} = \frac{extension}{natural\ length}x100\%$$

$$0.001\% = \frac{e}{2.5}x100\%$$

$$e = 2.5x10^{-4}m$$

$$1K = 2.5x10^{-4}m$$

$$4K = 2.5x10^{-4}x4$$

$$4K = 1x10^{-3}m$$

UNEB 2006 No 3

a) i) Define stress and strain (2 marks)

ii) Determine the dimensions of young's modulus (3 marks)

b) Sketch a graph of stress versus strain for a ductile material and explain its features (6 marks)

c) A steel wire of cross-section area 1mm^2 is cooled from a temperature of 60^{o}c to 15^{o}c, find the;

i. Strain (2marks)

ii. Force needed to prevent it from contracting young's modulus = 2x10^{11}Pa, coefficient of linear expansion for steel = 1.1x10^{-5}K^{-1} (3 marks)

d) Explain the energy changes which occur during plastic deformation (4 marks) **Ans (4.95x10^{-4}, 99N)**

UNEB 2005 No 2

a) Explain the terms

i. Ductility

ii. Stiffness

b) A copper wire and steel wire each of length 1.0m and diameter 1.0mm are joined and to end to form a composite wire 2.0m long, find the strain in each wire when the composite stretches by 2x10^{-3}m Young's modulus for copper and steel are 1.2 x10^{11} and 2.0x10^{11}Pa respectively. **Ans (1.25x10^{-3}, 7.5x10^{-4})**

UNEB 2003 No 3(d)

i) define the terms longitudinal stress and young's modulus of elasticity (2 marks)

ii) describe how to determine young's modulus for a steel wire. (07 marks)

UNEB 2001 No2

a) Define the following terms

i. Stress (1 mark)

ii. Strain (1 mark)

c) State the necessary measurements in the determination of young's modulus of a metal wire (2 marks)

d) Explain why the following precautions are taken during an experiment to determine young's modulus of a metal wire.

i. Two long, thin wires of the same material are suspended from a common support. (2 marks)

ii. The readings of the Vanier are also taken when the loads are gradually removed in steps. (1 mark)

CHAPTER 11: FLUID FLOW

A fluid element is a molecule of a fluid which follows the flow

A flowline is the path which an individual molecule in a fluid element describes

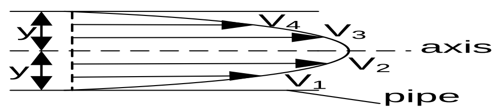
Why some fluids flow more easily than others

Fluid flow involves different parts of a fluid moving at different velocities. Different parts of the fluid therefore slide past each other in layers . There exists a frictional force between the layers of the fluid, which is the measure of the flow rate. The greater the a frictional force , the less easily it is for the liquid to flow and the lower the a frictional force , the more easily it is for the liquid to flow. Thus some fluids flow more easily than others.

11.1.1: LAMINAR AND TURBULENT FLOW

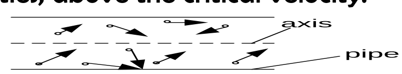
Laminar (steady/uniform) flow is the orderly flow of a liquid where flow lines are parallel to the axis of flow and equidistant layers from the axis of flow have the same velocity.

Laminar flow occurs at low velocities below the critical velocity.

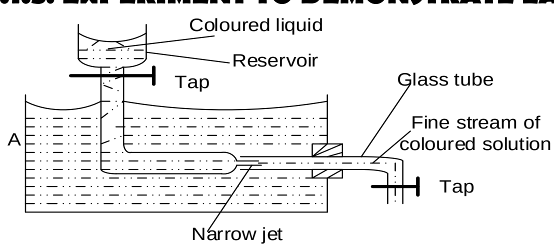


Turbulent flow is the disorderly flow of a liquid where flow lines are not parallel to the axis of flow and equidistant layers from the axis of flow have the varied velocities.

Turbulent flow occurs at high velocities, above the critical velocity.



11.1.3: EXPERIMENT TO DEMONSTRATE LAMINAR AND TURBULENT FLOW



- Taps are opened narrowly to allow cloured liquid to flow with low velocity. A fine stream is seen along the center of the narrow tube in an oderly flow and this illustrates laminar flow.
- When the taps are widely opened the stream of the coloured liquid breaks up and a coloured liquid spreads through the tube. This demonstrates turbulent flo

VISCOSITY

Viscosity is the frictional force between adjacent layers of a fluid.

Viscous drag is the frictional force experienced by a body moving in a fluid due to its viscosity.

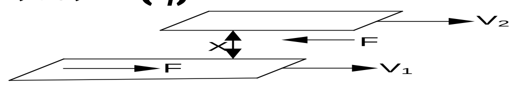
11.1.3: Effects of temperature on viscosity

- In liquids, viscosity is due to intermolecular forces of attraction between layers moving at different speeds. Increase in temperature reduces(weakens) intermolecular forces which increases molecular separation and speed, consequently viscosity in liquids decreases rapidly with increase in temperature
- In gases, viscosity is due to transfer of momentum . Molecules are further apart and have negligible intermolecular forces , molecules move randomly colliding with one another and continuously transferring momentum to the neighboring layers. Increasing the temperature of the gas increases the average speed (increases K.E) and make frequent collisions of the gas molecules hence increasing the transfer of momentum which results into increase in viscosity of the gas.

Differences between viscosity and solid friction

Solid friction	**Viscosity**
Independent of area of contact	Depends on area of contact
Independent of relative velocity between layers in contact	Directly proportional to velocity gradient
Independent of temperature but dependent on normal reaction	Depends on temperature

11.1.4: COEFFICIENT OF VISCOSITY (η)



Consider two parallel layers of a liquid moving with velocities V_1 and V_2 and separated by a distance x with area of contact between the layers A

The slower lower layer exerts a tangential retarding force F on the faster upper layer the lower layer its self-experiences an equal and opposite tangential force F due to the upper layer.

Velocity gradient between the layers $= \frac{Velocity\ change}{distance\ apart} = \frac{V_2 - V_1}{x}$

Definition

Velocity gradient is the change in velocity between two layers (points) per unit length of separation of the points.

Frictional force F between adjacent layers depends on

- Area of contact between the layers [F α A]
- Velocity gradient between layers [F α velocity gradient]

Therefore $F\ \alpha\ A\ x\ velocity\ gradient$

$F = \eta x\ A\ x\ Velocity\ gradient$

$\boldsymbol{\eta} = \frac{F}{A\ x\ Velocity\ gradient}$

Definition

Coefficient of viscosity is the frictional force acting on a unit area of a fluid when in a region of unit velocity gradient **OR**

Coefficient of viscosity is the tangential stress which one layer of a fluid exerts on another layer in contact with it when the velocity gradient between the layers is $1s^{-1}$.

Dimensions of η

$\eta = \frac{F}{A\ x\ Velocity\ gradient}$

$[\eta] = \frac{[F]}{[A]x[Velocity\ gradient]}$

$[\boldsymbol{\eta}] = \frac{M\,L\,T^{-1}}{L^2\left(\frac{L\,T^{-1}}{L}\right)}$

$[\boldsymbol{\eta}] = \boldsymbol{M\,L^{-1}\,T^{-1}}$

$Units\ of\ \eta = Nsm^{-2}$

11.1.5: Steady flow of a liquid through a pipe (poiseuille's formula)

Poiseuille derived an expression for the volume of a liquid flowing out of a pipe per second. He assumes that the flow was steady/laminar.

The volume of liquid flowing out of a pipe per unit time (V/t) dependents on;

- i. The coefficient of viscosity η of the liquid
- ii. The radius of the pipe r
- iii. The pressure gradient P/L causing the flow

$\frac{V}{t} \propto \eta^x\, r^y \left(\frac{P}{l}\right)^z$

$\frac{V}{t} = K\,\eta^x\, r^y \left(\frac{P}{l}\right)^z$............................x

$\frac{[V]}{[t]} = [K][\eta]^x\,[r]^y \left(\frac{[P]}{[l]}\right)^z$

K is a dimensionless constant

$L^3T^{-1} = (M\,L^{-1}T^{-1})^x\ L^y\ (M\,L^{-2}T^{-2})^z$

$L^3T^{-1} = M^{x+z}\ L^{y-x-2z}\ T^{-x-2z}$

For M, $0 = x + z$..........................1

For L, $3 = y - x - 2z$.........................2

For T, $-1 = -x - 2z$.......................3

From equation 1: $0 = x + z$

$x = -z$

Put into equation 3: $-1 = -(-z) - 2z$

$-1 = -z$

$z = 1$

$x = -1$

Put into equation 2: $3 = y - -1 - 2$

$3 = y - 1$

$y = 4$

$x = -1, y = 4, z = 1$

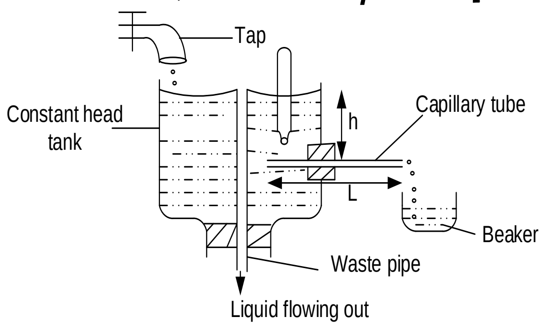
But from : $\frac{V}{t} = K\,\eta^x\, r^y \left(\frac{P}{l}\right)^z$

$\frac{V}{t} = \frac{K\,r^4 P}{\eta l}$

By experiment $K = \frac{\pi}{8}$

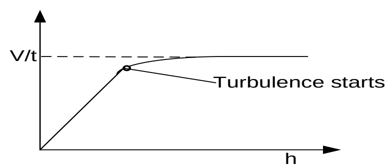
$\boxed{\frac{V}{t} = \frac{\pi\,r^4 P}{8\,\eta l}}$ - Poiseuille's formula

a: Measurement of η of a liquid by poiseuille's formula



- ❖ Measure and record the a constant head h .
- ❖ Measure and record volume V of liquid flowing through the capillary tube in time t
- ❖ Repeat several times by varying h to obtain a set of values for each volume v and calculate the volume per second$\left(\frac{V}{t}\right)$.
- ❖ Measure the length l of capillary tube, obtain the radius r of capillary tube by measuring the mass of a known length of mercury column or by column travelling microscope method
- ❖ Plot a graph of $\left(\frac{V}{t}\right)$ against h and find the slope, s of the graph.
- ❖ Calculate the coefficient of viscosity $\boldsymbol{\eta}$, from $S = \left(\frac{\pi r^4 \rho g}{8\eta l}\right)$

Theory



$$From \ \frac{V}{t} = \frac{\pi r^4 P}{8 \eta l}$$

But $P = h\rho g$ where $\boldsymbol{\rho}$is the density of the liquid

$$\frac{V}{t} = \left(\frac{\pi r^4 \rho g}{8\eta l}\right) h$$

Comparing with y = mx +c

Slope $S = \left(\frac{\pi r^4 \rho g}{8\eta l}\right)$

$$\boldsymbol{\eta} = \frac{\pi r^4 \rho g}{8 \ l s}$$

Note:

- ❖ The experiment must be carried out at a constant temperature to avoid changes in η
- ❖ Constant head apparatus is used to ensure that the rate of liquid flowing through the capillary tube is uniform. Since Poisseulle's formular holds for only laminar flow
- ❖ Great care is needed when measuring r because it appears in the calculation of η as r^4. This makes the % error in η due to an error in r four times the % error in r
- ❖ A capillary tube is used because r needs to be small so that h is large enough to be measured accurately

11.2.0: STOKE'S LAW AND TERMINAL VELOCITY

Stoke law states $\boldsymbol{F = 6\pi\eta\ rV}$

F- viscous drag

r-radius of the sphere

v- terminal Velocity of the sphere

$\boldsymbol{\eta}$ -Coefficient of viscosity of fluid

11.2.1: Derivation of Stoke's law

Stoke's suggested that any particle moving through a fluid experiences a retarding force called **viscous drag** due to the viscosity of the fluid. This force depends on the speed of the body V and acts in opposite direction to its motion

The viscous drag F on a spherical body depends

- ✓ On the radius (r) of the sphere
- ✓ Velocity V of the sphere
- ✓ Coefficient of viscosity η

F $\propto \eta^x V^y r^z$

F = $K\eta^x V^y r^z$...........(x)

[F] = $[K][\eta]^x [V]^y [r^z]$

K is a dimensionless constant

$MLT^{-2} = (ML^{-1}T^{-1})^x (LT^{-1})^y (L)^z$

$MLT^{-2} = M^x L^{y-x+z} T^{-x-y}$

For M: $x = 1$....................(1)

For L: $1 = y - x + z$

$y + z = 2$..................(2)

For T,: $-2 = -x - y$

$-2 = -1 - y$

$y = 1$

Put into eqn2: $y + z = 2$

$z = 1$

$x = 1, y = 1, z = 1$

From equation x: $F = K\eta^x V^y r^z$

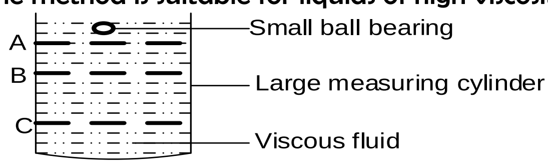
$F = k\,\eta\,Vr$

Experiment showed that $K = 6\pi$

$\boxed{F = 6\pi\eta\ rV}$ **-Stoke's law**

Measurement of η liquid by Stoke's law

The method is suitable for liquids of high viscosity such as glycerin and treacle



- ❖ Densities of the ball bearing and liquid ρ and σ respectively are obtained.
- ❖ Three reference marks A , B and C at equal distances are made on the sides of a tall transparent tube filled with the liquid.
- ❖ The ball is allowed to fall centrally through the liquid. The times t_1 and t_2 taken for the ball to fall from A to B and from B to C respectively are measured and noted .

When $t_1 = t_1 = t$, terminal velocity is obtained from

$V_o = \frac{AB}{t} = \frac{BC}{t} = \frac{AC}{2\,t}$......................[1]

- ❖ The and diameter d and hence radius r of the ball bearing is measured using a micrometer screw gauge.

Coefficient of viscosity is then calculated from Stoke's using

$\boldsymbol{\eta} = \frac{2\,r^2\,g\,(\rho-\sigma)}{9\,V_0}$....................................[2]

Note:

i) A measuring cylinder which is wide compared with the diameter of the ball bearing is used.

ii) Point C should be far away from the top of the tube so that the temperature remains constant.

iii) using a highly viscous liquid and a small ball bearing makes t large enough to be measured

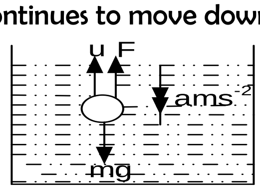
11.2.2: TERMINAL VELOCITY

Terminal velocity is the maximum constant velocity attained by a body falling through a viscous fluid.

EXPLANATION OF TERMINAL VELOCITY

Consider a sphere of radius,r falling from rest through a viscous fluid.

- ❖ The forces acting on the sphere are its weight W downwards , up thrust upwards U due to the displaced fluid and the viscous drag, F upwards due to viscosity of the fluid.
- ❖ Initially $W > U + F$ and the sphere accelerates downwards. As its velocity increases, viscous drag increases and eventually $W = U + F$ and net force is zero and sphere moves with constant velocity. The sphere continues to move down with a maximum constant velocity called ***terminal velocity***.



If σ and ρ re the densities of the fluid and sphere respectively, the;

At the terminal velocity: $Mg = U + F$......(1)

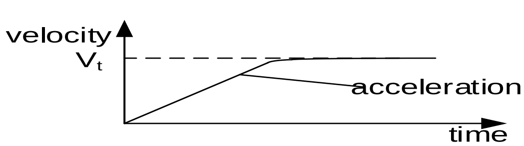
$\frac{4}{3}\pi r^3 \rho\, g = \frac{4}{3}\pi r^3 \sigma\, g + 6\pi\,\boldsymbol{\eta}\,rV_o$

$6\pi\,\boldsymbol{\eta}\,rV_o = \frac{4}{3}\pi r^3\ g(\rho - \sigma)$

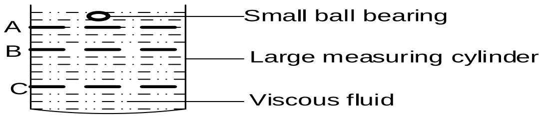
$V_o = \frac{4\,\pi\,r^3\,g\,(\rho-\sigma)}{3\,x6\pi\eta r}$

$V_o = \frac{2\,r^2\,g\,(\rho-\sigma)}{9\eta}$

A graph of velocity against time for an object falling in a fluid



Measurement of terminal veocity



- Densities of the ball bearing and liquid ρ and σ respectively are obtained.
- Three reference marks A , B and C at equal distances are made on the sides of a tall transparent tube filled with the liquid.
- The ball is allowed to fall centrally through the liquid. The times t_1 and t_2 taken for the ball to fall from A to B and from B to C respectively are measured and noted .

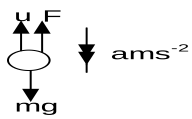
When $t_1 = t_1 = t$, terminal velocity is obtained from

$V_o = \frac{AB}{t} = \frac{BC}{t} = \frac{AC}{2\,t}$......................[1]

Numerical examples

1. A spherical raindrop of radius 2.0x10^{-4}m, falls vertically in air at 20^{o}c, if the densities of air and water are 1.3kgm^{-3} and 1x10^{3}kgm^{-3} respectively and the viscosity of air at 20^{o}c is 1.8x10^{-5}pa. Find the terminal velocity of the drop

Solution



At terminal velocity : $Mg = U + F$

$\frac{4}{3}\pi r^3 \rho_s\, g = \frac{4}{3}\pi r^3 \rho_f\, g + 6\pi\, \eta\, r V_o$

$$V_o = \frac{2\,r^2\,g\,(\rho_f - \rho_s)}{9\,\eta}$$

$$V_o = \frac{2x\,(2x10^{-4})^2 x9.81x\,(1x10^3 - 1.2)}{9x1.8x10^{-5}} = 4.84ms^{-1}$$

2. Calculate the terminal velocity of a rain drop of radius 0.2cm. Density of water 1000kgm^{-3} and density of air 1kgm^{-3} and coefficient of viscosity of air is 10^{-3}Pa

Solution

$$V_o = \frac{2\,r^2\,g\,(\rho_f - \rho_s)}{9\,\eta} = \frac{2x\,(0.2x10^{-2})^2 x9.81x\,(1000-1)}{9x1x10^{-3}} = 8.7ms^{-1}$$

3. Find the time taken for a particle of carbon of density 2300kgm-3 with radius 0.0001m to fall 2cm through air(coefficient of viscosity of air is 10^{-3}Pa). neglect air buoyance

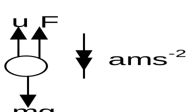
Solution

$$V_o = \frac{2\,r^2\,g\,(\rho_f - \rho_s)}{9\,\eta} = \frac{2x\,(0.001)^2 x9.81x\,(2300-0)}{9x1x10^{-3}} = 4.6x10^{-4}ms^{-1}$$

Time to fall 2cm= $\frac{2x10^{-2}}{4.6x10^{-4}} = 4348s$

4. A spherical oil drop of density 900kgm^{-3} and radius 2.5x10^{-6}m has a charge of 1.6x10^{-19}C. the drop falls under gravity between two plates
   i. Calculate the terminal velocity attained by the drop
   ii. What electric field intensity must be applied between the plates in order to keep the drop stationary (density air = 1kgm^{-3}, coefficient of viscosity of air = 1.8x10^{-3}Nm^{-2}s^{-1})

Solution



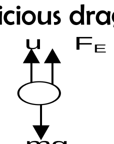
At terminal velocity: $Mg = U + F$

$\frac{4}{3}\pi r^3 \rho_s\, g = \frac{4}{3}\pi r^3 \rho_f\, g + 6\pi\eta\, r V_o$

$$V_o = \frac{2\,r^2\,g\,(\rho_f - \rho_s)}{9\eta}$$

$$V_o = \frac{2x\,(2.5x10^{-6})^2 x9.81x\,(900-1)}{9x1.85x10^{-5}} = 6.62x10^{-6}ms^{-1}$$

Since the sphere is moving down, the electric field must be applied upwards to keep it stationary and there will be no vicious drag



When it is stationary $Mg = U + F_E$

$$\frac{4}{3}\pi r^3 \rho_s\, g = \frac{4}{3}\pi r^3 \rho_f\, g + EQ$$

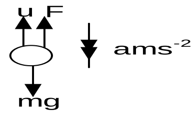
$$E = \frac{4\pi r^3 g\left(\rho_f - \rho_s\right)}{3xQ}$$

$E = \frac{4x\frac{22}{7}x(2.5x10^{-6})^3x9.81x(900-1)}{3x1.6x10^{-19}}$

$E = 3.60x10^6Vm^{-1}$

5. Find the terminal velocity of an oil drop of radius 2.5x10^{-6}m which falls through air. Neglecting the density of air.(Viscosity of air = 1.8x10^{-5}Nm^{-2}, density of oil = 900kgm^{-3})

Solution



At terminal velocity: $Mg = U + F$

$\frac{4}{3}\pi r^3\rho_s g = \frac{4}{3}\pi r^3\rho_f g + 6\pi \eta rV_o$

$V_o = \frac{2r^2 g(\rho_f - \rho_s)}{9\eta}$

But $\rho_f = 0kgm^{-3}$

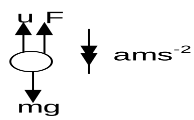
$V_o = \frac{2x(2.5x10^{-6})^2x9.81x(900-0)}{9x1.8x10^{-5}}$

$V_o = 6.81x10^{-4}ms^{-1}$

6. A metal ball of diameter 20mm is timed as it falls through oil at a steady speed, it takes 0.5s to fall through a vertical distance of 0.3m. Assuming that density of the metal is 7500kgm^{-3} and that of oil is 900kgm^{-3}, find
   i. The weight of the ball (2 marks)
   ii. The up thrust on the ball
   iii. The coefficient of viscosity of oil (03 marks

(Assume the viscous force = $6\pi\eta r$ V$_o$ where η is the coefficient of viscosity, r is radius of *the ball and V$_o$ is terminal velocity)*

Solution



i) Weight = $mg = \frac{4}{3}\pi r^3\rho_s g$

$W = \frac{4}{3}x\frac{22}{7}x(10x10^{-3})^3x7500x9.8 = 0.31N$

ii) Up thrust U = $\frac{4}{3}\pi r^3\rho_f g$

$= \frac{4}{3}x\frac{22}{7}x(10x10^{-3})^3x900x9.81$

U = $0.037N$

iii) At terminal velocity $Mg = U + F$

$0.31 = 0.037 + 6\pi\eta r V_o$

$\eta = \frac{0.31-0.037}{6\pi r V_o}$

but $V_o = \frac{0.3}{0.5} = 0.6m/s$

$\eta = \frac{0.31-0.037}{6x\frac{22}{7}x10x10^{-3}x0.6} = 2.414Nsm^{-2}$

Exercise:25

1. A small oil drop falls with terminal velocity of 4x10^{-4}ms^{-1} through air. Calculate the radius of the drop. What is the terminal velocity of oil drop if its radius is halved.
(viscosity of air = 1.8x10^{-5}Nm^{-2}s, density of oil = 900kgm^{-3}, neglect density of air) **An** [1.92x10^{-6}m, 1.0x10^{-4}ms^{-1}]
2. Calculate the terminal velocity of a rain drop of radius 0.2cm, density of air= 1.2kgm^{-3} and that of water =1000kgm^{-3} respectively and that the coefficient of viscosity of air is $9x10^{-3}Pa$. **An[8.7 ms^{-1}].**
3. A spherical rain drop of radius $2.0x10^{-4}$m falls vertically in air at 20°C. If the densities of air and water are 1.2kgm^{-3} and 1000kgm^{-3} respectively and that the coefficient of viscosity of air at 20°C is $1.8x10^{-5}Pa$ s, calculate the terminal velocity of the drop. **An[4.484 ms^{-1}].**
4. A metal sphere of radius 2.0x10^{-3}m and mass 3.0x10^{-4}kg falls under gravity, central down a wide tube filled with a liquid at 35°c, the density of the liquid is 700kgm^{-3}, the sphere attains a terminal velocity of magnitude 40x10^{-2}ms^{-1}. The tube is emptied and filled with another liquid at the same temperature and of density 900kgm^{-3}. When the metal sphere falls centrally down the tube, it is found to attain a terminal velocity of magnitude 25x10^{-2}ms^{-1}. Determine at 35°C, the ratio of the coefficient of viscosity of the second liquid to that of the first. **[an 1.640]**
5. In an experiment to determine the coefficient of viscosity of motor oil, the following measurements were made
   Mass of glass of sphere = 1.2x10^{-4}kg
   Terminal velocity of sphere = 5.4x10^{-2}ms^{-1}
   Diameter of sphere = 4.0x10^{-3},
   Density of oil = 860kgm^{-3}
   Calculate the coefficient of viscosity of the oil **[an 0.45Nsm^{-2}]**
6. A metal sphere of radius 3.0x10^{-3}m and mass 4.0x10^{-4}kg falls under gravity, central down a wide tube filled with a liquid at 25°C, the density of the liquid is 800kgm^{-3}, the sphere attains a terminal velocity of

magnitude $45cms^{-1}$. The tube is emptied and filled with another liquid at the same temperature and of density 100kgm^{-3}. When the metal sphere falls centrally down the tube, it is found to attain a terminal velocity of magnitude $20cms^{-1}$. Determine at 25°C, the ratio of the coefficient of viscosity of the second liquid to that of the first. **[An 2.09]**

7. A steel sphere of diameter 3.0x10^{-3}m falls through a cylinder containing a liquid x .When the sphere has attained a terminal velocity, it takes 1.08 s to travel between two fixed marks on the cylinder. When the experiment is repeated using another steel sphere of diameter 5.0x10^{-3}m with the cylinder containing liquid y, the time of fall between two fixed points is 4.8 s. if the density of liquid x is $1.26x10^3kgm^{-3}$, that of liquid y is$0.92x10^3kgm^{-3}$ and that of the steel ball is $7.8x10^3kgm^{-3}$, determine the ratio of the coefficient of viscosity of the liquid x to that of the liquid y, if the temperature remains constant throughout. **[An 0.77]**
8. Calculate the terminal velocities of the following rain drops falling through air
   (a) One with a diameter of 0.3cm
   (b) One with a diameter of 0.01mm
   (density of water=1000kgm^{-3}, and viscosity of air=1.0x10^{-3}Pas. neglect air buoyancy)
9. An explosion occurs at an altitude of 1000m where there is a constant horizontal wind speed of 10m/s. It is estimated that the smallest particles produced by the explosive have diameter of 0.01mm and density of 2000kgm^{-3}. Calculate
   (a) The time taken for the smallest particles to fall to the ground
   (b) The horizontal distance trvaelled from the point of the explosion
   (visocisity of air 1.8x10^{-5} Pas, density of air 1.2kgm^{-3})
10. Calculate the viscous drag on the drop of oil of radius 0.1mm falling through air at its terminal velocity
    (viscosity of air 1.8x10^{-5}Pas, density of air 850kgm^{-3})
11. Powdered chalk of density 2800kgm^{-3} is vigorously shaken up in a bottle containing 15cm depth of water. It is found that it is half an hour before all the chalk have finally settled to the bottom of the bottle. If the coefficient of viscosity of water is 1.1x110^{-3}Pas, find the diameter of the smallest chalk particle assumed to be spherical .
12. Compare the speed at which a steel ball of density 7800kgm^{-3} of radius 2mm will fall through treacle, with that at which an air bubble of density 1.3kgm^{-3} of radius 1mm will rise through the same liquid.(density of treacle=1600kgm^{-3})
13. Two spherical rain drops of equal size are falling through air at a velocity of 0.08ms^{-1}. If the drops join together forming a large spherical drop, what will be the new terminal velocity

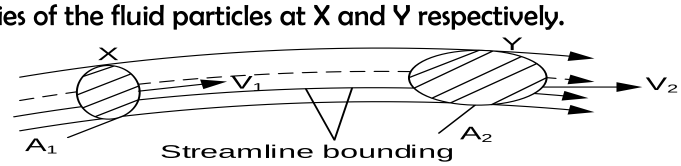
11.3.0: Equation of continuity

Consider a fluid undergoing steady flow and consider a section XY of a tube of flow with the fluid.

Let A_1 and A_2 be the cross-section areas of the tube of flow at X and Y respectively

$\rho_1$and ρ_2 be the densities of the fluid at X and Y respectively

V_1 and V_2 be the velocities of the fluid particles at X and Y respectively.



In a time interval Δt the fluid at X will move forward a distance $V_1\Delta t$. therefore, a volume $A_1V_1\Delta t$ will enter the tube at X. the mass of fluid entering at X in time Δt will be there be $\rho_1 A_1V_1\Delta t$

Similarly the mass leaving at Y in the same time is $\rho_2 A_2V_2\Delta t$

Since the mass entering at X is equal to mass leaving at Y

$$\rho_1A_1V_1\Delta t = \rho_2 A_2V_2\Delta t$$

For an incompressible fluid $\rho_1 = \rho_2$

$$\boxed{A_1V_1 = A_2V_2}\text{.....................................1}$$

Equation 1 is an equation of continuity for an incompressible fluid

Definition: An incompressible fluid is a fluid in which changes in pressure produce no change in the density of the fluid

11.3.1: WHY LIQUIDS FLOW FASTER IN CONSTRICTIONS

Volume flow per second is constant, so by the equation of continuity: $A_1V_1 = A_2V_2$

$V_2 = \frac{A_1}{A_2}V_1$ It implies that $A_2 \alpha \frac{1}{V_2}$ if $A_1 > A_2$ then $V_2 > V_1$

Hence the velocity at the wider part is less than that at the constructed part

11.3.2: BERNOULLI'S PRINCIPLE

It states that for a non-viscous incompressible fluid flowing steadily, the sum of the pressure plus the potential energy per unit volume plus kinetic energy per unit volume is constant at all points on a stream line.

i.e. $\boxed{P + ½\rho v^2 + \rho gh = a\ constant}$

P is the pressure with in the fluid

ρ is the density of the fluid

v is the velocity of the fluid

g is the acceleration due to gravity

h is height of the fluid (above reference line)

Assumptions

- ✓ The flow is laminar
- ✓ The fluid is incompressible and non viscous
- ✓ The pressure and velocity are uniform at any cross section of the tube

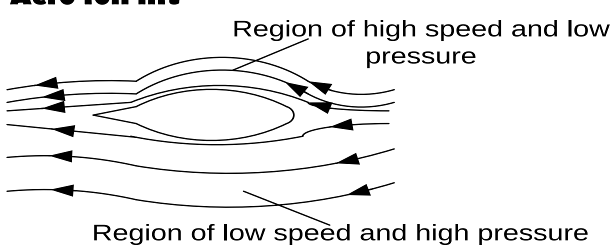
11.3.4: Application of Bernoulli's principle

It follows from Bernoulli's equation that whenever a flowing fluid speeds up, there is a corresponding decrease in the pressure and for the potential energy of the fluids. If the flow is horizontal, the whole of the velocity increase is accounted for by a decrease in pressure.

1. Sanction effect

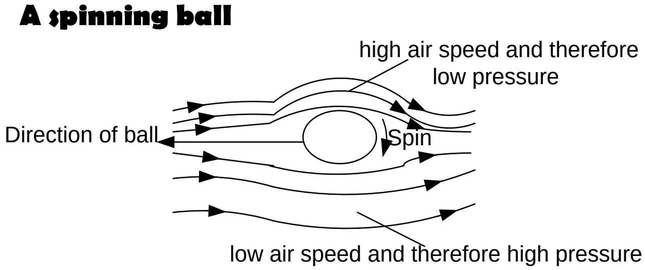
This is experienced by a person standing close to the platform at the station when a fast moving train passes. The fast moving air between the person and the train produces a decrease in pressure according to Bernoulli's principle . Behind the man the flow velocity is lower and pressure is higher. This pressure difference produces the resultant force whch pushes the person towards the train.

2. Aero foil lift



- ❖ An aero foil e.g. an air craft wing is shaped so that air flows faster along the top of the wings than below the wings.
- ❖ By Bernoulli's principle pressure below becomes greater than that above the wings.
- ❖ This pressure difference produces the resultant force called lift upwards force. It is this force which provides a force that lifts the plane off the ground at take off

3. A spinning ball



A ball such as a football, tennis or golf ball that is projected to travel through air experiences a sideways force which makes it curve in flight. This is because the spin drags air around with the ball such that air moves faster on one side of the ball than the other. The pressure difference causes a resultant force which makes the ball the curve as it spins.

4. Bunsen burner

The gas passes the narrow jet at high speed creating a low pressure region. Atmospheric pressure then pushes air in through the hole and the mixture flows up the tube to burn at the top

5. Carburetor

The air passage through a carburetor is partially constructed at the point where petrol and air are mixed. This increases the speed of air but lowers its pressure and permits more rapid evaporation of the petrol.

Examples

1. Water flows along a horizontal pipe of cross section area 30cm^2. The speed of water is 4ms^{-1} but this rises to 7.5m/s in constriction pipe. What is the area of this narrow part of the tube.

Solution

$A_1V_1 = A_2V_2$ | $30x10^{-4}x4 = A_2x7.5$ | $A_2 = 1.6x10^{-3}m^2$

2. Water leaves the jet of a horizontal horse at 10m/s. If the velocity of the water with in the horse is 0.4m/s. Calculate the pressure P with in the horse (density of water 1000kgm^{-3}) and atmospheric pressure 10^{5}Nm^{-2}

Solution

V_1 = 0.4m/s, P_1 = ?, 1000kg/m^3,
V_2 =10m/s, P = 10^5
P + ½ ρv^2 + ρgh = a constant
P_1 + ½ $\rho v_1{}^2$ = P_2 + ½ $\rho v_2{}^2$ for horizontal flow

P_1 + ½ x1000x0.4^2 = 10^5+ ½ x1000x10^2
P_1 = 1.5x10^5Pa

3. A fluid of density 1000kgm^{-3} flows in a horizontal tube. If the pressure the entry of the tube is 10^5Pa and at the exit is 10^3Pa, given that the velocity of the fluid at the entry is $8ms^{-1}$, calculate the velocity of the liquid at the exit.

Solution

$\boldsymbol{P + ½ \rho v^2 + \rho gh = a\ constant}$
P_1 + ½ $\rho v_1{}^2$ = P_2 + ½ $\rho v_2{}^2$ for horizontal flow

$10^5 + ½\, x1000x8^2 = 10^3 + ½\, x1000xV_2^2$
$V_2 = 16.25ms^{-1}$

4. An air craft design requires a dynamic lift of $2.4x10^4N$ on each square meter of the wing when the speed of the air craft through the air is 80ms^{-1}. Assuming that the air flows past the wing with streamline line flow and that the flow past the lower surface is equal to the speed of the air craft, what is required speed of the air over the upper surface of the wing if the density of the air is $1.29kgm^{-3}$.

Solution

P_1 + ½ $\rho v_1{}^2$ = P_2 + ½ $\rho v_2{}^2$ for horizontal flow
$P_2 - P_1 = \frac{1}{2}\rho(V_1^2 - V_2^2) = \frac{1}{2}x1.29x(V_1^2 - 80^2)$
lift force, $F = (P_2 - P_1)A$

$24000 = \left[\frac{1}{2}x1.29x(V_1^2 - 80^2)\right]x1$
$V_1 = 208.8ms^{-1}$

5. Air flows over the upper surface of the wings of an aero plane at a speed of 81ms^{-1} and past the lower surfaces of the wings at 57ms^{-1}. Calculate the lift force on the aero plane if it has a total wing area of 3.2m^2. (density of air = 1.3kgm^{-3})

Solution

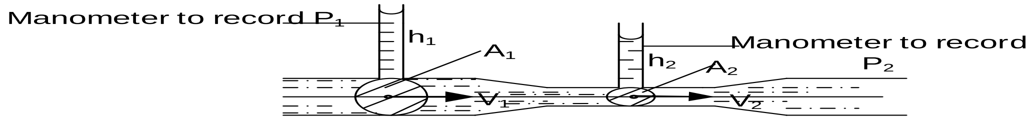
P_1 + ½ $\rho v_1{}^2$ = P_2 + ½ $\rho v_2{}^2$ for horizontal flow
$P_2 - P_1 = \frac{1}{2}\rho(V_1^2 - V_2^2) = \frac{1}{2}x1.3x(81^2 - 57^2)$
lift force, $F = (P_2 - P_1)A$

$F = \left[\frac{1}{2}x1.3x(81^2 - 57^2)\right]x3.2$
$F = 6.9x10^3N$

11.3.5: Measurement of fluid velocity

1. Venture meter

This is a device which introduces a constriction into a pipe carrying a fluid in order that the velocity of the fluid can be measured by measuring the resulting drop in pressure.



Consider the fluid to be non viscous, incompressible and of density ρ in a horizontal steady flow let the pressure and velocity be P_1 and V_1 at the main pipe and P_2 and V_2 at the constricted pipe along the same stream line

Applying Bernoulli's equation

$P_1 + ½ \rho v_1^2 = P_2 + ½ \rho v_2^2$ ....(1) (horizontal flow)

If the cross sectional areas at main and constructed equation of continuity.

$A_1V_1 = A_2V_2$

$V_2 = \frac{A_1V_1}{A_2}$

Put into equation 1

$P_1 + ½ \rho v_1^2 = P_2 + ½ \rho \left(\frac{A_1V_1}{A_2}\right)^2$

$$\boxed{P_1 - P_2 = \frac{1}{2}\rho\, v_1^{\,2}\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]}$$

Thus by measuring pressures P_1 and P_2 and knowing ρ, A_1, and A_2 it is possible to find the velocity of V_1 of the fluid in the un constricted (main) section of the pipe.

Note: $P_1 = \rho h_1 g$ and $P_2 = \rho h_2 g$

Examples

1.a)

a horizontal pipe of a diameter 36.0cm tapers to a diameter of 18.0cm at P. An ideal gas at a pressure of $2x10^5$Pa is moving along the wider part of the pipe at a speed of $30ms^{-1}$, the pressure of the gas at P is $1.8x10^5$Pa. Assuming the temperature of the gas remain constant calculate the speed of the gas at P.

b) For the gas in (a) recalculate the speed at P on the assumption that it can be treated as an incompressible fluid, and use Bernoulli's equation to calculate corresponding value for the pressure at P. Assume that in the wider part of the pipe the gas speed is still $30.0ms^{-1}$, the pressure is still $2.00x10^5$ Pa and at this pressure the density of the gas is $2.60kgm^{-3}$.

Solution

a) $P_1=2x10^5$Pa $d_1=36x10^{-2}$m, $v_1=30ms^{-1}$

$P_2=1.8x10^5$Pa $d_2=18x10^{-2}$m v_2=?

An ideal gas at constant temperature obeys Boyle's law.

$P_1V_1^{\ 1} = P_2V_2^{\ 1}$ --------------------------- [1]

volume $V_1^{\ 1} = A_1L_1$ and volume $V_2^{\ 1} = A_2L_2$

But L_1 = speed V_1 x t and L_2= speed V_2 x t

Put into equation 1 : $P_1A_1L_1t = P_2A_2L_2t$

$P_1\frac{\pi d_1^{\ 2}}{4}L_1t = P_2\frac{\pi d_2^{\ 2}}{4}L_2t$

$2x10^5 x\frac{\frac{22}{7}x(36x10^{-2})^2}{4}x30 = 2x10^5 x\frac{\frac{22}{7}x(18x10^{-2})^2}{4}xV_2$

$V_2 = 133.33m/s$

b) For an incompressible fluid

$A_1V_1 = A_2V_2$----------------- [2]

$P_1=2x10^5$Pa $d_1=36x10^{-2}$m $v_1=30ms^{-1}$

P_2=? $d_2=18x10^{-2}$m, v_2=?

$\frac{\pi d_1^{\ 2}}{4}V_1 = \frac{\pi d_2^{\ 2}}{4}V_2$

$\frac{22}{7}x\frac{(36x10^{-2})^2}{4}x30 = \frac{22}{7}x\frac{(18x10^{-2})^2}{4}xV_2$

$V_2 = 120m/s$

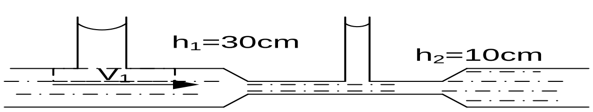
Using Bernoulli's equation

$P_1 + ½ \rho v_1^2 = P_2 + ½ \rho v_2^2$ for horizontal flow

$2x10^5 + ½ x2.6x30^2 = P_2 + ½ x2.6x120^2$

$P_2=1.825x10^5$Pa

2. A venturimeter consists of a horizontal tube with a constriction tube which replaces part of the piping system as shown below



If the cross-section area of the main pipe is $5.8x10^{-3}m^2$ and that of the constriction is $2.58x10^{-3}m^2$. Find the velocity V_1 of the liquid in the main pipe

Solution

$h_1 = 30x10^{-2}$, $h_2 = 10x10^{-2}m$, $\rho_1 = ?$ $\rho_2 = ?$,
$A_1 = 5.81x10\text{-}3m^3$, $A_2 = 2.58x10\text{-}3m^3$
$P_1 = h_1\rho g$ and $P_2 = h_2\rho g$
$P + ½ \rho v^2 + \rho gh$ = a constant
$P_1 + ½ \rho v_1^2 = P_2 + ½ \rho v_2^2$ for horizontal flow

$$\frac{1}{2}\rho\, v_1{}^2 + \rho g h_1 = \frac{1}{2}\rho\, v_2{}^2 + \rho g h_2$$

From equation of continuity

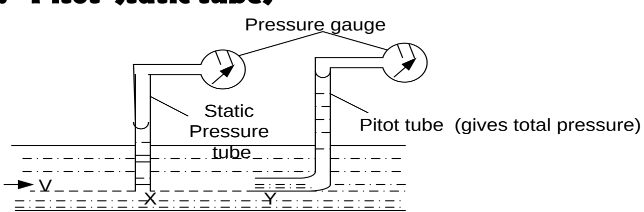
$A_1V_1 = A_2V_2$ $\therefore V_2 = \frac{A_1V_1}{A_2}$

$$\frac{1}{2}\rho\, v_1{}^2 + \rho g h_1 = \frac{1}{2}\rho\left(\frac{A_1V_1}{A_2}\right)^2 + \rho g h_2$$

$$30x10^{-2}x9.81 + \frac{1}{2}xV_1^2 = 10x10^{-2}x9.81 + \frac{1}{2}x\left(\frac{5.81x10^{-3}xV_1}{2.58x10^{-3}}\right)^2$$

$2.943 - 0.981 = 2.035612343V_1^2$
$V_1^2 = 0.963837739$
$V_1 = 0.982m/s$

3. Pitot-static tubes



The Pitot - static tube consists of two coaxial tubes, the pitot tube and the static tube. The gauge on the pitot tube measures the total pressure P_T, while that on the static tube measures the static presure P_S

By Bernoullis principle

Total pressure=static pressure+$d$ynamic pressure

$$P_T = P_s + \frac{1}{2}\rho V^2$$

$$\frac{1}{2}\rho V^2 = totalpressure - static\ pressure$$

$$V = \sqrt{\frac{2(P_T - P_s)}{\rho}}$$

- **Static pressure**

Static pressure at a point is the pressure that the fluid would have if it were at rest.

- **Dynamic pressure**

It is the pressure of a fluid due to its velocity

- **Total pressure**

It is the sum of the dynamic and static pressure.

Example

1. The static pressure in a horizontal pipe line is $4.3x10^4$Pa, the total pressure is $4.7x10^4$Pa and the area of cross-section is $20cm^2$. The fluid may be considered to be incompressible and non viscous and has a density of $10kgm^{-3}$. Calculate
   i. The flow velocity in the pipeline
   ii. The volume flow rate in the pipeline

Solution

Dynamic pressure=total pressure−static pressure
Dynamic pressure $= 4.7x10^4 - 4.3x10^4$
Dynamic pressure $= 0.4 \times 10^4$Pa
Dynamic pressure $= \frac{1}{2}\rho V^2$
$0.4 \times 10^4 = \frac{1}{2}x10^3 V^2$
V = 2.83m/s

ii) Volume flow rate $= \frac{volume}{time}$

$\frac{volume}{time} = \frac{A\,L}{time} = \frac{A\,v\,t}{t}$

$= 20x10^{-4}x\ 2.83$

$\frac{volume}{time} = 5.66x10^{-3}m^3s^{-1}$

2. Water flows steadily along a uniform flow tube of cross-sectional area $30cm^2$. The static pressure is $1.20x10^5Pa$ and the total pressure is $1.28x10^5Pa$. assuming that the density of water is $1000kgm^{-3}$, calculate the;
   (i) Flow velocity (ii) Volume flux
   (iii) Mass of water passing through a section of the tube per second

Solution

(i) $V = \sqrt{\frac{2(totalpressure - static\ pressure)}{\rho}}$

V= $\sqrt{\frac{2(1.28x10^5 - 1.20x10^5)}{1000}} = 4ms^{-1}$

(ii) volume per second = area x velocity

=30$x10^{-4}x4$

Volume flux = 0.012m^3s^{-1}

(iii) Mass per second= volume per second x ρ

$= 0.012x1000$

Mass per second= $12kgs^{-1}$

3. A pitot – static tube fitted with a pressure gauge is used to measure the speed of a boat at sea. Given that the speed of the boat does not exceed 10m/s and the density of sea water is 1050kgm^{-3}, calculate the maximum pressure on the gauge

Solution

Maximum pressure is the dynamic pressure

Dynamic pressure $= \frac{1}{2}\rho V^2$

= $\frac{1}{2}x1050x10^2$

Dynamic pressure =5.25x10^{4}Pa

Exercise: 26

1. Water flows speedily along a horizontal tube of cross-sectional area 25cm^2. The static pressure with in the pipe is 1.3x10^{5}pa and the total pressure 1.4x10^{5}pa. Calculate the velocity of the water flow and the mass of the water flow past a point in a tube per second. **[an 4.47m/s, 11.175kg/s]**
2. A lawn sprinkler has 20 holes each of cross sectional area 2x10^{-2}cm^2 and its connected to a horse pipe of cross sectional area 2.4cm^2, if the speed of the water in the horse pipe is 1.5m/s, estimate the speed of the water as it emerges from the holes. **[an9m/s]**
3. Water flows speedily along a uniform flow tube of cross section 30cm^2. The static pressure is 1.2x10^{5}Pa and the total pressure is 1.28x10^{5}Pa. Calculate the flow velocity and the mass of water per second flowing past a section of the tube. (Density of water is 1000kgm^{-3}**.)[an 4m/s, 12kg/s]**
4. Air flows over the upper surface of the wings of an aero plane at a speed of 120ms^{-1} and past the lower surfaces of the wings at 110ms^{-1}. Calculate the lift force on the aero plane if it has a total wing area of 20m^2. (density of air = $1.29kgm^{-3}$) **[an= 2.97x10^{4}N]**
5. What is the maximum weight of an air craft with a wing area of 50m^2 flying horizontally, if the velocity of air over the upper surface of wing is 150m/s and that over the lower surface is 140m/s
   (density of air = $\mathbf{1.29kgm^{-3}}$)
6. Water flows along a horizontal pipe of cross section 30cm^2. The speed of the water is 4m/s. But it rises to 7.5m/s is a construction in the pipe. What is the area of this narrow part of the

11.4.0: FLUIDS AT REST

11.4.1: DENSITY AND RELATIVE DENSITY

Density of a substance is defined as the mass per unit volume of a substance.

$$\rho = \frac{m}{v}$$

S.I unit's kgm^{-3}

Relative density

Definition

It is the ratio of the density of a substance to density of an equal volume of water at 4°c

It is at 4℃ because water has maximum density of 1000kgm^{-3} at that temperature

$$R.D = \frac{density\ of\ asubstance}{density\ of\ water\ at\ 4℃} = \frac{m_s/v_s}{m_w/v_s} = \frac{m_s}{m_w}$$

It can also be defined as the ratio of the mass of a substance to mass of an equal volume of water

$R.D = \frac{m_s}{m_w}$ for W = mg | $\frac{w_s/g}{w_w/g}$ | R.D $= \frac{w_s}{w_w}$

It can also be defined as the ratio of weight of a substance to weight of an equal volume of water.

Note: Relative density has no units.

11.4.2: ARCHIMEDE'S PRINCIPLE

It states that when a body is wholly or partially immersed in a fluid, it experiences an up thrust equals to the weight of the fluid displaced.

I.e. Up thrust = weight of fluid = apparent loss of weight of the object in a fluid.

Definition

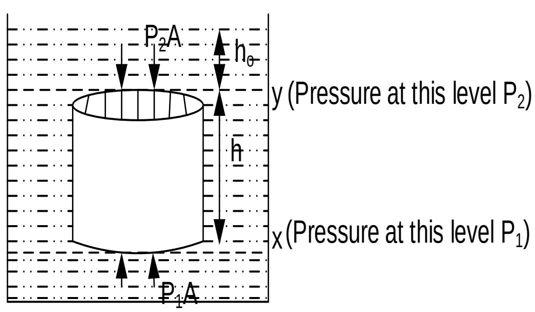
Up thrust is the apparent loss of weight of an object immersed in a fluid

Or

It is the resultant upward force on the body due to the fluid.

11.4.3: Verification of Archimedes' principle using a cylindrical rod

Consider a cylindrical rod of cross-sectional area A and height h immersed in a large quantity of a fluid of density ρ_f such that its top is at level Y, h_o meters below the surface of the fluid while its bottom is at level X shown below



Volume of fluid displaced = volume of cylinder= Ah

Mass of fluid displaced = A$h\rho_f$

Weight of fluid displaced = A$h\rho_f$ g.....................(i)

The fluid exerts forces of P_1A and P_2A on the bottom and top faces of the cylinder.

The up thrust (resultant upward force due to the fluid is therefore given by

$Upthrust\ = P_2A - P_1A$

$Upthrust = (h + h_o)\rho_f\ gA - h_o\rho_f\ gA$

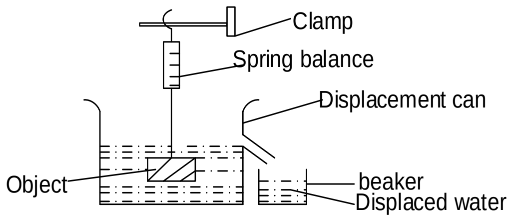
$Up\ thrust = Ah\rho_f$ g......................(ii)

From equation (i) and equation (ii), therefore;

$Upthrust\ =$ weight of fluid displaced

Question: Show that the weight of fluid displaced by an object is equal to up thrust on the object

11.4.4: Verification of Archimedes' principle using a spring balance.



- Fill the displacement can with water till water flows through the spout and wait until the water stops dripping.
- Weigh a solid object in air using a spring balance and record its weight W_a
- Place a beaker of known weight, W_b beneath the spout of the can.
- With the help of the spring balance, the solid object is carefully lowered into the water in the displacement can and wait until water stops dripping when it is completely immersed, its weight (apparent weight) is then read and recorded from the spring balance as W_w.
- Re weigh the beaker and the displaced water and record the weight as $W_{(b+w)}$
- If (W_a - W_w) = ($W_{(b+w)} - W_b$), then Archimedes's principle is verified

Theory

Let the weight of the empty beaker be W_b

Weight of water displaced = weight of (beaker +water) – weight of beaker

Weight of water displaced = $W_{(b+w)} - W_b$.................................1

Apparent loss of weight of object = weight of object in air – weight of object in water

Apparent loss of weight of the object = W_a - W_w

If (W_a - W_w) = ($W_{(b+w)} - W_b$), then Archimedes's principle is verified

11.4.5: Application of Archimedes' principle

It can be used to determine density and relative density of a solid and a liquid.

a) **Determination of density and relative density of a solid**

- Attach a sinker to the irregular solid and weigh them when the solid is outside but the sinker immersed in water (W_1)
- Weigh the solid and sinker when both are completely immersed in water (W_2)
- Up thrust in water = W_1 - W_2
- $R.D = \frac{W_1}{W_1 - W_2}$
- Density of solid = RD of solid x density of water

Example

1. An object suspended from the spring balance is found to have a weight of 4.92N in air and 3.87N when immersed in water. Calculate the density of the material from which the object is made of the density of water is 1000kgm^{-3}

Solution

$W_a = 4.92N, W_w = 3.87N$

$$R.D = \frac{W_a}{W_a - W_w} = \frac{4.92}{4.92 - 3.87} = 4.686$$

Density of substance = $RD\ x\ \rho$ of water

$= 4.686\ x\ 1000 =$ 4686kgm^{-3}

Exercise : 27

1. A piece of glass weighs 0.5N in air and 0.30N in water. Find the density of the glass. **An[2500kgm^{-3}]**

2. A spherical stone has a mass of 1.546kg, if its radius is 20cm. find the density of the stone in

   (i) $g\ cm^{-3}$ (ii) $kg\ m^{-3}$

   An [46.848 $g\ cm^{-3}$, 4.6848 $kg\ m^{-3}$]

3. What is the mass of the sphere of diameter 20cm if its relative density is 14.1 **An[59.22kg]**

4. A glass block weighs 25N in air. When wholly immersed in water, the block weighs 15N. calculate
   i. The up thrust on the block
   ii. The density of the glass in $kg\ m^{-3}$ **An[10N, 2500 $kg\ m^{-3}$]**

b) Determination of density and relative density of a liquid

- Weigh a solid (sinker) in air and record its weight W_a using a spring balance.
- Immerse the solid (sinker) wholly in water and record the apparent weight W_w
- Wipe the surface of the solid (sinker) with a piece of dry cloth and immerse it wholly in the liquid whose relative density is to be measured, read and record its apparent weight in the liquid W_L

Weight of water displaced (up thrust in water) = W_a - W_w

Weight of liquid displaced (up thrust in liquid) = W_a - W_L

Relative density $= \frac{upthrust\ in\ Liquid}{upthrus\ in\ water}$

- $R.D\ of\ the\ liquid\ = \frac{W_a - W_L}{W_a - W_w}$

Density of liquid = R.D of liquid x density of water

Example

1. A solid has a weight of 160N in air and 120N when wholly immersed in a liquid of relative density 0.8, determine the density of a solid

Solution

$$R.D\ of\ the\ liquid\ = \frac{W_a - W_L}{W_a - W_w}$$

$$0.8\ = \frac{160-120}{160 - W_w}$$

$$W_w =\ 110N$$

$$R.D\ of\ solid = \frac{W_a}{W_a - W_w}$$

$$R.D\ of\ solid = \frac{160}{110-110} =\ 3.2$$

Density of a solid = RD of solid $x\ \rho$of water

$= \ 3.2\ x\ 1000$

Density of a solid = 3200 kgm^{-3}

2. A piece of iron weighs 555N in air when completely immersed in water, it weighs 530N and weighs 535N when completely immersed in alcohol. Calculate the relative density of alcohol and the density of alcohol.

Solution

W_a = 555N W_w =530N W_L=535N

R.D of alcohol $= \frac{W_a - W_L}{W_a - W_w} = \frac{555-535}{555-530} = 0.8$

Density of alcohol = R.D of alcohol $x\ \rho$ of H_2O

$= 0.8\ x1000$

Density of alcohol = $800kgm^{-3}$

3. A string supports a solid iron of mass 0.18kg totally immersed in a liquid of density 800 $kg\ m^{-3}$. Calculate the tension in the string if the density of iron is 8000 $kg\ m^{-3}$

Solution

Weight of iron $= mg = 0.18x9.81 = 1.758N$

Volume of iron= $\frac{mass}{density} = \frac{0.18}{8000} = 2.25x10^{-5}m^3$

Mass of liquid displaced=2.25x$10^{-5}x$8000

$= 0.018kg$

Weight of the liquid displaced = $0.018x9.81$

Weight of the liquid displaced = 0.1766N

At equilibrium ; $mg\ =\ T + U$

$1.758 = T + 0.1766$

$T =\ 1.5892N$

4. A specimen of an alloy of silver and gold whose densities are 10.5gcm^{-3} and 18.9gcm^{-3} respectively, weigh 35.2g in air and 33.13 g in water. Find the composition by mass of the alloy assuming that there has been no volume change in the process of producing the alloy. Assume that the density of water is 1 g cm^{-3}

Solution

$m_s + m_g = 35.2$..................................1

R.D of alloy $= \frac{35.2}{35.2-33.13}$ =17

Density of alloy= R.D x density of water

Density of alloy= $17x1$ =17 g cm^{-3}

Volume of alloy= $\frac{m}{\rho} = \frac{35.2}{17} = 2.07cm^3$

Volume of alloy= $V_s + V_g$

Volume of alloy= $\frac{m_s}{\rho_s} + \frac{m_g}{\rho_g}$

2.07= $\frac{m_s}{10.5} + \frac{m_g}{18.9}$......................2

Solving 1 and 2 simultaneously

$m_g = 30.3g\ and\ m_s = 4.9g$

Exercise: 28

1. A block of mass 0.1kg is suspended from a spring balance when the block is immersed in water of density $1000kgm^{-3}$, the spring balance reads 0.63N. When the block is immersed in a liquid of unknown density the spring balance reads 0.7N, find
   i) Density of the solid
   ii) Density of the liquid **An [$2800kgm^{-3}$, $800kgm^{-3}$]**
2. An alloy contains two metals X and Y of densities $3.0x10^3kgm^{-3}$ and $5.0x10^3kgm^{-3}$ respectively. Calculate the density of the alloy if,
   (i) The volume of X is twice that of Y
   (ii) The mass of X is twice that of Y
   An[(i)=$3.7x10^3kgm^{-3}$(ii)= $3.5x10^3kgm^{-3}$]
3. An alloy contains two metals A and B, has a volume of $5.0x10^{-4}m^3$ and a density of $5.6x10^3kgm^{-3}$. The densities of A and B are $8.0x10^3kgm^{-3}$ and $4.0x10^3kgm^{-3}$ respectively. Calculate the mass of A and mass of B. **An [A= 1.6kg, B=1.2kg]**
4. A piece of glass has a mass 62 kg in air. It has a mass of 32kg when completely immersed in water and a mass of 6kg when completely immersed in an acid.
   (a) The glass (b) The acid in $kg\ m^{-3}$
   An[(a)=1550 $kg\ m^{-3}$ (b)= 1400 $kg\ m^{-3}$]
5. A body of mass 0.1kg and relative density 2 is suspended by a thread and completely immersed in a liquid of density $920kgm^{-3}$.
   i) Find the tension in the thread. **An[0.53N]**
   ii) If the thread breaks, what will be the initial acceleration? **An [$5.3ms^{-2}$]**
6. A tank contains a liquid of density $1200kgm^{-3}$. A body of volume $5x10^{-3}m^3$ and density $900kgm^{-3}$ is totally immersed in the liquid and attached to by a thread to the bottom of the tank. Find the tension in the thread. **An [14.72N]**
7. A block of metal weighs 50N in air and 25N in water
   (a) Determine the density of the metal in $kg\ m^{-3}$
   (b) Find the weight of the metal in paraffin whose relative density is 0.8
   An[2000 $kg\ m^{-3}$, 30N]

11.5.0: FLOATATION

A body floats in a liquid if its density is less than the density of the liquid.

11.5.1: Law of floatation

It states that a floating body displaces its own weight in the fluid in which its floating.

Experiment to verify the law of floatation

- ❖ Pour water in a displacement can until it over flows through the spout and wait until the water stops dripping
- ❖ Place a beaker under the spout. Gently place an object which floats on water and wait until water stops dripping from the spout
- ❖ Determine the weight of water displaced , W_1
- ❖ Repeat the procedure with another liquid which the object can float. If the weight of the liquid displaced is now W_2. Then W_1 = W_2 hence law of floatation

Note:

1. For a floating body
   - The weight of floating body =weight of fluid displaced =Up thrust
   - The mass of the floating body = the mass of the fluid displaced
   - A floating body sinks deeper in liquids of less density than in liquids of higher densities.
2. Density of a floating body = fraction submerged x density of liquid
3. Volume of displaced liquid = fraction submerged x volume of floating body.

Example

1. A solid weighs 237.5g in air and 12.5g when totally immersed in a fluid of density 0.9g/cm^3. Calculate
   a) Density of the solid.
   b) The density of the liquid in which the solid would float with 1/5 of its volume exposed above the liquid surface.

Solution

a) W_a =237.5g W_L =12.5g

Up thrust in liquid $= W_a - W_L = 237.5 - 12.5$

Up thrust in liquid (mass of liquid displaced) $=225$g

Volume of liquid displaced $= \frac{m}{\rho} = \frac{225}{0.9}$

Volume of liquid displaced =250cm^3

Volume of solid =250cm^3

Density of solid = $\frac{Mass\ of\ solid}{volume\ of\ solid} = \frac{237.5}{250}$

=0.95g/cm^3

b) If $\frac{1}{5}$ of its volume is exposed, then $\frac{4}{5}$ of its volume is submerged.

Volume of liquid = fraction x volume of the solid submerged

$= \frac{4}{5} x25$ =200cm^3

Mass of solid =237.5

Density of liquid = $\frac{237.5}{200}$ =1.19g/cm^3

OR

ρ of floating body = fraction submerged x ρ liquid

0.95 = $\frac{4}{5}$ x density of liquid

Density of liquid$= \frac{0.95x5}{4}$ =1.19gcm^{-3}

2. A solid of volume 10^{-4}m^3 floats in water of density 10^{3}kgm^{-3} with $\frac{3}{5}$ of its volume submerged
   i) Find the mass of the solid
   ii) If the solid floats in another liquid with $\frac{4}{5}$ of its volume submerged. What is the density of the liquid?

Solution

a) V =10^{-4} ρ_ω =1000kgm^{-3}

Volume submerged $= \frac{3}{5}$

Volume of water displaced $= \frac{3}{5}x$ volume of solid = $\frac{3}{5}x$ 10^{-4} $= 6x10^{-5}\ m^3$

mass of displaced water = volume of water displaced x density of water $= 6x10^{-5}x\ 1000 = 6x10^{-2}\ kg$

By law of floatation, mass of water displaced is equals to the mass of the solid

∴ Mass of solid = $6x10^{-2}\ kg$

b) Fraction submerged $= \frac{4}{5}$

Density of solid = $\frac{mass\ of\ solid}{volume\ of\ solid} = \frac{6\ x\ 10^{-2}}{10^{-4}}$ = 600kgm^{-3}

Density of solid = fraction submerged x density of liquid

600 $= \frac{4}{5}x$ density of liquid

Density of liquid = 750kgm^{-3}

Eercise: 29

1. A Ball with a volume of 32cm^3 floats on water with exactly half of the ball below the surface. What is the mass of the ball (density of water =1.0x10^{3}kgm^{-3}) **An [1kg]**
2. An object floats in a liquid of density 1.2x10^{3}kgm^{-3} with one quarter of its volume above the liquid surface. What is the density of the object. **An[900kgm^{-3}]**
3. A solid weighs 237.g in air and 212.5g when totally immersed in a liquid of density0.9gcm^{-3}. Calculate the;
   (i) Density of the solid
   (ii) Density of a liquid in which the solid would float with $^1/_5$ of tis volume exposed above the liquid surface. **An[9500 kgm^{-3}. 1190 kgm^{-3}].**
4. Object with a volume of 1.0x10^{-5}m^3 and density 4.0x10kgm^{-3} floats on water in a tank of cross sectional area 1.0x10^{-3}m^2
   a) By how much does the water level drop when the object is removed

b) Show that this decrease in water level reduces the force on the base of the tank by an equal amount to the weight of the (density of water = 1.0x10^{3}kgm^{-3}) **An [4x10^{3}kgm^{-3}]**

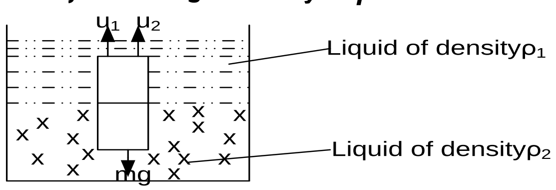
5. A block of wood floats in water of density 1000 kgm^{-3} with $^2/_3$ of its volume submerged . in oil it has $^9/_{10}$ of its volume submerged . find the densities of wood and oil

An[740.74 kgm^{-3}, 666.67 kgm^{-3}].

6. A piece of metal of mass 2.60g and density 8.4 g cm^{-3} is attached to a block of wax of 1.0 g and density 0.92 g cm^{-3}. When the system is placed in a liquid, it floats with wax just submerged. Find the density of the liquid. **An [1.13x10^{-6} g cm^{-3}]**

11.5.2: RELATION BETWEEN DENSITIES AND VOLUME FOR AN OBJECT FLOATING IN TWO LIQUIDS

Consider an object which floats at the interface between two immiscible liquids of density ρ_1 and ρ_2 with the object having a density of ρ



V_1 volume submerged to liquid of density ρ_1

V_2 e volume submerged to liquid of density ρ_2

U_1 the upthrust in liquid of density ρ_1

U_2 the upthrust in liquid of density ρ_2

Weight of object = Total upthrust

$$mg = U_1 + U_2$$
$$\rho(V_1 + V_2)g = \rho_1 V_1 g + \rho_2 V_2 g$$
$$\rho V_1 g + \rho V_1 g = \rho_1 V_1 g + \rho_2 V_2 g$$
$$\rho V_1 g - \rho_1 V_1 g = \rho_2 V_2 g - \rho V_2 g$$
$$\frac{V_1}{V_2} = \frac{\rho_2 - \rho}{\rho - \rho_1}$$

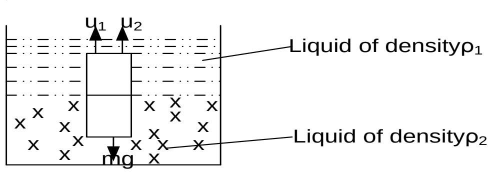
Note

When the object is replaced by a denser one, then the new object will sink much deeper and therefore V_1 decreases and V_2 increases by the same magnitude.

EXAMPLE: UNEB 2006 Q.4 (iii)

A block of wood floats at an interface between water and oil with 0.25 of its volume submerged in the oil. If the density of the wood is 7.3x10^{2}kgm^{-3} . Find the density of the oil.

Solution



V_1 = 0.75 V_2 = 0.25

ρ =730kgm^{-3} ρ_1 =1000kgm^{-3} ρ_2 = ?

$$\frac{V_1}{V_2} = \frac{\rho_2 - \rho}{\rho - \rho_1}$$

$$\frac{V_1}{V_2} = \frac{\rho_2 - \rho}{\rho - \rho_1}$$
$$\frac{0.75}{0.25} = \frac{\rho_2 - 730}{730 - 1000}$$
$$3x(730 - 1000) = \rho_2 - 730$$
$$\rho_2 = -810 + 730$$

ρ_2 = −80kgm^{-3}

ρ_2 =80kgm^{-3}

Applications of law of floatation

1- Balloons 2- Ships 3- Submarines

- **Balloons**

A balloon filled with a light gas such as hydrogen gas rises up because the weight of the displaced air is greater than the weight of the balloon plus its content. It's the net upward force (up thrust) which pushes the balloon upwards and the balloon continues rising until the up thrust acting on it becomes equal to the weight of the balloon plus its content then it begins floating.

$\therefore$ U = $W_b + W_h + W_L$

$V_a \rho_a$g = M_bg+ $V_h \rho_h$g+ M_Lg

U=Up thrust | W_h =weight of hydrogen | W_b =weight of balloon

W_L =weight of load M_b=mass of balloon M_L=mass of load	V_a =volume of air V_h=volume of hydrogen ρ_a =density of air	ρ_h =density of hydrogen

Note : Volume of air displaced = volume of balloon

$$V_a = V_b$$

EXAMPLES

1. A balloon has a capacity of 10m^3 and is filled with hydrogen. The balloon's fabric and the container have a mass of 1.25kg. Calculate the maximum mass the balloon can lift .

[ρ=0.089kgm^{-3}, ρ of air=1.29 kgm^{-3}]

Solution

V_b=10m^3 ρ_h =0.089, ρ_a =1.29 kgm^{-3}.

M_b=1.25 V_a=10m^3 V_b=10m^3

But up thrust = weight of balloon + weight of hydrogen +load

U= $W_b + W_h + W_L$

$V_a\,\rho_a$g = M_bg+ $V_h\rho_h$g+ M_Lg

10x1.29g=1.25g+10x0.089+ M_Lg

M_L =10.76kg

2. A hot air balloon has a volume of 500m^3. The balloon moves upwards at a constant speed in air of density 1.2kgm^{-3} when the density of the hot air inside it is 0.80kgm^{-3}.
   a) What is the combined mass of the balloon and the air inside it.
   b) What is the upward acceleration of the balloon when the temperature of the air inside it has been increased so that its density is 0.7kgm^{-3}.

Solution

V_b=500m^3 V_h=500m^3 V_a=500m^3

ρ_a =1.2kgm^{-3} ρ_h =0.8kgm^{-3}

U=W_b+W_h+W_L

$V_a\,\rho_a$g = M_bg+ $V_h\rho_h$g+ M_Lg

500x1.2g=(M_b+M_L)g +$M_h$xg

600 =(M_b+M_L+M_h)

Combined mass = 600kg

b) V_b=500m^3 V_h=500m^3 V_a=500m^3

ρ_a =1.2kgm^{-3}, ρ_h =0.8kgm^{-3}

At equilibrium : U=W_b+W_h+W_L

$V_a\,\rho_a$g = M_bg+ $V_h\rho_h$g+ M_Lg

500x1.2x9.81= (M_b+M_L)x9.81+500x0.8x9.81

(M_b+M_L)=200kg

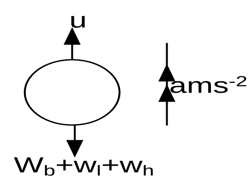
when ρ_h =0.7kg/m^3, V_h=500

W_h=$V_h\rho_h$g= $500x0.7x9.81 = 3433.5N$

(W_b+W_L)= $(M_b+M_L)x9.81$

W_b+W_L = $200x9.81$ =1962N

U= $V_a\,\rho_a$g = $500x1.2x9.81 = 5886$N



U − (W_b+W_h+W_L)= ma

$$5886 - (1962 + 3433.5) = 600xa$$

$$a = 0.82ms^{-2}$$

PRESSURE

The pressure acting on a surface is defined as the force per unit area acting normally on the surface

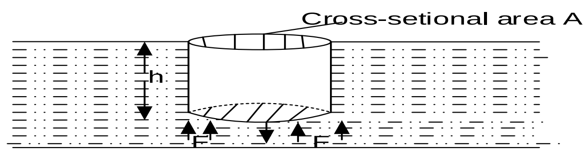
$$P=\frac{F}{A}$$

PRESSURE IN FLUIDS

The pressure in a fluid increase with depth, and all points at the same depth in the fluid are at the same pressure.

11.6.1: RELATION OF PRESSURE P WITH DEPTH h

Consider a cylindrical region of cross sectional area A and height h in a fluid of density ρ



The top of the cylinder is at the surface of the fluid and the vertical forces acting on it are its weight (mg) and an upward force F due to pressure p at the bottom of the cylinder.

The cylinder is in equilibrium and therefore

$F = mg$-------------------------[1]

But: $m = v\rho$ and $v = Ah$

F= Ahρg----------------------------- [4]

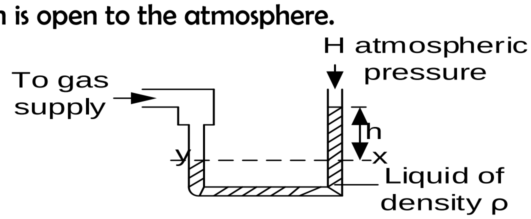
But P= $\frac{F}{A} = \frac{A\,h\,\rho\,g}{A}$

$P = h\rho g$

11.6.2: PRESSURE OF A GAS [U-TUBE MANOMETER]

This consists of a U-shaped tube containing a liquid. It is used to measure pressure.

The pressure to be measured (i.e. that of a gas) is applied to one arm of the manometer and the other arm is open to the atmosphere.



The gas pressure p is the same as the pressure at y

But pressure at y = pressure at x

$P = H + h\rho g$

Where $H = 1.01x10^5 Pa$

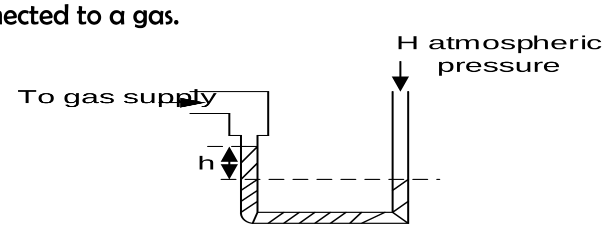
Or H=760mmHg

Or H=76cmHg

Note

The pressure recorded by the manometer (hρg) is known as gauge pressure. The actual pressure $(H + h\rho g)$ is called absolute pressure.

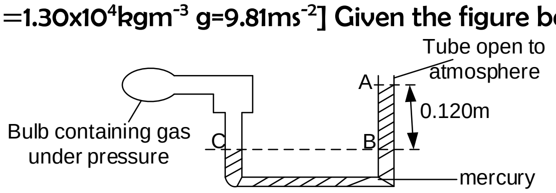
Suppose the level of the liquid in open limb of the manometer is lower than the level on the other side connected to a gas.



Pressure of gas $P = H - h\rho g$

Examples

1. Calculate the pressure of the gas in the bulb [Atmospheric pressure = $1.01x10^5$Pa] density of mercury =$1.30x10^4$kgm^{-3} g=9.81ms^{-2}] Given the figure below;



Solution

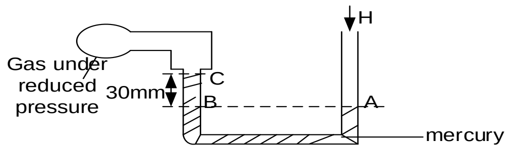
Pressure at C = pressure at B

Pressure at C = $H + h\rho g$

=$1.01x10^5$+$(0.12x1.36x10^4x9.81)$

Pressure of gas = $1.17x10^5$Pa

2. Using the diagram below, calculate the pressure of the gas in the bulb. (atmospheric pressure =760mmHg)

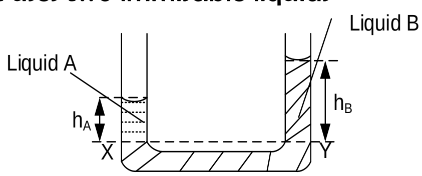


Pressure at B = pressure at A =760mmHg
Pressure at $C\ =\ (H-h)$
Pressure at C = $760-30\ = 730mmHg$
Gas pressure = $730mmHg$

Solution

11.6.3: DENSITY OF A LIQUID [U-TUBE MANOMETER]

It uses two immiscible liquids



The pressure P_x at X is equal to atmospheric pressure H plus the pressure exerted by the height h_A of liquid A i.e.

$P_x =$ H+ $h_A\,\rho_A$g

Where ρ_A is the density of liquid A

Similarly at Y

$P_y =$ H+ $h_B\,\rho_B$ g

Where ρ_B is the density of liquid B

Since x and y are at the same level

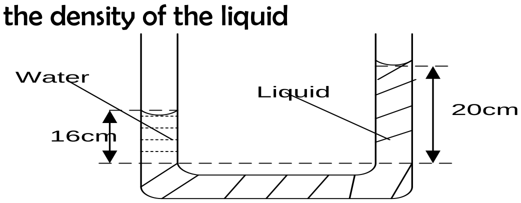
$P_x = P_y$

H+ $h_A\,\rho_A$g = H+ $h_B\,\rho_B$ g

$$\boxed{h_A\rho_A\ =\ \mathrm{h_B}\,\rho_B}$$

Example

Find the density of the liquid



Solution

$$h_w\rho_w\ =\ h_L\rho_L$$

$$\frac{16}{100}x1000 = \frac{20}{100}x\rho_L$$

$\rho_L =$ 800kgm^{-3}

UNEB 2016 Q.4

(a) (i) What is meant by **fluid element** and **flow line** as applied to fluid flow (02mk)

(ii) Explain why some fluids flow more easily than others. (03mk)

(b) (i) State **Bernoulli's principle** (01mk)

(ii) Explain how a pitot static tube works (04mk)

(c) Air flowing over the upper surface of an air craft's wings causes a lift force of 6400N. The air flows under the wings at a speed of 120m/s over an area of 28m^2. Find the speed of air flow over an equal area of the upper surface of the air of the air craft's wings. (density of air=$1.2kgm^{-3}$) **An** $\mathbf{121.6ms^{-1}}$ (4mk)

(d) **(i)** What is meant by **surface tension** and **angle of contact** of a liquid (02mk)

(ii) A water drop of radius 0.5cm is broken up into other drops of water of radius 1mm. Assuming isothermal conditions, find the total work done to break up the water drop. **An** $\mathbf{8.8x10^{-5}J}$ (04mk)

UNEB 2014Q.4

(a) Define coefficient of viscosity and state its units (02marks)

(b) Explain the origin of viscosity in air and account for the effect of temperature on it (05marks)

(c) Describe, stating the necessary precautions an experiment to measure the coefficient of viscosity of a liquid using Stoke's law (07marks)

(d) A steel ball bearing of diameter 8.0mm falls steadily through oil and covers a vertical height of 20.0cm in 0.56 s. if the density of the steel is 7800kgm^{-3} and that of oil is 900 kgm^{-3}. Calculate:

(i) Up thrust on the ball **An** $\mathbf{2.37x10^{-3}N}$ (03 marks)

(ii) Viscosity of oil **An** $\mathbf{0.674Nsm^{-2}}$ (03 marks)

UNEB 2013 Q.2

(a) Define terminal velocity. (01mark)

(b) Explain laminar flow and turbulent flow. (03marks)

(c) Describe an experiment to measure the coefficient of viscosity of water using Poiseulle's formula.

(d) (i) State Bernoulli's principle. (01marks)

(ii) Explain why a person standing near a railway line is sucked towards the railway line when a fast moving train passes. (03marks)

(e) A horizontal pipe of cross-sectional area 0.4 m^2, tapers to a cross-sectional area of 0.2 m^2. The pressure at the large section of the pipe is 8.0 x $10^4 N\,m^{-2}$ and the velocity of water through the pipe is 1.2 m s^{-1}. If atmospheric pressure is 1.01 x$10^5 N\,m^{-2}$, find the pressure at the small section of the pipe.

An[$9.884x10^4 N\,m^{-2}$,] (05marks)

UNEB 2012 Q 4

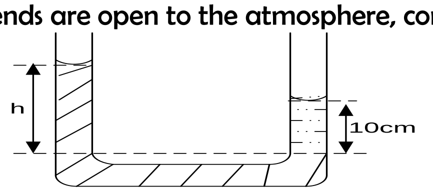
a) i) What is meant by the following terms steady flow and viscosity. (02marks)

ii) Explain the effect of increase in temperature on the viscosity of a liquid. (03marks)

b) i) Show that the pressure p exerted at a depth h below the free surface of a liquid of density ρ is given by $P = h\rho g$ (03marks)

ii) Define relative density (01mark)

iii) A U-tube whose ends are open to the atmosphere, contains water and oil as shown below.



Given that the density of oil is 800kgm^{-3}, find the value of h. **An [12.5cm]**

UNEB 2011 Q 3

a) i) What is meant by viscosity. (01mark)

ii) Explain the effect of temperature on the viscosity of a liquid. (03marks)

b) Derive an expression for the terminal velocity of a sphere of radius a, falling in liquid of viscosity η

c) Explain why velocity of a liquid at a wide part of tube is less than that at a narrow part. (2mks)

UNEB 2010 Q 3

a) Define viscosity of a fluid (01mark)

b) i) Derive an expression for the terminal velocity attained by a sphere of density δ, and radius a, falling through a fluid of density ρ and viscosity η (05marks)

ii) Explain the variation of the viscosity of a liquid with temperature. (02marks)

UNEB 2009 Q 4

a) i) State Archimedes principle (01mark)

ii) A tube of uniform cross sectional area of 4x10^{-3}m^2 and mass 0.2kg is separately floated vertical in water of density 1000kgm^{-3} and in oil of density 800kgm^{-3}. Calculate the difference in the lengths immersed. **An [1.25x10^{-2}m]** (04marks)

UNEB 2006 Q 4

a) i)State Archimedes principle (01mark)

ii) Describe an experiment to determine relative density of an irregular solid which floats in water.

UNEB 2005 Q 3

a) What is meant by the following terms

i) Velocity gradient (01mark)

ii) Coefficient of viscosity (01mark)

b) Derive an expression for the terminal velocity of a steel-ball bearing of radius r and density ρ falling through a liquid of density σ and coefficient of viscosity η. (05marks)

d) Explain with the aid of a diagram why air flow cover the wings of an air craft at take-off causes a lift.

UNEB 2003 Q 3

a) State the law of floatation . (01mark)

b) With the aid of a diagram describe how to measure the relative density of a liquid using Archimedes principle and the principle of moments. **An [refer to Abbot Pg 133]** (06marks)

c) A cross sectional area of a ferry at its water line is 720m^2. If sixteen cars of average mass 1100kg are placed on board, to what extra depth will the boat sink in the water. **An[2.4x10^{-2}m]** (04marks)

UNEB 2002 Q 3

a) i) Show that the weight of fluid displaced by an object is equal to the up thrust on the object. (5mks)

ii) A piece of metal of mass 2.60×10^{-3}kg and density 8.4×10^{3}kgm^{-3} is attached to a block of wax of mass 1.0×10^{-2} kg and density 9.2×10^{2}kgm^{-3}. When the system is placed in a liquid, it floats with wax just submerged. Find the density of liquid. (04marks)

b) Explain the

i) Terms laminar flow and turbulent flow (04marks)

ii) Effects of temperature on the viscosity of liquids and gases (03marks)

c) i) Distinguish between static pressure and dynamic pressure (02marks)

Solution

a)ii) By law of floatation, a floating body displaces its own weight

Mass of liquid displaced = $(2.60 \times 10^{-3} + 1.0 \times 10^{-2})$

= 1.26×10^{-2}kg

Volume of liquid displaced $= \frac{2.6x10^{-3}}{8.4x10^{3}} + \frac{1x10^{-2}}{9.2x10^{2}}$.

$= 1.12x10^{5}m^{3}$

ρ of liquid = $\frac{mass\ of\ liquid\ displaced}{volume\ of\ liquid\ displaced}$

$= \frac{1.26\ x\ 10^{-2}}{1.12\ x\ 10^{-5}}$

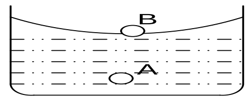
ρ of liquid $= 1.13x10^{3}kg\ m^{-3}$

CHAPTER 12: SURFACE TENSION

The surface of a liquid behaves like an elastic skin in a state of tension.
It is responsible for the following observations;

1- A needle floating on an undisturbed water surface though made of material which is denser than water
2- Some insects walk on water surface without sinking
3- Drops of water remaining suspended and becoming nearly spherical when falling from a tap
4- Mercury gathering into small droplets when spilt

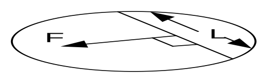
12.1.0: Molecular explanation for existence of surface tension



- Liquid molecules attract each other. In the bulk of the liquid the resultant force on any molecule such as A is zero.
- A surface molecule such as B is subjected to intermolecular forces of attraction below therefore potential energy of surface molecules exceeds that of the interior. Average separation of the surface molecules is greater than that of molecules in the interior. At any point on a liquid surface there is a net force away from that point and this makes the surface behave like an elastic skin in a state of tension. This accounts for surface tension.

Definition

Surface tension coefficient γof a liquid is defined as the force per unit length acting at right angles to one side of an imaging line drawn in the liquid surface.



$\gamma = \frac{F}{L}$

Units of γ are Nm^{-1}

Dimensions of γ

$\gamma = \frac{F}{L}$

$[\gamma] = \frac{[F]}{[L]} = \frac{M\,L\,T^{-2}}{L}$

$[\gamma] = M\,T^{-2}$

Other units of γ are kgs^{-2}

12.1.2: Factors affecting surface tension

i) Temperature

When the temperature of a liquid is increased, the liquid molecules gain kinetic energy and the molecules become more free to move and rush to the surface. The number of molecules in the surface increase, potential energy of the surface molecules is lowered and the separation of molecules decreases leading to a reduction in the intermolecular attraction, this reduces tension energy of molecules and hence surface energy tension is also reduced.

ii) Impurities

Impurities detergents and soap get between the molecules of the liquid reducing the intermolecular forces between the liquids and hence reducing surface tension

iii) Nature of the liquid

Different liquids have different surface tension

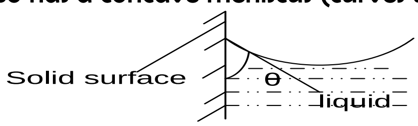
12.1.3: SHAPES OF LIQUID SURFACE

The surface of a liquid must be at right angles to the resultant force acting on it otherwise there would be component of this force parallel to the surface which would cause motion.
Normally a liquid surface is horizontal i.e. at right angles with the force of gravity but where it's in contact with the solid it's usually curved.
The particular form that this curvature takes is determined by the strengths of what are called the **cohesive** and **adhesive** forces.

Cohesive force is the attractive force exerted on a liquid molecules by the neighboring liquid molecules.
Adhesive forces is the attractive force exerted on a liquid molecule by the molecules in the surface of the solid.

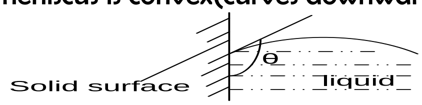
Consider a liquid in a container with vertical sides

- If the adhesive force is large comparative with the cohesive force, the liquid tends to stick to the wall and so has a concave meniscus (curves upwards).



e.g water and glass

- If the cohesive force is large compared with adhesive , the liquid surface pulls away from the wall and the meniscus is convex(curves downwards)



e.g mercury and glass

12.14: ANGLE OF CONTACT θ

This is the angle between the solid surface and the tangent plane to the liquid surface measured through the liquid.
From the diagrams above, the meniscus is concave when θ is less than 90^o and is convex when θ is greater than 90^o.
A liquid is said to wet a surface with which its angle of contact is less than 90^o.
The angle of contact of water and clean glass is **zero**, and that between mercury and clean glass is **13**7°. Thus water wets clean glass, mercury does not.
Addition of a detergent to a liquid lowers its surface tension and reduces the contact angle.

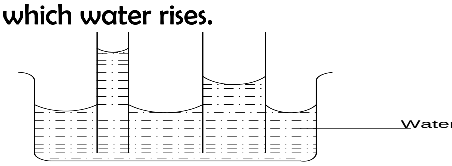
Measurement of angle of contact

A clean glass plate is placed at varying angles to a liquid until the surface on one side of the plate remains horizontal. The angle θ made between the horizontal surface and the plate is the angle of contact.

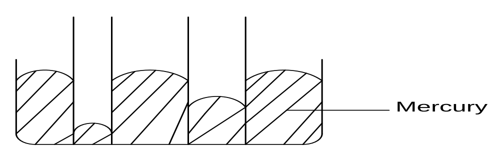
12.3.0: CAPILLARITY

When a capillary tube is immersed in water and the plane vertical with one end of water. Water rises to a height above the surface of water in the container. This is due to the fact that adhesive forces are greater than the cohesive forces.

The narrower the tube, the greater is the height to which water rises.



If the capillary tube is dipped inside mercury liquid is depressed below the outside level. This is because the cohesion of mercury is greater than the adhesion of mercury and glass.
The depression of the tube increases with decreases the diameter of the tube

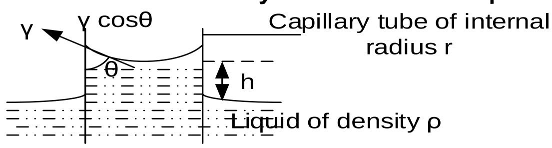


Definition
Capillarity: Is the rise or fall of a liquid in a capillary tube

12.3.1: Capillary rise

Around the boundary where the liquid surface meets the tube, surface tension forces exert a downward pull on the tube since they not balanced by any other surface tension forces.

The tube therefore exerts an equal but upwards force on the liquid which forces it to rise. The liquid stops rising when the weight of the raised column acting downwards equals to vertical component of the upward force exerted by the tube in the liquid.



Force acting upwards $F = \gamma cos\,\theta\, x\, L$

But L=2πr

F = γ cos θx2πr ------------[1]

Weight W= mg =Vρg

W =Ahρg= $\pi r^2 h\rho g$ ----------------- [2]

At equilibrium: $W = F$

$\pi r^2 h\rho g = \pi r$

h= $\frac{2\,\gamma\, cos\theta}{r\,\rho\, g}$

12.3.2: Capillary depression

Consider mercury inside a tube and the angle of contact θ

$$P_2 - P_1 = \frac{2\,\gamma\, cos\theta}{r}$$

But P_1 = H(atmospheric)

$$P_2 - H = \frac{2\,\gamma\, cos\theta}{r}$$

$P_2 = \frac{2\,\gamma\, cos\theta}{r} + H$ ------------------------ [1]

Also: $P_2 = H + h\rho g$---------------------- [2]

Equating

$$H + h\rho g = \frac{2\,\gamma\,\cos\theta}{r} + H$$

$$h\rho g = \frac{2\,\gamma\, cos\theta}{r}$$

$$\boxed{h = \frac{2\,\gamma\, cos\theta}{r\,\rho\, g}}$$

Example

1. A clean glass capillary tube of internal diameter 0.04cm is held with its lower end dipped in water contained in a beaker and with 12cm of the tube above the surface of water.
    i) To what height will water rise in the tube.
    ii) What will happen if the tube is now depressed until only 4cm of its length is above the surface. (γ of water =7.0x10^{-2}Nm^{-1}, ρ of water =1000kgm^{-3})

Solution

i) Using h = $\frac{2\,\gamma\,\cos\theta}{r\,\rho\, g}$

But for a clean glass of water θ =0

$$h = \frac{2\, x\, 7x\, 10^{-2}\, cos0}{\left(\frac{0.04\, x10^{-2}}{2}\right) x\ 1000\ x9.81} = 0.071\text{m}$$

ii) If only 4cm of the tube is left above the water surface, this length is less than h in part (i) above so water must change its angle of contact so that it can fit the 4cm

$$h = \frac{2\,\gamma\,\cos\theta}{r\,\rho\, g}$$

$$4x10^2 = \frac{2\text{ x } 7\text{x } 10^{-2}\cos0}{\left(\frac{0.04\text{ x}10^{-2}}{2}\right)\text{x } 1000\text{ x}9.81}$$

θ=55.9°

water forms a new surface with an angle of contact 56°

2. A U-tube is made with an internal diameter of one arm 2.0cm and the other 4mm and mercury is poured in the two tubes. If the angle of contact of mercury with glass after exposure to air is 160°. What will be the difference in level of surface in the tubes, take surface tension of mercury as 0.0472Nm^{-1}

Solution

r_1=2x10^{-3}m r_2=1x10^{-2}m ρ =13600kgm^{-3}(density of mercury)

γ=0.0472 θ=180-160° θ=20°(we subtracted to obtain a positive value of the $cos\theta$)

Note: we only subtract for angles greater than 90°

$h_1 = \frac{2\,\gamma\,\cos\theta}{r_1\,\rho\, g} = \frac{2x0.047Cos20}{2x10^{-3}x13600x9.81}$ =3.32x10^{-4}m

$h_2 = \frac{2\,\gamma\,\cos\theta}{r_2\,\rho\, g} = \frac{2x0.047Cos20}{1x10^{-3}x13600x9.81}$ =6.65x10^{-5}m

Difference =h_1- h_2

$$= 3.32x10^{-4} - 6.65x10^{-5}$$
$$= 2.655x10^{-4}m$$

Exercise: 30

1. A liquid of density 1000kg$m^{-3}$and surface tension $7.26x10^{-2}Nm^{-1}$, dipped in it is a capillary tube with a bore radius of 0.5mm. If the angle of contact is 0° determine,
   i) the height of the column of the liquid rise
   ii) if the tube is pushed until its 2cm above the level of the liquid, explain in what happen

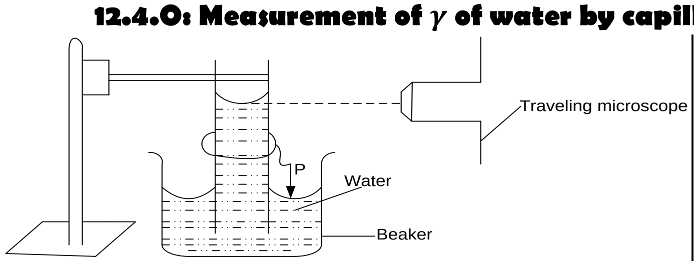
   An[2.96x10^{-2}m, 47.5°]
2. The two vertical arms of manometer containing water, have different internal radii of 10^{-3}m and $2x10^{-3}$m respectively. Determine the difference in height of the two liquids levels when the arms are open to the atmosphere. (surface tension and density of water are $7.2x10^{-2}Nm^{-11}$ and 10^3kgm^{-3} respectively)

   An[7.14x10^{-3}m]
3. The end of a clean glass capillary tube having internal diameter 0.6mm is dipped into a beaker containing water, which rises up the tube to a vertical height of 5.0cm above the water surface in the beaker. Calculate the surface tension of water (Density of water =1000kgm^{-3}, g=10ms^{-2}).
   What would be the difference if the tube were not perfectly clean so that the water did not wet it, but had an angle of contact of 30° with the tube surface.

   An[7.5x10Nm^{-1}, the water would rise to only 4.3cm]
4. A capillary tube which is clean is immersed in water of surface tension $7.2x10^{-2}Nm^{-1}$ and water rises 6.2cm in the capillary tube. What will be the difference in the mercury level, if the same capillary tube is immersed in the mercury (surface tension of mercury = 0.84 Nm^{-1}, angle of contact between mercury and glass =140°, ρ of mercury =$1.36x10^4kgm^{-3}$, ρ of water =10^3kgm^{-33}) **An[h=4.2cm]**
5. Mercury is poured into glass U-tube with vertical limbs of diameters 2.0mm and 12.0mm respectively. If the angle of contact between mercury and the glass is 140° and the surface tension of mercury is 0.52Nm^{-1}, calculate the difference in the levels of mercury. (density of mercury is $1.36x10^4kgm^{-3}$)
   An($4.9x10^{-3}m$)
6. A U-tube with limbs of diameter 7mm and 4mmm contains water of surface tension $7x10^{-2}Nm^{-1}$, angle of contact $0°$ and density 1000kgm^{-3}. Find the difference in the levels. $An\ 3.1mm$
7. A glass U-tube is such that the diameter of one limb 4.0mm while that of the other is 8.0mm. the tube is inverted vertically with the open ends below the surface of water in a beaker. Given that surface tension of water is $7.2x10^{-2}Nm^{-1}$, angle of contact between water and glass is zero, and that density of water is 1000kgm^{-3}. What is the difference between the heights to which water rises in the two limbs.
   $An\ 7.34mm$
8. Calculate the height to which the liquid rises in the capillary tube of diameter 0.4mm placed vertically inside
   (i) A liquid of density $800kgm^{-3}$ and surface tension $5x10^{-2}Nm^{-1}$ and angle of contact $30°$
   (ii) Mercury of angle of contact 139° and surface tension 0.52Nm^{-1}

   An[0.032m, 0.0294m]

12.4.0: Measurement of γ of water by capillary tube method



- A clean capillary tube is dipped in water as shown and a wire p which is bent is tied along the capillary tube with a rubber band.
- When the tube is dipped into water, the wire p is adjusted so that its top just touches the surface of the water.
- A travelling microscope is focused on the water meniscus in the capillary tube and the reading noted, say h_1.
- The beaker is then removes and the travelling microscope is focused on the tip of the wire p and scale reading h_2 is noted.
- The height of the water rise, h is calculated from h=/h_1-h_2/.
- The capillary tube is removed and its diameter and hence radius , r is determined by using a travelling microscope. The surface tension can be obtained from ;

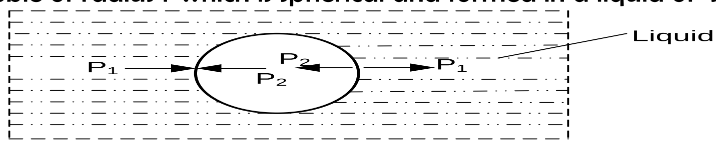
  $\gamma = \frac{h\,r\,\rho\,g}{2\cos\theta}$ for clean glass of water $\theta=0°$

12.2.0: PRESSURE DIFFERENCE ACROSS A SPHERICAL INTERFACE

The pressure inside a soap bubble is greater than the pressure of the air outside the bubble. If this were not so, the combined effect of the external pressure and the surface tension forces in the soap film would cause the bubble to collapse, similarly the pressure inside an air bubble in a liquid exceeds the pressure in the liquid and the pressure inside a mercury drop is greater than that outside it.

12.2.1: Pressure difference across an air bubble

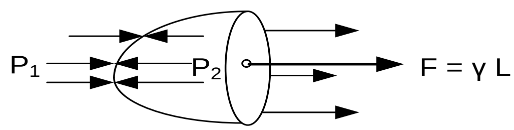
Consider an air bubble of radius r which is spherical and formed in a liquid of surface tension γ



P_1 = External pressure on the bulb due to the liquid

P_2 = internal pressure of air in the bubble

Considering half of the bubble. The remaining half experiences surface tension force due to the other half and this force acts towards the right.



For the bubble to maintain its shape the, internal pressure should be bigger than the external pressure.

At equilibrium; Force due to P_2 = force due to P_1 + surface tension

$$AP_2 = AP_1 + \gamma L$$

$$\pi r^2 P_2 = \pi r^2 P_1 + 2\pi r \gamma$$

$$\pi r^2 (P_2 - P_1) = 2\pi r \gamma$$

$$\boxed{P_2 - P_1 = \frac{2\gamma}{r}}$$

OR Excess pressure $= \frac{2\gamma}{r}$

Note:

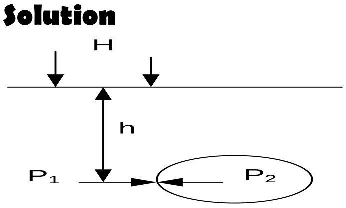
The pressure inside an air bubble is greater than that outside, otherwise the combined effect of the external pressure and the surface tension forces in the air bubble to collapse.

The same case can be extended to a soap bubble.

Example

Calculate the pressure inside a spherical air bubble of diameter 0.1cm blown at depth of 20cm below the surface of a liquid of density $1.26x10^3 kgm^{-3}$ and surface tension 0.064Nm^{-1}.(height of mercury barometer is 0.76m, and density of mercury is $13.6x10^3 kgm^{-3}$).

Solution



$P_1 = H + h\rho g$

$P_1 = 0.76x13.6x10^3x9.81 + \frac{20}{100}x1.26x10^3x9.81$

P_1=101643Pa

Excess pressure of air bubble $= \frac{2\gamma}{r}$

$P_2 - P_1 = \frac{2\gamma}{r}$

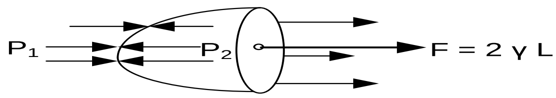
$P_2 - 101643 = \frac{2\ x\ 0.064}{0.05\ x\ 10^{-2}}$

$P_2 = 1.02x10^5$Pa

12.2.2: Excess pressure (pressure difference) for a soap bubble

For a soap bubble of radius r, there are two surfaces of liquid in contact with air (the air inside the bubble and air outside the bubble). Therefore the total length of surface in contact with air is 2L such that surface tension force.

$$F=2L\gamma$$



At equilibrium : Inside force due to P_2 = external force due to P_1 +surface tension force

$$AP_2=AP_1+2\gamma L$$

$$\pi r^2P_2=\pi r^2P_1+4\pi r\ \gamma$$

$$\pi r^2(P_2-P_1)=4\pi r\gamma$$

$$\boxed{P_2-P_1=\frac{4\gamma}{r}}$$

Excess pressure = $\frac{4\gamma}{r}$

Example

A soap bubble has a diameter of 4mm. calculate the pressure inside it, if the atmospheric pressure is 10^5Nm^{-2}, and that the surface tension of soap solution is $2.8x10^{-2}Nm^{-1}$

Solution

$P_2-P_1=\frac{4\gamma}{r}$

$P_2-10^5=\frac{4\,x2.8x10^{-2}}{2\,x\,10^{-3}}$

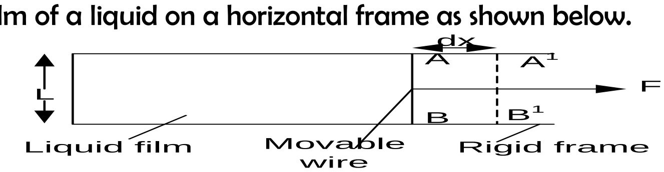
$P_2=1.0006x10^5Pa$

12.1.1: FREE SURFACE ENERGY (σ)

It is defined as the work done in increasing area of the surface by $1m^2$ under isothermal conditions . Units of σ are Jm^{-2} or Nm^{-1}

Relation between surface tension and surface energy

Consider stretching a thin film of a liquid on a horizontal frame as shown below.



If AB is moved a distance dx to A^1B^1, then surface tension $\gamma=\frac{F}{2l}$

surface energy (σ) is given by;

$$\sigma.=\frac{F\,x\,dx}{2\,L\,x\,dx}=\frac{2\,L\,\gamma\,dx}{2\,L\,dx}$$

$$\sigma=\gamma$$

∴ free surface energy =surface tension

Example

1. Calculate the work done against surface tension force on blowing a soap bubble of diameter 15mm , if the surface tension of the soap solution is $3.0x10^{-2}Nm^{-1}$.

Solution

$\gamma=\frac{work\ done}{increase\ in\ S.A}$

Work done = γ x increase in surface area = $\gamma x(2x4\pi r^2)$

$=3.0x10^{-2}x2x4x\frac{22}{7}x\left(\frac{15x10^{-3}}{2}\right)^2$

Work done = $4.241x10^{-5}J$

Increases in surface area is multiplied by 2 for both the upper and lower surface of a soap bubble.

2. Calculate the change in surface energy of a soap bubble when its radius decreases from 5cm to 1cm, given that the surface tension of soap solution is $2x10^{-2}Nm^{-1}$

Solution

$\gamma = \frac{work\ done}{increase\ in\ S.A}$

Work done $= \gamma\ x$ increase in surface area $= \gamma x(2x4\pi r^2)$

5cm bubble: Work done $= 2.0x10^{-2}x2x4x\frac{22}{7}x(5x10^{-2})^2 = 1.257x10^{-3}$J

1cm bubble: Work done $= 2.0x10^{-2}x2x4x\frac{22}{7}x(1x10^{-2})^2 = 5.027x10^{-5}$J

Change in surface energy $= 1.257x10^{-3} - 5.027x10^{-5} = 1.207x10^{-3}$J

3. A liquid drop of diameter 0.5cm breaks up into 27 tiny droplets all of the same size. If the surface tension of the liquid is 0.07Nm^{-1} calculate the resulting change in energy.

Solution

Diameter of big drop, D= 0.5cm $\therefore R = 0.25cm = 2.5x10^{-3}m$

Volume of big drop$= \frac{4}{3}\pi R^3 = \frac{4}{3}\pi(2.5x10^{-3})^3$

Volume of 27 tiny droplets$= 27x\frac{4}{3}\pi r^3$

$$27x\frac{4}{3}\pi r^3 = \frac{4}{3}\pi(2.5x10^{-3})^3$$

$$\boldsymbol{r} = 8.3x10^{-4}m$$

Work done $= \gamma\ x$ increase in surface area$= \gamma x(4\pi r^2)$

Big drop: Work done$= 0.07x4x\frac{22}{7}x(2.5x10^{-3})^2 = 5.5x10^{-6}$J

27 drop lets: Work done$= 27x0.07x4x\frac{22}{7}x(8.3x10^{-4})^2 = 1.637x10^{-5}$J

Change in surface energy $= 1.637x10^{-5} - 5.5x10^{-6} = 1.087x10^{-5}$J

4. Calculate the work done in breaking up a drop of water of radius 0.5cm in to tiny droplets of water each of radius 1mm assuming isothermal conditions, given that surface tension of water is $7x10^{-2}$ Nm^{-1}.

Solution

Radius of big drop, R= 0.5cm $= 5x10^{-3}m$ and Radius of n tiny droplets, r= 1mm $= 1x10^{-3}m$

Volume of big drop$= \frac{4}{3}\pi R^3 = \frac{4}{3}\pi(5x10^{-3})^3$

Volume of n tiny droplets$= nx\frac{4}{3}\pi r^3 = nx\frac{4}{3}\pi(1x10^{-3})^3$

$$nx\frac{4}{3}\pi(1x10^{-3})^3 = \frac{4}{3}\pi(5x10^{-3})^3$$

$$\boldsymbol{n} = 125\ droplets$$

Work done $= \gamma\ x$ increase in surface area$= \gamma x(4\pi r^2)$

Big drop: Work done$= 7x10^{-2}\ x4x\frac{22}{7}x(5x10^{-3})^2 = 2.2x10^{-5}$J

125 drop lets: Work done$= 125x7x10^{-2}\ x4x\frac{22}{7}x(1x10^{-3})^2 = 1.1x10^{-4}$J

Change in surface energy $= 1.1x10^{-4} - 2.2x10^{-5} = x1.09x10^{-4}$J

EXERCISE: 31

1. A spherical drop of mercury of radius 2mm falls to the ground and breaks into 10smaller drops of equal size. Calculate the amount of work that has to be done.
(Surface tension of mercury = 4.72 x 10^{-1} Nm^{-1}) **An[2.74x10^{-5}J]**
2. Calculate the excess pressure within a bubble of air of radius 0.1mm in water given that the surface tension of air is $7.27x10^{-2}Nm^{-1}$. **An(1454Pa)**
3. What is the excess pressure inside a spherical soap bubble of radius 5cm if the surface tension of the soap film is $3.5x10^{-2}Nm^{-1}$. What is the work done in blowing the bubble

Relationship between surface area and shape of a drop

For any given volume, a sphere is the shape with minimum surface area. Hence minimum surface energy therefore the most stable and this explains why small droplets from a tap and rain are spherical in shape.

Why small mercury droplets are spherical and larger one flatten out

A small drop takes on a spherical shape to minimize the surface energy which tends to be greater than the gravitational potential energy. Therefore, the gravitational potential force cannot distort the spherical shape due to the very small mass of tiny droplets.
A large drop flattens out in order to minimize the gravitational potential energy, which tends to exceed the surface energy. Due to its large weight, gravitational force distorts the spherical shape of large drops.
The shape of the drop must agree with the principle that the sum of gravitational potential energy and surface energy must be a minimum

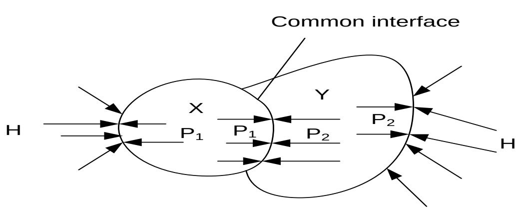
COMBINED BUBBLES

CASE 1

A soap bubble x of radius r_1, and another bubble y of radius r_2, are brought together so that the combined bubble has a common interface of radius R. show that

$R = \frac{r_1\ r_2}{r_2 - r_1}$

Solution



Excess pressure on x

$P_1 - H = \frac{4\gamma}{r_1}$ ------------------------------ [1]

Excess pressure on y

$P_2 - H = \frac{4\gamma}{r_2}$ ------------------------------ [2]

Equation –equation 2 gives

$P_1 - P_2 = \frac{4\gamma}{r_1} - \frac{4\gamma}{r_2}$

$P_1 - P_2 = \frac{4\gamma}{r_1} - \frac{4\gamma}{r_2}$ ------------------------ [3]

Excess pressure at the interface

$P_1 - P_2 = \frac{4\gamma}{R}$ ------------------------------ [4]

Equating equation 3 and equation 4

$\frac{4\gamma}{R} = \frac{4\gamma}{r_1} - \frac{4\gamma}{r_2}$

$\frac{1}{R} = \frac{1}{r_1} - \frac{1}{r_2}$

$\frac{1}{R} = \frac{r_1 - r_2}{r_1 r_2}$

$R = \frac{r_1\ r_2}{r_2 - r_1}$

Example

1. A soap bubble x of radius 0.03m and another bubble y on radius,0.04m are brought together so that the combined bubble has a common interface of radius r. calculate r

   Solution

   Using $r = \frac{r_1\ r_2}{r_2 - r_1} = \frac{0.03x\ 0.04}{0.04 - 0.03}$ =0.12m

2. Two soaps bubble A and B of radii 6cm and 10cm respectively coalesce so that the combined bubble has a common interface . calculate the radius of curvature of this common surface and hence the pressure difference. Given that surface tension of soap is $2.5x10^{-2}Nm^{-1}$

   Solution

   Using $r = \frac{r_1\ r_2}{r_2 - r_1} = \frac{0.06x\ 0.1}{0.1 - 0.06}$ =0.15m

   pressure difference $= \frac{4\gamma}{r} = \frac{4x\ 2.5x10^{-2}}{0.15} = 0.667Pa$

CASE 2

Two bubbles of a soap solution of radii r_1 and r_2 of surface tension γ and pressure P coalence under isothermal conditions to form one bubble. Find the expression for the radius of the bubble formed.

Solution

Let R be the radius of the new bubble | A_2 be the surface area of bubble with radius r_2

A_1 be the surface area of bubble with radius r_1 | A be the surface area of bubble with radius R

Under isothermal conditions, work done in enlarging the surface area of a bubble is given by

$$2\gamma A = 2\gamma A_1 + 2\gamma A_2$$

$$2\gamma 4\pi R^2 = 2\gamma 4\pi\, r_1{}^2. + 2\gamma 4\pi\, r_2{}^2$$

$$R^2 = r_1{}^2 + r_2{}^2$$

$$R = \sqrt{r_1{}^2 + r_2{}^2}$$

Examples

1. Two soap bubbles have radii of 3cm and 4cm, the bubbles are in a vacuum and they combine to form a single larger bubble. Calculate the radius of this bubble

Solution

$$R = \sqrt{r_1{}^2 + r_2{}^2} = \sqrt{3^2 + 4^2} = 5cm$$

2. Two soap bubbles of radii 2cm and 4cm respectively coalesce under isothermal conditions. If the surface tension of the soap solution is $2.5x10^{-2}$

Solution

$$R = \sqrt{r_1{}^2 + r_2{}^2} = \sqrt{(2x10^{-2})^2 + (4x10^{-2})^2} = \sqrt{20x10^{-4}}m$$

pressure difference $= \frac{4\,\gamma}{\mathrm{r}} = \frac{4x\ 2x10^{-2}}{\sqrt{20x10^{-4}}} = 1.789Pa$

EXERCISE: 32

1. A soap bubble whose radius is 12mm becomes attracted to one of radius 20mm. Calculate the radius of curvature of the common interface. **An[30mm]**
2. Two soap bubbles of radii 2.0cm and 4.0cm respectively coalesce under isothermal conditions. If the surface tension of the soap solution is $2.5x10^{-2}Nm^{-1}$. Calculate the excess pressure inside the resulting soap bubble. **An[2.36Pa]**

UNEB 2017 Q.4

(c) A capillary tube is held in a vertical positon with one end dipping in a liquid of surface tension γ and density ρ. If the liquid rises to a height, h derive an expression for h in terms of γ, ρ and radius r of the tube assuming the ange of contact is zero. (04mks)

(d) A mercury drop of diameter 2.0mm falls vertically and on hitting the ground, it splits into two drops each of radius 0.5mm. If the surface tension of the mercury is 0.52Nm^{-1} calculate the resulting change in surface energy **An** ($2.289x10^{-5}$J) (05mks)

(e) State the effect of temperature on surface tension of a liquid. (01mk)

UNEB 2015 Q.4

(a) (i) Distinguish between **surface tension** and **surface energy** (01mk)

(ii) Show that surface energy and surface tension are numerically equal (03mk)

(iii) Explain why water dripping out of a tap does so in a spherical shapes (03mk)

(b) Two soaps bubbles of radii 2.0 cm and 4.0 cm respectively coalesce under isothermal conditions. If the surface tension of the soap solution is $2.5x10^{-2}Nm^{-1}$, calculate the excess pressure inside the resulting soap bubble **An(2.24Pa)** (04mk)

(c) (i) State **Bernoulli's principle** (01mk)

(ii) Explain how wind at a high speed over speed over the roof of a building can cause the roof to be ripped of the building (03mk)

(iii) An aero plane has a mass of 8000kg and total wing span 8.0m^2. When moving through still air, ratio of it's velocity to that of the air at its lower surface is 1.0, while the ratio of its velocity o that of air above its wings is 0.25. At what velocity will the aero plane be able to just lift off the ground (density of air=$1.3kgm^{-3}$) **An(31.72m/s)** (5mk)

UNEB 2009 Q.4

C) i)Define surface tension in terms of work (1mk)

ii)Use the molecular theory to account for the surface tension of liquid (4mk)

iii)Explain the effect of increasing temperature of a liquid on its surface tension (4mk)

iv) Calculate the excess pressure inside a soap bubble of diameter 3.0cm if the surface tension of the soap solution is $2.5x10^{-2}Nm^{-1}$. **An[6.67Pa]** (2mk)

UNEB 2008 Q.3

a) i) Define surface tension (01mark)

ii) Explain the origin of surface tension (03marks)

iii) Describe an experiment to measure the surface tension of a liquid by the capillary method (06marks)

UNEB 2002 Q.4

a) Define the term surface tension in terms of surface energy (01mark)

b) i) Calculate the work done against surface tension is blowing a soap bubble of diameter 15mm, if the surface tension of the soap solution is $3.0x10^{-2}Nm^{-1}$ **An** [$4.24x10^{-5}$J] (03marks)

ii) A soap bubble of a radius r_1 is attached to another bubble of radius r_2. If r_1 is less than r_2. Show that the radius of curvature of the common interface is $\frac{r_1\ r_2}{r_2 - r_1}$ (05marks)

UNEB 2001 Q.3

a) Define surface tension and derive its dimension (3mk)

b) Explain using the molecular theory the occurrence of surface tension (4mk)

c) Describe an experiment to measure surface tension of a liquid by the capillary tube method (6mk)

d) i)Show that the excess pressure in a soap bubble is given by $P = \frac{4\gamma}{r}$

ii) Calculate the total pressure within a bubble of air of radius 0.1mm in water, if the bubble is formed 10cm below the water surface and surface tension of water is $7.27x10^{-2}Nm^{-1}$. [Atmospheric pressure =$1.01x10^5$Pa]

An $1.03x10^5$Pa

SECTIONB: HEAT AND THERMO DYNAMIC

CHAPTER1: THERMOMETRY

Heat is the amount of energy which moves from hotter to colder region.

Temperature is a number that expresses the degree of hotness of a body on a given scale.

Temperature is measured using a thermometer which has a scale on it.

Thermometers use a physical property which is called thermometric property to measure temperature.

Definition A thermometric property is a physical property which varies linearly and continuously with temperature.

1.1: QUALITIES OF A GOOD THERMOMETRIC PROPERTY

- It should vary linearly with temperature
- It should vary continuously with temperature
- It should be measurable over a wide range of temperature
- It should be sensitive to temperatures changes

TYPES OF THERMOMETERS AND THEIR THERMOMETRIC PROPERTY

Thermometer	Thermometric property
Liquid in glass	Length L of liquid column
Thermocouple	E.M.F " E"
Resistance eg Platinum	Electrical resistance "R" of a wire
Constant Volume gas	Pressure "P" of a fixed mass of a gas
Constant pressure gas	Volume "V" of afixed mass of a gas
Pyrometer	Wavelength λ(quality)

1.1.0: FIXED POINT

This is temperature at which a substance changes states under specific conditions.

1.1.1: ICE POINT

Ice point is temperature at which pure ice can exist in dynamic equilibrium with pure water at standard atmospheric pressure of 760mmHg. Ice point corresponds to 0°C

1.1.2: STEAM POINT

This is temperature at which pure water can exist in dynamic equilibrium with pure vapour at standard atmospheric pressure (760mmHg). Steam point corresponds to 100°C

1.1.3: TRIPLE POINT OF WATER

This is a temperature at which pure ice, pure water and pure vapour can exist together in dynamic equilibrium.

The triple point of water is chosen at fixed point and is defined as 273.16 K.

1.2.1: TYPES OF TEMPERATURE SCALE

Centigrade or Celsius temperature scale

Kelvin or absolute temperature or abnormal or thermodynamic temperature scale

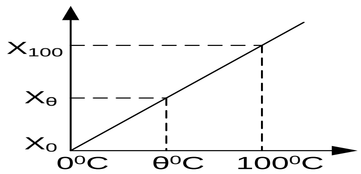
1.2.2: CENTIGRADE/ CELSIUS TEMPERATURE SCALE

Is a temperature scale which uses ice point (0°C) as it lower fixed point and steam point (100°C) as its upper fixed point

1.2.3: STEPS IN SETTING UP CELSIUS TEMPERATURE SCALE

- Choose a thermometric properly of substance and let it be X
- Measure the value of the property at ice point, steam point and let values be X_0, X_{100} respectively.
- Measure the value of the properly at unknown temperature θ and let it be X_θ
- Unknown temperature is determined from $\theta = \left(\frac{X_\theta - X_0}{X_{100} - X_0}\right) x100°C$

A graph of property value against temperature.



Slope $=\frac{\Delta y}{\Delta x}$

$\frac{X_{100}-X_\theta}{100-0}=\frac{X_\theta-X_0}{\theta-0}$

$\theta=\left(\frac{X_\theta-X_0}{X_{100}-X_0}\right)$ X100°C

Equation above is a defining equation of Celsius scale of temperature

Definition of a celcious scale of temperature for different thermometers

Thermo couple

θ= $\left(\frac{E_\theta-E_0}{E_{100}-E_0}\right)$ X100°C

Platinum resistance

θ= $\left(\frac{R_\theta-R_0}{R_{100}-R_0}\right)$ x100°c

Constant volume gas

θ= $\left(\frac{P_\theta-P_0}{P_{100}-P_0}\right)$ x100°c

Constant pressure gas

θ= $\left(\frac{V_\theta-V_0}{V_{100}-V_0}\right)$ x100°c

Liquid in glass

θ= $\left(\frac{L_\theta-L_0}{L_{100}-L_0}\right)$ x100°c

1.2.4: KELVIN / THERMODYNAMIC TEMPERATURE SCALE

This is a temperature scale which uses triple point of water as a fixed point.

Kelvin is defined as $\frac{1}{273.16}$ of the thermodynamic temperature of the triple point of water

Steps to establish Kelvin scale

- ✓ Select thermometric property X of substance
- ✓ Measure the property at triple point of water, let it be X_{tr}
- ✓ Measure the property at un known temperature T, let it be X_T
- ✓ Assuming a linear variation of X with temperature then the un known temperature can be determined from

$$\boxed{T=\frac{X_T}{X_{tr}}x273.16K}$$

Definition of a thermodynamic scale of temperature for different thermometers

Thermo couple

$T=\frac{E_T}{E_{tr}}x273.16K$

Platinum resistance

$$T=\frac{R_T}{R_{tr}}x273.16K$$

Constant volume gas

$$T=\frac{P_T}{P_{tr}}x273.16K$$

Constant pressure gas

$T=\frac{V_T}{V_{tr}}x273.16K$

Liquid in glass

$$T=\frac{L_T}{L_{tr}}x273.16K$$

1.2.5: DISAGREEMENT OF TEMPERATURE SCALES

Different thermometers give different readings when measuring temperature of the same body except at fixed points where they must agree and this is because different thermometric properties <u>vary differently</u> with temperature but <u>agree at fixed points</u>.

Example

1) A resistance thermometer has a resistance of 21.42Ω at ice point, 29.10 Ω at steam point and 28.11 Ω at un known temperature θ. calculate θ on scale of this thermometer.

Solution

θ= $\left(\frac{R_\theta-R_0}{R_{100}-R_0}\right)$ x100°C | θ= $\left(\frac{28.11-21.42}{29.10-21.42}\right)x100℃$ | θ=87.11°C

2) The resistance of the wire is measured at ice point, steam point and at un known temperature θ and the following values were obtained 2.00 Ω, 2.48 Ω, 2.60 Ω respectively. Determine θ

θ= $\left(\frac{R_\theta-R_0}{R_{100}-R_0}\right)$ x100°C | θ= $\left(\frac{2.60-2.00}{2.48-2.00}\right)$ x100 | θ =125℃

3) The length of mercury column is 2.00cm at ice point, 2.73cm at steam point.

(i) What temperature on the mercury in glass thermometer corresponds to the value of 8.43cm?

(ii) When the above temperature is measured on gas thermometer scale it correspond to a value of 1020°C.Explain the discrepancy

Solution

i) L_θ=2.00 L_θ=8.43, L_{100}=2.73	θ= $(\frac{L_\theta-L_0}{L_{100}-L_0})$ x100°C θ= $(\frac{8.43-200}{2.73-2.00})$ x100°C	θ= 880.8°C

(ii) Different thermometric properties vary differently with temperature but agree at fixed points

4) A particular resistance thermometer has resistance of 30Ω at ice point, 41. 58Ω at steam point and 34.58Ω when immersed in a boiling liquid. A constant volume gas thermometer gives readings, $1.333x10^5$Pa, $1.821x10^5$Pa and $1.528x10^5$Pa at the same temperatures. Calculate the temperature at which the liquid is boiling on scale of;

(i) Resistance thermometer (ii) Gas thermometer .

Solution

i) R_0=30 Ω R_θ=31.58 Ω, R_{100}=41.58 Ω $\theta = (\frac{R_\theta-R_0}{R_{100}-R_0})$ x100°C= $[\frac{34.58-30}{41.58-30}]$ x100°C θ=39.55°C ii) P_θ=$1.333x10^5$Pa P_{100}=$1.821X10^5$Pa P_θ=$1.628x10^5$Pa	θ= $(\frac{P_\theta-P_0}{P_{100}-P_0})$ x100°C θ= $(\frac{1.528x10^5-1.333x10^5}{1.821x10^5-1.333x10^5})$ x100°C θ= 39.959°C

Example on triple point of water or Kelvin scale

5) Pressure recorded by constant volume thermometer at Kelvin temperature T is given by $4.8x10^4Nm^{-2}$. Calculate T if the pressure at triple point of water is $4.2x10^4Nm^{-2}$

Solution

T=$\frac{P_T}{P_{tr}}$x273.16K P_T=4.8x10Nm^{-2}	P_{tr} =42x10^4Nm^{-2} T= $\frac{4.8x10^4}{4.2x10^4}$x273.16K	T= 312.18K

6) The resistance of platinum wire at triple point of water is 5.16Ω. what will be the resistance at 100°C

Solution

T= $\frac{R_T}{R_{tr}}$x273.16K	(273+100)= $\frac{R_T}{5.16}$X273.16	R_T= 7.045Ω

Determining temperature on a scale of one thermometer as read by another

1) The resistance, $R_\theta$of a particular resistance thermometer at Celsius temperature θ as measured by a constant volume gas thermometer is. $R_\theta = 50 + 0.17\theta + 3x10^{-4}\,\theta^2$ Calculate the temperature as measured on a scale of a resistance thermometer which corresponds to a temperature of 60°C at a gas thermometer.

Solution

θ= $(\frac{R_\theta-R_0}{R_{100}-R_0})$ x100°C $R_\theta = 50 + 0.17\theta + 3x10^{-4}\,\theta^2$ $R_0 = 50 + 0.17x0 + 3x10^{-4}x\,0^2$ $R_0 = 50$	R_{100}=50+0.17x100+3x10^{-4}x100^2 R_{100}= 70Ω R_{60}=50+0.17x60+3x10^{-4}x60^2 R_{60}=61.28Ω	θ= $(\frac{61.28-50}{70-50})$ x100°C θ= 56.4°C

2) The value of property X of certain substance X_t is given by $X_t = X_0 + 0.5t + 2x10^{-4}t^2$

Where t =temperature in °C measured in gas thermometer scale. What will be the Celsius temperature at 50°C on this thermometer scale?

Solution

X_{100}=X_0+52 X_0=X_0	X_{50} =X_0+25.5	θ= $(\frac{X_{50}-X_0}{X_{100}-X_0})$ x100°C

θ= $(\frac{X0+25.5-X0}{X0+52-X0})x100°C$

θ= $(\frac{25.5}{52})$ x100°C

θ= 49.04°C

3) The resistance of the wire as measured by gas thermometer varies with temperature θ according to the equation. $R_\theta = R_0\ (1 + 50\alpha\theta + 200\alpha\theta^2)$.Determine temperature on resistance thermometer that corresponds to 40°Con the gas scale

Solution

R_{100}= R_0 (1+50xαx100+200α100^2)

R_{100} = R_0 [1+α(2005000)]

R_0=R_0

R_{40}= R_0 (1+50αx40+200xα40^2)

R_{40}= R_0 [1+α (322000)]

θ= $[\frac{R_{40}-R_0}{R_{100}-R_0}]$ x100°C

θ= $(\frac{R0[1+\alpha(322000)]-R0}{R0[1+\alpha(2005000)]R0})$

θ= $[\frac{322000}{2005000}]$ x100°C

θ= 16.059°C

Exercise: 33

1) The resistance of the element in a platinum resistance thermometer is 6.75 Ω at triple point of water and 7.166 Ω at room temperature. What is the temperature of the room on a scale of resistance thermometer?. state one assumption you have made. **An[290K]**

2) A particular constant –volume gas thermometer registers a pressure of 1.937x10^{4}Pa at the triple point of water and 2.618x10^{4}Pa at the boiling of a liquid. What is the boiling point of the liquid according to this thermometer? **An[369.2K]**

3) The resistance of platinum thermometer is 2.04Ω at ice point and 3.02Ω at the steam point.
   i) What should be the temperature of platinum wire so as to have a resistance of 9.24Ω?
   ii) If a constant-pressure thermometer had been used, the same temperature would correspond to 1040°C. Explain the deviation. **An**[734.7°C]

4) The resistance R of platinum wire at temperature θ°C as measured by mercury-in-glass thermometer is given by; $R_\theta = R_0(1 + a\theta + b\theta^2)$ where $a = 3.8x10^{-3}K^{-1}$ and $b = -5.6x10^{-7}K^{-2}$. Calculate the temperature of platinum thermometer corresponding to 200°C on glass scale. **An**[197°C]

5) The resistance R of platinum wire at temperature θ°C as measured by a constant volume thermometer is given by; $R_\theta = R_0(1 + 8000\alpha\theta - \alpha\theta^2)$ where α is a constant. Calculate the temperature of platinum thermometer corresponding to 400°C on glass scale. **An**[384.8°C]

1.3.0: TYPES OF THERMOMETERS

a)-Liquid in glass thermometer;

measurement of temperature using a liquid in glass thermometer

- Place the bulb in pure melting ice and the length of the mercury column in capillary tube, L_0 is measured and recorded
- Place the bulb in steam from boiling water and the length of the mercury column in capillary tube, L_{100} is measured and recorded
- Place the bulb in contact with the body of an unknown temperature θ and the length of mercury column L_θ is measured and recorded
- Unknown temperature is determined from $\theta = \left(\frac{L_\theta - L_0}{L_{100} - L_0}\right)x100°C$

Advantages of a Liquid in Glass Thermometer

- It is easy to use
- It is very cheap
- It is very portable
- It has direct readings

Disadvantages of a Liquid in Glass Thermometer

- It has small range of temperature
- It is not very accurate
- Its fragile so care is needed
- It is not very sensitive
- It can not measure temperature at a point
- It can not meaures rapidily changing temperatures

N.B:

A liquid in glass thermometer is not very accurate because of the following;

1. Parallax errors which contribute about $\pm 0.1°C$
2. Non uniformity of the bore of capillary tube limits accuracy to about 0.1°C
3. The glass contracts and expands and takes long hours to recover its correct size and shape and therefore spoils the calibration

Reasons why mercury is used as thermometric property .

- It doesn't wet the glass
- It is opaque
- It expands uniformly
- It is a good conductor of heat

Reasons why water is not used as thermometric property

- It wets the glass
- It is not opaque
- It is a bad conductor of heat
- It is has non uniform expansitivity.

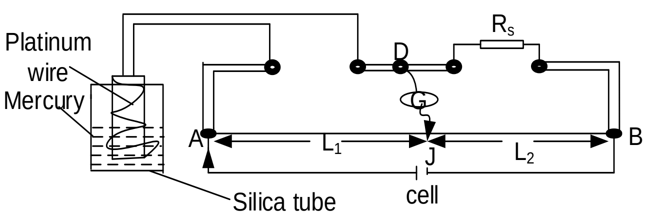
b)-RESISTANCE THERMOMETER [PLATINUM RESISTANCE THERMOMETER]

A resistance thermometer uses resistance(R) of a metal wire as a thermometric property.

QUALITIES OF A METAL TO BE USED IN A RESISTANCE THERMOMETER

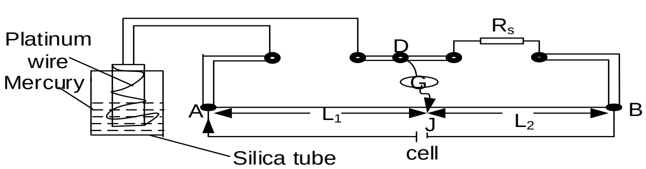
- Material of the wire should have a high temperature co-efficient of resistance (R) so that a small change in temperature causes a measurable change in resistance.
- The variation of resistance with temperature should be linear. Platinum is chosen to be used because it satisfies above 2 conditions.

MEASUREMENT OF CELCIUS SCALE TEMPERATURE OF A BODY USING PLATINUM RESISTANCE THERMOMETER



- A standard resistor R_S is connected to the right hand gap and silica tube leads on the left hand gap of a meter bridge
- With the silica tube immersed in ice, J is adjusted along the slide wire until G reads zero. The length l_1 and l_2 are read and recorded, $R_0 = \frac{l_1}{l_2}R_s$
- The above procedure is repeated with silica tube separately in steam and unknown temperature θ and resistances R_{100} and R_θ respectively calculated
- Unknown temperature, $\theta = (\frac{R_\theta - R_0}{R_{100} - R_0})$ x100°C

MEASUREMENT OF A BSOLUTE TEMPERATURE OF A BODY USING PLATINUM RESISTANCE THERMOMETER



- A standard resistor R_S is connected to the right hand gap and silica tube leads on the left hand gap of a meter bridge
- With the silica tube immersed in amixture of ice, pure vapour and pure water, J is adjusted along the slide wire until G reads zero. The length l_1 and l_2 are read and recorded, $R_{tr} = \frac{l_1}{l_2}R_s$
- The above procedure is repeated with silica tube in unknown temperature T and resistance R_T calculated
- Unknown temperature, $T = \frac{R_T}{R_{tr}}x273.16K$

ADVANTAGES OF PLATINUM RESISTANCE THERMOMETER

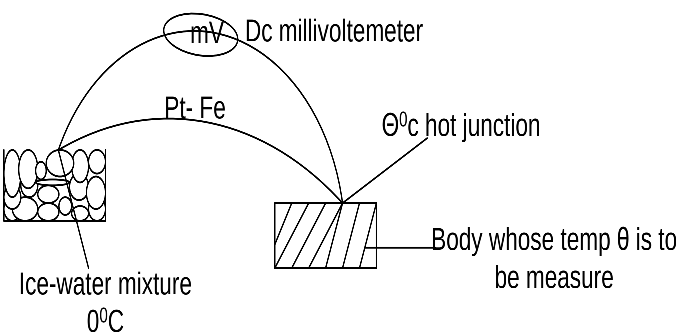
- It is used for measuring small unit temperature.
- It is very accurate. It is because the resistance of platinum wire varies linearly with temperature.
- It has a wide range of temperature i.e. from -200℃ to 1200℃
- It is very sensitive to small unit temperatures.

DISADVANTAGES OF PLATINUM RESISTANCE THERMOMETERS

- It cannot measure very rapidly changing temperature. This is because it has low -thermal conductivity and high heat capacity.
- It cannot measure temperature at a point due to size of silica tube.
- Its heavy and not portal

C) -THERMO COUPLE THERMOMETER

When two wires of different materials are joined together to form two junctions and their junctions maintained at different temperatures, a small E.M.f is created between the junctions. These effects is called thermo electric or **see beck effect** and such an arrangement gives a thermo couple.



- One junction is placed on the water-ice mixture and the other junction is put in steam and the Emf set up is m measured on milivoltmeter E_{100}
- With the other junction still in the water-ice mixture, and the other junction now put in contact with a body of unknown temperature,θ and the Emf set up is m measured on milivoltmeter E_θ
- The temperature of the body can then be calculated from

$$\theta = \left(\frac{E_\theta}{E_{100}}\right) \times 100°C$$

ADVANTAGES OF THERMO COUPLE

- It measures temperature at a point e.g. temperature of crystal since the wires can be made thin.
- It is used to measure rapidly changing temperatures. This is because of its small heat capacity and high thermal conductivity.
- It is portable
- It has a wide range of temperature between -250℃ to 1600℃ and this can be achieved by using different metals.
- It can be used to determine direct readings if connected to galvanometer which has been calibrated to read temperatures directly.

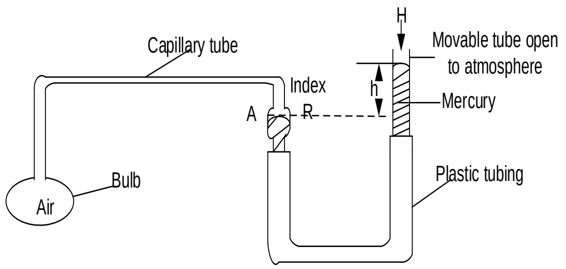
DISADVANTAGES OF THERMO COUPLE

- It cannot measure slowly changing temperatures.
- It is inaccurate because $E.m.f$ doesn't vary linearly with temperature.

N.B an $E.m.f$ can be generated from junction if.

- ✓ The junctions are made from different metals.
- ✓ The junctions are kept at different temperatures.

d)-CONSTANT VOLUME GAS THERMOMETER



- The bulb with air is immersed in a substance whose temperature is required.
- The substance warms up the bulb and the gas (air) expands forcing mercury up in a movable tube.
- By adjusting the plastic tubing up and down, the level in A is restored keeping the volume constant.
- The difference in mercury levels h is determined and the thermometer reading H due to atmosphere in the open limb is recorded
- The total pressure, P_θ exerted by the gas at temperature, θ is obtained from $P_\theta = H + h$.
- The pressure is then measured at the point P_o, at steam point P_{100}, by the same procedure
- Therefore the celecius temperature, θ on this thermometer is obtained from

$$\theta = \left(\frac{P_\theta - P_0}{P_{100} - P_0}\right) x100℃$$

ADVANTAGES OF CONSTANT VOLUME GAS THERMOMETER

- It is very sensitive
- It has wide range of temperature from-270℃ to 1500℃
- It is very accurate since the pressure of fixed mass of gas at constant volume varies linearly with temperature.
- It is used as a standard to calibrate other thermometer e.g. thermo couple thermometer.

DISADVANTAGES OF CONSTANT VOLUME GAS TERMOMETER

- It is bulky i.e. is not portable.
- It has no direct readings; therefore it requires skills to be read it.
- It cannot measure rapidly changing temperatures as the bulb needs time to reach steady states.

Corrections in a constant volume gas thermometer include;

- The temperature of the gas in the dead space because its temperature lies between that of the bulb and the room temperature.
- Thermal expansion of the bulb
- The capillary effect at the mercury surface.

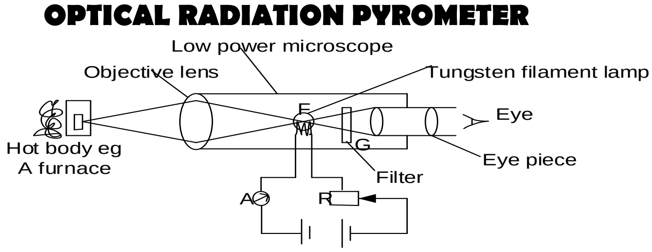
e)-PYROMETERS

They are used to measure very high temperatures e.g. temperature of furnace

They are divided into two;

- Total radiation pyrometer which responds to total radiation i.e. heat and light produced.
- Optical radiation pyrometer which responds to only light produced.

OPTICAL RADIATION PYROMETER



- A hot body whose temperature is to be measured is focused by objective lens so that its image of the object lies in the <u>same plane</u> as the fillament.
- The light from both the filament and the body pass through red filter and viewed by the eye.
- If the image of the hot body is brighter than the filament, the filament appears dark on bright background.
- If the filament is brighter than the image of the hot body, the filament appears bright on a dark background.
- Using the reheostat R, the current through filament is adjusted until the filament cannot be distinguished in the background. At that point, the temperature of hot body is then equals that of the filament. And this temperature can then be read from the ammeter (previously calibrated in ℃.

UNEB 2017 Qn5

(a) (i) State the thermometric property used in the constant-volume gas thermometer (1marks)

(ii) Give **two** characteristics of a good thermometric property (02marks)

(b) (i) Describe the steps taken to set up a celcius scale of temperature for a mercury-in-glass thermometer (04marks)

(ii) State four disadvantages of mercury-in-glass thermometer. (02marks)

(c) Describe with the aid of a labellled diagram the operation of an optical pyrometer. (06marks)

(d) When oxygen is withdrawn from a tank of volume $50l$, the reading of a pressure gauge attached to the tank drops from $21.4x10^5Pa$ to $7.8x10^5Pa$. If the temperature of gas remaining in the tank falls from $30°C$ to $10°C$, calculate the mass of oxygen withdrawn. **An(828.8g)** (05marks)

UNEB 2015 Qn5

(e) (i) State four desirable properties a material; must have to be used as a thermometric substance

(ii) State why scales of temperature based on different thermometric property may not agree

UNEB 2014 Qn7

(e) (i) Two thermometers are used to measure the temperature of a body. Explain the temperature values may be different (02marks)

(ii) A platinum resistance thermometer has a resistance of 5.42 Ω at triple point of water. Calculate its resistance at a temperature of 50.0°C $An[6.41\ \Omega]$ (02marks)

UNEB 2011 Qn 5

(b) (i)Define the term thermometric property and give four examples (02marks)

(ii) State two qualities of a good thermometer property (01marks)

(c) (i)With reference to the a liquid in glass thermometer, describe the steps involved in setting up a Kelvin scale of temperature (03marks)

(ii) State one advantage and disadvantage of the resistance thermometer. (01mk)

(d)A resistance thermometer has a resistance of 21.42Ω at ice point, 29.10Ω at steam point and 28.11Ω at some unknown temperature θ. Calculate θ on the scale of this thermometer. **An[87.11°C]** (03mk)

UNEB 2007 Qn 5

(a) (i) Define a thermometric property and give two examples (02marks)

(ii) When is the temperature **0 K** attained (02marks)

(b) (i) With reference to a constant-volume gas thermometer define temperature on the Celsius scale

(ii) State two advantages and two disadvantages of constant-volume gas thermometer. (02marks)

(c) (i) Define the triple point of water (01mark)

(ii) Describe how you would measure the temperature of a body on thermodynamic scale using a thermo couple. (03marks)

UNEB 2005 Qn 5

(a) (i)What is meant by the term fixed points in thermometry. Give two examples of such points

(ii) How is temperature on a Celsius scale defined on a platinum resistance thermometer?

(b) Explain the extent to which thermometer based on different properties but calibrate using the same fixed points are likely to agree when used to measure a temperature

(i) Near one of the fixed points (02marks)

(ii) Midway between the two fixed points (02marks)

(d) What are the advantages of a thermocouple over a constant volume gas thermometer in measuring temperature.

Solution

b)i) They may agree, because for points near the fixed points the values of the thermometric properties vary almost in step for points close to the fixed points.

ii) They may not agree for temperature midway between fixed points the different thermometric properties vary differently with temperature.

UNEB 2004 Q5

(a) What is meant by

(i) Thermometric property (01mark)

(ii) Triple point of water (01mark)

(b) (i)Describe the steps taken to establish a temperature scale (05marks)
(ii) Explain why the thermometers may give different values for the same unknown the temperature.
(c) (i)Describe with the aid of a diagram, how a constant volume gas thermometer may be used to measure temperature (06marks)
(ii) State three corrections that need to be made when using the thermometer in c(i) above.
(iii)State and explain the sources of in accuracies in using mercury-in-glass thermometer.

In Accuracies rise Because

The none uniformity of the capillary tube from which the thermometer was made. This causes equal changes in volume of the liquid not producing equal changes in the length of the liquid column.

UNEB 2000 Q7

(a) (i) State the desired properties a material must have to be used as a thermometric liquid substance.
(ii) Explain why scales of temperature based on different thermometer properties may not agree
(b) (i) Draw a labelled diagram to show the structure of a simple constant volume gas thermometer.
(ii) Describe how a simple-constant volume gas thermometer can be used to establish a Celsius scale of temperature. (05marks)
(iii) State the advantage and disadvantage of mercury in glass thermometer and a constant volume gas (03marks)
(c) The resistance of the element of a platinum resistance thermometer is 4Ω at the point and 5.46Ω at the steam point. What temperature on the platinum resistance scale would correspond to a resistance of a 9.84Ω **An[400°C]** (03marks)

CHAPTER2: CALORIMETRY

The heat energy of a system is its internal energy and it can be either heat capacity or latent heat.

2.1.0: HEAT CAPACITY AND SPECIFIC HEAT CAPACITY

- **Specific heat capacity** of substance is quantity of heat required to raise the temperature of 1kg mass of substance by 1kelvin.
  Its S.I units are joules per kilogram per Kelvin [$Jkg^{-1}K^{-1}$].
- **Heat capacity** is the amount of heat required to raise the temperature of any mass of the a substance by 1Kelvin.
  Its units are joules per Kelvin [JK^{-1}]
  The heat gained Q or lost by the substance is given by

$$Q = mass\ x\ S.H.C\ x\ temp\ change$$

$$\boxed{\boldsymbol{Q = m\, c\, \Delta\, \theta}}$$

Where $\Delta\theta = \theta_1 - \theta_2$ $\quad c = S.H.C$

Heat capacity = mass x S.H.C, which implies

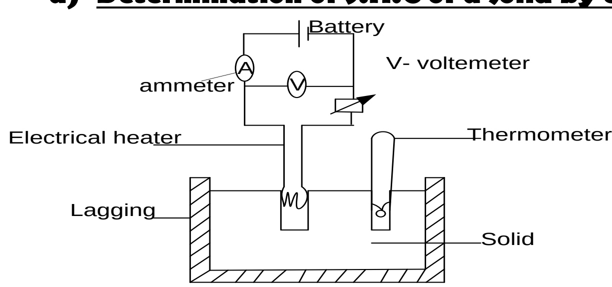
$$\boxed{\mathrm{Q = Heat\ capacity\ x\ temperature\ change}}$$

EXERCISE:33

1) Calculate the quantity of heat required to raise the temperature of a metal block with a heat capacity of 23.1J°C^{-1} by 30.0°C. **An [693J]**
2) An electrical heater supplies 500J of heat energy to a copper cylinder of mass 32.4g. Find the increase in temperature of the cylinder (specific heat capacity of copper = 385Jkg^{-1} °C^{-1}) **An[40.1°C]**
3) How much heat must be removed from an object with a heat capacity of 150J°C^{-1}, in order to reduce its temperature from 80.0°C to 20.0°C. **An [$9x10^3$J]**

2.1.2: METHODS OF DETERMINING S.H.C

a) Determination of S.H.C of a solid by electrical method



- A solid block of a metal is drilled with two holes, one for thermometer and other for an electric heater filled with mercury for good thermal contact
- The mass, m of the block is found and its initial temperature θ_1 recorded.
- A suitable steady current is switched on and stop clock is started simultaneously
- Ammeter and voltage readings I and V from the voltmeter are noted.
- When the temperature has risen appreciably, the current is stopped and the time, t of heating is noted and also the final temperature θ_2 is read and recorded.
- Assuming no heat loss to the surrounding, heat supplied by the heater = heat gained by the block.

$$Ivt = mC[\theta_2 - \theta_1]$$

- Therefore the specific heat capacity, C of the metal is got from

$$C = \frac{\mathrm{Ivt}}{\mathrm{m}[\theta_2 - \theta_1]}$$

Examples

1. A steady current of $12\ A$ and $p.d$ of $240\ V$ is passed through a block of mass $1500g$ for $1\frac{1}{2}\ minutes$. If the temperature of the block rises from $25°C$ to $80°C$. Calculate;

(i) S.H.C of the block
(ii) The heat capacity of 4 kg mass of the block

Solution

i) $t = 1\frac{1}{2}\ minutes = 1\frac{1}{2}x60s = 90\ s,$

$m\ =\ 1500g\ = \frac{1500}{1000} = 1.5\ kg$

$Q = m\ C\ \ \Delta\theta$

$I\ V\ t = m\ C\ \ \Delta\theta$

$12x240x90 = 1.5x\ C\ \ (80 - 25)$

$C = \frac{12x240x90}{1.5x55}$

$C = 3141.82J\ kg^{-1}\ K^{-1}$

$ii)\qquad H = m\ C$

$H = 4\ x\ 3141.82$

$H = 12567.28\ J\ \ K^{-1}$

2. A heater rated 2 k W is used for heating the solid of mass 6 kg, if its temperature rises from 30°C to 40°C. In 12 s, find the S.H.C of the solid.

Solution

$Q = m\ C\ \ \Delta\theta$

$I\ V\ t = m\ C\ \ \Delta\theta$

$Px\ t = m\ C\ \ \Delta\theta$

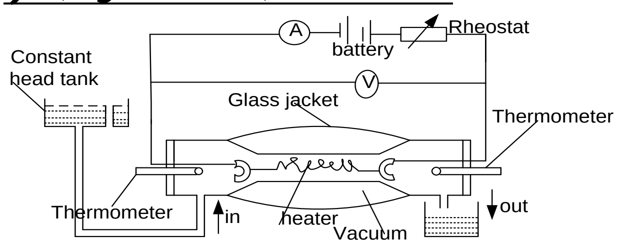
$2x1000x12 = 6x\ C\ \ (40 - 30)$

$C = \frac{2x1000x12}{6x10}$

$C = 400\ J\ kg^{-1}\ K^{-1}$

b)-Determination of S.H.C of a liquid

i)-Using continuous flow method



- A steady flow of the liquid is set and system left to run until thermometers indicate steady temperatures.
- The inflow temperature θ_1 and out flow temperature θ_2 are read and recorded
- The Ammeter reading I_1 and Voltmeter reading V_1 are read and recorded
- The mass m_1 which flows per second is measured and recorded
- At steady state $I_1V_1 = m_1\mathrm{c}(\theta_2 - \theta_1) + h$-- [1] where h is rate of heat loss to surrounding.
- The experiment is repeated for different flow rate. The current and voltage are adjusted until the inflow and outflow temperatures are the same as before
- The Ammeter reading I_2 and Voltmeter reading V_2 are read and recorded
- The new mass m_2 which flows per second is measured and recorded
- At steady state $I_2V_2 = m_2\mathrm{c}(\theta_2 - \theta_1) + h$ - [2]

Therefore specific heat capacity of a liquid, c is got from

$$C= \frac{I_2V_2 - I_1V_1}{(m_2 - m_1)(\theta_2 - \theta_1)}$$

MERITS OF CONTINUOUS FLOW METHOD

- The heat capacity of apparatus is not required since at steady states, the apparatus does not absorb any more heat.
- No cooling correction is required since the heat lost to the surrounding is taken care by repeating the experiment.
- The temperature to be measured θ_1 and θ_2 are constant at steady state.
- They can therefore be measured at leisure and accurately using platinum resistance thermometer.
- There are no heat losses by convection since apparatus has vacuum.

DEMERITS OF CONTINUOUS FLOW METHOD

- It can't be used to determine S.H.C of solid
- It requires a large quantity of liquid and therefore it is expensive

Questions

1) In the flow method to determine the S.H.C of the liquid, the following two sets of results were obtained.

	Experiment 1	Experiment 2
P.d across water (V)	5.0	3.0
Current through heater (A)	0.3	0.2
Temperature of liquid at inlet (^oC)	25	25
Temperature of liquid at outlet (^oC)	41	41
Mass of liquid (kg)	0.15	0.07
Time taken (s)	200	120

a) Calculate the S.H.C of the liquid

b) Heat lost per second

Solution

a) $I_1V_1 = m_1c(\theta_2 - \theta_1) + h$

$I_2V_2 = m_2c(\theta_2 - \theta_1) + h$

C= $\frac{I_2V_2 - I_1V_1}{(m_2-m_1)(\theta_2-\theta_1)}$

C= $\frac{5.0x0.3 - 3.0x0.2}{\left(\frac{0.15}{200}-\frac{0.07}{120}\right)(41-25)} = 3.3x10^2 Jkg^{-1}K^{-1}$

b) $I_1V_1 = m_1c(\theta_2 - \theta_1) + h$

$5.0x0.3 = \frac{0.15}{200}x330x(41-25) + h$

$h = -2.55J$

2) In continuous flow experiment it was found that when applied $p.d$ was 12.0V, current 1.5A, a rate of flow of liquid of 50.0g/minute cause the temperature of inflow liquid to differ by 10°C. When the $p.d$ was increased to 16.0V with current of 1.6A, the rate of flow of 90.0g/minute was required to produce the same temperature difference as before. Find ;

(i) S.H.C of the liquid

(ii) Rate of heat loss to the surrounding

Solution

$I_1V_1 = m_1c(\theta_2 - \theta_1) + h$

$I_2V_2 = m_2c(\theta_2 - \theta_1) + h$

C= $\frac{I_2V_2 - I_1V_1}{(m_2-m_1)(\theta_2-\theta_1)} = \frac{12x1.5 - 16x1.6}{\left(\frac{50x10^{-3}}{60}-\frac{90x10^{-3}}{60}\right)(10)}$

$C = 1.14x10^3 Jkg^{-1}K^{-1}$

ii) $I_2V_2 = m_2c(\theta_2 - \theta_1) + h$

$16x1.6 = \frac{90x10^{-3}}{60}x1.14x10^3x10 + h$

$h = 8.50 watts$

3) Water flow at rate of 0.15kg/minute through a tube and is heated by a heater dissipating 25.2W. The inflow and outflow temperature are 15.2°C and 17.4°C respectively. When the rate of flow is increased to 0.232kg/minute and rate of heating to 37.8W. The inflow and outflow temperature are not altered. Find;

i) S.H.C of water

ii) Rate of loss of heat in the tube

solution

$I_1V_1 = m_1c(\theta_2 - \theta_1) + h$

$I_2V_2 = m_2c(\theta_2 - \theta_1) + h$

C= $\frac{I_2V_2 - I_1V_1}{(m_2-m_1)(\theta_2-\theta_1)} = \frac{25.2 - 37.8}{\left(\frac{0.15}{60}-\frac{0.232}{60}\right)(17.4-15.2)}$ =4200$Jkg^{-1}K^{-1}$

ii) $I_1V_1 = m_1c(\theta_2 - \theta_1) + h$

25.2 = $\frac{0.15}{60}$x4200x$(17.4 - 15.2)$+h

h=2.21watts

4) In an experiment to measure specific heat capacity of water, stream of water flows at rate of 5gs^{-1}over an electrical heater dissipating 135W and temperature rise of 5K is observed. On increasing the rate of flow to 10g$s^{-1}$the same temperature rises is produced with dissipation of 240W.

Solution

$I_1V_1 = m_1c(\theta_2 - \theta_1) + h$

$I_2V_2 = m_2c(\theta_2 - \theta_1) + h$

C= $\frac{I_2V_2 - I_1V_1}{(m_2-m_1)(\theta_2-\theta_1)}$

C= $\frac{240 - 135}{(10x10^{-3} - 5x10^{-3})(5)}$

C= 4200 $Jkg^{-1}K^{-1}$

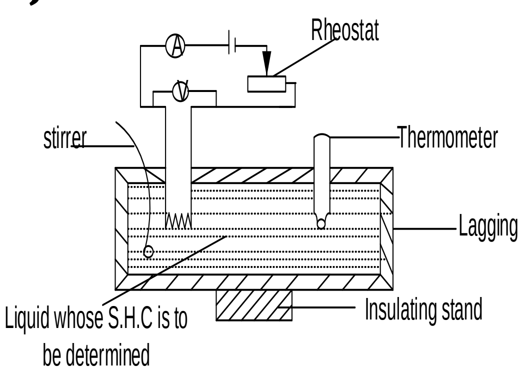
EXERCISE: 34

1) In an electrical constant flow experiment to determine the specific heat capacity of a liquid, heat is supplied to the liquid at a rate of 12W. When the rate of flow is 0.060kgmin^{-1}, the temperature rise along the flow is 2.0K. Use these figures to calculate a value for the specific heat capacity of the liquid. If the true value of the specific heat capacity is 5400Jkg^{-1}K^{-1}, estimate the percentage of heat lost in the apparatus. **An [6000 Jkg^{-1}K^{-1} 11%]**

2) When water was passed through a continuous flow calorimeter the rise in temperature was from 16 to 20°C, the mass of water flowing was 100g in one minute, the $p.d$ across the heating coil was 20V and the current was 1.5A. Another liquid at 16.0°C was then passed through the calorimeter and to get the same change in temperature, the $p.d$ was changed to 13V, the current to 1.2A and the rate of flow to 120g in one minute. Calculate the S.H.C of the liquid if the S.H.C of water is 4200 $Jkg^{-1}°C^{-1}$
An[1700 Jkg^{-1}K^{-1}]

3) With a certain liquid, the inflow and outflow temperatures were maintained at 25.20°C and 26.51°C respectively for a p.d of 12.0V and current 1.50A, the rate of flow was 90g per minute, with 16.0V and 2.00A, the rate of flow was 310g per minute. Find the S.H.C. of the liquid and also the power lost to the surrounding. **An [2910 Jkg^{-1}K^{-1}, 12.3W]**

ii)Electrical method



❖ When $d.c$ is switched on for time t, the temperature of the liquid and calorimeter changes from θ_1 to θ_2.

❖ The resistant is then adjusted to get a suitable value of I and V when the mixture is uniform after stirring. Assuming that there is not heat gained by the thermometer, then there is no heat lost to the surrounding.

❖ The electric energy supplied by heater=heat gain by calorimeter and liquid.

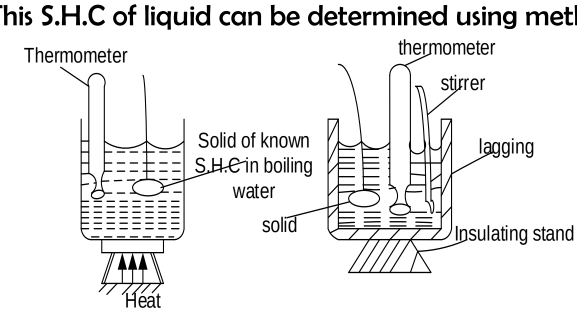
$$Ivt = M_L C_L (\theta_2-\theta_1) + M_C C_C (\theta_2-\theta_1)$$

$$C_L = \frac{Ivt - M_C C_C(\theta_2-\theta_1)}{M_L(\theta_2-\theta_1)}$$

M_C = mass of calorimeter
M_L = mass of liquid
C_c = S.H.C of calorimeter, C_L = S.H.C of liquid

iii)USING METHOD OF MIXTURES

This S.H.C of liquid can be determined using method of mixture as follows



➢ The solid of mass M_s and S.H.C C_s in boiling water at temperature θ_1 is transferred to liquid of mass M_L whose S.H.C [C_L] is to be determined in calorimeter of mass M_c and S.H.C C_c both at temperature θ_2.

➢ The mixture is stirred uniformly until final steady temperature θ_3 is obtained

➢ Assuming there is no heat gained by the stirrer and thermometer and no heat is lost to the surrounding.

➢ Heat lost by solid= heat gained by calorimeter +heat gained by liquid

$$M_S C_S (\theta_1-\theta_3) = M_L C_L (\theta_2-\theta_3) + M_c C_c(\theta_2-\theta_3)$$

$$C_L = \frac{M_S C_S(\theta_1-\theta_3) - M_c C_c(\theta_2-\theta_3)}{M_L(\theta_2-\theta_3)}$$

PRECAUTIONS TAKEN IN DETERMINING S.H.C BY METHOD OF MIXTURES

- The solid should be transferred as soon as possible to liquid in calorimeter.
- The liquid in calorimeter should be well stirred to ensure uniformly of temperature.
- The calorimeter should be supported on an insulated stand and should also be lagged to reduce heat loss by conduction.
- The calorimeter should be well polished to minimize heat loss by radiation.

DISADVANTAGES OF METHODS OF MIXTURE

- Some heat is lost to the surrounding
- Some heat is absorbed by stirrer and thermometer.
- Some heat losses by conduction and convection

N.B: heat losses that cannot be eliminated can be catered for by a cooling correction

Examples

1. What is the final temperature of the mixture if 100g of water at $70°C$ is added to 200g of cold water at $10°C$. And well stirred (Neglect the heat absorbed by the container and S.H.C of water is 42000 $J\ kg^{-1}\ K^{-1}$) .

Solution

Heatlost by hot water = heat gained by cold water

$$M_H C_H(\theta_1 - \theta_3) = M_C C_C(\theta_2 - \theta_3)$$

$$\frac{100}{1000} x\ 4200 x(70 - \theta) = \frac{200}{1000} x4200x(\theta - 10)$$

$$0.1x(70 - \theta) = 0.2x(\theta - 10)$$

$$7 - 0.1\theta = 0.2\theta - 2$$

$$\theta = 30°C$$

2. The temperature of 500g of a certain metal is raised to 100°C and it is then placed in 200g of water at 15°C. If the final steady temperature rises to 21°C, calculate the S.H.C of the metal.

Solution

Heat lost by metal = heat gained by water

$$M_m C_m(\theta_1 - \theta_3) = M_w C_w(\theta_2 - \theta_3)$$

$$\frac{500}{1000} x\ C_m x(100 - 21) = \frac{200}{1000} x4200x(21 - 15)$$

$$0.5xC_m x\ 89 = 0.2x4200x6$$

$$C_m = \frac{0.2x4200x6}{0.5x89} = 128\ J\ kg^{-1}\ K^{-1}$$

3. The temperature of a piece of copper of mass 250g is raised to 100°C and it is then transferred to a well– lagged aluminum can of mass 10.0g containing 120g of methylated spirit at 10.0°C. calculate the final steady temperature after the spirit has been well stirred. Neglect the heat capacity of the stirrer and any losses from evaporation. (S.H.C of copper, aluminum and spirit respectively= $400\ J\ kg^{-1}\ K^{-1}, = 900\ J\ kg^{-1}\ K^{-1}, = 2400\ J\ kg^{-1}\ K^{-1}$)

Solution

Heat lost by copper = heat gained by aluminum + heat gained by spirit

$$M_C C_C(\theta_1 - \theta_3) = M_A C_A(\theta_2 - \theta_3) + M_S C_S(\theta_2 - \theta_3)$$

$$0.25x400(100 - \theta) = 0.1x900(\theta - 10) + 0.12x2400(\theta - 10)$$

$$10000 - 100\theta = 297\theta - 2970$$

$$\theta = \frac{12970}{397} = 32.7°C$$

4. A liquid of mass 200g in a calorimeter of heat capacity 500 $J\ K^{-1}$ is heated such that its temperature changes from 25°C to 50°C. Find the S.H.C of the liquid if the heat supplied was 14,000J.

Solution

Heat supplied = heat gained by liquid + heat gained by calorimeter

$$Q = M_L C_L(\theta_2 - \theta_3) + M_C C_C(\theta_2 - \theta_3)$$

$$14000 = 0.2xC_L(50 - 25) + 500x(50 - 25)$$

$$14000 = 5xC_L + 12500$$

$$C_L = 300\ J\ kg^{-1}\ K^{-1}$$

5. A metal of mass 0.2kg at 100°C is dropped into 0.08kg of water at 13°C contained in calorimeter of mass 0.12kg and S.H.C 400 $J\ kg^{-1}\ K^{-1}$. The final temperature reached is 35°C. Determine the S.H.C of the solid.

Solution

Ms=0.2kg	θ_2=15°C	Cw =4200Jkg^{-1}K^{-1}
θ_1=100°C	Mc=0.12	θ_3 =35°C
Mw=0.08kg	Cc=400Jkg^{-1}K^{-1}	

Heat lost by the solid=heat gained by calorimeter + heat gained by water

$MsCs$ (θ_1-θ_2) = $McCc$(θ_3-θ_2)+Mw Cw (θ_3-θ_2)

0.2x Cs (100-35)= 0.12x 400 (35-15)+0.08x4200(35-15)

13Cs $= 960 + 6120$

Cs=590.769 $J\ kg^{-1}\ K^{-1}$

6. Hot water of mass 0.4kg at 100°C is poured into calorimeter of mass 0.3kg and S.H.C 400Jkg^{-1}K^{-1} and contains 0.2kg of a liquid at 10°C. The final temperature of mixture is 40°C determines the S.H.C of a liquid.

Solution

Mw=0.4kg	Cc=400 $J\ kg^{-1}\ K^{-1}$	θ_3= 40°C
θ_1=100°C	M_L=0.2kg θ_2=10°C	
Mc=0.3kg		

Heat lost by the hot water =heat gained by the colorimeter +heat gain by liquid

$MwCs$ (θ_3-θ_1)= $McCc$(θ_3-θ_2)+ M_LC_L (θ_3-θ_2)

0.4x 4200$(100-40)$=0.3x400$(40-10)$ +0.2$xC_L(40-10)$

$100800 = 3600 + 6C_L$

C_L=16200 $J\ kg^{-1}\ K^{-1}$

7. A 15W heating coil is immersed in 0.2kg of water and switched on for 560 seconds during which time; the temperature rises by 10°C. When water was replaced by some volume of another liquid of mass 0.15kg, the power required for same time is 8.3W. Calculate the S.H.C of the liquid.

Solution

$Ivt = M_LC_L\Delta\theta$

8.3x 560=0.15xC_Lx10

C_L =[$\frac{8.3x560}{0.15x10}$]

C_L =3.1x10^3 $J\ kg^{-1}\ K^{-1}$

Assumption, same temperature rise occurs.

8. When a block of metal of mass 0.11kg and S.H.C 400 $J\ kg^{-1}\ K^{-1}$is heated to 100°C and quickly transferred to a calorimeter containing 0.2kg of a liquid at 10°C, the resulting temperature is 13°C. On repeating the experiment with 0.4kg of the liquid in the same container at same temperature of 10°C, the resulting temperature is 14.5°C. Calculate;
   a) S.H.C of the liquid
   b) Thermal capacity of the container.

Solution

$M_S = 0.11kg$, Cs=400 $J\ kg^{-1}\ K^{-1}$	θ_2=10°C
$\theta_1 = 100$°C θ_2=10°C θ_3=18°C	θ_3=14.5°C
M_L=0.2kg M_L=0.4kg	

Heat lost by solid = heat gained by liquid + heat gained by container

$MsCs$ (θ_1-θ_3) =M_LC_L (θ_3-θ_2) + $McCc$ (θ_3-θ_2)

0.11x400(100-18) =0.2xC_L (18-10) + H(18-10)

3608 = 1.6C_L+$8H$ ....(1)

$MsCs$ (θ_1-θ_3) = M_LC_L (θ_3-θ_2) + $McCc$ (θ_3-θ2)

$0.11x400(100-14.5) = 0.4\ x$ C_L $(145-10) + H\ (14.5-10)$

3762 =1.8 C_L + $4.5H$ ....(2)

Solving equation1 and equation2 simultaneously

C_L = 1925 $J\ kg^{-1}\ K^{-1}$

H= 66 $J\ K^{-1}$ [thermal capacity of the container]

EXERCISE :35

1) 400g of a liquid at a temperature 70°C is mixed with another liquid of mass 200g at a temperature of 25°C. Find the final temperature of the mixture, if the S.H.C of the liquid is 4200 $J\ kg^{-1}\ K^{-1}$.
An[=55°C]

2) 60 kg of hot water at 82°C was added to 300 kg of cold water at 10°C. Calculate the final temperature of the mixture (S.H.C of water =4200 $J\ kg^{-1}\ K^{-1}$) **An[=22°C].**

3) Calculate the final steady temperature obtained when 0.8 kg of glycerine at 25°C is put into a copper calorimeter of mass 0.5 kg at 0°C (S.H.C of copper =400 $J\ kg^{-1}\ K^{-1}$, $S.H.C\ of\ glycerine = 250\ J\ kg^{-1}\ K^{-1}$). **An[12.5°C]**

4) A copper block of mass 250g is heated to a temperature of 145°C and then dropped into a copper calorimeter of mass 250g which contains 2500m^3 of water at 20°C. Calculate the final temperature of water. (S.H.C of copper = 400 $J\ kg^{-1}\ °C^{-1}$, S.H.C of water = 4200 $J\ kg^{-1}\ °C^{-1}$). **An[30°C]**

5) The temperature of heat which raises the temperature of 0.1 kg of water from 25°C to 60°C is used to heat a metal rod of mass 1.7 kg and S.H.C of the rod was 20°C. Calculate the final temperature of the rod. **An [48.8°C]**

6) A piece of copper of mass 100g is heated to 100°C and is then transferred to a well lagged copper can of mass 50g containing 200g of water at 10°C. Neglecting heat loss, calculate the final steady temperature of water after it has been well stirred. Take S.H.C of copper and water to be 400 $J\ kg^{-1}\ K^{-1}$ and 4200 $J\ kg^{-1}\ K^{-1}$ respectively. **An[14**°C]

7) A block of metal of mass 0.5kg initially at a temperature of 100°C is gently lowered into an insulated copper container of mass 0.05kg containing 0.9kg of water at 20°C. Neglecting heat loss, calculate the specific het capacity of the metal block. (Take S.H.C of water to be 400 $J\ kg^{-1}\ K^{-1}$and 4200 $J\ kg^{-1}\ K^{-1}$respectively. **An[506.6** $J\ kg^{-1}\ K^{-1}$**]**

8) A heating coil is placed in thermal flask containing 0.6kg of water for 600s. The temperature of water rises by 25°C during this time. Water is replaced by 0.4kg of another liquid. And the same temperature rise occurs in 180s. Calculate the S.H.C of the liquid given that S.H.C of water is 4200 $J\ kg^{-1}\ K^{-1}$. State any assumption. **An [1890** $J\ kg^{-1}\ K^{-1}$**]**

9) Copper calorimeter of mass 120g contains 100g of paraffin at 15°C. If 45g of aluminum at 100°C is transferred to the liquid and the final temperature is 27°C. Calculate the S.H.C of paraffin [S.H.C of aluminum and copper are 1000 and 400 $J\ kg^{-1}\ K^{-1}$ respectively**]. Ans. 2.4 x10^{3}Jkg^{-1}K^{-1}**

10) A steady current of 12 A and $p.d$ of 240V is passed, through a block of mass 1500g for 1 ½ minutes. If the temperature of the block rises from 25°C to 80°C, calculate
    (i) The specific heat capacity of the block
    (ii) The heat capacity of 4kg mass of the block**. An [3141.82** $J\ kg^{-1}\ K^{-1}$**, 12567.28** $J\ K^{-1}$**]**

11) A liquid of mass 250g is heated to 80°C and then quickly transferred to a calorimeter of heat capacity 380JK^{-1} containing 400g of water at 30°C. If the maximum temperature recorded is 55°C and specific heat capacity of water is 4200 $J\ kg^{-1}\ K^{-1}$. Calculate the S.H.C of the liquid. **An [8240** $J\ kg^{-1}\ K^{-1}$**]**

12) 500g of water is put in a calorimeter of heat capacity 0.38Jk^{-1} and heated to 60°C. It takes 2minute for the water to cool from 60°C to 55°C. When the water is replaced with 600g of a certain liquid, it takes 1 ½ minute for the liquid to cool from 60°C to 55°C. Calculate the S.H.C of the liquid. **An [2624.8** $J\ kg^{-1}\ K^{-1}$**]**

13) When a metal cylinder of mass 2.0x10^{-2}kg and specific heat capacity 500 $J\ kg^{-1}\ K^{-1}$ is heated by an electrical heater working at a constant power, the initial rate of rise of temperature is 3.0Kmin^{-1}. .After a time the heater is switched off and the initial rate of fall of temperature is 0.3Kmin^{-1}. What is the rate at which the cylinder gains heat energy immediately before the heater is switched off? **An[0.45W]**

14) A copper block has a conical hole bored in it into which a conical copper plug just fits. The mass of the block is 376g and that of the plug is 18g. The block and plug are initially at room temperature 10°C and almost completely surrounded by a layer of insulating material. The plug is removed from the block, cooled to a temperature of -196°C and then quickly inserted into the block again. The temperature of the block falls to 3°C and then slowly rises. Calculate the value of the mean specific heat capacity of copper (in the range -196°C to 3°C) obtained by ignoring heat flow into the block from the surrounding. (S.H.C of copper to the temperature range 3°C to 10°C is 380 $J\ kg^{-1}\ K^{-1}$). **An [279** $J\ kg^{-1}\ K^{-1}$**]**

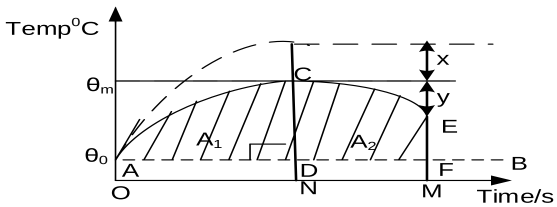
2.1.3: COOLING CORRECTION

Is the number of degree Celsius that should be added to the observed maximum temperature to cater for heat losses during rise or fall.

OR

Is the extra temperature that is added to the observed maximum temperature to compensate for the heat lose to the surrounding.

2.1.4: DETERMINATION OF COOLING CORRECTION OF A POOR CONDUCTOR E.G. RUBBER



- Pour a liquid in a calorimeter and place it on a table. Place a thermometer into the liquid and after some time record the temperature of the surrounding θ_0
- Gently place the heated solid into the liquid and stirr
- Temperature of mixture is recorded at different time interval until the temperature of the mixture has fallen by about 1°C below the observed maximum temperature θ_m.
- A graph of temperature against time is plotted.
- Draw a line AB through θ_0 parallel to the time axis
- Draw a line CD through θ_m parallel to the temperature axis
- Draw a line EF beyond CD parallel to the temperature axis and note y
- Areas A_1 and A_2 are estimated by counting squares of the graph paper.
- The cooling correction x, then determined from. $\frac{A_1}{A_2} = \frac{x}{y}$ $\therefore$ $x = \frac{A_1}{A_2}y$ and added to θ_m

2.1.5: NEWTON'S LAWS OF COOLING

It states that under conditions of forced convection, the rate of heat loss is directly proportioned to excess temperature over the surrounding

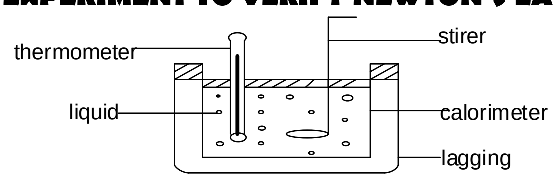
$\frac{dQ}{dt} \propto (\theta - \theta_R)$,

$\frac{dQ}{dt} = -k(\theta - \theta_R)$,

But $\frac{dQ}{dt} = mc\frac{d\theta}{dt}$

$\therefore$ $\frac{d\theta}{dt} = -k(\theta - \theta_R)$

EXPERIMENT TO VERIFY NEWTON'S LAW OF COOLING



- Hot water in a calorimeter is placed near an open window.
- Temperature θ of the water is recorded at equal time interval for about 20 minutes.
- A graph of temperature θ against time t is plotted.
- Different slopes at different temperatures $\theta_1, \theta_2, \theta_3$ are determined.
- For each temperature the excess temperature, $\theta - \theta_R$ is calculated, where θ_R is room temperature
- A graph of slope against excess temperature is plotted
- A straight line graph through the origin verifies Newton's law of cooling.

2.1.6: HEAT LOSS AND TEMPERATURE CHANGE

The rate of heat loss also depends on;

- Excess temperature $(\theta - \theta_R)$,
- Surface area of the body
- The nature of the surface of the body i.e. Dull surface lose heat faster than shining

A body having a uniform surface area and uniform temperatures, heat loss per second is given by $\frac{d\theta}{dt}$.

Since $Q = m\,l\Delta\theta$

$\frac{dQ}{dt} = -mc\frac{d\theta}{dt}$

$\frac{d\theta}{dt} = -\frac{dQ}{dt}/mc$

But $m = \rho v$

$\frac{d\theta}{dt} = \frac{dQ}{dt}/\rho vc$

$\frac{d\theta}{dt} = \frac{1}{\rho vc}\frac{dQ}{dt}$

Question: Explain why a small body cools faster than larger bodies of the same material.

Rate of heat loss $\propto \frac{surface\ area}{volume}$. This implies that heat loss$\propto \frac{1}{length}$. Since $\frac{d\theta}{dt} = -{}^{1}/_{mc}\frac{dQ}{dt}$ and mass$\propto\ volume$, a small body cools faster than a large body

92.2.0: LATENT HEAT

This is the amount of heat required for the substance to change state at constant temperature.

Why temperature remains constant during change of state (phase)

- During melting (change of state from solid to liquid), the heat energy supplied is used to weaken the intermolecular forces and increase separation between molecules. This increases the potential energy of the molecules but the mean kinetic energy of the molecules remain constant . Further increase in separation between molecules causes the regular patterns to collapse as the solid changes to a liquid, until the process is complete the temperature remains constant.
- During boiling (change from liquid to vapour state) the heat supplied is used to break the intermolecular forces and increases separation between molecules. This increases the potential energy of the molecules but the mean kinetic energy of the molecules remain constant . Also some of the energy is used in doing work during expansion against atmospheric pressure, hence no temperature change occurs.

Significance of latent heat on regulation of body temperautre

On a hot day the body sweats. Evaporation occurs at the surface of the body. The temperature of the sweat falls to maintain evaporation. Latent heat is constantly drawn from the body and the body cools.

LATENT HEAT OF FUSION

This is heat required to change any mass of substance from solid to a liquid at constant temperature.

SPECIFIC LATENT HEAT OF FUSION

Is the quantity of heat required to change **1kg** mass of a solid to a liquid at **constant temperature**. It is measured in Jkg^{-1}

LATENT HEAT OF VAPOURIZATION

Is the quantity of heat required to change any mass of substance from liquid to gas at a constant temperature.

SPECIFIC LATENT HEAT OF VAPOURIZATION

Is the quantity of heat required to change **1kg** mass of liquid to gas at **a constant temperature.** It is measured in Jkg^{-1}

2.2.1: WHY LATENT HEAT OF VAPOURIZATION IS HIGHER THAN LATENT HEAT OF FUSION

- In fusion, heat is required to weaken the intermolecular bonds accompanied with a small increase in volume hence negligible work done against atmospheric pressure.
- While in vaporization, heat is required to break intermolecular attractions and form a gas followed by a large increase in volume and more work is done against atmospheric pressure in expanding the gas.

Example

1. Ice has a mass of 3 kg. Calculate the heat required to melt it at 0°C. (S.L.H of fusion = $3.36x10^5 Jkg^{-1}$).

Solution

$Q = m\ l = 3\ x\ \ 3.36x10^5 = 1.008x10^6\ J$

2. Find the heat required to change 2 kg of ice at 0°C into water at 50°C.(S.L.H of fusion of ice = 3.36$x10^5$Jkg^{-1}, $S.H.C\ of\ water = 4200\ J\ kg^{-1}\ K^{-1}$).

Solution

Ice at 0°C → Water 0°C + Water 50°C

$Q = m\,l + m\,C\,\Delta\theta$

$Q = 3\ x\ \ 3.36x10^5 + 2x4200x(50-0)$

$Q = 1.008x10^6\ + 4.2x10^6 = 1.092x10^6\ J$

3. An ice making machine removes heat from water at a rate of 20 Js^{-1}. How long will it take to convert 0.5 kg of water at 20 °C to ice at 0°C°C. (S.L.H of fusion of ice = 3.36$x10^5$Jkg^{-1}, S.H.C of water=4200$J\ kg^{-1}\ K^{-1}$).

Solution

water 20°C → Water 0°C + Ice 0°C

$Q = m\,C\,\Delta\theta + \ m\,l$

$P\,x\,t\ = m\,C\,\Delta\theta + \ m\,l$

$20\ x\ t = 0.5x4200x(20-0) + 0.5x\ \ 3.36x10^5$

$20\ t = 42000\ + 168000$

$t = \frac{210000}{20}\ = 1.05x10^4$ s

4. A calorimeter with heat capacity of $80J°C^{-1}$contains 50g of water at 40°C what mass of ice at 0°C needs to be added in order to reduce the temperature to 10°C. Assume no heat is lost to the surround (S.H.C of water = 4200J$kg^{-1}°C^{-1}$, S.L.H of the of ice = 3.4x10^{5}Jkg^{-1}).

Solution

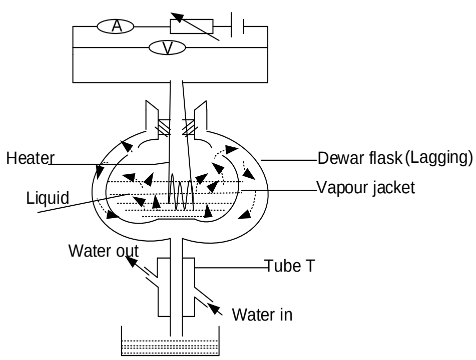
$$\begin{pmatrix} Heat\ lost\ by \\ calorimeter \\ from \\ 40°C\ to\ 10°C \end{pmatrix} + \begin{pmatrix} Heat\ lost\ by \\ water \\ from \\ 40°C\ to\ 10°C \end{pmatrix} = \begin{pmatrix} Heat\ gained \\ by\ ice \\ at\ 0°C \end{pmatrix} + \begin{pmatrix} Heat\ gained \\ by\ melting \\ ice \\ from\ 0°C\ to\ 10°C \end{pmatrix}$$

Mc Cc (40-10) + Mw Cw (40-10) = M_1 L+ C+M_1 $C_{1}x$ (10-0)

$80\ x\ 30\ + \frac{50}{1000}\,x4200x30\ =$ M_1 (3.4x10^5+$4200x10$)

M_1 = 0.023kg   Mass of ice required = 23g

2.2.2: DETERMINATION OF THE S.L.H OF VAPOURIZATION (L_v) OF LIQUID BY a)ELECTRIC METHOD [DEWAR FLASK METHOD]



- Switch k is closed and liquid is heated until it starts boiling
- A stop clock is started and mass m_1 of liquid collected in a time in a time t. noted
- The Ammeter reading, $I_1$and Voltmeter reading V_1 are recorded.
- At steady state, $I_1\ V_1\ t\ = m_1$x $l_V + h$ ...(1) where $h = ht$ heat lost to surrounding
- The Rheostat is adjusted and a new Ammeter reading I_2 and Voltmeter reading V_2 are recorded
- New mass m_2 of the liquid collected in the same time t is obtained

$I_2\ V_2\ t\ = m_2$x $l_V + h$ ....(2)

The specific latent heat of vapourization is obtained from

$$\boxed{L_V = \frac{(I_2V_{2-}\,I_1V_1)t}{(M_{2-}\,M_1)}}$$

EXAMPLES

1) When electrical energy is supplied at a rate of 12W to a boiling liquid $5.0x10^{-3}$ Kg of the liquid evaporates in 30 min .On reducing the electrical power to 7W, $1.0x10^{-3}$ Kg of the liquid evaporates in the same time. Calculate;

a) S.L.H of vapouration

b) Power loss to the surrounding

Solution

$I_1\ V_1\ t\ = m_1 \text{x}\ l_V + h,\ I_2\ V_2\ t\ = m_2 \text{x}\ l_V + h$

$$L_V = \frac{(I_2V_2 - I_1V_1)t}{(M_2 - M_1)} = \frac{(7-12)x30x60}{(1x10^{-3} - 5x10^{-3})}$$

$Lv = 2.25x10^6 Jkg^{-1}$

b) $I_1\ V_1\ = \frac{m_1}{t} \text{x}\ l_V + h$

$12\ = \frac{5x10^{-3}}{30x60} x\ 2.25x10^6 + h$

h= $5.75W$

2) An experiment to determine S.L.H of vapourization of alcohol using dewar flask gave the following results.

Experiment 1	Experiment 2
V_1=7.4V	V_2=10.0V
I_1=2.6A	I_2=6.6A
m_1=5.8x10^{-3}kg	m_2=11.3x10^{-3}kg
t_1=300s	t_2=300s

a) Find S.L.H of vapourization of alcohol

b) Heat lost to surrounding per unit time.

Solution

a) $I_1\ V_1\ t\ = m_1 \text{x}\ l_V + h,$

$I_2\ V_2\ t\ = m_2 \text{x}\ l_V + h$

$$L_V = \frac{(I_2V_2 - I_1V_1)t}{(M_2 - M_1)} = \frac{(10x6.6 - 7.4x2.6)x300}{(11.3x10^{-3} - 5.8x10^{-3})}$$

$Lv = 2.55\text{x}10^6\ Jkg^{-1}$

b)- $I_1\ V_1\ = \frac{m_1}{t} \text{x}\ l_V + h$

$7.4x2.6\ = \frac{5.8x10^{-3}}{300} \text{x}\ 2.55\text{x}10^6 + h$

h= $30W$

3) When electrical power is supplied at rate of 12W, mass of liquid of $8.6x10^{-3}$ kg evapourates in 30minutes. On reducing the power to 7W, $5x10^{-3}$kg of the liquid evapouration in same time. Calculate;

(i) S.L.H of evapouration of liquid. **An $2.25x10^6$ Jkg^{-1}**

(ii) Power lost to the surrounding. **An $1Js^{-1}$**

4) In an experiment to determine S.L.H.V of a liquid using Dewar flask in the following results were obtained.

Voltage V(V)	Current I(A)	Mass collected in 300s/g
7.4	2.6	5.8
10.0	3.6	11.3

Calculate the power of the heater to evaporate 3.0g of water in 2 minutes.

Solution

$I_1\ V_1\ t\ = m_1 \text{x}\ l_V + h,$

$I_2\ V_2\ t\ = m_2 \text{x}\ l_V + h$

$$Lv = \frac{I_2V_2 - I_1V_1}{M_2 - M_1} = \frac{10x3.6 - 7.4x2.6}{(11.3-5.8)x\frac{1}{300}x10^{-3}}$$

$Lv =\ 9.14x\ 10^5\ Jkg^{-1}$

Put into equation (2)

$I_2\ V_2\ t\ = m_2 \text{x}\ l_V + h$

$10x\ 3.6\ = \frac{11.3}{300} x10^{-3} x9.14x10^5\ +\ h$

$h\ =\ 1.57w$

$I_3\ V_3 = M_3\ Lv + h$

$P_3 = (\frac{3x10^{-3}}{2x60} x\ 9.14x\ 10^5) + 1.57$

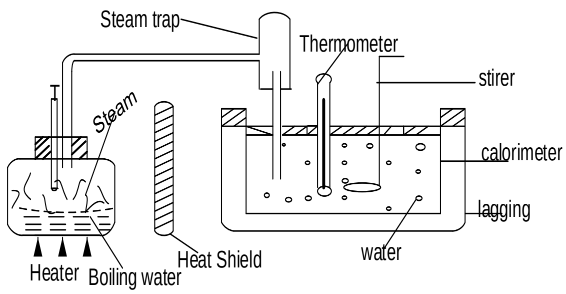
$P_3 =\ 24.42W$

5) In an experiment to determine S.L.H.V of a liquid using Dewar flask in the following results were obtained.

Voltage V(V)	Current I(A)	Mass collected in 400s/g
10.0	2.00	14.6
11.2	250	30.6

Calculate the heat lost to surrounding 400s.**An(5080J)**

b)DETERMINATION OF S.L.H.V BY METHOD OF MIXTURE



- The mass m_1 of water and the calorimeter is measured and noted
- The initial temperature, θ_1 of water in the calorimeter is noted
- Steam from boiling water is then passed into the water in the calorimeter through a steam trap.
- After a measurable temperature rise, the final temperature, θ_2 of the water in calorimeter is measured and noted.
- The new mass, m_2 of the water in the calorimeter is again measured and the mass, m_s of condensed steam is calculated from $m_s = m_2 - m_1$
- Temperature θ_3 of steam is measured by thermometer T and recorded
- The mass m_c of the empty calorimeter is obtained by weighing

$$\begin{pmatrix}\text{Heat given by}\\ \text{steam condensing}\end{pmatrix} + \begin{pmatrix}\text{heat given by}\\ \text{condensed water}\\ \text{from } \theta_3 \text{ to } \theta°\text{C}\end{pmatrix} = \begin{pmatrix}\text{heat taken}\\ \text{by calorimeter}\end{pmatrix} + \begin{pmatrix}heat\ taken\\ by\ water\end{pmatrix}$$

$m_s l_v + m_s C_w (\theta_3 - \theta_2) = (m_c C_c + m_w C_w)(\theta_3 - \theta_2)$

C_w = S.H.C of water

m_w = mass of water $where\ m_w = m_1 - m_c$

C_c = S.H.C of calorimeter

- l_v can be obtained

EXAMPLE

1) An electric kettle with a 2.0kW heating element has a heat capacity of 400JK. 1.0kg of water at 20°C is placed in the kettle. The kettle is switched on and it is found that 13 minutes later the mass of water in it is 0.5kg. Ignoring heat losses calculate a value for the specific latent heat of vaporization of water. (specific heat capacity of water is 4200 $Jkg^{-1}K$)

Solution

$Pt = M_f C_f (\theta_2 - \theta_1) + M_w C_w (\theta_2 - \theta_1) + MsL$

$2x1000x13x60 = 400\,(100-20) + 1x4200[100-20] + (1-0.5)\,L$

$l = 2.38x10^6 Jkg^{-1}$

2) An electrical heater rated 500W is immersed in liquid of mass 2.0kg contained in large thermal flask of heat capacity 840Jkg^{-1} at 28°C. Electrical power is supplied to heater for 10minutes. If S.H.C of liquid is $2.5x10^3\ Jkg^{-1}K^{-1}$. Its **S.L.H.V** is $8.54x10^3\ Jkg^{-1}k$ and its boiling point is 78°C. Estimate the amount of liquid which boils off.

Solution

Heat supplied by heater = heat gained by flask + heat gained by liquid + heat used for evaporating the liquid.

$Ivt = M_f C_f (\theta_2 - \theta_1) + M_L C_L (\theta_2 - \theta_1) + MsLv$

$500x10x60 = 840\,(78-28) + 2x2.5x10^3\,[78-28] + M_s\,(8.54x10^3)$

Ms = 0.936kg

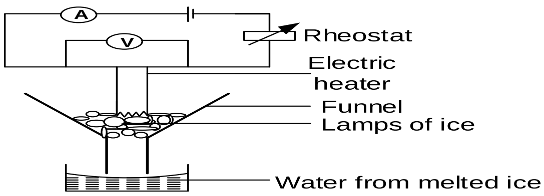
Excersise:36

1) Ice at 0°C is added to 200g of water initially at 70°C in a vacuum flask. When 50g of ice is added and has all melted, the temperature of the flask and content is 40°C. When further 80g of ice has been added and has been melted, the temperature of the whole becomes 10°C. Calculate the S.L.H of fusion of the neglecting any heat loss of surrounding. **Ans $3.78x10^5\ Jkg^{-1}$**
2) A calorimeter of mass 20g and specific heat capacity 800 $J\,kg^{-1}K^{-1}$ contains 500 g of water at 30 °C. Dry steam at 100°C is passed through the water in the calorimeter until the temperature of water rises to 70°C. If the specific latent heat of vaporization of water is 2260000 $J\,kg^{-1}$, calculate the mass of steam condensed
3) A calorimeter of mass 35.0 g and specific heat capacity 840 $J\,kg^{-1}K^{-1}$ contains 143.0 g of water at 7 °C. Dry steam at 100°C is bubbled through the water in the calorimeter until the temperature of water rises to 29°C. If the mass of steam condensed is 5.6 g, find the specific latent heat of vaporization of water
4) A copper container of heat capacity 60 $J\,kg^{-1}$ contains 0.5 kg of water at 20 °C. Dry steam is passed into the water in the calorimeter until the temperature of water rises to 50°C. Calculate the mass of steam condensed

Explain why specific latent heat of vaporization of water is higher at 20°C than at 100°C

- ❖ At 20°C the molecules of the liquid are closer together than at 100°C. The intermolecular forces of attraction are stronger at 20°C than at 100°C.
- ❖ More energy is required to break the bonds at 20°C than the heat needed at 100°C

c) DETERMINATION OF S.L.H.F OF ICE BY ELECTRICAL METHOD



- ❖ The rheostat is adjusted until suitable values of I and V are obtained
- ❖ The heat supplied by the heater is used to melt the ice and water, and water from melted ice is collected and weighed per unit time.

$$\begin{pmatrix}\text{Heat supplied by}\\ \text{heater per second}\end{pmatrix} + \begin{pmatrix}\text{heat absorbed from}\\ \text{surrouding per second}\end{pmatrix} =$$

latent heat absorbed by ice

$IV + h = ML_F$ .... (1)

- ❖ The experiment is repeated with values of I_1V_1 and M_1 is also determined by

$I_1V_1 + h = M_1L_F$.............. (2)

- ❖ l_f can be obtained from

$$\boxed{L_f = \frac{I\,V - I_1V_1}{M - M_1}}$$

Exercise: 37

1) Calculate the heat required to melt 200g of ice at 0°C . (S.L.H of ice= $3.4 \times 10^5 Jkg^{-1}$) **An 6.8×10^4 J**
2) Calculate the heat required to turn 500g of Ice at 0°C into water at 100°C. (S.L.H of ice= $3.4 \times 10^5 Jkg^{-1}$ S.H.C of water = $4200 Jkg^{-1}$) **An [$3.8 \times 10^5 J$]**
3) Calculate the heat given out when 600g of steam at 100°C condenses to water at 20°C [S.L.H of steam = 2.26×10^6 Jkg^{-1}, S.H.C of water = 4200 Jkg^{-1}]. **An [1.56×10^6J]**
4) 1kg of vegetables, having a specific heat capacity 2200 Jkg^{-1} at a temperature 373K are plugged into a mixture of ice and water at 273K. How much is melted.
   [S.L.H of fusion of the = 3.3×10^5 Jkg^{-1}] **An [0.67kg]**
5) 3kg of molten lead (melting point 600K) is allowed to cool down until it has solidified. It is found that the temperature of the lead falls from 605K to 600K in 10s, remains constant at 600K for 300s, and then fall to 595K in a further 8. 4s. Assuming that the rate of loss of energy remains constant and that the specific heat capacity of solid lead is $140 Jkg^{-1}K^{-1}$. Calculate.
   (a) Rate of loss of energy from the lead.
   (b) The specific latent heat of fusion of lead.
   (c) The specific heat capacity of liquid lead
   An [250W, $2.5 \times 10^4 Jkg^{-1}$, $167 Jkg^{-1}K^{-1}$]
6) 0.02kg of ice and 0.10kg water at 0°C are in a container. Steam at 100°C is passed in until all the ice is just melted. How much water is now in the container?
   S.L.H of steam = $2.3 \times 10^6 Jkg^{-1}$, S.L.H of ice = $3.4 \times 10^5 Jkg^{-1}$,
   S.H.C of water = $4.2 \times 10^3 Jkg^{-1}K^{-1}$ **An [0.1225kg]**
7) When apiece of ice of mass 6×10^{-4}kg at a temperature of 272K is dropped into liquid nitrogen boiling at 77K in a vacuum flask $8 \times 10^{-4} m^3$ of nitrogen, measured at 294K and 0.75m mercury pressure are produced. Calculate the mean specific heat capacity of ice between 272K and 77K. Assume that the S.L.H of vaporization of nitrogen is $2.13 \times 10^5 Jkg^{-1}$ and that the density of nitrogen at S.T.P is $1.25 kgm^{-3}$ **.An $1.67 \times 10^3 Jkg^{-1}K^{-1}$**
8) Wet clothing at a temperature of 0°C is hung out to dry when the air temperature is 0°C and there is a dry wind blowing. After some time, it is found that some of the water has evaporated and the remainder has frozen. What is the source of the energy required to evaporate the water. Estimate the proportion of the water originally in the clothing which remains as ice, state any assumptions you make.
   (S.L.H of fusion of ice at 273K = 333k Jkg^{-1}
   S.L.H of vaporization of water at 273K = 2500k Jkg^{-1}) **An [88%]**

9) In a factory heating system water enters the radiators at 60°C and leaves at 38°C. The system is replaced is by one in which steam at 100°C is condensed in the radiators, the condensed steam leaving at 82°C. What mass of steam will supply the same heat as 1.00kg of hot water in the first instance.(The S.L.H of vapourisation of water is 2.26 x10^6Jkg^{-1} at 100°C. The S.H.Cof water is 4.2x10^3Jkg^{-1}°C^{-1}) **An [0.0396kg]**

10) A beaker containing ether at a temperature of 13°C is placed in a large vessel in which the pressure can be reduced so that the ether boils, this results in cooling of the remaining ether. What proportion of the ether has evaporated when the temperature of the remainder has been reduced to 0°C (assume no interchange of heat between the ether and its surrounding)

(Mean S.H.C of ether over the temperature range 0-13°C = 2.4x10^3Jkg^{-1}°C^{-1})

(Mean S.L.H of vapourisation of ether in the temperature range 0-13°C = 3.9x10^5Jkg^{-1}°C^{-1})

An[7.4%]

11) In an express coffee machines, steam at 100°C is passed into milk to heat it. Calculate

(i) The energy required to heat 150g of milk from room temperature (20°C) to 80°C.

(ii) The mass of steam condensed. **An [3.6x10^6J, 15.8g]**

UNEB 2016 Q5

(a) (i) Define **specific latent heat of fusion** (01mark)

(ii) State the effect of impurities on melting point. (01mark)

(b) Explain why there is no change in temperature when a substance is melting (04marks)

(c) With the aid of a diagram, describe the continuous flow method of measuring the specific heat capacity of a liquid (06marks)

(d) In an experiment to determine the specific heat of fusion of ice, a heating coil is placed in a filter funnel and surrounded by lumps of ice. The following two sets of readings were obtained.

V(V)	4.0	6.0
I(A)	2.0	3.0
Mass of water m(g) collected in 500 s	14.9	29.8

Calculate the;

(i) Specific latent heat of fusion of ice. **An [3.36x10^5Jkg^{-1}]** (04marks)

(ii) Energy gained in the course of obtaining the first set of readings **An [500J]** (03marks)

(e) Why are two sets of readings necessary in (d) above. (01mark)

UNEB 2015 Q5

(c) Describe with the aid a diagram an experiment to determine specific latent heat of vaporization of steam using the method of mixtures (07marks)

(d) A 600W electric heater is used to raise the temperature of a certain mass of water in a thermos flask from room temperature to 80°C. The same temperature rise is obtained when steams from a boiler is passed into an equal mass of water at room temperature in the same time. If 16g of water were being evaporated every minute in the boiler, find the specific latent heat of vaporisation of steam, assumption no heat loses. **An(2.26x10^6 Jkg^{-1})** (04marks)

UNEB 2014 Q7

(a) Define specific latent heat of vaporisation (01mark)

(b) With the aid of a labelled diagram, describe an experiment to measure the specific latent of vaporisation of a liquid using an electrical method (07mark)

(c) Explain the effect of pressure on boiling point of a liquid (02mark)

(d) A liquid of specific heat capacity $2.8x10^3Jkg^{-1}K^{-1}$ and specific latent hate of vaporisation $9.00x10^5Jkg^{-1}$ is contained in a flask of heat capacity 800JK^{-1} at a temperature of 32°C. An electric heater rated 1 kW is immersed in 2.5kg of the liquid and switched on for 12 minutes, calculate the amount of liquid that boils off, given that boiling point of the liquid is 80°C

$An(3.84x10^{-1}kg)$ (06mark)

UNEB 2013 Q5

(a) Define

(i) Specific heat capacity (01mark)

(ii) Specific latent heat of vaporization of a liquid (01mark)

(b) With the aid of a labelled diagram, describe an electrical method of determining the specific heat capacity of a solid (07marks)

(c) An electrical heater rated 48W, 12V, is placed in a well insulated metal of mass 1.0kg at a temperature of 18℃. When the power is switched on for 5minutes, the temperature of the metal rises to 34℃. Find the specific heat of the metal. **An (900 $Jkg^{-1}K^{-1}$,)** (04marks)

(d) (i) State **Newton's law of cooling** (01marks)

(ii) Use Newton's law of cooling to show that

$$\frac{d\theta}{dt} = -k(\theta - \theta_R)$$

Where $\frac{d\theta}{dt}$ is the rate of fall of temperature and θ_R is the temperature of the surrounding

(e) Explain why evaporation causes cooling (03marks)

UNEB 2012 Q5

(a) (i) Define the terms specific heat capacity and specific latent heat of fusion (2mk)

(ii)Explain the changes that take place in the molecular structure of substances during fusion and vaporization. (04marks)

c) With the aid of a labelled diagram describe an experiment to determine the S.H.C of a liquid using the continuous flow method (08marks)

d) Steam at 100℃ is passed into a copper calorimeter of mass 150g containing 340g of water at 15℃. This is done until the temperature of the calorimeter and its content is 71℃. If the mass of the calorimeter and its contents is found to be 525g. Calculate the specific latent heat of vaporization of water.

Solution

Mass of calorimeter Mc = 150g

Mass of water Mw = 340g

Mass of steam Ms = 525 – (150+340) = 35g

$$\begin{pmatrix}\text{Heat supplied}\\ \text{by steam}\\ \text{at }100℃\end{pmatrix} + \begin{pmatrix}Condensing\ steam\\ from\\ 100℃\ to\ 71℃\end{pmatrix} = \begin{pmatrix}\text{heat gained by}\\ \text{calorimeter}\\ \text{from}\\ 15℃\text{ to}71℃\end{pmatrix} + \begin{pmatrix}heat\ gained\ by\\ water\ from\\ 15℃\ to\ 71℃\end{pmatrix}$$

Ms Lv + MsCs (100-71) = McCc (71-15) + Mw Cw (71-15)

$\frac{35}{1000}$Lv + $\frac{35}{1000}$x 4200x29 = $\frac{150}{1000}$ x400x56 + $\frac{340}{1000}$ x 4200 x56

Lv= 2.26X10^6 Jkg^{-1}

UNEB 2011 QN. 6

a) Define S.H.C of a substance and states its units (02marks)

b) (i) Describe how S.H.C of a liquid can be obtained by the continuous flow method (07marks)

(ii) State one disadvantage of this method (01mark)

c) An electric kettle rated 1000W, 240V is used on 220Vmains to boil 0.52kg of water. If the heat capacity of the kettle is 400Jkg^{-1} and the initial temperature of the water is 20℃how long will the water take to boil. **An[246s]** (04marks)

UNEB 2009 QN 5

(b) (i) Define specific heat capacity of a substance (01mark)

(ii)Hot water at 85℃ and cold water at 10℃ are run into a bath at a rate of 3.0x$10^{-2}m^3min^{-1}$ and V respectively. At the point of filling the bath the temperature of the mixture of water was 40°C. Calculate the time taken to fill the bath if its capacity is 1.5m^3 (05marks)

(c) The specific latent heat of fusion of a substance is significantly different from its specific latent heat of vaporization at the same pressure. Explain how the difference arises (04marks)

(d) Explain in terms of S.H.C why water is used in a car radiator than any other liquid. (02marks)

Solution

Let M_h = be mass of hot water

Mc = be mass of cold water

Heat supplied by hot water = heat gained by cold water

M_h C (85-40) = Mc (40-10)

M_h = $\frac{30}{45}$ Mc .... (1)

Let t be the time taken to fill

But M_h = ρ x volume

M_h = ρx (3x10^{-2}) t ....,(2)

Also Mc = $\rho\ xV\ t$ ....(3)

Put equation (2) and equation (3) to equation (1)

ρx (3x10^{-2}) t = $\frac{30}{45}\rho$vt
V= 4.5x10^{-2} m^3 min^{-1}
If the total volume = 1.5m^3

If the volume of cold and hot water at filling temperature are V_1 and V_2 respectively.
V_1+V_2 = 1.5m^3
3x10^{-2} t + 4.5x10^{-2} t = 1.5
t=20minutes

c) Water has a very high S.H.C hence a small amount of water can absorb a lot of heat energy. Other liquids have low S.H.C so a large amount of these liquids are needed to carry away the heat consequently this wound require a larger radiator which is un economical.

UNEB 2008 Q 5

(a) Define the following terms
   (i) S.H.C of vaporization of a liquid (01mark)
   (ii) Coefficient of thermal conductivity (01mark)
(b) With the aid of a well labelled diagram describe an experiment to measure the S.L.H of vaporization of water by an electrical method (07marks)
(c) An appliance rated 240V, 200W evaporates 20g of water in the 5minutes. Find the heat loss if S.L.H of vaporization is 2.26x10^{6}Jkg^{-1} (03marks) **An[14800J]**
(d) Explain why at a given external pressure a liquid boils at a constant temperature (4marks)
(e) With the aid of a suitable sketch graph explain the temperature distribution a long a lagged and un lagged metal rods, heated at one end (04marks)

UNEB 2007 Q 6

(a) (i) Define latent heat (01mark)
   (ii)Explain the significance of latent heat in regulation of body temperature (3marks)
(b) (i)Using kinetic theory, explain boiling of a liquid. (03marks)
   (ii)Describe how you would determine the S.L.H of vaporization of water using the method of mixtures.
   (iii)Explain why latent heat of vaporization is always greater than that of fusion (02marks)

Solution

UNEB 2006 Q 6

(a) (i) Define S.H.C of a substance (01mark)
   (ii) State three advantages of the continuous flow method over the method of mixtures in the determination of S.H.C of a liquid (03marks)
(b) In a continuous flow experiment, a steady difference of temperature of 1.5^{0}C is maintained when the rate of liquid flow is 45gs^{-1} and the rate of electrical heating is 60.5W. On reducing the liquid flow rate to 15gs^{-1}, 36.5W is required to maintain. Calculate the;
   (i) S.H.C of the liquid (04marks)
   (ii) Rate of heat loss to the surrounding (3marks)**An [533.3Jkg^{-1}k^{-1}, 24.5W]**
(c) (i) Describe an electrical method for the determination of the S.H.C of a metal (06marks)
   (ii) State the assumptions made in the above experiment (02marks)
   (iii) Comment about the accuracy of the result of the experiment in C (i) above (01mark)

Solution

C (i) assumption
- ❖ There is no heat loss to the surrounding
- ❖ The quantity of heat gained by the thermometer and the heater is negligible
- ❖ The volume of the metal is constant hence no work is done against the atmospheric pressure.

ii) Due to heat loss to the surrounding, it implies that more heat was supplied than as required to causes the observed temperature change. Hence the value of C is greater than the actual value.

UNEB 2005 QN 5

(c) The continuous flow method is used in the determination of the S.H.C of the liquid.
   (i) What are the principle advantages of this method compared to the method of mixture
   (ii) In such a method, 50g of water is collected in 1minute, the voltmeter and ammeter readings are 12.0V and 2.50A respectively. While the inflow and outflow temperatures are 20°Cand 28°C respectively. When the flow rate is reduced to 25gmin^{-1}, the voltmeter and ammeter read 8.8V and 1.85A respectively while the temperatures remain constant. Calculate the S.H.C of water (5marks)
   An[4.116x10^{3}Jkg^{-1}k^{-1}]

UNEB 2002 QN 7

(a) (i) Define S.H.C of a substance (01mark)

(ii) State how heat losses are minimized in Calorimetry

(b) (i) What is meant by a cooling correction (02marks)

(ii) Explain how the cooling correction may be estimated in the determination of the heat capacity of a poor conductor of heat by the method of mixtures. (05mks)

(iii)Explain why a small body cools faster than a larger one of the same material. (04marks)

(c) Describe how you would determine the S.H.C of a liquid by the continuous flow method. (07marks)

UNEB 2001 QN 7

(a) Explain why temperature remains constant during change of phase (04marks)

(b) Describe with the aid of a labelled diagram, an electrical method for determination of S.L.H of vaporization of a liquid. (07marks)

(c) Water vapour and liquid water are confirmed in a air tight vessel. The temperature of the water is raised until all the water has evaporated, draw a sketch graph to show how the pressure of the water vapour changes with temperature and account for its main features (06 marks)

UNEB 1999 Q7

(a) Define S.H.C (01 mark)

(b) Describe an electrical method of measuring S.H.C of a metal. (06 marks)

(c) In a continuous flow calorimeter for measurement of S.H.C of a liquid, $3.6x10^{-3}m^3$ of a liquid flows through the apparatus in 10 minutes. When electrical energy is supplied to the heating coil at the rate of 44W, a steady difference of 4k is obtained between the temperature of the out-flowing and inflowing liquid. When the flow rate is increased to $4.8x10^{-3}m^3$ of liquid in 10 minutes, the electrical power required to maintain the temperature difference is 58W. Find the

(i) S.H.C of the liquid (06 marks)

(ii) Rate of loss of heat to the surrounding (02 marks)

[Density of liquid = $800kgm^{-3}$]

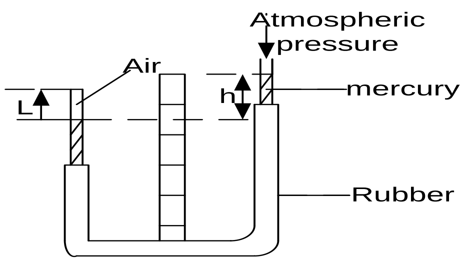
CHAPTER3: GAS LAWS

Boyle's law:

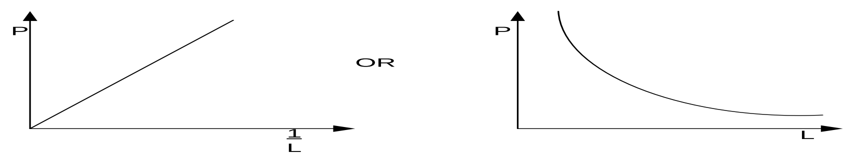
it states that the pressure of fixed mass of a gas is inversely proportional to its volume at constant temperature i.e.

P $\propto \frac{1}{v}$ | PV = constant | $\boxed{P_1V_1 = P_2V_2}$

EXPERIMENT TO VARIFY BOYLES LAW



- A fixed mass of the gas is trapped inside J tube of uniform cross section using mercury.
- Measure and record the atmospheric pressure H using a barometer
- Adjust the flexible tube by lowering or raising the open end.
- Measure and record the difference in mercury levels h
- Record the length l of the air column trapped in the closed tube
- Obtain the air pressure, $P = H \pm h$.
- Repeat the procedure and obtain a series of values P and l , $l \propto volume$
- Plot a graph of P against $\frac{1}{l}$ and a straight line graph passing through origin verifies Boyles law



This verifies Boyle's law

1. A given mass of a gas has a volume of 100 cm^3 at 75 $N\,m^{-2}$. At what pressure is it when the volume decreases to 60cm^3

Solution

$P_1V_1 = P_2V_2$
$75\,x\,100 = P_2\,x\,60$

$P_2 = \frac{75\,x\,100}{60}$

$P_2 = 125N\,m^{-2}$

2. The cylinder of an exhaust pump has a volume of $25cm^3$. If it is connected through a valve to a flask of volume $225cm^3$ contianing air at a pressure of 75cmHg, calculate the pressure of the air I the flask after two strokes of the pump, assuming that the temperature of the air remains constant (04marks)

An(60.8cmHg)

Solution

1st stroke: $P_1V_1 = P_2V_2$
$75\,x\,225 = P_2\,x\,(225 + 25)$
$P_2 = 67.5cmHg$
2nd stroke: $P_2V_2 = P_3V_3$
$67.5\,x\,225 = P_3\,x\,(225 + 25)$

1st stroke: $P_1V_1 = P_2V_2$
$75\,x\,225 = P_2\,x\,(225 + 25)$
$P_3 = 60.8cmHg$

Alternatively

$$\boldsymbol{P_1} = \left(\frac{V_1}{V_1 + V_2}\right)^{\boldsymbol{n}} \boldsymbol{P}$$

$$\boldsymbol{P_1} = \left(\frac{225}{225 + 25}\right)^{\mathbf{2}} \mathbf{75}$$

$$\boldsymbol{P_1} = 60.8cmHg$$

CHARLES LAW:

It states that the volume of fixed mass of gas is directly proportional to its absolute temperature at constant pressure i.e

V∝T
$\frac{V}{T}$ = constant

$$\boxed{\frac{V_2}{T_2} = \frac{V_1}{T_1}}$$

Absolute zero temperature (0K) is the temperature attained when molecules slow down and acquire their minimum possible energy.

Molecular explanation for existence of absolute zero temperature

When a gas is cooled, its molecules loose kinetic energy continuously since it depends directly on temperature. As molecules loose kinetic energy they move closer into close proximity until when they cease to have kinetic energy. At this point the gas is said to occupy a negligible volume and its temperature at this point is called the absolute zero temperature and the pressure the gas exerts on the walls of the container occupied is negligible.

Example

1. When the temperature of a gas is at 0°C, its volume is 75 cm^3. Find its volume when the gas is heated up to 273°C.

Solution

$V_1 = 75\ cm^3$, $V_2 = ?$,

$$\frac{V_2}{T_2} = \frac{V_1}{T_1}$$

$$\frac{V_2}{273 + 273} = \frac{75}{0 + 273}$$

$V_2 = 150cm^3$

2. The volume of a fixed mass of a gas at 27°C is 150 cm^3. What is its temperature at 200cm^3

Solution

$V_1 = 150\ cm^3, V_2 = \ 200cm^3,$

$$\frac{V_2}{T_2} = \frac{V_1}{T_1}$$

$$\frac{200}{T_2} = \frac{150}{27 + 273}$$

$$T_2 = \frac{300\ x\ 200}{150}$$

$T_2 = 400\ K$

Temperature = 400 -273

=127°C

PRESSURE LAW/ GAY LUSSAC LAW

It states that the pressure of fixed mass of gas is directly proportional to its absolute temperature at constant volume i.e.

P∝T

$\frac{P}{T}$ = constant

$$\boxed{\frac{P_1}{T_1} = \frac{P_2}{T_2}}$$

EXAMPLE

1. The pressure of a gas is 75$N\ m^{-2}$ at −73°C. What is its pressure when a gas is heated up to 127°C

Solution

$P_1 = 75\ N\ m^{-2}, P_2 = ?,$

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

$$\frac{75}{-73 + 273} = \frac{P_2}{127 + 273}$$

$P_2 = 150\ N\ m^{-2}$

2. A sealed flask contains a gas at a temperature of 27°C and a pressure of 90 kPa. If the temperature rises to 127°C. What will be the new pressure?

Solution

$P_1 = 90\ kPa, P_2 = ?, \frac{P_1}{T_1} = \frac{P_2}{T_2}$

$$\frac{90}{27 + 273} = \frac{P_2}{127 + 273}$$

$P_2 = 120\ kPa$

3.1: EQUATION OF STATE

This an equation relating pressure, volume and temperature.

$$\boxed{\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}}$$

Examples

1. When the pressure of 1m^3 of a gas at −73 °C is increased to 3 times its original value, the temperature becomes 27 °C. Find the new volume of the gas

Solution

$P_1 = P\ N\ m^{-2}, V_1 = 1m^3,$

$P_2 = 3\ P\ ,\ V_2 = ?,$

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$\frac{P\ x\ 1}{-73 + 273} = \frac{3\ PxV_2}{27 + 273}$$

$V_2 = 0.5m^3$

2. A litre of gas at 0°C and $10^5\ Nm^{-2}$ pressure is suddenly compressed to $\frac{1}{4}$ of its volume and its temperature rises to 273°C. Calculate the resulting pressure of the gas.

Solution

$P_1 = 10^5\ Nm^{-2}, V_1 = 1\ l,$
$P_2 = ?\ ,\ V_2 = \frac{l}{4},$

$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

$\frac{10^5\ x\ 1}{273} = \frac{P_2 x\ ^1/_4}{546}$

$P_2 = 800000\ Nm^{-2}$

Note:

At standard temperature and pressure (s.t.p) a gas has an absolute temperature and normal pressure ie. T = 273 K, P = 76 cmHg

Example

1. 240 cm^3 of oxygen gas was collected when a temperature is 20°C at a pressure of 50 cmHg. Calculate its volume at s.t.p.

Solution

$P_1 = 50\ cmHg, V_1 = 240\ cm^3,$
$P_2 = 76\ cmHg\ ,\ V_2 = ?,$

$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

$\frac{50\ x\ 240}{20 + 273} = \frac{V_2\ x\ 76}{273}$

$V_2 = 147.12\ cm^3$

2. The volume of hydrogen at 273°C is 10 cm^3 at a pressure of 152 cmHg. What is its volume at s.t.p

Solution

$P_1 = 152cmHg, V_1 = 10\ cm^3,$
$P_2 = 76\ cmHg\ ,\ V_2 = ?,$

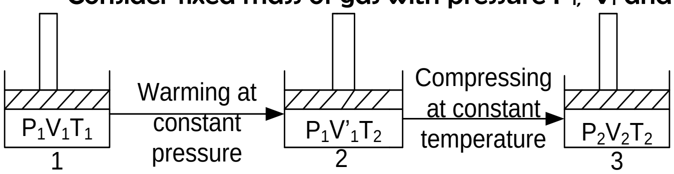
$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

$\frac{152\ x\ 10}{273 + 273} = \frac{V_2\ x\ 76}{273}$

$V_2 = 10\ cm^3$

Derivation of equation of state

Consider fixed mass of gas with pressure P_1, V_1 and temperature T_1 enclosed in piston cylinder system.



Moving from 1 to 2, Charles law applies

$\frac{V_1}{T_1} = \frac{V_1'}{T_2}$

$V_1' = V_1 \frac{T_2}{T_1}$ .... (1)

Moving from 2 to 3, Boyle's law apples

$P_1 V_1' = P_2 V_2$ .... (2)

Putting V_1 into equation (2)

$P_1 V_1 \frac{T_2}{T_1} = P_2 V_2$

$\boxed{\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}}$

$\frac{PV}{T}$ = Constant

PV= constant XT

$PV = nRT$

$\boxed{PV = n\,R\,T}$

This is an equation of state or ideal gas equation.

Where n= number of moles of gas

n= $\frac{mass\ given\ (m)}{relative\ molecular\ mass(M)}$

R= molar gas constant [8.31Jmol⁻¹k⁻¹]

To determine R, we consider the standard condition at $s.t.p$

Volume at $s.t.p$ = $22.4x10^{-3}$ m^3

Pressure at $s.t.p$ = $1.01325x10^5Nm^{-2}$

Temperature at $s.t.p$ =273K

Number of mole n = 1

$PV = nRT$

R= $\frac{PV}{nT}$

R= $\frac{1.01325x10^5\ x\ 22.4x10^{-3}}{1X273}$

R= $8.31Jmol^{-1}K^{-1}$

EXAMPLES

1) A gas is confined in the container of volume $0.1m^3$ at pressure of $1.0x10^5Nm^{-2}$ And temperature of 300K. If the gas is found to be ideal gas, calculate the density of the gas [Rmm=32]

Solution

PV= nRT ∴n= $\frac{pv}{RT}$

n=$\frac{1.0x10^5x0.1}{8.31x300}$=

4.01moles

But n=$\frac{m}{M}$

4.01= $\frac{mass}{32x10^{-3}}$

Mass = 0.128kg | But $\rho = \frac{M}{V} = \frac{0.128}{0.1}$ | ρ= 1.28kg/m^{-3}

2) Calculate the molecular mass of hydrogen of the density of hydrogen at s.t.p is $0.09kgm^{-3}$

Solution

$Pv = \frac{m}{M}RT \therefore M = \frac{mRT}{Pv}$ but $m = \rho v$

$M = \frac{\rho v RT}{Pv}$

$M = \frac{0.09x8.314x273}{1.013x10^5} = 2.02x10^{-3}kg$

Calculation involving loss of gas

1) Oxygen gas is contained in cylinder of volume 1x10^{-2} m^3 at temperature of 300K and pressure 2.5x0^{5}Nm^{-2}. After some oxygen is used at constant temperature, pressure falls to 1.3x10^5 Nm^{-2}. Calculate the mass of oxygen used.

Solution

$V_1 = 1x10^{-2}$ m^3

$T_1 = 300K$, $P_1 = 2.5x10^5Nm^{-2}$

$P_1V_1 = \frac{m}{M}RT_1 \quad \therefore m_1 = \frac{P_1V_1M}{RT_1}$

$m_1 = \frac{2.5x10^5x1x10^{-2}x32x10^{-3}}{8.31x300}$ = 0.032kg

$m_2 = \frac{1.3x10^5x1x10^{-2}x32x10^{-3}}{8.31x300}$ = 0.0166kg

Therefore mass of oxygen = $[m_1 - m_2]$ kg

= [0.032-0.0166] kg

= 0.0154 kg

2) A cylinder of gas has mass of gas 10kg and pressure of 8 atmospheres at 27℃ when some gas is used in cold room at -3℃. The remaining gas in the cylinder at its temperature has a pressure of 6.4 atmospheres. Calculate the mass of the gas used.

Solution

m_1 = 10kg $\quad m_2$ = ?

P_1 = 8atm $\quad P_2$ = 6.4atm

T_1 = 27+273 = 300K $\quad T_2$= (-3+ 273) = 270K

$Pv = \frac{m}{M}RT \quad \therefore M = \frac{mRT}{PV}$

$M = \frac{10x\,8.31x300}{8xv}$ ……… (1)

$M = \frac{m_2x8.31x270}{6.4xv}$ ……… (2)

Equating equation (1) to (2)

$\frac{10x8.31x300}{8V} = \frac{m_2x8.31x270}{6.4V}$

m_2 = 8.89kg

Therefore mass of gas = $m_1 - m_2$

= [10-8.89] kg

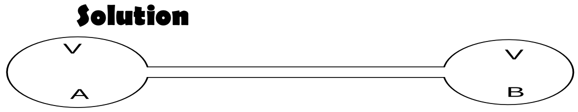
= 1.11kg

Connected containers

In closed containers the total number of molecules remains constant

1) Two glass bulbs of equal volume are joined by another tube and are filled with a gas at $s.t\,p$. When one of the bulbs is kept in melting ice and another place in as hot bath the new pressure is 877.6mmHg. Calculate the temperature of bath

Solution



P_A = 760mmHg $\quad P_B$ = 760mmHg

T_A = 273k $\quad T_B$ = 273K

Since cylinders are enclosed, the number of moles is both cylinders before cooling will be the same after cooling (heating).

$n_A + n_B = n_A^1 + n_B^1$

$\frac{P_AV_A}{RT_A} + \frac{P_BV_B}{RT_B} = \frac{P_A'V_A'}{RT_A'} + \frac{P_B'V_B'}{RT_B'}$

$P'_A = P'_B$ = 877.6mmHg

T'_A = (0+273) = 273K $\quad T'_B$ =?

$\frac{760xV}{8.31x273} + \frac{760xV}{8.31x273} = \frac{877.6xV}{8.31x273} + \frac{877.6xV}{8.31xT_B'}$

$\frac{642.4}{2268.63} = \frac{877.6}{8.31T_B'}$

T'_B =372.95K

3) Two containers A and B of volume 3x10^{3}cm^3 and 6x10^{3}cm^3 respectively contain helium gas at a pressure of 1.0x10^3 pa and temperature 300K. Container A is heated to 373K while container B is cooled 273K. Find the final pressure of the helium gas.

Solution

V_A = 3x10^{3}cm^3 $\quad P_A$ = 1.0x10^{3}pa $\quad T_A$ = 300K $\quad V_B$ =6x10^{3}cm^3

P_B = 1.0x10^{3}Pa $\quad T_B$ = 300K

T'_A = 373K $\quad T'_B$ = 273K

$n_A + n_B = n_A^1 + n_B^1$

$\frac{P_AV_A}{RT_A} + \frac{P_BV_B}{RT_B} = \frac{P_A'V_A'}{RT_A'} + \frac{P_B'V_B'}{RT_B'}$

$$\frac{1.0x10^3x3x10^3}{8.31x300}+\frac{1.0x10^3x6x10^3}{8.31x300}=\frac{P_A'x3x10^3}{8.31x373}+\frac{P_B'x6x10^3}{8.31x273}$$

$P_A' = P_B'$=P

$916461 = 2493\,(819000 + 2238000P)$

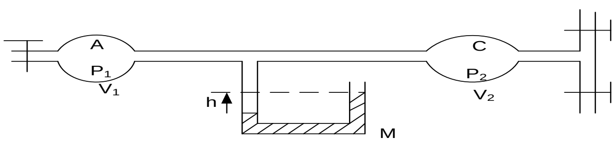
$P = 999.3Pa$

3.2: DALTONS LAW OF PARTIAL PRESSURE

It states that the total pressure of a mixture of gases that do not react chemically is the sum of partial pressure of the constituents

DEFINITION. Partial pressure of gas is the pressure the gas would exert if it was to occupy the whole container alone.

3.2.1: EXPERIMENT TO DEMONSTRATE DALTON'S LAW



- The apparatus above can be used to study the pressure of mixture of gases
- A is a bulb of volume V_1 containing air at atmospheric pressure P_1
- C is another bulb of volume V_2 containing carbon dioxide at atmospheric pressure P_2
- When the bulbs are connected by opening the taps, the gases mix and reach the same pressure P

$$P = \frac{P_1V_1}{V_1+V_2}+\frac{P_2V_2}{V_1+V_2}$$

EXAMPLE

1) Two containers A and B of volume 3x10^{3}cm^3 and 6x10^{3}cm^3 respectively contain helium gas at pressure 1x10^{3}Pa and temperature 300K. Container A is heated to 373Kwhile container B is cooled to 273K. Find the final pressure of the helium gas.

Solution

P = $\frac{P_1V_1}{V_1+V_2}+\frac{P_2V_2}{V_1+V_2}$

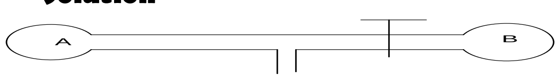
$P_2 = P_1$

P= $\frac{1x10^3x3x10^{-3}}{3x10^{-3}+6x10^{-3}}$ + $\frac{1x10^3x6x10^{-3}}{3x10^{-3}+6x10^{-3}}$

P= 1000Nm^{-2}

2) Two bulbs A of volume 100cm^3 and B of volume 50cm^3 connected to freeway tap which enables them to be filled with gas or evacuated. Initially bulb A is filled with an ideal gas at 10^{0}Cto pressure of 3.0x10^{5}Pa. Bulb B is filled with an I deal gas at 100℃ to a pressure of 1.0x10^{5}Pa . Two bulbs and connected with A maintained at 10^{0}C and B at 100^{0}C .Calculate the pressure at equilibrium

Solution



P = $\frac{P_AV_A}{V_A+V_B}+\frac{P_BV_B}{V_A+V_B}$

P= $\frac{3x10^5x100}{100+50}$ + $\frac{1x10^5x50}{100+50}$

P= 2.33x10^5 Pa

Alternatively

$n_A+n_B = n_A^1+n_B^1$

$\frac{P_AV_A}{RT_A}+\frac{P_BV_B}{RT_B}=\frac{P_A'V_A'}{RT_A'}+\frac{P_B'V_B'}{RT_B'}$

$(\frac{3x10^5x100}{831x283})+(\frac{1x10^5x50}{831x373}) = \frac{px100}{8.31x283}+\frac{px50}{8.31x373}$

P = 2.33x10^5 Pa

3) Two cylinder A and B of volume V and 3V respectively are separately filled with gas. The cylinders are connected with tap closed with pressure of gas A and B being P and 4P respectively. When tap is open, the common pressure becomes 60Pa. Find P

Solution

$P = \frac{P_AV_A}{V_A+V_B}+\frac{P_BV_B}{V_A+V_B}$

$P = \frac{PxV}{V+3V}+\frac{4Px3V}{V+3V}$

$60 = \frac{PV}{V+3V}+\frac{4Px\,3V}{V+3V}$

$P = 18.46Pa$

EXERCISE: 5

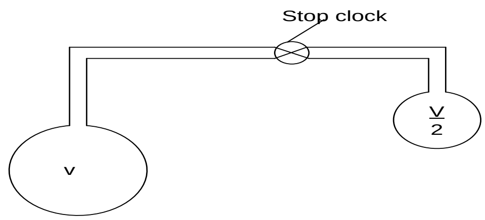
1) Nitrogen gas under an initial pressure of 5.0x10⁶Pa at 15°C is contained in cylinder of volume 0.040cm³. After a period of three years the pressure has fallen to 2.0x10⁶Pa at the since temperature because of leakage.

[Assume molar mass of nitrogen= 0.028kgmol⁻¹][R= 8.3Jmol⁻¹K⁻¹, NA= 6.0x10²³mol⁻¹] Calculate;

(i) The mass of gas originally present in the cylinder.
(ii) The mass of gas which escaped from the cylinder in there years
(iii) The average number of nitrogen molecules which escaped from the cylinder per second.[Take one year the be equal to 3.2x10⁷s]

2) Carbon dioxide is contained in a cylinder whose volume is $2x10^{-3}m^3$at 330K and$3.0x10^5Nm^{-2}$. The pressure falls to $2.5x10^5Nm^{-2}$after some of the gas is used at constant temperature. Calculate the mass of the gas used given that molecular mass of carbon dioxide is 44g. **An(0.00161kg)**

3) A volume of gas V at a temperature T_1 and a pressure P is enclosed in a sphere. It is conn3cted to another sphere of volume $\frac{V}{2}$ by a tube and stop clock is closed



If the stop clock is opened the temperature of the gas in the second sphere becomes, T_2. The first sphere is maintained at a temperature, T_1. Show that the final pressure P^1 within the apparatus is $P^1 = \frac{2PT_2}{2T_2+T_1}$

4) Two identical bulbs are joined with a thin glass tube and filled with air which is initially at 20°C. What will the pressure in the apparatus become if one bulb is immersed in steam and the other in melting ice?

3.3.0: IDEAL GAS

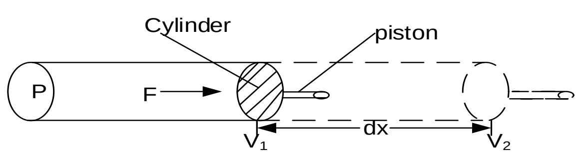
It is a gas which obeys the Boyles law under all conditions

3.3.1: PROPERTIES OF IDEAL GAS

- The internal energy of an ideal gas is entirely kinetic energy and depends only on its temperature and on number of atom in its molecule.
- Inter molecular forces of attraction are negligible
- The volume of molecules is negligible compared to the volume of the container
- The collision between any particles is assumed to be elastic;
- Duration of collision is negligible compared with time between collision

3.4.1: Work done by the gas in expansion at constant pressure

For an ideal gas enclosed in a cylinder by a frictionless piston of area of cross-section A. gas expands by pushing piston by dx



Force on piston, F=PA

Work done during expansion gas $dw = Fdx$

$dw = PAdx$

$\therefore dw = Pdv$ since $dv = Adx$

$\int_0^w dw$ = $\int_{v_1}^{v_2} Pdv$

W = $\int_{v_1}^{v_2} Pdv$ .... (A)

W = $\int_{v_1}^{v_2} Pdv$

W = P$[v]_{v_1}^{v_2}$ = P $[V_2 - V_1]$ ....(B)

Generally :The external work done in expanding gas at constant pressure

W= PΔV

NB: A piston is used such that the gas expands at constant pressure

3.4.2: THE 1ST LAW OF THERMO DYNAMICS

The **1st law states** that the total energy in a closed system is conserved.

When we warm gas so that it expands, the heat (ΔQ) appears partly as an increase in internal energy (Δu) and partly as external work done (Δw).

$$\boxed{\Delta Q = \Delta u + \Delta w}$$

But $\Delta w = P\Delta V$

$$\Delta Q = \Delta u + P\Delta V$$

ΔQ = heat supplied

Δu = increase in internal energy

Δw = work done

Examples

1) A gas in a cylinder has pressure of $2.0x10^5$Pa. The piston has an area of $3x10^{-3}m^2$ and it is pulled out slowly through distance of 10mm.Fi*nd the external work done by the gas*

Solution

$\Delta W = P\Delta V$

$\Delta v = A\Delta L$

$W = 2x10^5 x\ 3x10^{-3} x10x10^{-3}$

W= 6Joules

2) 1kg of water is converted to steam at temperature of 100℃ and pressure of $1.0x10^5$Pa. if the density of steam is $0.58kgm^{-3}$ and S.L.H.V of water is $2.3x10^6Jkg^{-1}$. Calculate the;

(i) External work done (ii) The internal energy

Solution

i) $\Delta w = P\Delta v = P(\frac{Ms}{\rho_s} - \frac{Mw}{\rho_w})$.

$\Delta w = 1x10^5 \left(\frac{1}{0.58} - \frac{1}{1000}\right) = 172300J$

(ii) $\Delta Q = ml_v = 1x2.3x10^6$

$\Delta Q = 2.3x10^6J$

$\Delta Q = \Delta u + \Delta w$

$\Delta u = 2.3x10^6 - 172300 = 2.1277x10^6$

3.4.3: Internal energy U of the gas

The internal of a gas consists of kinetic energy due to the motion of the particles and potential energy due to the intermolecular forces. The total sum of kinetic energy and potential energy of the particles of the gas is the internal energy of the gas

(i) The internal of an **ideal gas** is the kinetic energy of the gas due to its thermal motion of molecules. The magnitude of this internal energy depends on temperature and the number of atoms in its molecules

(ii) The internal energy of **a real gas** has two components;

- Kinetic energy component due to **thermal motion** of its molecules
- Potential energy component which is due to its **inter molecular forces**

3.5.0: SPECIFIC HEAT CAPACITIES OF GASES

Gases unlike solids and liquids have a number of specific principle heat capacities

- For gases a small increase in temperature will also produce a large increase in both pressure and volume. So to study how pressure varies with temperature, volume must be kept constant and to study how volume changes with temperature, pressure must be kept constant.
- For solid and liquid, the change in pressure can be neglected.

In particular, there are two principle heat capacities;

(i) Specific heat capacity at constant pressure

(ii) Specific heat capacity at constant volume

a) S.H.C AT CONSTANT VOLUME

This is the amount of heat required to change temperature of 1kg mass by 1Kelvin at constant volume. It is denoted by c_v(c-small) and it is measured in $Jkg^{-1}K^{-1}$

b) **S.H.C AT CONSTANT PRESSURE**

This is amount of heat required to change temperature of 1kg mass by 1 Kelvin at constant pressure. It is denoted by c_p (c-small) and it is measured in $Jkg^{-1}K^{-1}$

c) MOLAR HEAT CAPACITY AT CONSTANT VOLUME

Is the amount of heat required to change the temperature of 1mole of gas by 1 Kelvin at constant volume?

It is denoted by Cv (C-capital). It is measured in $Jmol^{-1}K^{-1}$. $C_v = c_vM$ Where M= molar mass

d) MOLAR HEAT CAPACITY AT CONSTANT PRESSURE

Is the amount of heat required to change the temperature of 1 mole of gas by 1 Kelvin at constant pressure?

It is denoted by Cp (C-capital) and it is measured $Jmol^{-1}K^{-1}$. $Cp = c_pM$

3.5.1: DIFFERENCES BETWEEN MOLAR HEAT CAPACITIES [$Cp - Cv = R$]

From 1st law of thermodynamics: $\Delta Q = \Delta u + \Delta w$

At constant pressure: $nC_P\,\Delta T = \Delta u + P\,\Delta V$ .... (1)

For an ideal gas equation $P\Delta V = nR\Delta T$

$nC_P\,\Delta T = \Delta u + nR\Delta T$

At constant volume $nC_V\Delta T = \Delta u + 0$ since $P\Delta V = 0$

$nC_P\,\Delta T = nC_V\Delta T + nR\Delta T$

$C_P = C_V + R$

$$\boxed{C_P - C_V = R}$$

Example

The S.H.C of oxygen at constant volume is $719Jkg^{-1}K^{-1}$. If the density of oxygen at S.T.P is $1.429kgm^{-3}$. Calculate the S.H.C of oxygen at constant pressure (04marks)

Solution

$PV = \frac{m}{M}RT$ But $m = v\rho$

$$M = \frac{\rho RT}{P} = \frac{1.429x8.31x273}{1.01x10^5} = 0.0324kg$$

But Cp –Cv =R where Cp and Cv are molar heat capacities

$Mc_p - Mc_v = R$ where c_p and c_v are S.H.C are constant pressure and volume respectively

$c_p = \frac{8.31+0.0324x719}{0.0324}$

$c_p = 977.9\ Jkg^{-1}k^{-1}$

NOTE:

Cp is always greater than Cv because when heat is supplied at constant pressure, it is used for increasing internal energy and doing external work in expansion to keep pressure constant. While when heat is supplied at constant volume. It is only used for increasing internal energy. Therefore molar heat capacity of an ideal gas at constant pressure is more than that at constant volume hence to get the same temperature rise, more heat must be supplied

Explain why $Cp - Cv = R$ is negligible for gases but not solids and liquids.

The volume of solid and liquids change very little when heated at constant pressure compared with the volume changes for gases for the same temperature change. Thus solids and liquids do very little work against atmospheric pressure. Therefore there is very little difference in energy when they expand and when they are not allowed to expand

NOTE: $\frac{C_P}{C_V} = \gamma$

EXAMPLES

1) A gas has volume of $0.02m^3$ at pressure of $2x10^5Pa$ and temperature of 27°C. It is heated at constant pressure until its volume increases to $0.03m^3$. Calculate;
   (i) The external work done
   (ii) The new temperature of the gas
   (iii) The increase in internal energy of gas, if its mass is 16g and molar heat capacity at constant volume is 0.8 $Jmol^{-1}K^{-1}$ and its molar mass is 32g.

Solution

i) $W = P\Delta V = 2x10^5\ (0.03 - 0.02)$

W= 2000 Joules

ii) At constant pressure : $\frac{V_1}{T_1} = \frac{V_2}{T_2}$

$$\frac{0.02}{300} = \frac{0.03}{T_2}$$

$T_2 = 450K$

iii) $u\Delta = nCv\Delta T = \frac{m}{M}\ Cv\Delta T$ | $\Delta u = \frac{16}{32}$ x 0.8 x (450-300) $= 60\,J$

2) A cylinder contains 4moles of oxygen gas at temperature of 27°C. The cylinder is fitted with frictionless piston which maintains constant pressure of 1.5×10^5Pa. The gas is heated until temperature increases to 127°C. Calculate;
   (i) The amount of heat supplied to gas
   (ii) What is the change in internal energy of gas?
   (iii) What is the work done by the gas (Cp = 29.4J$mol^{-1}K^{-1}$, R= 8.31 J$mol^{-1}K^{-1}$)

Solution

i) $\Delta Q_P = nCp\Delta T = 4x29.4\,x\,(400-300)$

ii) $\Delta Q_P = 11760\,J$

$\Delta u = nCv\Delta T$

But $\boldsymbol{C_P - C_V = R}$

$\Delta u = 4x(29.4-8.31)(400-300)$

$\Delta u = 8436\,J$

iii) $\Delta Q = \Delta u + \Delta w$

$\Delta w = 11760 - 8436 = 3324\,J$

3) 10moles of gas initially at 27°C is heated at constant pressure of 1.01×10^5Pa. As volume increases from $0.250m^3$ to $0.375m^3$. Calculate the increase in internal energy (assume Cp=28.5J/mol/K)

Solution

At constant pressure: $\frac{V_1}{T_1} = \frac{V_2}{T_2}$

$\frac{0.250}{300} = \frac{0.375}{T2}$

$T_2 = 450K$

$\Delta u = nCv\Delta T$

$\Delta u = 10x$ (Cp-R) (T_2-T_1)

$\Delta u = 10x\,(28.5-8.31)\,(450-300)$

$\Delta u = 30285\,J$

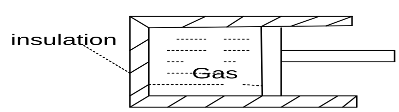
EXERCISE:37

1) Nitrogen gas is trapped in the container by movable piston. If temperature of gas is raised from 0°C to 50°C at constant pressure of 4.0×10^5Pa and total heat added is 3.0×10^4 J. Calculate the work done by the gas [Cp=29.1J$mole^{-1}K^{-1}$, $\frac{C_P}{C_V}$= 1.4] **(Ans 8.57×10^3J).**

2) An ideal gas with volume of $0.1m^3$ expands at a constant pressure of $1.5x10^5Pa$ to treble its volume. Calculate the work done by the gas **An(3 $x10^5J$)**

3) At a temperature of 100°Cand a pressure of 1.01×10^5Pa, 1.00kg of steam occupies $1.67m^3$ but the same mass of water occupies only $1.04\times10^{-3}m^3$. The S.L H of vaporization of water at 100°C is 2.26×10^6Jkg^{-1}. For a system consisting of 1.00kg water changing to steam at 100°C and 1.01×10^5Pa find;
   (a) The heat supplied to the system
   (b) The work done by the system
   (c) The increase in internal energy of the system.**An [2.26×10^6J, 1.69×10^5 J, 2.09×10^6J]**

4) Some gas, assumed to behave ideally, is contained within a cylinder which is surrounded by insulation to prevent loss of heat as shown below.



Initially the volume of gas is $2.9\times10^{-4}m^3$, its pressure is 1.04×10^5Pa and its temperature is 314K.
   (a) Use the equation of state for an ideal gas to find the amount in moles of gas in the cylinder.
   (b) The gas is then compressed to a volume of $2.9\times10^{-5}m^3$ and its temperature rises to 790K. Calculate the pressure of the gas after its compression.
   (c) The work done on the gas during the compression is 91J. Use the first law of thermodynamics to find the increase in the internal energy of the gas during the compression.
   (d) Explain the meaning of internal energy as applied to this system and use your result in (c) to explain why a rise in the temperature of the gas takes place during the compression.[Molar gas constant =8.3J$k^{-1}mol^{-1}$] **An [1.2×10^{-2}, 2.6×10^6Pa, 91J]**

5) The specific latent heat of vaporization of a particular liquid at 130°C and a pressure of 2.60×10^5Pa is 1.84×10^6Jkg^{-1}. The specific volume of the liquid under these conditions is $2\times10^{-3}m^3kg^{-1}$ and that of the vapour is $5.66\times10^{-1}m^3kg^{-1}$. Calculate;
   (a) The work done and

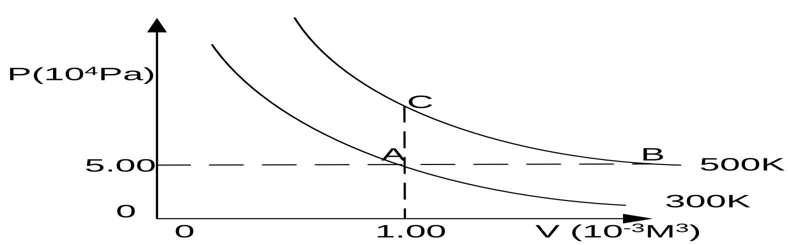
(b) The increase in internal energy when 1.00kg of the vapour is formed from the a liquid under these conditions. **An[1.47×10^5J, 1.69×10^6J]**

6) A mass of 0.35kg of ethanol is vaporized at its boiling point of 78°C and a pressure of 1.0×10^5Pa. At this temperature, the specific latent heat of vaporization of ethanol is 0. 95×10^6Jkg^{-1} and the densities of the liquid and vapour are 790kgm^{-3}and 1.6kgm^{-3} respectively. Calculate;
   (i) The work done by the system.
   (ii) The change in internal energy of the system. **An[2.2×10^4J, 3.1×10^5 J].**

7) (a)A cylinder fitted with an apparatus which can move without friction contains 0.05moles of monatomic ideal gas at a temperature of 27°C and a pressure of 1.0×10^5Pa .The cylinder is calibrated to determine the boiling point of a liquid of boiling point 350K. Calculate;.
   (i) The volume (ii) The internal energy of the gas
   (b) The temperature of the gas in (a) is raised to 77°C, the pressure remaining constant. Calculate;
   (i) The change in internal energy (ii) The external work done
   (iii) The total heat energy supplied (molar gas constant 8.3Jmol^{-1}k^{-1})
   An ([1.2×10^{-3}m^3, 1.9×10^2J] [31J, 21J, 52J])

8) (a)A quantity of 0.2moles of air enters a diesel engine at a pressure of 1.04×10^5Pa and at a temperature of 297K. Assuming that air behaves as an ideal gas, find the volume of this quantity of air. **An[4.75×10^{-3}m^3]**
   (b) The air is then compressed to one twentieth of this volume, the pressure having risened to 6.89×10^6Pa. Find the new temperature. **An[984K]**
   (c) Heating of the air then takes place by burning small quantity of fuel in it to supply 6150J. This is done at a constant pressure of 6.89×10^6Pa and the volume of air increases and the temperature rises to 2040K. find;
   (i) the molar heat capacity of air at constant pressure
   (ii) The volume of air after burning the fuel
   (iii) The work done by the air during this expansion
   (iv) The change in the internal energy of the air during this expansion.
   (Molar gas constant = 8.31JK^{-1}mol^{-1})**An[29.1JK^{-1}mol^{-1}, 4.92×10^{-4}m^3, 1.76x103J, 4.39×10^3J]**

9) The diagram shows curves relating pressure, P and volume V for a fixed mass of an ideal monatomic gas at 300K and 500K. The gas is in a container fitted with a piston which can move with negligible friction.



(a) Give the equation of state for n moles of an ideal gas, defining the symbols used. Show by calculation that;
   (i) The number of moles of gas in the container is 2.01×10^{-2}
   (ii) The volume of the gas at B on the graph is 1.67×10^{-3}m^3, R= 8.31Jmol^{-1}k^{-1}

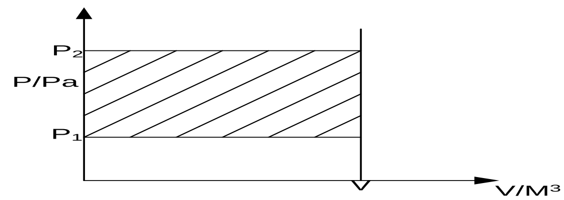
10) A steel pressure vessel of volume 2.2×10^{-2}m^3 contains 4.0×10^{-2}kg of a gas at a pressure of 1.0×10^5Pa and temperature 300K. An explosion suddenly releases 6.48×10^4J of energy, which raises the pressure instantaneously to 1.0×10^6Pa. Assuming no loss of heat to the vessel, and ideal gas behavior calculate;
   (a) The maximum temperature attained
   (b) The two principal specific heat capacities of the gas.
   (c) What is the velocity of sound in this gas at a temperature of 300K?
   An[3000K, 600Jkg^{-1}K^{-1}, 783 Jkg^{-1}K^{-1}, 268ms^{-1}]

11) (a)A vessel of volume 1.0×10^{-2}m^3 contains an ideal gas at a temperature of 300K and pressure 1.5×10^5Pa. Calculate the mass of a gas given that the density of the gas at a temperature 285K and pressure 1.0×10^5Pa is 1.2kgm^{-3}.

(b) 750J of heat energy is suddenly released in the gas causing an instantaneous rise of pressure to 1.8×10^{5}pa. Assuming ideal gas behavior and no loss of heat to the containing vessel, calculate the temperature rise and hence the specific heat capacity at constant volume of the gas. **An[1.7×10^{-2}kg, 60K, 7.3×10^{2}J$kg^{-1}K^{-1}$]**

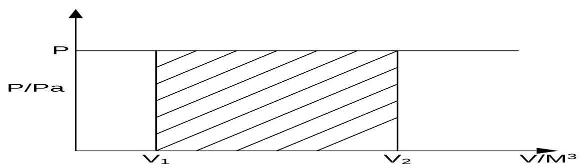
3.6.0: ISOVOLUMETRIC PROCESS [VOLUME CONSTANT]

This is the process which occurs at constant volume. The conditions for it to occur is that the gas must be contained in a sealed vessel. I.e. $\Delta w = 0$ since $\Delta v = 0$



3.6.1: ISOBARIC PROCESS (PRESSURE CONSTANT)

This is the process which occurs at constant pressure. The condition for it to occur is that the gas must be enclosed in the cylinder with frictionless movable piston.
At any instant the pressure of the gas is equal to external pressure.



Δw = P(V₂-V₁)

Δw = area under graph

3.7.0: ISOTHERMAL AND ADIABATIC PROCESS
a)ISOTHERMAL PROCESS

Is the change (expansion or compression) which occurs at constant temperature?
For an isothermal change PV = constant. Heat must be supplied at the same rate as the gas is doing its work

$$\Delta Q = \Delta u + \Delta w$$

But $\Delta u = nCv\Delta T$ but $\Delta T = 0$ $\Delta u = 0$

$$\therefore \Delta Q = \Delta w \ldots\ldots\ldots\ldots\ldots\ldots \text{(x)}$$

Equation (x) above implies that in an isothermal change all heat supplied to gas must be used to do external work.

REVERSIBLE ISOTHERMAL CHANGE:

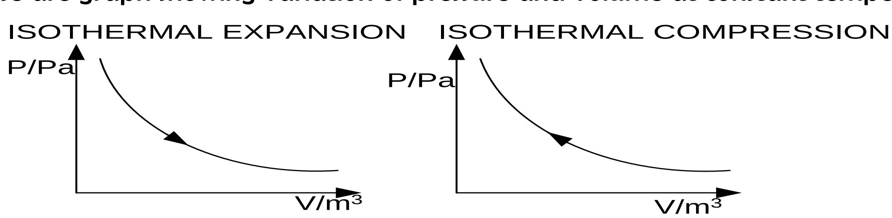
It's defined as, a change that occurs at constant temperature and can be made to go in the reverse direction by an infinitesimal change in the conditions causing it to take place

3.7.1: CONDITIONS FOR ISOTHERMAL PROCESS

- ❖ The gas must be contained in cylinder with very thin, highly conducting walls so that heat can easily be transferred to a gas.
- ❖ The gas cylinder must be surrounded by constant temperature bath
- ❖ The process must be carried slowly to allow enough time for heat transfer.

ISOTHERMALS

These are graph showing variation of pressure and volume at constant temperature.



3.7.2: EQUATION FOR AN ISOTHERMAL PROCESS

Consider an isothermal expansion of the gas from V_1 to V_2, then using the equation of state.

$PV = nRT$ $\quad\quad$ $i.e$ $\boxed{P_1V_1 = P_2V_2}$

All isothermals obey Boyle's law

3.7.3: WORK DONE (ΔW) IN AN ISOTHERMAL EXPANSION

Consider an isothermal expansion from V_1 to V_2

$$\Delta w = P\Delta v$$

$\int_o^w \Delta w = \int_{V1}^{V2} \mathrm{P\Delta v}$

W= $\int_{v1}^{v2} \mathrm{P\Delta v}$

But $PV = nRT$

P= $\frac{\mathrm{nRT}}{V}$

W= $\int_{V1}^{V2} \frac{\mathrm{nRT}}{V}$ dv

W= $nRT \int_{V_1}^{V_2} \frac{1}{V}$ dv

W= $nRT\, [1nV]_{V_1}^{V_2}$

W= nRT (1nV_2 -1nV_1)

$\boxed{W = n\,R\,T\,In\frac{V_2}{V_1}}$ OR $\boxed{W = P_1\,V_1 In\frac{V_2}{V_1}}$ OR $\boxed{W = P_2\,V_2 In\frac{V_2}{V_1}}$

b) ADIABATIC PROCESS (ΔQ = 0)

An adiabatic process is a change (expansion or compression) in which there is no heat exchange between the gas and the surrounding.

Using the 1st law of thermal dynamics.

$\Delta Q = \Delta u + \Delta w$ $\quad$ But ΔQ = 0

Therefor$e\ \Delta u = -\Delta w$ .... (xx)

- Equation (xx) shows that, in an adiabatic process the external work done in expanding the gas is at expense of internal energy and this result into cooling of the gas.

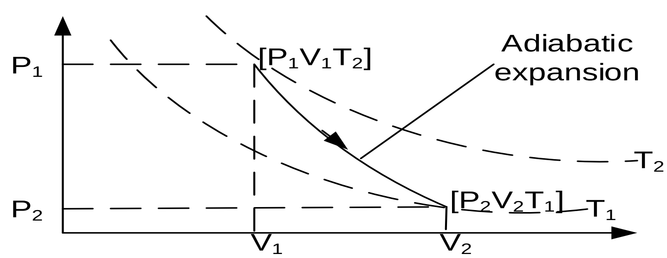
Question; explain why an adiabatic expansion results into cooling of the gas.

During an adiabatic expansion, no heat is supplied to the gas. Molecules of the gas strike the receding piston and bounce off with reduced velocities hence lower kinetic energies. Since the absolute temperature is proportional to mean kinetic energy of the molecules, the gas cools during expansion

3.7.4: CONDITION FOR ADIABATIC PROCESS

- The gas must be contained in thick walled poorly conducting vessel
- The process must be carried out rapidly such that no heat leaves or enter system.

P- V GRAPH FOR ADIABATIC PROCESS



Reversible adiabatic change

This is a change in which there is **no** heat exchange between gas and surrounding and can be made to go in a reverse direction with an infinitesimal change in the condition causing the process .

3.7.5: EQUATION FOR ADIABATIC PROCESS

From the 1st law of thermal dynamics

ΔQ = $\Delta u + \Delta w$ .... (1)

But $\Delta u = C_v\Delta T$ for 1mole of gas And $\Delta w = P\Delta V$

Putting these into equation1

$$\Delta Q = C_v\Delta T + P\Delta V$$

But for an a adiabatic process ΔQ = 0

Therefore $C_v \Delta T + P\Delta V = 0$ ....(2)

$Pv = RT$ for 1mole of an ideal gas

Differentiating it partially, gives

$$P\Delta V + V\Delta P = R\Delta T$$

$P\Delta V = R\Delta T - V\Delta P$ ....(3)

Putting equation (3) into (2), gives

$C_v \Delta T + R\Delta T - V\Delta P = 0$ .....(4)

But $C_P - C_v = R$

$$C_v \Delta T + (C_P - C_v)\Delta T - V\Delta P = 0$$

$$C_P \Delta T - V\Delta P = 0 \;\ldots\ldots\ldots\ldots(5)$$

From equation (2)

$$C_v \Delta T + P\Delta V = 0$$

$\Delta T = \frac{-P\Delta v}{C_v}$

Putting ΔT into equation (5)

$$C_P x\left(-\frac{P\Delta V}{C_v}\right) - V\Delta P = 0$$

$\frac{C_P}{C_v} P\Delta V + V\Delta P = 0$

$\frac{C_P}{C_v} = \gamma$

$\gamma P\Delta V + V\Delta P = 0$

Driving all through by PV

$\frac{\gamma \Delta V}{PV} + \frac{V\Delta P}{PV} = 0$

$\frac{\gamma \Delta V}{V} + \frac{\Delta P}{P} = 0$

Integrating all sides

$\gamma \int \frac{\Delta V}{V} + \int \frac{\Delta P}{P} = constant$

$\gamma 1nV + 1nP = constant$

$$1nV^{\gamma} + InP = Inc$$

$InPV^{\gamma} = Inc$

PV^{γ} = Constant

$$\boxed{P_1 V_1^{\gamma} = P_2 V_2^{\gamma}}$$

3.7.6: RELATIONSHIP BETWEEN TEMPERATURE 'T' AND VOLUME 'V' FOR AN ADIABATIC PROCESS

From PV^{γ} = Constant

But from ideal gas equation

PV = RT for 1 mole of gas

$\therefore P = \frac{RT}{V}$

$\frac{RT}{V} . V^{\gamma}$ = Constant

$RTV^{\gamma-1}$ = Constant

$TV^{\gamma-1} = \frac{Constant}{R}$

$TV^{\gamma-1}$ = Constant

$$\boxed{T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}}$$

WORK DONE (ΔW) IN AN ADIABATIC EXPANSION

$\Delta Q = \Delta u + \Delta w$ But $\Delta Q = 0$

Therefore $\Delta u = -\Delta w$

$$\Delta u = C_v \Delta T$$

$\Delta w = -nC_v(T_2 - T_1)$.......(1)

$$C_P - C_v = R$$

$$\frac{C_p}{C_v} - \frac{C_v}{C_v} = \frac{R}{C_v}$$

$$\gamma - 1 = \frac{R}{C_v}$$

$C_v = \frac{R}{\gamma - 1}$..............(2)

From $PV = nRT$

$T_2 = \frac{P_2 V_2}{nR}$............(3)

$$T_1 = \frac{P_1 V_1}{nR} \ldots\ldots\ldots\ldots(4)$$

Putting 2, 3, 4 into 1

$$\Delta w = -n\frac{R}{\gamma - 1}\left(\frac{P_2 V_2}{nR} - \frac{P_1 V_1}{nR}\right)$$

$$\Delta w = -\frac{(P_2 V_2 - P_1 V_1)}{\gamma - 1}$$

$$\boxed{\Delta w = \frac{(P_2 V_2 - P_1 V_1)}{1 - \gamma}}$$

EXAMPLES

1) An ideal gas at 18°C is compressed adiabatically until its volume is halved. Calculate the final temperature of gas (assume S.H.C of gas at constant pressure and volume are 2100$Jkg^{-1}K^{-1}$ and 1500 $Jkg^{-1}K^{-1} respectively$)

Solution

T_1 = (18+273) = 291K

T_2 =? , V_1 = V, $V_2 = \frac{V}{2}$

$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$

But $\gamma = \frac{C_P}{C_v} = \frac{2100}{1500} = 1.4$

$291xV^{1.4-1} = T_2 X\left(\frac{V}{2}\right)^{1.4-1}$

$291 \text{ x } V^{0.4} = T_2 \text{ x } \left(\frac{V}{2}\right)^{0.4}$

$291\text{x } V^{0.4} = T_2 \text{ x} \frac{V^{0.4}}{2^{0.4}}$

$T_2 = 291\text{x}2^{0.4}$

T_2 = 383.916K

Therefore T_2 = (383.976 - 273)°C

T_2 = 110.976°C

2) A mass of an ideal gas of volume 200m³ at 144K expands adiabatically to temperature of 137K. Calculate its new volume. [Take $\gamma = 1.4$]

Solution

T_1 = 144K, V_1 = 200m^3, T_2 = 137K, V_2 = ?

$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

144x $(200)^{1.4-1}$ = 137x $V_2^{1.4-1}$

1198.87 = 137X$V^{0.4}$

$V_2^{0.4}$ = 8.75092

$V_2 = (8.75092)^{\frac{1}{0.4}}$

V_2 = 226.53m^3

3) The temperature of 1 mole of helium gas at pressure of 1.0x10^{5}Pa increases from 20^{o}C to 100℃ when the gas is compressed adiabatically. Find the final pressure of gas. [$\gamma = 1.67$]

Soluion

$P_1V_1 = nRT_1$

1x10^5 V_1 = 8.31x293

V_1 = 0.0243m^3

$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$293x0.0243^{(1.67-1)} = 373V_2^{(1.67-1)}$

$293x0.0243^{0.67} = 373V_2^{0.67}$

0.0631 = $V_2^{0.67}$

$V_2 = (0.0651)^{\frac{1}{0.67}}$ = 0.0169m^3

$P_1V_1^{\gamma} = P_2V_2^{\gamma}$

1x 10^{5}x $0.0243^{1.67}$= P_2x$0.0169^{1.67}$

201.355 = P_2x$0.0169^{1.67}$

P_2=1.83x10^{5}Pa

4) a)(i)What is meant by isothermal and adiabatic?

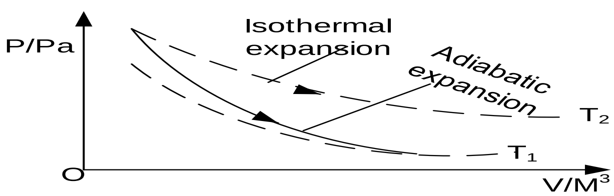
(ii)Using the same axes and starting from same point, sketch P.V diagram to illustrate changes in a)i) above.

(b)An ideal gas is trapped in cylinder by a movable piston. Initially it occupies a volume of 8x10^{-3}m^3 and exerts pressure of 108kPa. The gas undergoes an isothermal expansion until its volume is 27x10^{-3}m^3. It is then compressed adiabatically to the original volume of the gas.

(i) Calculate the final pressure of the gas

(ii) Sketch a well labeled diagram for the two stages of gas on P-V diagram. [The ratio of principal molar heat capacity of gas is 5:3]

Solution



b)(i) *V_1 = 8x10^{-3} , P_1 = 108X10^{3}Pa, T_1 =?*

Isothermal expansion $P_1V_1T_1 \rightarrow P_2V_2T_1$

$P_1V_1 = P_2V_2$

108x10^{3}x10^3 = P_2x 27x10^{-3}

$P_2 = \frac{108x10^3x10^3}{27x10^{-3}} = 32000Pa$

ii) **Adiabatic** $P_2\ V_2\ T_1 \rightarrow P_3\ V_1\ T_2$

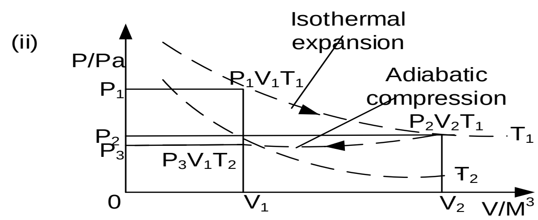
$P_1V_1^{\gamma} = P_2V_2^{\gamma}$

32000x(27x10^{-3})$^{1.67}$ = P_3 (8x10^{-3}) $^{(1.67)}$

76.829 = 0.000314891P_3

$P_3 = \frac{76.829}{0.000314891}$ = 2.439x10^{5}Pa

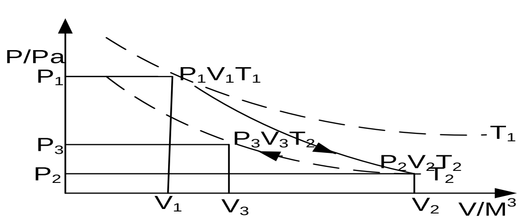
(ii)



5) Amass of air occupying initially a volume 2x10^{-3}m^3 at a pressure of 760mmHg and temperature 20℃ is expanded a diabatically and reversibly to twice its volume and then compressed isothermally and reversibly to volume of 3x 10^{-3}m^3.Find the temperature and pressure, assume that $\gamma = 1.4$

Solution

V_1 = 2x10^{3}m^3, P_1= 760mmHg, T_1= (20^{o}C+273) $= 293K$, Adiabatically V_2= 2x2x10^{-3}m^3



Adiabatic $P_1\ V_1\ T_1 \rightarrow P_2V_2T_2$

$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

293 x(2x10^{-3})$^{1.4-1}$ = T_2 x2x2x10$^{-3(1.4-1)}$

$T_2 = \frac{24.39838}{0.1098} = 222.09K$

$P_1V_1^{\gamma} = P_2V_2^{\gamma}$

760 x (2x10^{-3})$^{1.4}$ = P_2x (4x10^{-3}) $^{1.4}$

0.1265 = 0.000439 P_2

P_2 = 287.8mmHg

Isothermal $P_2\ V_2\ T_2 \rightarrow P_3\ V_3\ T_2$

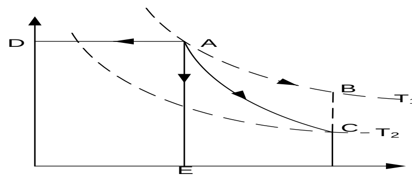
Isothermal obey Boyles law

$P_2 V_2 = P_3 V_3$
$287.8x4x10^3 = P_3x3x10^{-3}$
$P_3 = 383.7mmHg$

Therefore final temperature is 222K and final pressure is 383.7mmHg.

6) Show on the same graph starting on the same point P_1V_1 on P-V sketch curve for a fixed mass of an ideal gas undergoing the following process.
   (i) Isothermal process
   (ii) Adiabatic process
   (iii) Isovolumetric process
   (iv) Isobaric process

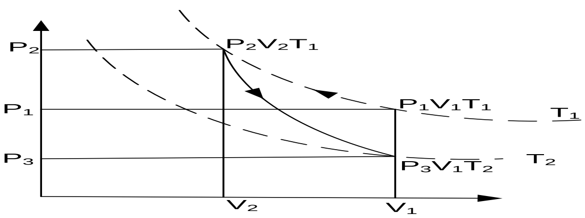
Solution



AB= isothermal expansion
AC = adiabatic expansion
AE = isovolumetric
AD = isobaric

7) A vessel contains $2.5x10^{-3}m^3$ of an ideal gas at pressure of $8.3x10^4Nm^2$ and temperature of 35^oC. the gas is compressed isothermally to volume of $1.0x10^{-3}m^3$. It is then allowed to expand adiabatically to the original volume ($\gamma = 1.4$). Calculate ;
   (i) Find temperature of the gas
   (ii) Work done during isothermal compression of the gas.

Solution



$V_1 = 25x10^{-3}m^3$, $T_1 = 35+273 = 308K$,
$P_1 = 8.5x10^4Nm^{-2}$, Isothermal $V_2 = 1.0x10^{-3}m^3$

i) **Isothermal** $P_1 V_1T_1 \rightarrow P_2 V_2 T_1$

$P_1 V_1 = P_2 V_2$
$8.5x10^4 \times 2.5x10^{-3} = P_2 \times 1.0x10^{-3}$
$P_2 = 2.125x10^5Pa$

Adiabatic $P_2 V_2 T_1 \rightarrow P_3 V_1 T_2$

$P_2V_2^{\gamma} = P_3V_1^{\gamma}$
$P_3 = \frac{2.125x10^5x(10^{-3})^{1.4}}{2.5x(10^{-3})1.4}$
$P_3 = 50.917x10^3 Pa$
$T_1V_2^{\gamma-1} = T_2V_1^{\gamma-1}$
$308 \times 1.0x10^{-3(1.4-1)} = 2.5x10^{-3(1.4-1)} T_2$
$19.433 = 0.091 T_2$
$T_2 = 213.48K$

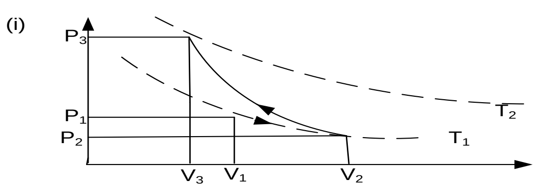
ii) $\Delta w = - P_1V_1 1n\frac{V1}{V2}$

$\Delta w = - 8.5x10x2.5x10^{-3} 1n(\frac{2.3x10^{-3}}{1.0x10^{-3}})$
$\Delta w = -212.5 \text{ x}1n2.5 = -195$Joules
The negative sign implies work done on the gas

8) A gas having a temperature of 27^oC volume of $30000cm^3$ and pressure of 80cmHg expands isothermally to double its volume. The gas is then adiabatically compressed to half its original volume.
   (i) Represent these changes on P-V sketch
   (ii) Calculate final pressure and temperature of gas ($\gamma = 1.4$)

Solution

(i)



(ii) $T_1 = 27+237 = 300K$
$V_1 = 3000x\ 10^{-6}m^3$, $V_1 = 3x10^{-3}m^3$
$P_1 = 80cmHg$
Isothermally $V_2 = 2V_1 = 6x10^{-3}$

Isothermal $P_1V_1T_1 \rightarrow P_2V_2T_1$

$P_1V_1 = P_2V_2$
$80x3x10^{-3} = P_2x6x10^{-3}$
$P_2 = \frac{80x3x10^{-3}}{6x10^{-3}} = 40cmHg$

Adiabatic: $P_2V_2T_2 \rightarrow P_3 V_3T_2$

But $V_3 = \frac{1}{2}V_1 = 1.5x\ 10^{-3}m^{-3}$
$P2V_2^{\gamma} = P_3V_3^{\gamma}$
$40x6x10^{-3(1.4)} = P_3x1.5\ x10^{-3(1.4)}$
$P_3 = \frac{0.01514}{0.0000946} = 278.57cmHg$

$T_1V_2^{\gamma-1} = T_2V_3^{\gamma-1}$

$T_2 = 300\ (\frac{6x10}{1.5x10}) = 522.3K$

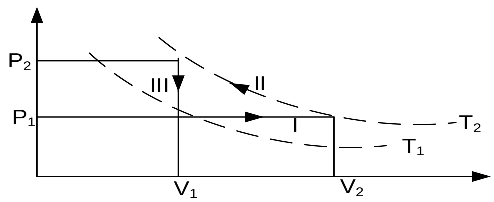
Final pressure = 278.5cmHg and final temperature = 522.3K

9) A cylinder with piston containing 1 mole of gas at pressure of $1x10^5$Pa with temperature of 300K. The gas is heated at constant pressure until its volume doubles. It is then compressed isothermally back to its original volume and finally it is cooled at constant volume to the original state.

(i) Represent the above process on P-V diagram.

(ii) Calculate the total work done in the above processes

Solution



n= 1mole V_1= $22.4x10^{-3}m^3$

P_1= 1x 10 ^{3}Pa T_1 = 300K

Isobaric: $P_1 = P_2$

$V_2 = 2V_1 = 44.8x10^{-3}m^3$

Isothermally: temperature constant

$V_3 = V_1 = 22.4x10^{-3}m^3$

Isovolumetric: V_4=V_1

$P_1V_1T_1 \rightarrow P_1V_2T_2$

Isobars obey Charles law: $\frac{V_1}{T_1} = \frac{V_2}{T_2}$

$\frac{22.4x10}{300} = \frac{44.8x10^{-3}}{T2}$

$T_2 = \frac{13.44}{0.0224} = 600K$

Isothermal compression $P_1V_2T_2 \rightarrow P_2V_1T_2$

$P_1V_2 = P_2V_1$

$1x10^5x\ 44.8x10^{-3} = P_2\ x22.4x10^{-3}$

$P = \frac{4480}{0.0224} = 2x10^5$Pa

Work done in (i)

Isobaric $\Delta w = P_1(V_2 - V_1)$

$\Delta w = 1x10^5(44.8x10^{-3} - 22.4x10^{-3})$

Δw = 2240Joules

Work done in (ii)

Isothermal $\Delta w = -P_1V_2\ 1n\ \frac{V1}{V2}$

$\Delta w = -10^5x44.810^{-3}\ 1n\ (\frac{22.4x10^{-3}}{44.8x10^{-3}})$

$\Delta w = 3.105x10^3$Joules

Work done in (iii)= 0

because there is no volume change Total work done = work done in i and ii.

$= 2240+3.105x10^3$

$= 5.345x10^3$ Joules

EXERCISE: 38

1) A gas is confined in the container of volume $0.1m^3$ at pressure of $1x10^5$Pa and temperature of 300K. if the gas is assumed to be ideal. Calculate the density of gas (RMM of the gas is 32)

2) Air at 20°C is allowed to expand adiabatically until its pressure has fallen to one-third of its original value. What is the final temperature of the air if γ =1.4

3) A certain volume of helium at 15°C is expanded adiabatically until its volume is trebled. Calculate the temperature of the gas immediately after the expansion has taken place γ =1.67

4) An ideal gas at 27°C and at pressure of 760mm of mercury is compressed isothermally until its volume is halved. It is then expanded reversibly and adiabatically to twice its original volume. Calculate the final pressure and temperature of the gas if γ =1.4 **An(1520mm of mercury, 172K)**

5) Air is contained in a cylinder by a frictionless gas tight piston.

(a) Find the work done by the gas as it expands from a volume of $0.015m^3$ to a volume of $0.027m^3$ at a constant pressure of $2.0x10^5$Pa

(b) Find the final pressure if, starting from the same initial conditions as in (a) and expanding by the same amount, the change that occurs

(i) Isothermally (ii) adiabatically

$(\gamma of air = 1.40)$ **An[$2.4x10^3$J, $1.1x10^5$Pa, $8.8x10^4$Pa]**

6) The cylinder in fig1 below holds a volume V_1 = 1000cm^3 of air at an initial pressure P_1 = 1.10x10^{5}Pa and temperature T_1 = 300K. Assume that air behaves like an ideal gas.

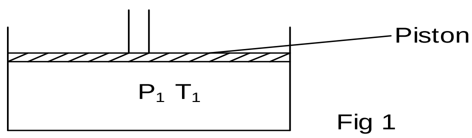


Fig 1

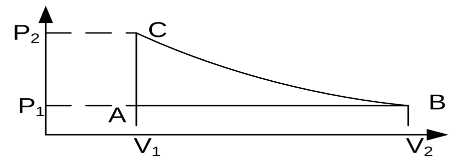


Fig 2

Fig 2 shows a sequence of changes imposed on the air in the cylinder.

AB – the air is heated to 375K at constant pressure. Calculate the new volume V_2.

BC – the air is compressed isothermally to volume V_1. Calculate the new pressure P_2.

CA – the air cools at constant volume to pressure P_1. State how a value for the work done on the air during with sequence of change may be found from the graph in fig 2 .

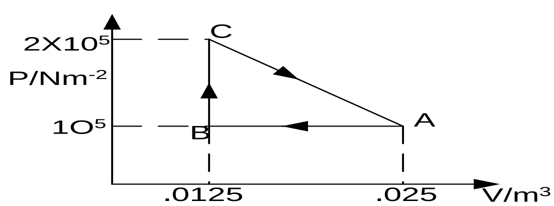
An1250cm^3, 1.38x10^{5}Pa

7) A vessel of volume 8.00x10^{3}m^3 contains an ideal gas at a pressure of 1.14x10^{5}Pa. A stop cock in the vessel is opened and the gas expands a adiabatically, expelling some of its original mass, until its pressure is equal to that outside the vessel (1.0x10^{5}Pa). The stop cock is then closed and the vessel is allowed to stand until the temperature returns to its original value in this equilibrium state, the pressure is 1.06x10^{5}Pa.

(a) Explain why there was a temperature change as a result of adiabatic expansion
(b) Find the volume which the mass of gas finally left in the vessel occupied under the original conditions.
(c) Sketch a graph showing the way in which the pressure and volume of the mass of gas finally left in the vessel changed during the operations described above.
(d) What is the value of γ, the ratio of the principal heat capacities of the gas .

An [7.44x10^{-3}m^3, 1.66]

8)



The diagram represents an energy cycle where by a mole of an ideal gas is firstly cooled at constant pressure (A→B) then heated at constant volume (B→C) and then returned to its original state (C→A)

(i) Calculate the temperature of the gas at A, at B and at C
(ii) Calculate the heat given out by the gas in the process A→B
(iii) Calculate the heat absorbed in the process B→C
(iv) Calculate the net amount of work done in the cycle
(v) Calculate the net amount of heat transferred in the cycle

[R=8.3Jmol^{-1}k^{-1} Cv = $\frac{5}{2}$ R] **An[301.2K at A and C, 150.6K at B, 4375J, 3125J, 625J, 625J]**

9) A quantity of ideal gas whose ratio of principal molars heat capacities is $\frac{5}{3}$ has temperature 300K, volume 64x10^{-3}m^3 and pressure 243kPa. It is made to undergo the following three changes in order

A: reversible adiabatic compression to a volume 27x10^{-3}m^3

B: reversible isothermal expansion back to 64x10^{-3}m^3

C: a return to the original state

(i) calculate the pressure on completion of process A
(ii) Calculate the temperature at which process B occurs
(iii) Describe process C

10) 1g of hydrogen at $s.t.p$ has it's volume halfed by an adiabatic change. Calculate the change in internal energy of the gas.[R = 8.31J $mol^{-1}K^{-1}$, γ= 1.4]. **An [905.79J]**

UNEB 2011 Q 6

di) Distinguish between isothermal and a adiabatic changes (02marks)

ii)An ideal gas at 18°C is compressed a adiabatically until the volume is halved.

Calculate the final temperature of the gas.

(Assume specific heat capacities of the gas at constant pressure and volume are 2100J$kg^{-1}K^{-1}$ and 1500J$kg^{-1}K^{-1}$ respectively) **An[383.98k]** (4marks)

UNEB2010 Q.6

a)i)State the difference between isothermal and adiabatic expansion of a gas

ii)Using the same axes and point, sketch the graph of pressure verses volume for a fixed mass of gas undergoing isothermal and a adiabatic change (3marks)

b)Show that the work W done by a gas which expands reversibly from V_0 to V_1 is given by W= $\int_{v0}^{v1} pdv$ (4marks)

c)i)State two differences between real and ideal gases

ii)Draw labeled diagram showing P-V isothermal for a real gas above and below the critical temperature (3mark)

d)Ten moles of a gas initially at 27°C and heated at a constant pressure 1.0x10^5 Pa and the volume increased from 0.250cm^3 to 0.375m^3. Calculate the increases in internal energy [assume Cp = 28.5J$mol^{-1}K^{-1}$] (6mark) **An [3.012x10^4J]**

UNEB 2009 Q.6

a)i)State Boyle's law (01mark)

ii)Describe an experiment that can be used to verify Boy led law. (06mark)

c)i)What is meant by eversible process

ii)State the conditions necessary for isothermal and adiabatic process to occur

d)A mass of an ideal gas of volume 2000m^3 at 144K expands adiabatically to a temperature of 137K. Calculate the new volume (take γ = 1.40) (3mark) **An [226.47cm^3]**

UNEB 2009 Q.6

a)i)State Boyle's law (01mark)

ii)Describe an experiment that can be used to verify Boy led law

c)i)What is meant by a reversible process

ii)State the conditions necessary for isothermal and adiabatic process to occur

d)A mass of an ideal gas of volume 200m^3 at 144K expands adiabatically to a temperature of 137K. calculate the new volume (take γ = 1.40)**An[226.47cm^3]**

UNEB 2008 Q.6

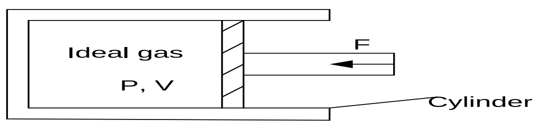
a)Describe an experiment to verify Newton's law of cooling

c)ii)Nitrogen gas is trapped in a container by a movable piston. If the temperature of the gas raised from 0°C to 50°C at a constant pressure of 4x10^5Pa and the total heat added is 3x10^4J. Calculate the work done by the gas. **An[8.57x10^3]**

(Molar heat capacity of nitrogen at constant pressure is 29.1J$mol^{-1}k^{-1}$ $\frac{C_P}{C_V} = 1.4$)

UNEB 2007 Q,7

a)



A fixed mass of an ideal gas confined in a cylinder by a friction less piston of cross section area A . The piston is in equilibrium under the action of a force F as shown above. Show that the work done W by the gas when it expands from V_1 to V_2 is given by W= $\int_{v1}^{v2} pdv$

b)State the first law of thermodynamics and use it to distinguish between isothermal and adiabatic changes in a gas.

c)The temperature of one mole of helium gas at a pressure $1.0x10^5$Pa increases from 20^oC to $100^oC$when the gas is compressed adiabatically. Find the final pressure of the gas (take $\gamma = 1.67$**)**

An [$1.83x10^5$Pa]

UNEB 2005 Q.6

a)i)What is meant by isothermal and adiabatic changes (02marks)

ii)Using the same axes, and starting from the same point, sketch a P-V diagram to illustrate the change in a(i) (02marks)

b)An ideal gas is trapped in a cylinder by a movable piston. Initially it occupies a volume of $8x10^{-3}m^3$ and exerts a pressure of 108kPa. The gas volume is $27x10^{-3}m^3$. It is then compressed adiabatically to the original volume of the gas

i)Calculate the final pressure of the gas (06marks)

ii)Sketch and label the two stages of the gas on a P-V diagram [the ratio of principal molar heat capacities of the gas = 5:3] **An [$2.43x10^3$Pa]** (02marks)

c)i)Define molar heat capacities at constant pressure. (01mark)

ii)Derive the expression C_P-C_v = R for 1mole of a gas (05mark)

iii)In what ways does a real gas differ from an ideal gas (01mark)

UNEB 2004 Q.7

b)A gas is confined in a container of volume $0.1m^3$ at a pressure of $1.0x10^5Nm^{-2}$ and a temperature of 300K. if the gas is assumed to be ideal calculate the density of the gas (05marks)[the relative molecular mass of the gas is 32]**An [$7.71x10^{23}kgm^{-3}$]**

c)What is meant by

i)Isothermal change (01mark)

ii)Adiabatic change (01mark)

d)A gas at a pressure of $1.0x10^6$Pa is compressed adiabatically to half its volume and then allowed to expand isothermally to its original volume. Calculate the final pressure of the gas. [assume the ratio of the principal specific heat capacities $\frac{C_P}{C_V} = 1.4$] (05marks) **An[$1.32x10^6$Pa]**

UNEB 2003 Q.5

a)i)Define molar heat capacity of a gas at constant volume. (1mark)

ii)The S.H.C of oxygen at constant volume is $719Jkg^{-1}K^{-1}$. If the density of oxygen at S.T.P is $1.429kgm^{-3}$. Calculate the S.H.C of oxygen at constant pressure (04marks) **An ($977.9Jkg^{-1}K^{-1}$)**

UNEB 2002 Q.5

d)i)What is meant by a reversible isothermal change (02marks)

ii)State the conditions for achieving a reversible isothermal change .(02marks)

e)An ideal gas at 27^oC and at a pressure of $1.0x10^5$Pa is compressed reversibly and isothermally until its volume is halved. It is then expanded reversibly and adiabatically to twice its original volume. Calculate the final pressure and temperature of the gas if γ =1.4 **An [$2.9x10^4$Pa]**

UNEB 2001 Q.6

a)i)Explain what happens when a quantity of heat is applied to a fixed mass of gas (02marks)

ii)Derive the relation between the principal molar heat capacities Cp and Cv for an ideal gas (05marks)

b)i)What is an adiabatic process (1mark)

ii)A bicycle pump contains air at 290K. The piston of the pump is slowly pushed in until the volume of the air pump. The outlet is then sealed off and the piston suddenly pulled out to full extension. If no air escapes. Find its temperature immediately after pulling the piston (take $\frac{cp}{cv} = 1.4$) **An[152.3K]**

CHAPTER4: KINETIC THEORY OF GASES

Brownian motion

It's a continuous random and haphazard motion of fluid particles caused by repeated collision of particles exerting a resulting force on each other which changes in a magnitudes and direction

Kinetic theory of matter states that Matter is made up of small particles called molecular atoms that are in continuous random motion and the speed of movement of the particles is directly proportional to temperature.

Explain why gas fills container in which it is placed and exerts pressure on the walls using kinetic theory of gases.

- A gas contains molecules with a negligible intermolecular forces and are free to move in all directions. As they move they collide with each other and with the walls of the container. The movement makes them fill the available space and the collisions with the walls constitute the pressure exerted on the wall

Explain using kinetic theory why the pressure of fixed mass of gas rises when its temperature is increased at constant volume.

- When gas temperature increases, the average kinetic energy of molecules increases, they make more frequent collisions with the walls of the container. This implies greater pressure of the gas. In addition pressure increases as a result of a higher rate of change of momentum at each collision.

Explain using kinetic theory why the pressure of fixed mass of gas rises when its volume is decreased at constant temperature.

- When the volume occupied by the gas is reduced, the molecules take less time to move between the walls as the distance is reduced. The number of collisions per unit time per unit area increases, hence pressure increases at contant temperature.

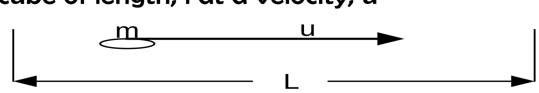
4.1: DERIVATION OF EXPRESSION OF PRESSURE EXERTED ON CONTAINER BY THE GAS ($P = \frac{1}{3}\rho\overline{C^2}$)

In deriving this expression, the following assumptions are considered;

- Intermolecular forces of attraction are negligible
- Molecules make perfectly elastic collisions
- The volume of molecules is negligible compared to the volume of container.
- The duration of collision is negligible compared with time between collisions.

Derivation of expression $P = \frac{1}{3}\rho\overline{C^2}$

Consider a molecule of mass, m moving in a cube of length, l at a velocity, u



Change in momentum $= mu - (-mu) = 2mu$

Rate of change in momentum$= \frac{2mu}{t}$

But time, t between collisions$= \frac{2L}{u}$

Force on the wall by molecule, $F_1 = \frac{2mu_1}{\left(\frac{2L}{u_1}\right)} = \frac{m{u^2}_1}{l}$

For N molecules, force on the wall,F

$$F = \frac{m{u^2}_1}{l} + \frac{m{u^2}_2}{l} + \cdots\cdots \frac{m{u^2}_N}{l}$$

Pressure, $P = \frac{F}{A} = \frac{m}{l^3}(u_1^2 + u_2^2 + \cdots\cdots u_N^2)$

since $A = l^2$

$$\overline{U^2} = \frac{U_1^2 + U_2^2 \ldots\ldots\ldots + U_N^2}{N}$$

$$\therefore N\,\overline{U^2} = U_1^2 + U_2^2 \ldots\ldots + U_N^2$$

$P = \frac{Nm\overline{U^2}}{L^3} = \rho\overline{U^2}$ since $\rho = \frac{Nm}{L^3}$

The molecules do not show any preferences in moving parallel to any direction.

$\overline{C^2} = \overline{U^2} + \overline{V^2} + \overline{W^2}$ and $\overline{U^2} = \overline{V^2} = \overline{W^2}$

$\overline{C^2} = 3\overline{U^2} \quad \therefore \quad \overline{U^2} = \frac{1}{3}\overline{C^2}$

$$\boxed{P = \frac{1}{3}\rho\overline{C^2}}$$

Since density, $\rho = \frac{Nm}{V}$ where m is mass of one moleceule

$P = \frac{1}{3}\frac{Nm}{V}\overline{C^2}$

$$\boxed{PV = \frac{1}{3}Nm\overline{C^2}}$$

4.1.1: RELATIONSHIP BTN MEAN KINETIC ENERGY AND ABSOLUTE TEMPERATURE

From: $PV = \frac{1}{3}Nm\overline{C^2}$.....................1

For an ideal gas: PV = nRT 2

$\frac{1}{3}Nm\overline{C^2}$= nRT

$m\overline{C^2}$=$\frac{3nRT}{N}$

Multiplying both side by ½: ½ $m\overline{C^2}$ = ½ X $\frac{3nRT}{N}$

Mean $K.e$ = $\frac{3}{2}\frac{nRT}{N}$

But for 1mole of gas N = N_A

$$\boxed{Mean\ K.e = \frac{3}{2}\frac{RT}{N_A}}$$

OR ½ $m\overline{C^2}$ =$\frac{3}{2}\frac{RT}{N_A}$

OR ½ $m\overline{C^2} = \frac{3}{2}kT$

where k is Boltzmann constant

From above equation

Mean kinetic energy ∝ temperature

$i.e$ $\overline{C^2}$ ∝ T

$\sqrt{\overline{C^2}}$ ∝ $\sqrt{T}$

N.B: The number of molecules N is $\boxed{N = nNA}$

The mass of molecules $M = mN$

Where m is mass of one molecule

OR $M = mn$N$_A$

EXAMPLES

1) Calculate the rms of the gas molecules and the speed of sound in the atmosphere of Jupiter given that the speed of sound in the gas is 0.682ms^{-1} , and the atmosphere of Jupiter contains mainly methane gas. (Temperature of Jupiter atmosphere is -130°C) molecular weight of methane 16.04gmol^{-1} and the gas constant R = 8.31Jmol^{-1}K^{-1}).

Solution

T = -130 + 273 = 143K

½ $m\overline{C^2}$ =$\frac{3}{2}\frac{RT}{N_A}$

$\overline{C^2}$ =$\frac{3}{M}\frac{RT}{N_A}$

But mN_A = $16.4x10^{-3}$kgmol^{-1}

$\sqrt{\overline{C^2}} = \sqrt{\frac{3x8.31x143}{16.04x10^{-3}}}$ =**4.71X**10^2ms^{-1}

speed of sound in atmosphere = $0.682x4.71x10^2$

= 321.2ms^{-1}

2) Given that density of oxygen is $0.098kgm^{-3}$at a pressure of 1.0 x 10^5Nm^{-2}. Calculate the root mean square speed of oxygen

Solution

$$\sqrt{\overline{C^2}} = \sqrt{\frac{3\,P}{\rho}}$$

$$\sqrt{\overline{C^2}} = \sqrt{\frac{3\ x1x10^5}{0.098}} = 1749.64ms^{-1}$$

3) Calculate the rms speed of molecule of an ideal gas at 130°C, given that the density of the gas at pressure of 1.0 x 10^5$Nm^{-2}$and temperature of 0°C is 1.43kgm^{-3}

Solution

$\overline{C_1^2}$= ?, P$_1$ = 1.0 x 10^3,

T$_1$ = 273K, ρ = 1.43$kgm^{-3,}$

$\overline{C_2^2}$= ? T$_2$ = 403K

$P_1 = \frac{1}{3}\rho\overline{C_1^2}$

$$\sqrt{\overline{C_1^2}} = \sqrt{\frac{3P_1}{\rho_1}}$$

$$\sqrt{\overline{C_1^2}} = \sqrt{\frac{3x1.0x10^5}{1.43}}$$

$$\sqrt{\overline{C_1^2}} = \sqrt{209.79x10^3}$$

$$\frac{\sqrt{\overline{C^2}_1}}{\sqrt{\overline{C_2^2}}} = \frac{\sqrt{T_1}}{\sqrt{T_2}}$$

$$\frac{\sqrt{209.79x10^3}}{\sqrt{\overline{C_2^2}}} = \frac{\sqrt{273}}{\sqrt{403}}$$

$$\sqrt{\overline{C_2^2}} = 556.4878m/s$$

4) Calculate the root mean square speed of the molecules of hydrogen at 27°C given that the density of hydrogen at pressure of 1.0 x 10^{5}Nm^{-2} and a temperature of 0°C is 0.09kgm^{-3}.

Solution

$\overline{C_1^2}$ = $\frac{3P_1}{\rho}$

$\overline{C_1^2}$ = $\frac{3x1x10^5}{0.09}$

$\overline{C_1^2}$ = 3.333 x 10^{6}ms^{-1}

$\frac{\sqrt{\bar{C^2}_1}}{\sqrt{\bar{C_2^2}}} = \frac{\sqrt{T_1}}{\sqrt{T_2}}$ | $\frac{\sqrt{3.333X10^6}}{\sqrt{\bar{C_2^2}}} = \frac{\sqrt{273}}{\sqrt{300}}$ | $\sqrt{\bar{C_2^2}} = 1.91389x10^3 m/s$

EXERCISE:39

1) The density of nitrogen at s.t.p is 1.251kgm^{-3}. Calculate the root mean square velocity of nitrogen molecules **An(493m/s)**
2) A mole of an ideal gas at 300K is subjected to a pressure of 10^5Pa and it's volume is 0.025m^3 calculate
   (a) the molar gas constant R | (b) the Boltzmann constant k
   (c) the average translational kinetic energy of a molecule of the gas
   (N_A = 6.0 x 10^{23}mol^{-1}) **An (8.3Jk^{-1}mol^{-1}, 1.4 x 10^{-23}JK^{-1}, 6.3 x 10^{-21}J)**
3) A vessel of volume 1.0 x 10^{-3}m^3 contains helium gas at a pressure of 2.0 x 10^5Pa when the temperature is 300K. lative atomic mass of helium = 4, the Avogadro constant = 6.0 x 10^{23}mol^{-1}, R = 8.3Jmol K^{-1}
   (a) What is the mass of helium in the vessel
   (b) How many helium atoms are there is the vessel
   (c) Calculate the $r.m.s$ speed of the helium atoms.**An(0.32g, 4.8 x 10^{22}, 1.4 x 10^3ms^{-1})**
4) What would be the total kinetic energy of the atoms of 1kg of neon gas at a pressure of 10^5Pa and temperature 293K, given that the density of neon under these conditions is 828gm$^{-3.}$ What would be the total kinetic energy of the atoms of 1kg of neon gas at 300K. Hence determine the specific heat capacity of neon at constant volume. **An[1.81(2) x 10^5J, 1.85(5) X 10^5J 6.1 X 10^2Jkg^{-1}K^{-1}]**
5) Some helium (molar mass = 0.004kgmol^{-1}) is contained in a vessel of volum 8 x 10^{-4}m^3 at a temperature of 300K. The pressure of the gas is 200kPa. Calculate
   (i) The mass of helium present
   (ii) the internal energy (the translational kinetic energy of the gas molecules)
   (molar gas constant = 8.3JK^{-1}mol^{-1}) **An [2.57 x 10^{-4}kg 240J]**
6) **A** cubical container of volume0.10m^3 contains Uranium hexafluoride gas at a pressure of 1.0 x 10^6Pa and a temperature of 300K. Assuming that the gas is ideal determine;
   (i) the number of moles of gas present given that universal gas constant R = 8.3JK^{-1}mol^{-1}.
   (ii) the mass of gas present, given that it's relative molecular mass is 352.
   (iii) the density of the gas
   (iv) the $r.m.s$ speed of the molecules **An (40.2, 14.1kg, 141kgm^{-3}, 146ms^{-1})**
7) Helium gas is contained in a cylinder by a gas tight piston which can be assumed to move without friction. The gas occupies a volume of 1.0 x 10^{-3}m^3 at a temperature of 300K and a pressure of 1.0 x 10^5Pa
   (a) calculate;
      (i) the number of helium a toms in the container
      (ii) the total kinetic energy of the helium atoms. **An(2.4 x 10^{22}, 150J)**
   (b) Energy is now supplied to the gas in such away that the gas expands and the temperature remains constant at 300K. State and explain what changes, if any will have occurred in the following quantities
      (i) the internal energy of the gas | (ii) the $r.m.s$ speed of the helium atoms
      (iii) the density of the gas (the Boltzmann constant = 1.4 x 10^{-23}JK^{-1})
8) Use the following data to calculate the root mean square speed of helium molecules at 2000°C
   Mass of one mole of helium = 4g, Molar gas constant = 8.31Jmol^{-1}K^{-1} **An[3.76 x 10^3ms^{-1}]**
9) A cylinder of volume 0.080m^3 contains oxygen at a temperature of 280K and a pressure of 90kPa. (NA = 6.0 x 10^{23}mol^{-1} R = 8.3JK^{-1}mol^{-1} and molar mass of oxygen M = 0.032kgmol^{-1}). Calculate
   (i) the mass of oxygen in the cylinder
   (ii) the number of oxygen molecules in the cylinder.
   (iii) the R.M.S speed of the oxygen molecules
   An (9.9 x 10^{-2}kg, 1.9 x 10^{24}, 4.7 x 10^2ms^{-1})
10) Helium gas occupies a volume of 0.04m^3 at a pressure of 2 x 10^5 Pa and temp of 300K. Calculate;
   (i) the mass of helium | (ii) the rms speed of its molecules
   (iii) the rms at 432K, when the gas is heated at constant pressure to this temperature
   (iv) the rms of hydrogen molecule at 432k (Rmm of helium and hydrogen, 4 and 2 respectively R = 8.31Jmol^{-1}K^{-1}) An[**12.8359g, 1368ms^{-1}, 1643ms^{-1}, 2324ms^{-1}**]

4.2: DEDUCTIONS OF KINETIC THEORY

1.Boyles law:

It states that for a fixed mass of gas, the volume is inversely proportioned to pressure at constant temperature.

i.e PV = a constant

From kinetic theory

PV = ⅓ N$m\overline{C^2}$

Multiply both sides by ½

½ PV = ⅓N x ½$m\overline{C^2}$

But ½ $m\overline{C^2} \propto T$

For a fixed mass of gas N is constant

If T is constant, Therefore PV = constant

2. Charles's law:

It states that the volume of fixed mass of a gas is directly proportional to absolute temperature at constant pressure.

PV = ⅓N$m\overline{C^2}$

Multiplying both sides by ½

½ PV = ⅓N x ½ $m\overline{C^2}$

Making V the subject

V = $\frac{2}{3}\frac{N}{P}$X$\frac{1}{2}m\overline{C^2}$

But ½ m $\overline{C^2}$ ∝T

For a fixed mass of a gas N is constant,

Hence V∝ T

4. Daltons law of partial pressure:

It states that partial pressure of a mixture of gases which do not react chemicall is the sum of the partial pressure of component gases.

Note: partial pressure of gas, is the pressure the gas would have if it is to occupy the whole container alone.

$$\boldsymbol{P = \frac{1}{3}\rho \overline{C^2}}$$

Since density, ρ = $\frac{Nm}{V}$ where m is mass of one molecuele

$$PV = \frac{1}{3}Nm\overline{C^2}$$

$$\therefore N = \frac{3PV}{m\overline{C^2}}$$

If the gas has two co mponents 1 and 2

$N_1 = \frac{3P_1V}{m_1\overline{C^2}_1}$ and $N_2 = \frac{3P_2V}{m_2\overline{C^2}_2}$

N = $N_1 + N_2$

$$\frac{3PV}{m\overline{C^2}} = \frac{3P_1V}{m_1\overline{C^2}_1} + \frac{3P_2V}{m_2\overline{C^2}_2}$$

At constant temperature

½ $m\overline{C^2}$ = ½$m_1\overline{C^2}_1$ = ½ $m_2\overline{C^2}_2$

Hence P = P_1 + P_2

4.2: REAL GASES

Real gases obey ideal gas equation ($PV = nRT$) only when they are at very low pressure and at high temperatures.

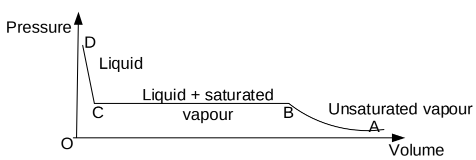
4.2.1: PROPERTIES OF REAL GASES

- Intermolecular forces of attraction and repulsion are not negligible
- Volumes of molecules are not negligible compared to volume of container
- The collision in real gases are inelastic
- They do not obey gas laws and equations

Note: At high temperature and low pressure real gases behave like ideal gases.

- At high temperature the average kinetic energy of the molecules is high and intermolecular seperation increases, intermolecular forces are so weak such that they become negligible and thus the gas behaves like an ideal gas.
- At low pressure for a fixed number of molecules, volume increases. So the molecules will occupy a negligible volumes compared with that of the container. Hence the gas will behave like an ideal one.

4.2.2: Pressure against volume curve for a real gas compressed below critical temperature



- In region AB, there is unsaturated vapour wich fairly obeys Boyle's law at low pressures.
- At higher pressures (BC), some of the vapour condenses and we have liquid plus saturated vapour but pressure remains constant as volume reduces
- At much higher pressures (CD), all the vapour condenses into a liquid and there is a very small change in volume for a large pressure increase.

Definition Critical temperature of gas is the temperature above which the gas can not be liquefied by mere compression.

4.2.3: VANDER-WAAL EQUATION

Vander Waal modified the ideal gas equation by taking into account two of assumption made by kinetic theory to be valid.

The two assumptions include;

- The intermolecular forces of attraction between molecules may not be negligible.
- The volume of molecules may not be negligible as compared to volume V occupied by the gas.

1. In real gas the intermolecular forces of attraction are not negligible. Therefore the observed pressure is actually less than the pressure in the ideal case by an amount $\frac{a}{V^2}$ called pressure defect
2. The factor b accounts for the fact that the molecules of a gas have a finite volume that is not negligible compared o the volume of the gas. It accounts for the volume avalaible for the motion of molecules called covolume.

Therefore Vander Waal's equation is given by

$$\boxed{\mathbf{(P + \frac{a}{V^2})(V - b) = nRT}}$$

Co-volume is the free space in which the molecules of a gas can move.

4.3: VAPOURS

A vapour is gaseous state of substance below its critical temperature. A vapour can either be saturated or unsaturated

A gas is a gaseous state of substance above it's critical temperature

Supper saturated vapor is one whose rate of evaporation exceeds its rate of condensation.

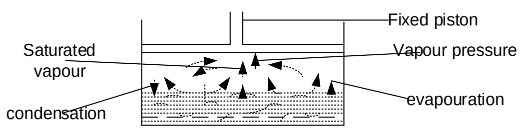
4.3.1: SATURATED AND UNSATURATED VAPOUR

- A saturated vapour is one which is in dynamic equilibrium with it's own liquid. Saturated vapours do not obey gas laws
- Unsaturated vapour is one which is not in dynamic equilibrium with it's own liquid. Un saturated vapours approximately obey gas laws

4.3.2: SATURATED VAPOUR PRESSURE (S.V.P)

S.V.P of a liquid is the maximum constant pressure exerted by the vapour in dynamic equilibrium with its liquid

1:Explanation of occurrence of S.V.P using kinetic theory



- Consider a liquid confined in the container with fixed piston. The liquid molecules are moving randomly with mean kinetic energy determined by liquid temperature. The most energetic molecules have sufficient K.e to overcome the attraction by other molecules and leave the surface of liquid to become vapour molecules by a process of **evaporation** .
- The molecules of the vapour are also moving randomly with a mean kinetic. The vapour molecule collide with walls of the vessel giving rise to vapour pressure and others bombard the surface of the liquid and re-enter the liquid by **condensation**.
- A state of dynamic equilibrium is attained when the rate of condensation equals to rate of evapouration. At this point the density of vapour and hence vapour pressure is maximum and constant at that temperature of the vapour and this is called S.V.P.

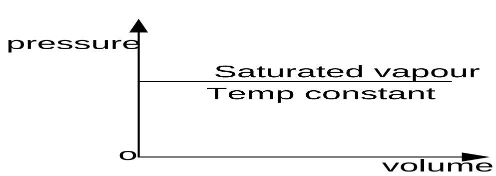
NB:

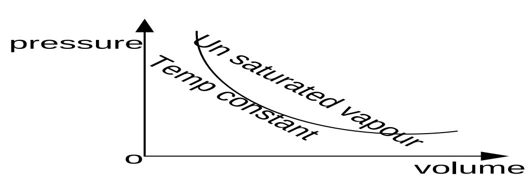
- The rate of evapouration depend on temperature of the liquid
- The rate of condensation depends on density of vapour
- Vapour pressure depends on density of the vapour
- Saturated vapour pressure depends on density of the vapour

2:Effect of volume change on S.V.P at constant temperature

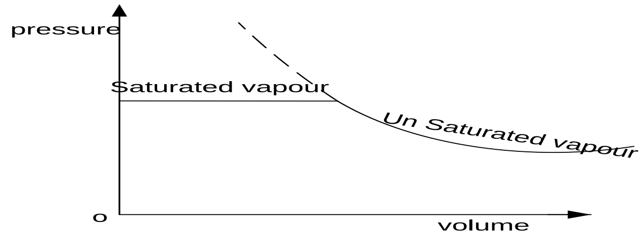
- When the volume of saturated vapour is decreased at constant temperature, the density of vapour increases and the rate of condensation increases.
- As a result more molecules return to the liquid than leave it. The number of molecules in the vapour continue to fall until dynamic equilibrium is again restored with SVP having the **original value**.

NB: Volume change at constant temperature has no effect on SVP



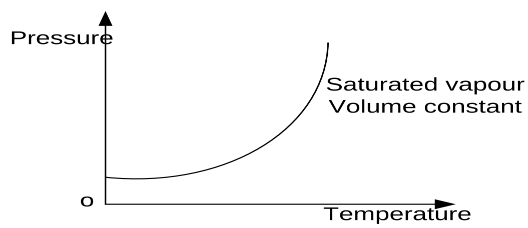


Saturated vapours do not obey Boyle's law, unsaturated vapour obey Boyle's law

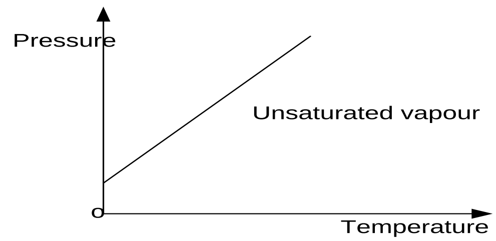


3:Effects of increasing temperature on SVP at constant volume

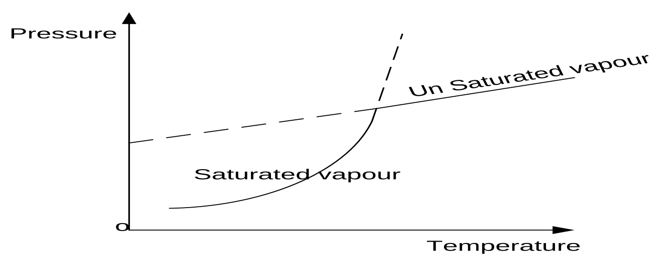
If a liquid is in dynamic equilibrium with its vapour, an increase in temperature increases the mean kinetic energy of molecules and hence evapouration rate increases. The vapour density increases, implying increase in the rate of condensation until a dynamic equilibrium is restored. There are now more molecules in the vapour phase than previously that are moving faster and hence higher pressure.



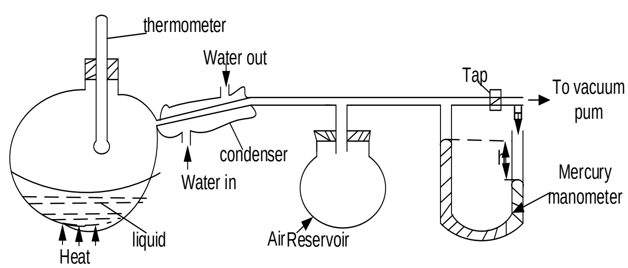
Saturated vapour increases almost exponentialy with temperature. Saturated vapour donot obey Pressure law



Pressure increases linearly with temperature. Unsaturated vapours obey pressure law

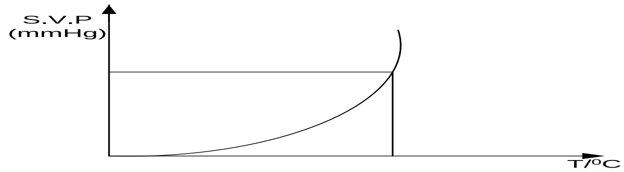


4.5.6: EXPERIMENT TO VERIFY VARATION OF SVP WITH TEMPERATURE



- The pressure above the water is set to any desired value (below or above) atmospheric pressure using a vacuum pump.
- The tap is closed and the liquid is heated until it boils.
- The temperature θ of the vapour is determined using a thermometer and noted.
- The difference, h in mercury levels is noted from the manometer.
- The pressure, p of the vapour;
  $P = H \pm h$ where H is barometric height
- The procedure is repeated for different pressures P and corresponding temperature θ noted.
- A graph of P against θ is plotted and SVP of the liquid at a particular temperature can be obtained

A graph of SVP against temperature is plotted



From the graph it can be concluded that SVP increase with temperature

Note: Never apply ($\frac{P\,V}{T}$ = constant) to saturated vapours however, it can be applied to unsaturated vapours

- If it is a mixture of gas and unsaturated vapour, apply the equation of state to the mixture.
- If it is a mixture of gas and saturated vapour, apply Dalton law of partial pressure to separate the pressure of the gas (Pg) from SVP (P_s) and apply the equation of state to gas alone
- Pressure of a mixture of gases (P) = pressure of gas (Pg) + SVP (P_s)

$$P = P_g + P_s$$

$$\boxed{P_g = P - P_s}$$

EXAMPLES

1) A closed vessel contains air saturated with water at 77°C. The total pressure in vessel is 1000mmHg. Calculate the new pressure in the vessel if the temperature is reduced to 27°C. [SVP of water at 77°C = 314mmHg, SVP of water at 27°C = 27mmHg]

Solution

$$P_g = P - P_s$$

P_{g1} = 1000 – 314 = 686mmHg

$Pg_2 = (P_2 - 27)$

$$\frac{\text{Pg1}}{\text{T1}} = \frac{\text{Pg2}}{\text{T2}}$$

$$\frac{686}{77+273} = \frac{P_{2-}27}{27+273}$$

P_2 = 615mmHg

2) A closed vessel of fixed volume contain air and water, the pressure in vessel are 20°C and 75°C are 737.5mmHg and 1144mmHg respectively. Some of the water remains a liquid at 75°C. If SVP of water are 20°C is 17.5mmHg. Find it's value at 75°C.

Solution

$$P_g = P - P_s$$

Pg_1 = (737.5 – 17.5)= 720mmHg

$Pg_2 = 1144 - Ps_2$

$$\frac{\text{Pg1}}{\text{T1}} = \frac{\text{Pg2}}{\text{T2}}$$

$$\frac{720}{20+273} = \frac{1144-\text{Ps2}}{75+273}$$

$P_{s2} = 288.8mmHg$

3) A narrow tube of uniform bore closed at the end has air trapped by small drop of water. If the atmospheric pressure 760mmHg and saturated vapour pressure of air at 10°C and 30°C are 10mmHg and 40mmHg respectively. Calculate the length of column of air at 30°C, if it is 10cm at 10°C.

Solution

Pg_1 = 760 – 10 = 750mmHg

Pg_2 = (760-40) = 720mmHg

$$\frac{P_{g1}\,V_1}{T_1} = \frac{P_{g2}\,V_2}{T_2}$$

$$\frac{P_{g1}L_1\,A}{T_1} = \frac{P_{g2}\,L_2A}{T_2}$$

$$\frac{750x10}{10+273} = \frac{720xL_2}{30+273}$$

L_2 = 11.15cm

4) A volume of $4.0x10^{-3}cm^3$ air is saturated with water vapour at 100°C. The air is cooled at 20°C at constant pressure of $1.33x10^5Pa$. calciuliate the volume of air after cooling if S.V.P of water at at 20°C is $2.3x10^3Pa$. (Atmospherice pressure $= 1.01x10^5Pa$)

Solution

$$\frac{P_{g1}\,V_1}{T_1} = \frac{P_{g2}\,V_2}{T_2}$$

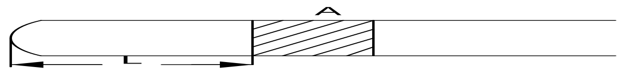
$$\frac{(1.33x10^5 - 1.01x10^5)x4x10^{-3}}{100+273} = \frac{(1.33x10^5 - 2.3x10^3)xV_2}{20+273}$$

$V_2 = 7.693x10^{-4}cm^3$

A volume

EXERCISE :40

1) State the relation between pressure and volume at constant temperature for
   (a) an ideal gas (b) a saturated vapour



A long uniform horizontal capillary tube sealed at one end and open to the air at the other contains air trapped behind a short column of water A. The length L of the trapped air column at temperature 300K and 360K is 10cm and 30cm respectively. Given that the vapour pressure of water at the same temperature are 4kPa and 62kPa respectively. Calculate the atmospheric pressure. **An(1.01 x 10^5Pa)**

2) A sealed vessel contains a mixture of air and water vapour in contact with water. The total pressure in the vessel at 27°C and 60°C are respectively 1.0 x 10^5Pa and 1.3 x 10^5Pa. If the saturated vapour pressure of water at 60°C is 2.0 x 10^4Pa what is it's value at 27°C (1Pa = 1Nm^{-2}). **An[9 x 10^2Pa]**

3) The saturation vapour pressure of water is 6 x $10^4$$Nm^{-2}$ at a temperature 360K and 0.3 x $10^4$$Nm^{-2}$ at temperature 300K. A vessel contains only water vapour at a temperature of 360K and pressure 2 x $10^4$$Nm^{-2}$. It may be assumed that unsaturated water vapour behaves like an ideal gas. If the vapour were to remain unsaturated what would be the pressure in the vessel at 300K. What is the actual pressure at this temperature and what fraction, if any of the vapour has condensed.
   An[1.7x$10^4$$Nm^{-2}$, 3.0x$10^3$$Nm^{-2}$,82%]

4) A horizontal tube of uniform bore closed at one end has some air trapped by small quantity of water. The length of air column is 20cm at 12°C. Find stating any assumption made the length of air column when the temperature is increased to 38°C. [SVP of H_2O at 12°C and 38°C are 105mmHg and 49.5mmHg respectively, atmospheric pressure = 75.0cmHg]. **An (23.04)**

4.5.1: BOILING

This is defined as the process by which a liquid turns to vapor at constant temperature (boiling point)
Boiling point of liquid is the constant temperature at which saturated vapour pressure is equal to external atmospheric pressure.

4.5.2: Explanation of boiling using kinetic theory

- Molecules of a liquid though moving randomly have attractive forces between them. When a liquid is heated molecules move faster and forces of attraction are weakened until they overcome at the boiling point temperature.
- At boiling point the saturated vapour pressure of the liquid is equal to the external pressure (atmospheric pressure plus hydrostatic pressure plus the pressure due to surface tension). The liquid molecules with enough energy escape from the bulk to the atmosphere

Effect of pressure on boiling point of a liquid

Increase of pressure raises the boiling point. Boiling takes place when SVP just exceeds external pressure. SVP increases with temperature so increase external pressure and therefore increase in boiling point

Effect of altitude on boiling point of a liquid

Boiling takes place when SVP just exceeds external pressure. Atmospheric pressure reduces with incease in altitude, therefore boiling point of a liquid decreases with increase in altitude,

Question: Explain why at a given external pressure a liquid boils at a constant temp.

A liquid boils when saturated vapour pressure is equal to the external pressure. But since the saturated vapour pressure is dependent on the temp of the liquid, then it implies that for a given external pressure the boiling will occur at a constant temperature.

Question: Explain why the temperature of a liquid does not change when the liquid is boiling.

At boiling point, there is change in state to vapour and all the heat supplied is used to do work by breaking the molecular bonds of the liquid. The temperature will not change until all the bonds are broken

NB:

- Water can be made to boil at temperature less than 100 ℃ by boiling it at higher altitude or boiling it when it is free of impurities.
- Addition of impurities raise the boiling point of a liquid since impurities absorb some of the supplied heat making the liquid to boil at a higher temperature than its normal boiling point thus faster cooking.

4.4.1: EVAPOURATION

This is the process by which a liquid become a vapour and leaves a liquid surface.

It can take place at all temperatures and only at the surface but it is greatest when the liquid is at it's boiling point.

4.4.2: Explanation using kinetic theory

- Evaporation occurs when the most energetic molecules at the liquid surface escape.
- The molecules that remain are those with low kinetic energy. Since mean kinetic energy of the molecules is directly proportional to absolute temperature, the liquid cools

4.4.3: Ways of increasing evapouration

- Increasing surface area of liquid
- Increasing temperature of the liquid
- Reducing air pressure above the liquid
- Causing a drought to remove vapour molecule before they have any chance to retain the liquid.

4.5.4: Differences between evapouration and boiling

- Boiling occurs through out the volume of the liquid while evapouration occurs at the surface.
- A liquid boils at single temp for any given external pressure whereas evaporation occurs at any temperature.

Melting

This is defined as the process by which a solid turns to liquid at constant temperature called melting point i.e.

Melting point is constant temperature at which a solid substance liquidizes at constant atmospheric pressure

Question: Explain why the temperature of a solid does not change when the solid is melting.

During melting, the heat energy supplied is used to weaken the intermolecular forces and increase separation between molecules. This increases the potential energy of the molecules but the mean kinetic energy of the molecules remain constant consequently the temperature remaining constant.

NB:

- Skaters glide/slide easily over ice because the work done against friction is transferred into internal energy which makes ice to melt forming a thin film of water between the blades of the skate and ice which eases the gliding.

- A weighed wire passes through a block of ice with out cutting it into two pieces because increased pressure due to weights on the wire lower the melting point of ice as water no longer under pressure refreezes as it gives out latent heat.
- Impurities like salt lower the melting point of a solid e.g. freezing point of pure ice is 0 ℃ but that for impure ice is less than 0 ℃.

Related explanations

- Metallic utensils being good conductors of heat, they absorb heat (from food) which would be carried away by the volatile liquid to the cooling fins thus delaying the refrigerating process. Such utensils are not recommended to be used in refrigerators.
- Milk in a bottle wrapped in a wet cloth cools faster than that placed in a bucket exposed to a drought. This is because the wet cloth speeds up the rate of evaporation thus more cooling.
- It advisable for a heavily perspiring person to stand in a shade other than drought because drought speeds up evaporation thus faster cooling which may lead to over cooling of the body and eventually this over cooling may lower the body's resistance to infections.
- When taking a bath using cold water, the individual feels colder on a very shiny day than on a rainy day because on a shiny day, the body is at high temperatures such that on pouring cold water on the body, water absorbs some of the body's heat thus its cooling. Yet on a rainy day the body is at a relatively low temperature implying that less heat is absorbed from it when cold water is poured on it.
- Two individuals; **A** (suffering from serious malaria) and **B** (normal) taking a bath of cold water at the same time of the day, **A** feels colder than **B** because the sick person's body is at relatively higher temperature than of a normal person. When cold water is poured on the sick person's body, much heat is absorbed from it compared to that absorbed from a normal person thus more coldness.
- Two normal identical individuals; **A** (takes a bath of water at 35 ℃) and **B** (takes a bath of water at 25 ℃) after the bath, **A** experience more coldness than **B**. This is because Water at 35 ℃ raises the body's temperature more than that at 25 ℃. This means that after the bath, the individual who takes a bath of water at 35 ℃ looses more heat to the surrounding than what one who takes a bath of water at 25 ℃ would loose to it.
- Water bottles are made of plastic other than glass and not fully filled because when water cools, it expands such that ice takes up a bigger volume. The un filled space is to cater for increase in volume on solidification and the bottle is made plastic to with stand breaking due to increase in volume
- A cloudy film forms on screens of cars being driven in rain because of the condensation of the excess water vapor in atmospheric moist air as a result of exceeding its dew point.

UNEB 2017 Q.6

3. (i) What is meant by **Boilin point** (01mark)
   (ii) Explain why boiling point of a liquid increases with increase in the external pressure (04marks)
4. (i) Explain how the pressure of a fixed mass of a gas can be increased at
   - Constant temperature. (03marks)
   - Constant volume. (03marks)
5. (i) Sketch a pressure versus volume curve for a real gas undergoing compression. (02marks)
   (ii) Explain the main features of the curve in (c)(i) above (03marks)
6. The cylinder of an exhaust pump has a volume of $25cm^3$. If it is connected through a valve to a flask of volume $225cm^3$ contianing air at a pressure of 75cmHg, calculate the pressure of the air I the flask after two strokes of the pump, assuming that the temperature of the air remains constant (04marks)

An(60.8cmHg)

UNEB 2016 Q.6

(a) (i) State**Dalton's law of partial pressures** (01mark)

(ii) The kinetic theory expression for the pressure P, of an ideal gas of density ρ, and mean square speed, $\overline{c^2}$ is $P = \frac{1}{3}\rho \overline{C^2}$. Use the expression to deduce Dalton's law (05marks)

(b) (i) What is meant by **isothermal** process and **adiabatic** process. (02marks)

(ii) Explain why a diabatic expansion of a gas causes cooling. (03marks)

(c) A gas at a temperature of 17°C and pressure of $1.0x10^5Pa$ is compressed isothermally to half its original volume. It is then allowed to expand adiabatically to its original volume.

(i) Sketch on a P-V curve the above processes. (02marks)

(ii) If the specific heat capacity at constant pressure is 2100$Jmol^{-1}K^{-1}$ and at constant volume is 1500$Jmol^{-1}K^{-1}$, find the final temperature of the gas. **An(219.8K)** (04marks)

(d) (i) What is meant by **a saturated vapour** (01mark)

(ii) Explain briefly the effect of altitude on the boiling point of a liquid (02marks)

UNEB 2015 Q.6

(a) Define the following terms

(i) Absolute zero (01mark)

(ii) Cooling correction (01mark)

(b) (i) State **Daltons law of partial pressure** (01mark)

(ii) The kinetic theory expression for the pressure P, of an ideal gas of density ρ, and mean square speed, $\overline{c^2}$ is $P = \frac{1}{3}\rho \overline{C^2}$. Use the expression to deduce Dalton's law (05marks)

(c) Explain clearly the steps taken to determine the cooling correction when measuring the specific heat capacity of a poor conductor by method of mixtures (07marks)

(d) The density of air at 0 and pressure of 101kPa is 1.29kgm^{-3}. Calculate pressure of 200kPa

An(Not possible) (05marks)

UNEB 2014 Q.5

(a) (i) State **two** differences between **saturated** and **unsaturated** vapours (02marks)

(ii) Sketch graphs of pressure against temperature for an ideal gas and for saturated water vapour originally at 0°C (03marks)

(b) The specific heat capacity of oxygen at constant volume is 719$Jkg^{-1}K^{-1}$ and its density at standard temperature and pressure is 1.49kgm^{-3}. Calculate the specific heat capacity of oxygen at a constant pressure $\boldsymbol{An(977.9Jkg^{-1}K^{-1})}$ (04marks)

(c) (i) With the aid of a labelled diagram, describe an experiment to determine saturated vapour pressure of water (05marks)

(ii) State how the experimental setup in (c) (i) may be modified to determine a saturated vapour pressure above atmospheric pressur3e (01mark)

(d) (i) Define an ideal gas (01mark)

(ii) State and explain the conditions under which real gases behave as ideal gas (04marks)

UNEB 2013 Q.6

(a) The pressure, P, of an ideal gas is given by $P = \frac{1}{3}\rho \overline{C^2}$**,** where ρ is the density of the ideal gas and $\overline{C^2}$ it's mean square speed.

(i) Show clearly the steps taken to derive this expression. (06marks)

(ii) State the assumptions made in deriving this expression. (02marks)

(b) Sketch the pressure versus volume curve for a real gas for temperatures above and below the critical temperature. (03marks)

(c) For one mole of a real gas, the equation of state is

$$\mathbf{(P + \frac{a}{V^2})(V - b) = RT}$$

Explain the significance of the terms $\frac{a}{V^2}$ and b (02marks)

(d) A balloon of volume 5.5$x\ 10^{-2}m^3$ is filled with helium to a pressure of 1.10$x\ 10^5$ N m^{-2} at a temperature of 20°C. Calculate the;
   (i) Number of helium atoms in the balloon **A n[1.496$x10^{24}$]** (03marks)
   (ii) Net force acting on the square metre of material of the balloon if the atmospheric pressure is 1.01$x\ 10^5\ N\ m^{-2}$ **An(9.0x10^3N)** (04marks)

UNEB 2012 Q. 6

(a) (i) Define saturated vapour pressure (01mark)
   (ii)Describe with the aid of a diagram, how saturated vapour pressure of a liquid can be determined at a given temperature. (06marks)
(b) Use the kinetic theory to explain the following observations.
   (i) Saturated vapour pressure of a liquid increases with temperature (03marks)
   (ii) Saturated vapour pressure is not affected by a decrease in volume at constant pressure
(c) When hydrogen gas is collected over water the pressure in the tube at 15°C and 75°C are 65.5cm and 105.6cm of mercury respectively. If the saturated vapour pressure at 15°C is 1.42cm of mercury, find the value at 75°C.
(d) Explain why the molar heat capacity of an ideal gas at constant pressure differs from the molar heat capacity at constant volume. (03marks).

Solution

d) From $P_g = P - P_s$
P_{g1} = 65.5 – 1.42 = 64.08cmHg
P_{g2} = 105.6 – P_{s2}

$\frac{Pg1}{T1} = \frac{Pg2}{T2}$

$\frac{64.08}{15+273} = \frac{105.6-Ps2}{75+273}$

P_{s2} = 28.17cmHg

The pressure of saturated vapour at 75 is 28.17cmHg

UNEB 2009 Q.6)

(b) Explain the following observations using the kinetic theory
   (i) A gas fills any container in which it is placed and exerts a pressure on it's walls
   (ii) The pressure of a fixed mass of a gas rises when it's temperature is increased at a constant volume.

UNEB 2008 Q.6

(b) (i) Distinguish between a real and an ideal gas (03marks)
   (ii) Derive the expression P = ⅓$\rho \overline{C^2}$ for the pressure of an ideal gas of density ρ and mean square speed $\overline{C^2}$ (06marks)
(c) (i) Explain why the pressure of a fixed mass of gas in a closed container increases when temperature of the container is raised. (02marks)

UNEB 2007 Q.7

(d) With the aid of a P-V diagram, explain what happens when a real gas is compressed at different temperatures. (04marks)
(e) The root mean square speed of the molecules of a gas is 44.72ms^{-1}. Find the temperature of the gas, if it's density is 9 x 10^{-2}kgms^{-1} and the volume is 42m^3 **An (T = $\frac{303.2K}{n}$) n is no of moles**

UNEB 2006 Q.5

(a) Define saturated vapour pressure (SVP) (01marks)
(b) Use the kinetic theory of matter to explain the following observations
   (i) Saturated vapour pressure of a liquid increases with temperature. (03marks)
   (ii) saturated vapour pressure is not affected by a decrease in volume at constant
(c) Describe how the saturated vapour pressure of a liquid at various temperatures can be determined
(d) (i) State Dalton's law of partial pressure. (01marks)
   (ii) A horizontal tube of uniform bore, closed at one end, has some air trapped by a small quantity of water. The length of the enclosed air column is 20cm at 12°C. Find stating any assumption made the length of the air column when the temperature is raised to 38°C. (SVP of water at 12°C and 38°C are 105mmHg and 49.5mmHg respectively, atmospheric pressure = 75cmHg)**An (23.04cm)** (5marks)

UNEB 2003 Q.7

(a) (i) What is meant by kinetic theory of gases (03marks)

(ii) Define an ideal gas (01marks)

(iii) State and explain the condition under which real gases behave as ideal gases

(b) (i) Describe an experiment to show that a liquid boils only when it's saturated vapour pressure is equal to the external pressure (05marks)

(ii) Explain how cooking at a pressure 76cm of mercury and a temperature of 100°C may be achieved on top of high mountains (03marks)

(c) (i) Define root – mean – square speed of molecules of a gas (01marks)

(ii) The masses of hydrogen and oxygen atoms are $1.66\ x\ 10^{-27}kg$ and $2.66\ x\ 10^{-26}$kg respectively. What is the ratio of the root mean square speed of hydrogen to that of oxygen molecules at the same temperature. **An (4:1)**

Solution

c) (ii)

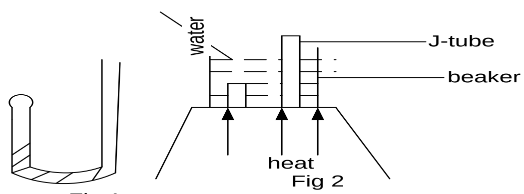


Fig 1 Fig 2

At high altitude you can cook at a pressure of 76cmHg and a temperature of 100°C by use of a pressure cooker that has a safety valve that opens when the saturated vapour pressure in side the cooker is 76cmHg. This valve ensures that the saturated vapour pressure can not exceed 76cmHg and consequently the temperature of the contents being boiled can not exceed 100°C

UNEB 2003 Q.5

(b) Indicate the different states of a real gas at different temperature on a pressure versus volume sketch graph. (03marks)

(c) (i) In deriving the expression P = ⅓$\rho\overline{C^2}$ for the pressure of an ideal gas, two of the assumptions made are not valid for a real gas. State the assumptions. (2mk)

(ii) the equation of state of one mole of a real gas is $(\mathbf{P} + \frac{\mathbf{a}}{\mathbf{V^2}})\ (\mathbf{V} - \mathbf{b})\ =\ \mathbf{nRT}$

Account for the terms $\frac{\mathbf{a}}{\mathbf{V^2}}$ and **b** (02marks)

(d) Use the expression P = ⅓$\rho\overline{C^2}$for the pressure of an ideal gas to denve Dalton's law of partial pressures.

(e) Explain with the aid of a volume versus temperature sketch graph, what happen to a gas cooled at constant pressure from room temperature to zero Kelvin (4mk)

UNEB 2002 Q.2

(a) State the assumptions made in the derivation of the expression P = ⅓$\rho\overline{C^2}$for pressure of an ideal gas

(b) Use the expression in (a) above to deduce Dalton's law of partial pressure (03marks)

(c) Describe an experiment to determine the saturation vapour pressure of a liquid (06marks)

CHAPTER5: HEAT TRANSFER

There are 3 ways of heat transfer namely;

- Conduction
- Radiation
- Convection

5.1: CONDUCTION

This is the process of heat transfer through a substance from region of high temperature to low temperature without the bulk movement of the molecules.

It is mainly due to collision between atoms that vibrate about their fixed positions

5.1.2: MECHANISMS OF HEAT CONDUCTION

a) IN NON METALLIC SOLIDS AND FLUIDS (poor conductors).

When one end of a poor conductor is heated, atoms at the hot end vibrate with increased amplitudes, collide with neigboruring atoms and lose energy to them. The neighbouring atoms also vibrate with increased amplitudes, collide with adjacent atoms and lose energy to them. In this way, heat energy is transmitted from one end to the other.

b) IN METALS (good conductors).;

- Metals have free electrons. When heated the electrons at the hot end gain more energy and transfer energy as they collide with atoms in solid lattice.
- The mechanism of heat transfer by atomic vibrations also occurs in good conductors but its effect is much smaller

Question: Explain why metals are better conductor than non metallic solids.

In metals heat is carried by inter atomic vibration just like in non- metallic solid. But in addition to this, metals have free electrons in their lattice that move with very high velocity when heated since they are light. So they pass on their heat energy due to collision with the atoms in metallic lattice and this occurs at faster rate

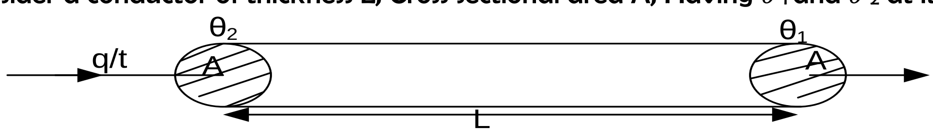
5.1.3: THERMAL CONDUCTIVITY (K)

Thermal conductivity is the rate of heat flow at right angles to the opposite faces of 1m^3 of material when temperature difference across the faces is 1 Kelvin,

S.I unit of K is $W\ m^{-1}k^{-1}$

OR Is the rate of heat flow through material per unit cross-sectional area per unit temperature gradient

Consider a conductor of thickness L, Cross sectional area A, Having θ_1 and θ_2 at its end. (θ_2>θ_1)



The rate of heat flow per second is directly proportional to the cross sectional area and the temperature difference but inversely proportional to thickness i.e

$$\frac{Q}{t} \propto \frac{A(\theta_2 - \theta_1)}{L} \qquad \frac{Q}{t} = \frac{KA(\theta_2 - \theta_1)}{L}$$

K is called co efficient of thermal conductivity of given material which depends on nature of material.

5.1.5: FACTORS ON WHICH RATE OF HEAT FLOW ($\frac{Q}{t}$) DEPENDS.

- It depends on cross sectional area A
- It depends on temperature gradient between faces ($\frac{\theta_2 - \theta_1)}{l}$
- It depends on nature of material (thermal conductivity K)

Definition: Temperature gradient of a conductor is the ration of the difference in temperature between the ends of the conductor tot the length of the conductor

Why at steady state the rate of thermal energy transfer is the same in both layer

No heat is lost to the surrounding as it flows form inner to outer surface. The temperature gradient across the composite surface remains constant

Examples

1. An aluminum plate of cross section area 300cm^2 and thickness 5cm has one side maintained at 100°C by steam and another side by 30°C .The energy passes through the plate at a rate of 9kW.Calculate the coefficient of thermal conductivity of aluminum.

Solution

$$K = \frac{L\frac{Q}{t}}{A(\theta_2 - \theta_1)}$$

$$K = \frac{5x10^{-2}x9000}{300x10^{-4}x(100-30)}$$

K = 214.29Wm^{-1}K^{-1}

2. Calculate the rate of loss of heat through a window glass of thickness 6mm and area $2m^2$. If the temperature difference between the two sides is 20℃. Thermal conductivity of glass =0.8W$m^{-1}K^{-1}$

Solution

$$\frac{Q}{t} = \frac{K_b A(\Delta\theta)}{L_b}$$

$$\frac{Q}{t} = \frac{0.8x2(20)}{6x10^{-3}}$$

$$\frac{Q}{t} = 5.3x10^3 W$$

3. Calculate the quantity of heat conducted through $2m^2$ of a brick wall 12cm thick in 1 hour, if the temperature on one side is 18℃ and on the other side is 28℃. Thermal conductivity of brick 0.13Wm^{-1}K^{-1}

Solution

$$\frac{Q}{t} = \frac{K_b A(\Delta\theta)}{L_b}$$

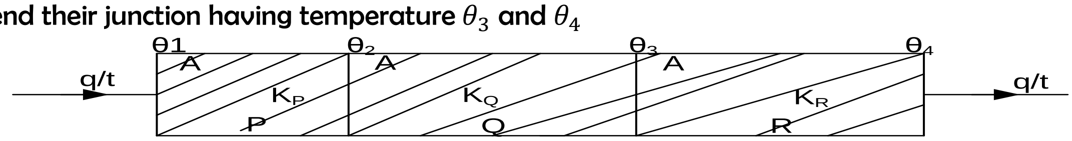
$$Q = \frac{0.13x2(28-18)}{12x10^{-2}}x1x3600$$

$$Q = 1.56x10^5 J$$

5.1.6: HEAT FLOW THROUGH SEVERAL SURFACES

i) Surface in Series

Consider 3 plates PQR of thermal conductivity K$_P$, K$_Q$, K$_R$ respectively whose ends are maintained at θ_1 to $\theta_2$end their junction having temperature θ_3 and θ_4



$$\frac{Q}{t} = \frac{K_P A(\theta_1 - \theta_2)}{L_P} = \frac{K_Q A(\theta_2 - \theta_3)}{L_Q} = \frac{K_R A(\theta_3 - \theta_4)}{L_R}$$

EXAMPLES

1) A sheet of rubber and a sheet of card board, each 2mm thick, are pressed together and their outer faces are maintained respectively at 0℃ and 25℃. If the thermal conductivities of rubber and cardboard are respectively 0.13 and 0.05 W m^{-1} K^{-1}, find the quantity of heat which flows in 1 hour across the composite sheet of area 100cm^2

Solution

$$\frac{Q}{t} = \frac{K_R A(\theta - 0)}{L_R} = \frac{K_B A(25 - \theta)}{L_B}$$

$$\frac{Q}{t} = \frac{0.13A(\theta - 0)}{2x10^{-3}} = \frac{0.05A(25 - \theta)}{2x10^{-3}}$$

$$\frac{0.13A(\theta - 0)}{2x10^{-3}} = \frac{0.05A(25 - \theta)}{2x10^{-3}}$$

$$\theta = 7℃$$

$$Q = \frac{0.13x100x10^{-4}(\theta - 0)}{2x10^{-3}}x1x60x60$$

$$Q = 1.64x10^4 J$$

2) Two brick walls each of thickness 10cm are separated by air gap of thickness10cm, the outer faces of brick walls are maintained at 20° C and 5°C respectively. Calculate temperature of inner surface of walls. Compare the rate of heat loss through the layer of air with heat through a single brick wall (thermal conductivity of air 0.02Wm^{-1}K^{-1} and that of bricks 0.6W m^{-1} K^{-1})

Solution

$$\frac{Q}{t} = \frac{K_bA(20-\theta_1)}{L_b} = \frac{K_aA(\theta_1-\theta_2)}{L_a} = \frac{K_bA(\theta_2-5)}{L_b}$$

$$\frac{K_bA(20-\theta_1)}{L_b} = \frac{K_aA(\theta_1-\theta_2)}{L_a}$$

$$\frac{0.6(20-\theta_1)}{10x10^{-2}} = \frac{0.02x(\theta_1-\theta_2)}{10x10^{-2}}$$

$0.62\theta_1 - 0.02\theta_2$ =.......... 1

$$\frac{K_aA(\theta_1-\theta_2)}{L_a} = \frac{K_bA(\theta_2-5)}{L_b}$$

$$\frac{0.02(\theta_1-\theta_2)}{10x10^{-2}} = \frac{0.6x(\theta_2-5)}{10x10^{-2}}$$

$0.02\theta_1 - 0.62\theta_2 = -3$ 2

Solving expression (1) and (2) simultaneously.

θ_1 = 19.5°C

θ_2 = 5.5°C

(ii) $\frac{Q}{t} = \frac{K_bA(20-\theta_1)}{L_b} = \frac{0.6(20-19.5)A}{10x10^{-2}}$

=3A

$\frac{Q}{t} = \frac{K_aA(\theta_1-\theta_2)}{L_a} = \frac{0.02(19.5-5.5)A}{10x10^{-2}}$

=2.8A

$= \frac{3A}{2.8A}$

=3:2.8

3) A window of height 1m and width 1.5m contain double glazed unit of two single glass plates each of thickness 4.0mm separated by air gap of 2.0mm.Calulate the rate at which heat is conducted through the window if the temperature of external surface of glass are 20°C and 30°C respectively.(Thermal conductivity of glass and air are 0.72Wm^{-1}K^{-1} and 0.025Wm^{-1}K^{-1} respectively).

Solution

$$\frac{Q}{t} = \frac{K_gA(30-\theta_1)}{L_g} = \frac{K_aA(\theta_1-\theta_2)}{L_a} = \frac{K_gA(\theta_2-20)}{L_g}$$

$$\frac{K_gA(30-\theta_1)}{L_g} = \frac{K_aA(\theta_1-\theta_2)}{L_a}$$

$$\frac{0.72A(30-\theta_1)}{4x10^{-3}} = \frac{0.025A(\theta_1-\theta_2)}{2x10^{-3}}$$

$12.5\theta_2 - 192.5\theta_1 = -5400$ 1

$$\frac{K_gA(30-\theta_1)}{L_g} = \frac{K_gA(\theta_2-20)}{L_g}$$

$30 - \theta_1 = \theta_2 - 20$

$\theta_1 + \theta_2 = 50$ 2

Solving expression 2 and 1 simultaneously

θ_1= 29.4°C, θ_2 = 20.61°C

$$\frac{Q}{t} = \frac{K_gA(30-\theta_1)}{L_g}$$

$$\frac{Q}{t} = \frac{0.72x1.5(30-29.4)}{4x10^{-3}}$$

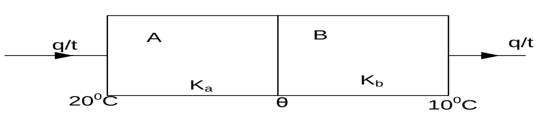
$\frac{Q}{t}$ = 164.7Js^{-1}

4) A wall 6m by 3m consists of two layers A and B of thermal conductivities 0.6Wm^{-1}K^{-1} and 0.5Wm^{-1}K^{-1} respectively. The thickness of layer is 15.0cm.The inner surface of layer A is at temperature of 20°C while outer layer B is at temperature of 10°C.Calculate

(i) The temperature of interface of A and B.

(ii) The rate of heat through wall.

Solution



$$\frac{Q}{t} = \frac{K_aA(20-\theta)}{L_a} = \frac{K_bA(\theta-10)}{L_b}$$

$$\frac{0.6A(20-\theta)}{15x10^{-2}} = \frac{0.025A(\theta-10)}{15x10^{-2}}$$

$6(20-\theta) = 0.5(\theta-10)$

θ = 15.45°C

ii) $\frac{Q}{t} = \frac{K_aA(20-\theta)}{L_a}$

$= \frac{0.6A(20-\theta)}{15x10^{-2}}$

$= \frac{0.6x6x3(20-15.45)}{15x10^{-2}}$

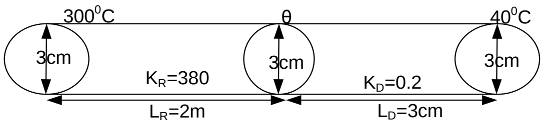
$\frac{Q}{t}$ = 324Js^{-1}

5) A copper rod 2m long and of diameter 3cm is lagged. One end is maintained at 300°C, the other end is placed against 3cm thick card board disk of same diameter as the rod. The free end of disk is maintained at 40°C .Calculate;

(i) Steady state temperature at copper card board junction.

(ii) Quantity of heat flowing against junction in 10 minutes.
(Thermal conductivity of copper and card board are 380 and 0.2$Wm^{-1}K^{-1}$respectively).

Solution



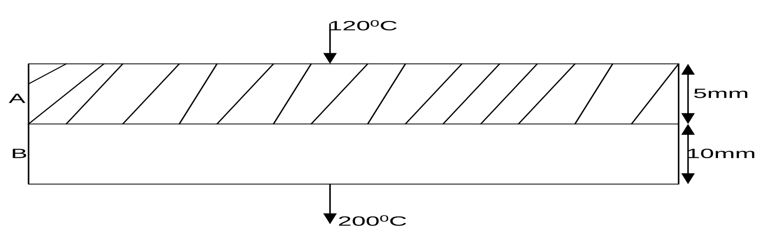
$$\frac{Q}{t} = \frac{K_R A(300-\theta)}{L_R} = \frac{K_b A(\theta - 40)}{L_b}$$
$$\frac{380(300-\theta)}{2} = \frac{0.2(\theta-40)}{3x10^{-2}}$$
$$57000 - 190\theta = 6.667\theta - 266.667$$
θ = 291.19°C

$$\frac{Q}{t} = \frac{K_R A(300-\theta)}{L_R}$$
Area = $\frac{\pi d^2}{4}$

$$\frac{Q}{t} = \frac{380x\left[\frac{\frac{22}{7}(3x10^{-2})^2}{4}\right](300-291.19)}{2}$$
$$\frac{Q}{t} = 1.183Js^{-1}$$
$$Q = 1.183xt = 1.183x10x60 = 709.8J$$

EXERCISE: 41

1) A well lagged composite metal bar of uniform cross section area 2cm^2 is made by joining 40cm rod of copper to 25cm rod of Aluminium. The extreme ends of the bar are maintained respectively at 100° C and 0°C respectively. Calculate;
   (i) The temperature of junction of two rods.
   (ii) Rate of heat flow
   (Thermal conductivity of copper and Aluminum is 386 and 210 $Wm^{-1}k^{-1}$ respectively).
   An (i) 53.5°C (ii) 8.9745J
2) A rectangular room 12m by 10m has vertical walls 4m high to support the roof. The walls and a roof are 25cm thick and are made of material 0f thermal conductivity 0.25$Wm^{-1}K^{-1}$.Tthe door and window covers area 16m^2 and are made of glass of thickness 5mm and thermal conductivity 1.2$Wm^{-1}K^{-1}$,If the room is maintained at constant temperature above that of its surrounding. Calculate the percentage heat loss by conduction through the doors and window. Heat losses through the floor may be neglected. **An(93.7%)**
3) A concrete floor of a hall has dimensions of 10.0m by 8.0m.It is covered with carpet of thickness 2.0cm.The temperature inside the hall is 22°C while that of the surrounding just below the concrete is 12°C.Thermal conductivity of concrete and carpet are 1 and 0.05$Wm^{-1}k^{-1}$ respectively and thickness of concrete is 10cm.Calculate
   (i) Temperature at the interface of concrete and Carpet
   (ii) The rate at which flow through the floor. **An(14° C , 1600W)**
4)



The metal conductors A and B each of radius 20cm and thickness 5mm and 10mm respectively are placed in contact as shown above. The upper surface of A and lower surface of B are maintained at temperature of 120°C and 200° C respectively. Calculate;
   (i) Temperature of interface
   (ii) Rate of hard flow through A **An(138.9°C, 99.75 x 10^{3}w)**
   (Thermal conductivities of A and B are 210 and 130 $Wm^{-1}K^{-1}$ respectively)

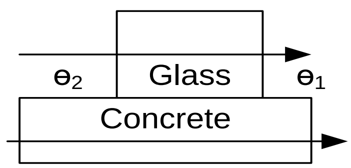
5) A composite slab is made of two materials A and B of thickness 6cm and 3cm respectively and thermal conductivities 120, 80 $Wm^{-1}K^{-1}$ respectively. If the external surfaces of A and B are kept at 70℃ and 20℃ respectively.

(i) Calculate the temperature of the junction of the materials if the slabs are uniform
(ii) Find the rate of heat flow through a unit area of the slab **An(41.43°C, 57140W)**

ii) Surface in parallel

1) A small green house consists of $34m^2$ of glass of thickness 3.0mm and $9.0m^2$ of concrete wall of thickness 0.080m. On a sunny day, the interior of the green house receives a steady 25kW of solar radiation. Estimate the difference in temperature between inside and outside of the green house. The temperature inside and outside may be assumed uniform and heat transfers downwards into the ground inside the green house may be neglected.(Thermal conductivity of glass and concrete are 0.85 $Wm^{-1}K^{-1}$, 1.5 $Wm^{-1}K^{-1}$ respectively)

Solution



$$Total\ heat = heat\ flow\ through\ glass + heat\ flow\ through\ concret$$

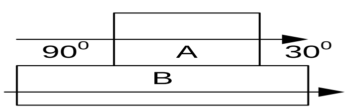
$$\frac{Q}{t} = \frac{K_G A(\theta_{2-}\theta_1)}{L_G} + \frac{K_C A(\theta_{2-}\theta_1)}{L_C}$$

$$25000 = \frac{0.85x34(\theta_{2-}\theta_1)}{0.003} + \frac{9x1.5(\theta_{2-}\theta_1)}{0.08}$$

$$\theta_{2-}\theta_1 = 2.55℃$$

2) Two perfectly lagged metal bars A and B, each of length 20cm, are arrange in parallel, with their hot ends maintained at 90°C and their cold ends at 30°C. If the cross sectional area of each bar is 2.5cm^2, find the rate of heat flow through the parallel bars. (Thermal conductivity of A and B are 400 $Wm^{-1}K^{-1}$, 200 $Wm^{-1}K^{-1}$ respectively)

Solution



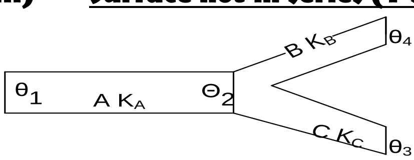
$$Total\ heat = heat\ flow\ through\ A + heat\ flow\ through\ B$$

$$\frac{Q}{t} = \frac{K_G A(90-30)}{L_G} + \frac{K_C A(90-30)}{L_C}$$

$$\frac{Q}{t} = \frac{400x2.5x10^{-2}x60}{0.2} + \frac{200x2.5x10^{-2}x60}{0.2}$$

$$\frac{Q}{t} = 45W$$

iii) Surface not in series (Y shaped)



$$\frac{Q}{t} = \frac{K_A A(\theta 1-\theta_2)}{L_A} = \frac{K_B A(\theta_{2-}\theta_3)}{L_B} + \frac{K_C A(\theta_{2-}\theta 4)}{L_C}$$

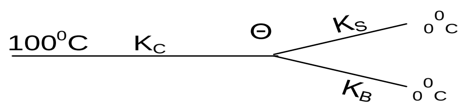
EXAMPLE

Rods of copper, brass and steel are welded together to form Y-Shaped figure. The cross sectional area of each rod is $2cm^2$ .The end of copper rod maintained at 100°C and the ends of brass and steel rod at 0°C, assume that there is not heat loss from surface of rod and that length of rods are 46cm, 13cm and 12cm respectively. Calculate the;

(i) temperature of junction.
(ii) heat current in the copper rod

(thermal conductivities of copper, brass and steel are respectively 385$Wm^{-1}K^{-1}$, 109$Wm^{-1}K^{-1}$ and 50.2$Wm^{-1}K^{-1}$

Solution



$\frac{Q}{t} = \frac{K_C A(100-\theta)}{L_C} = \frac{K_B A(\theta-0)}{L_B} + \frac{K_S A(\theta-0)}{L_S}$

$$\frac{385(100-\theta)}{0.46} = \frac{109(\theta-0)}{0.13} + \frac{50.2(\theta-0)}{0.12}$$

8369565−836.9565 θ=418.33 θ + 838.46 θ

θ = 39.97℃

$$\frac{Q}{t} = \frac{K_C A(100-\theta)}{L_C}$$

$\frac{Q}{t} = \frac{3852x10^{-4}(100-39.97)}{0.46} =$ **10.05 JS⁻¹**

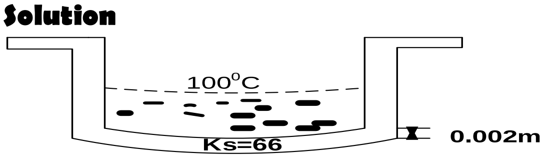
5.1.7: RELATIONSHIP BETWEEMN RATE OF HEAT FLOW AND LATENT HEAT OF VAPOURISATION.

$\frac{Q}{t} = ML.$ Where M= Mass per unit time L= Latent heat of Vapourisation

EXAMPLE

1) An Iron saucepan containing water which boils steadily at 100°C stands on a hot plate and heat is conducted through the base of the pan of area 4m^2 and uniform thickness 2 x 10^{-3}m. If water evaporate at a rate of 0.09 kg/min. Calculate the surface temperature of out side surface of the pan.(Thermal conductivity of Iron = 66Wm^{-1}K^{-1} and Lv = 2.2 x 10^6 JK^{-1})

Solution



$\frac{Q}{t}$ = ML= $\frac{K_S A(\theta-100)}{L_S}$

$\frac{0.09}{60}$ x 2.2 X10^6= $\frac{66x0.04(\theta-100)}{2x10^{-3}}$

θ = 100.025℃

2) A copper kettle has Circular base of radius 10cm and thickness 3mm, the upper surface of base is covered with a uniform layer of soot 1mm thick. Kettle contains water which is boiled to boiling point by an electrical heat. In steady state 5g of steam are produced each minute. What is the temperature of the lower surface of the base assuming that heat conduction from the side of the kettle can be ignored (thermal conductivity of copper and soot respectively are 390Wm^{-1}K^{-1} and 13.0Wm^{-1}K^{-1} and Lv= 2.26 x 10^{6}Jkg^{-1}.)

Solution

$\frac{Q}{t}$ = ML = $\frac{5x10^{-3}}{60}$ x 2.26 x 10^{+6} $= 188.333$

$$\frac{Q}{t} = \frac{K_S A(\theta_2-\theta_1)}{L_S} = \frac{K_k A(\theta_{1-}100)}{L_k}$$

188.333 = $\frac{13\pi x(10x10^{-2})^2(\theta_2-\theta_1)}{1x10^{-3}}$

0.188333 = 0.4084$\theta_2 - 0.4084\theta_1 \ldots\ldots..1$

Also: 188.333 = $\frac{390\pi x(10x10^{-2})^2(\theta_1-100)}{3x10^{-3}}$

0.564999 = 12.2522θ_1 - 12.2522x100.....2

$\theta_1 = 100.46$

Put into eqn1

0.188333 = 0.4084$\theta_2 - 0.4084x100.46$

θ_2 = 105.06℃

EXERCISE: 42

1) Water contained in an aluminum kettle on a stove steadily boiling away at 100° C at a rate of 3.68 x 10^{-4}kgs^{-1}.The base has an area of 6.0 x 10mm and thickness 4mm.Calculate;
   (i) The rate of heat flow through the base **An (882J)**
   (ii) The temperature of lower surface of the base. **An (102.6°C)**

   [Thermal conductivity of Aluminium 210 Wm^{-1}K^{-1}, $S.L.v$ of H_2O= 2.26 X 10^{6}Jkg^{-1})

2) One end of a perfectly lagged metal bar of length 10cm is kept at 100℃ while the other end is in contact with ice. Find the rate at which the ice melts if the thermal conductivity of the metal is 400 Wm^{-1}K^{-1} and its cross-sectional area is $5x10^{-4}m^2$ and specific latent heat of fusion of ice is $3.36x10^5 Jkg^{-1}$. **An**(5.95$x10^{-4}kgs^{-1}$)

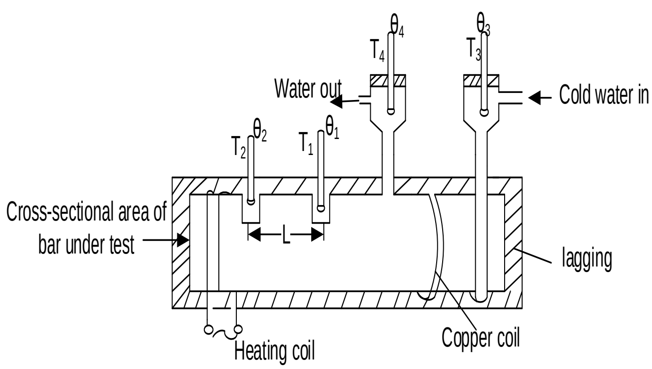
3) One end of a perfectly lagged copper bar of length 12cm is kept in boiling water while the other end is in contact with melting ice. Find the;
   (i) Energy flow per second through the bar
   (ii) Mass of ice which melts in 15s
   (if the thermal conductivity of the copper is 350 $Wm^{-1}K^{-1}$ and its cross-sectional area is $1.5cm^2$ and specific latent heat of fusion of ice is $3.34x10^5Jkg^{-1}$). **An**($48.1W, 2.16g$)

4. A layer of boiler scale deposits on the inside of boiler, in order to maintain same rate of heat flow. What will be the temperature difference between the exposed surface of the boiler If the deposit is 5mm thick (SLv .H of water 2.27 x 10^6 Jkg^{-1}, thermal conductivity of boiler scale 4.7 $Wm^{-1}K^{-1}$ **AN. (359.6°C)**

5. Ice is forming on the surface of a pond. When it is 4.6cm thick, the temperature of the surface of the ice in contact with air is 260K, while the surface in contact with the water is at temperature 273K. calculate the;
   (i) rate of heat per unit are from the water
   (ii) Rate at which the thickness of the ice is increasing
   (if the thermal conductivity of the ice is 2.3 $Wm^{-1}K^{-1}$ and specific latent heat of fusion of ice is $3.25x10^5Jkg^{-1}, density\ of\ water = 1000kgm^{-3}$). **An**(6.5$x10^2Wm^{-2}, 2.0x10^{-3}mms^{-1}$)

a) DETERMINATION OF THERMAL CONDUCTIVITY K OF A GOOD CONDUCTOR OF HEAT E.G AMETAL LIKE COPPER USING SEARLE'S METHOD

Searle's is best suited for a good conductor because it achieves measurable temperature gradient and measurable heat flow and this can be obtained by good conductor.

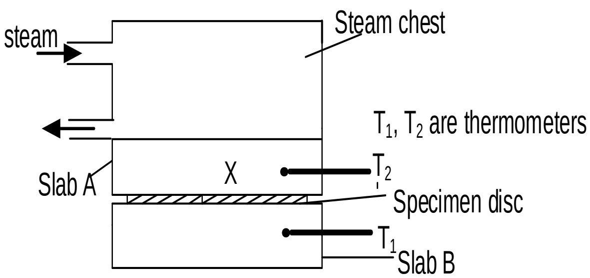


- A long copper bar of cross-sectional area A is used.
- It carries a heater at one end and copper coil soldered at the other end.
- Two thermometers are inserted in the holes drilled in the bar at a known separation l
- The holes are smeared with smeared with mercury for good thermal contact
- Water is allowed to flow through the copper coil and the heater is switched on.
- When the thermometers read steady temperatures θ_1, θ_2, θ_3 and θ_4 are recorded from thermometers T_1, T_2, T_3 and T_4 respectively.
  $\frac{Q}{t} = \frac{KA(\theta_2-\theta_1)}{l}$ where k is thermal conductivity of copper metal
- The mass m of water flowing out per second through the coil is determined.
  $\frac{Q}{t} = mc(\theta_4 - \theta_3)$ where c is specific heat capacity of water
- Therefore thermal conductivity, k of a good conductor is got from

$$\boxed{K = \frac{MCL(\theta_4-\theta_3)}{A(\theta_2-\theta_1)}}$$

b) DETERMINATION OF THERMAL CONDUCTIVITY (K) OF A POOR CONDUCTOR E.G RUBBER, GLASS USING CHEST OR LEE DISK METHOD.

For a poor conductor, the material has to be made thin so that a measurable temperature gradient can be obtained



- ❖ A sample in the form of a disc of small thickness t and diameter, D is used.
- ❖ The thin disc is sand wiched between two metal slabs A and B each carrying a thermometer.
- ❖ Steam is passed through the steam chest until the thermometers record steady temperatures θ_2 and θ_1 which are recorded.

$$\frac{Q}{t} = \frac{KA(\theta_2-\theta_1)}{t} \text{......................1}$$

- ❖ The sample is withdrawn and block B is heated directly when in contact with A until its temperature is about 10°C above θ_1.
- ❖ The steam chest is removed and the disc is placed on top of slab B.
- ❖ the temperature of the slab B is recorded at suitable time intervals.
- ❖ A cooling curve is plotted and the slope s of the graph at θ_1 is determined.
- ❖ The mass, m of slab B of specific heat capacity, c is determined.

$$\frac{Q}{t} = mcs$$

- ❖ Thermal conductivity, k of the disc is got from

$$mcs = \frac{k\,\pi d^4}{4}\frac{(\theta_2-\theta_1)}{t}$$

Precautions

- ➢ Sample is a thin disc
- ➢ Faces of the disc are highly polished and clean
- ➢ A thin layer of grease is smeared on the faces for good thermal contact

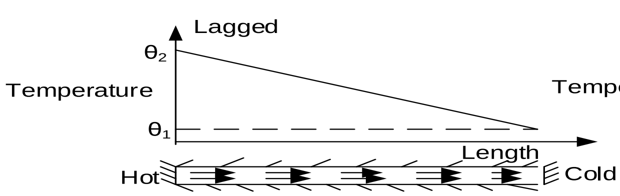
5.1.9: VARIATION OF TEMPERATURE ALONG A BAR WHICH IS :

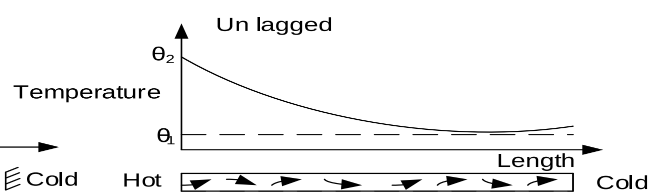
1. Lagged.

When a metal bar is fully lagged, no heat is lost to the surrounding, rate of heat flow along the bar is the same hence temperature fall along the bar is uniform

2.Un lagged

When a metal bar is fully unlagged, heat is lost to the surrounding, rate of heat flow along the bar is not the same hence temperature gradient along the bar decreases with distance from the hot end to cold end.

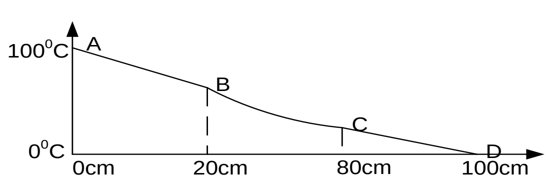




EXAMPLE

1) Two end of metal bar of length 1m are perfectly lagged up to 20cm from either end. The end of the bar is maintained at 100 and 0°C respectively.
   (i) Sketch a graph temp vs distance along the bar
   (ii) Explain the features of graph in (i) above

Solution



- ❖ Along AB, the bar is lagged and therefore the rate of heat flow is uniform along.
- ❖ Along BC, the bar is un lagged and therefore the heat is lost to the surroundings hence the rate of heat flow decreases as you move from B to C.
- ❖ Along CD, the bar is lagged and therefore the rate of heat flow is uniform along this section. However is it at lower rate that at AB since the heat was lost to surrounding along BC.

EXERCISE: 43

3) An ideally lagged composite bar 25cm long consists of a copper bar 15cm long joined to an aluminum bar 10cm long and of equal cross sectional area. The free end of the copper is maintained at 100°C and the free end of aluminum at 0°C. Calculate the temperature gradient in each bar when steady state conditions have been reached. (thermal conductivity of copper=390Wm^{-1} °C^{-1}, thermal conductivity of aluminum= 210Wm^{-1}°C^{-1}). **An [copper = $3x10^2$ °C m^{-1}, aluminum= $5.5x\ 10^2$ °C m^{-1}]**

4) If a copper kettle has a base of thickness 2.0mm and area 3.0x10^{-2}m^2, estimate the steady difference in temperature between inner and outer surface of the base which must be maintained to enable enough heat to pass through so that the temperature of 1kg of water rises at the rate of 0.25Ks^{-1}. Assume that there are no heat losses, the thermal conductivity of copper = 3.8x10^{2}Wm^{-1}K^{-1} and the specific heat capacity of water = 4.2x10^{3}Jkg^{-1}K^{-1}. After reaching the temperature of 373K, the water is allowed to boil under the same conditions for 120 seconds and the mass of water remaining in the kettle is 0.948kg. Deduce a value for the S.L.H of vaporization of water. **An[0.2°C, 2.4x10^{6}Jkg^{-1}]**

5) A cubical container full of hot water at a temperature of 90°C is completely lagged with an insulating material of thermal conductivity 6.4x10^{-2} Wm^{-1} °C^{-1}. The edge of the container are 1.0m long and the thickness of the lagging is 1.0cm. estimate the rate of flow of heat through the lagging if the external temperature of the lagging is 40°C. Mention any assumptions you make in deriving your result. Discuss qualitatively how your result will be affected if the thickness of the lagging is increased considerably assuming that the temperature of the surrounding air is 18°C. **An [1.9x10^{3}W]**

6) A thin walled hot water tank, having a total surface area 5m^2, contains 0.8m^3 of water at temperature of 350K. it is lagged with a 50mm thick layer of material of thermal conductivity 4x10^{-2}Wm^{-1}K^{-1}. The temperature of the outside surface of the lagging is 290K. What electrical power must be supplied to an immersion heater to maintain the temperature of the water at 350K. (Assume the thickness of the copper walls of the tank to be negligible). What is the justification for the assumption that the thickness of the copper walls of the tank may be neglected? (Thermal conductivity of copper=400 Wm^{-1}K^{-1})
If the heater were switched off, how long would it take for the temperature of the hot water to fall 1K. (Density of water =1000kgm^{-3}, specific heat capacity of water=4170 Jkg^{-1}K^{-1}) **An [240W, 232min]**

7) A window pane consists of a sheet of glass of area 2.0m^2 and thickness 5.0mm. if the surface temperature are maintained at 0°C and 20°C. Calculate the rate of flow of heat through the pane assuming a steady state is maintained. The window is now double glazed by adding a similar sheet of glass so that a layer of air 10mm thick is trapped between the two panes. Assuming that the air is still, calculate the ratio of the rate of flow of heat through the window in the first case to that in the second (conductivity of glass =0.80 Wm^{-1}K^{-1}, conductivity of air =0.025 Wm^{-1}K^{-1})An**[6400W, 66:1].**

8) An iron pan containing water boiling steadily at 100°C and stands on a hot-plate and heat conducted through the base of the pan evaporates 0.09kg of water per minute. If the base of the pan has an area of 0.04m^2 and a uniform thickness of 2x10^{-3}m, calculate the surface temperature of the underside of the pan. [Thermal conductivity of iron
=66 Wm^{-1}K^{-1} and S.L.H of evaporation of water at 100°C =2.2 x10^{6}Jkg^{-1}] **An[102.5°C]**

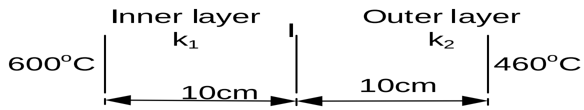
9) (a) A sheet of glass has an area of 2.0m^2 and a thickness of 8.0x10^{-3}m. The glass has a thermal conductivity of 0.80 Wm^{-1}K^{-1}. Calculate the rate of heat transfer through the glass when there is temperature difference of 20K between its faces. **An[4.0kW]**
(b) A room in a house is heated to a temperature 20K above that outside. The room has 2m^2 of windows of glass similar to the type used in(a). Suggest why the rate of heat transfer through the glass is much less than the value calculated above.
(c) Explain why two sheets of similar glass insulate much more effectively when separated by a thin layer of air than when they are is contact.

10) Outline an experiment to measure the thermal conductivity of a solid which is a poor conductor, showing how the results is calculated from the measurements
Calculate the theoretical percentage change in heat loss by conduction achieved by replacing a single glass window by a double glass separated by 10mm of air. In each case the glass is 2mm thick (The ratio of the thermal conductivities of glass to air is 3:1)

Suggest why, in practice the change would be much less than that calculated. **An[94%]**

11) A double glazed window consists of two panes of glass each 4mm thick separated by a 10mm layer of air. Assuming the thermal conductivity of glass to be 50 times greater than that of air, calculate the ratios:
    (a) Temperature gradient in the glass to that in air gap
    (b) Temperature difference across one pane of the glass to temperature difference across the air gap. **An[0.02, 0.008]**

12) The diagram shows a furnace wall which is constructed of two types of brick. The temperatures of the inner and outer surfaces of the wall are 600 °C and 460 °C respectively, as shown in the diagram. The value of the thermal conductivity, k_1, for the inner layer of the furnace wall is $0.8Wm^{-1}K^{-1}$ and that of the outer layer, k_2, is $1.6Wm^{-1}K^{-1}$



(i) Explain why, in steady state, the rate of thermal energy transfer must be the same in both layers
(ii) Determine the temperature at the interface, I, between the layers. **An(507 °C)**
(iii) Sketch and label a graph which shows the variation of temperature with distance a cross the wall

5.2: RADIATION

Thermal radiation is a means of heat flow from hot places to cold places by means of electromagnetic waves.

Radiation emitted by a hot body is a mixture of different wavelength. The amount of radiation for a given wavelength depends on the temperature of the body.At lower temperature, the body emits mainly infrared and at high temperatures the body emits ultraviolet, visible in addition to infrared

5.2.1:Infrared radiations

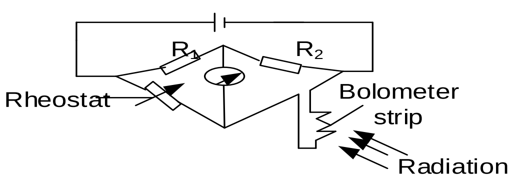
Infrared is part of electromagnetic spectrum extending from 0.7μm to about 1mm

5.2.2:Properties of infrared radiation(electromagnetic radiations)

- Move at a speed of light ($3x10^8ms^{-1}$)
- It can be reflected and refracted just like light
- Cause an increase in temperature when absorbed by matter
- It can cause photo electric emission surface
- It affects special types of photographic plates and it enables pictures to be taken in dark
- It is absorbed by glass but is transmitted by rock salt and quartz

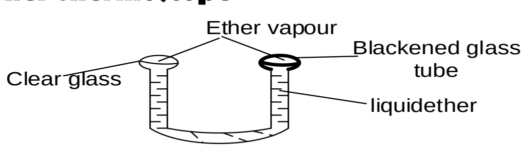
5.2.3:Detection of infrared radiations

Bolometer



- A bolometer is connected to a wheatstone bridge circuit and its resistance measured.
- The radiation is allowed to fall on the bolometer which is then absorbed and the temperature increases
- The new resistance of the bolometer is also measured.
- An increase in resistance obtained detects infrared radiations

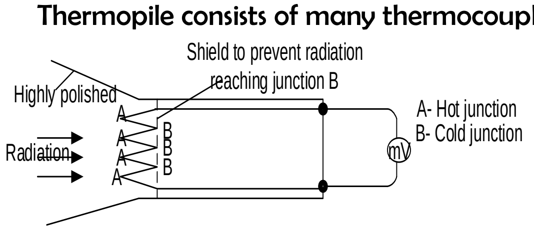
Ether thermoscope



- A mixture of air and ether vapour is tapped in a tube partly filled with liquid ether.
- When infrared radiations fall on the apparatus, the liquid ether rises into the clear bulb while the level falls in the blackened bulb. This is because the blackened bulb absorbs more than the clear bulb.
- This shows that a blackened surface is a better absorber of thermal radiations than a shiny polished surface and therefore detects infrared radiation

Thermopile

Thermopile consists of many thermocouples connected in series



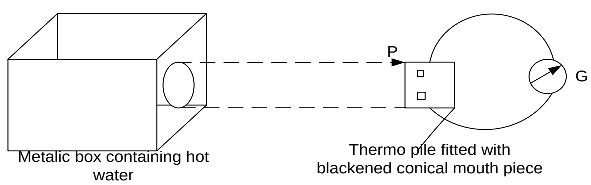
- Radiation falling on junction A is absorbed and temperature rises above that of junction B.
- An $E.m.f$ is generated and is measured by mill voltmeter connected directly to the thermopile and deflects as a result.

5.2.4: PREVOST'S THEORY OF HEAT EXCHANGE

Its states that, when a body is in thermodynamic equilibrium with its surrounding, its rate of emission of radiations to the surrounding is equal to its rate of absorption of radiations from the surrounding.

It is concluded in provosts theory that a good absorber of radiation, must also be a good emitter otherwise its temperature would rise above that of its surrounding.

5.2.5: EXPERIMENT TO DETERMINE WHICH SURFACE ARE GOOD ABSORBERS AND POOR ABSOBERS OF HEAT RADIATION

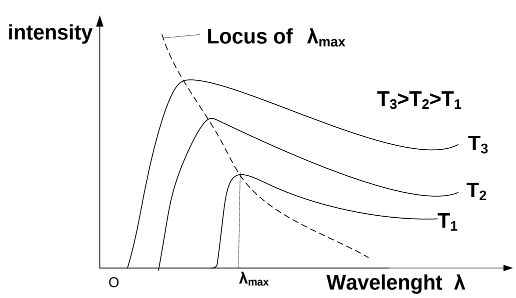


- A metal cube whose sides have a variety of finishes dull black, white highly polished is used
- The metal cube is filled with water and water is kept boiling at by a constant supply of heat
- A thermopile is made to face the various finishes of the cube at equal distances and each time the deflection on the galvanometer noted.
- The galvanometer deflection is greatest when the thermopile fasces the dull black surface and less when it is facing the highly polished surface
- This means that a highly polished surface is a poor radiator and the dull black surface is the better radiator.

5.2.6: BLACK BODY RADIATION

A black body radiation is the radiation whose quality (wave length) depends only on the temperature of the body.

Spectral curves for black body radiation



Special features of the curve

- As the temperature increases, the intensity for every wavelenght increases but the intensity for a shorter wavelength increases more rapidly
- At each temperature, there is a maximum intensity for a particular wavelength.
- λ_{max} decreases as temperature increases

Wein's displacement law

It states that the wavelength λ_{max}, for which the radiation emitted by a black body has maximum intensity is inversely proportional to the absolute temperature of the body

i.e $\lambda_{max} \propto \frac{1}{T}$ or $\boxed{\lambda\text{max T} = \text{constant}}$

wein's displacement constant $= 2.9x10^{-3}mK$

Examples

(i) Calculate the wavelength of the radiation emitted by a black body at $15x10^6$K

Solution

$\lambda\text{max T} = 2.9x10^{-3}$

$\lambda\text{max} = \frac{2.9x10^{-3}}{15x10^6}$

$\lambda\text{max} = 1.93\text{x}10^{-10}\text{m}$

(ii) Calculate the wavelength of the radiation emitted by a black body at 2.7K **An(1.07mm)**

Questions

1. Draw spectral curves for three different temperature and use them to explain;
   (i) Explain what happens to total energy radiated by a black body as temperature increases
   (ii) Explain how Wein's displacement law is used to explain colour changes of hot metal object as temperature is raised

- As the temperature increases, the relative intensity (energy) at each wave length increases (the body becomes brighter) but the increase is much the rapid for shorter wave length. (the colour of the body changes).
- The appearance of the body depends on the position of λ. The body changes its colour when cold (λ_{max} in the red region of visible spectrum) to yellow hot, white hot (λ_{max} in the middle spectrum visible) and eventually to blue hot (λ_{max} in blue region)
- The area under each spectral curve = intensity E or energy emitted per second per meter squared or power per meter squared.

Why center of fire appears white

This is because temperature is highest at the center of the fire and this corresponds to the energy intensity where all wavelength radiations are emitted. The combination of all the colors at this temperature makes the fire appear white

Question. State black body radiation laws. (Weins displacement law and Stefan-Boltzmann's law)

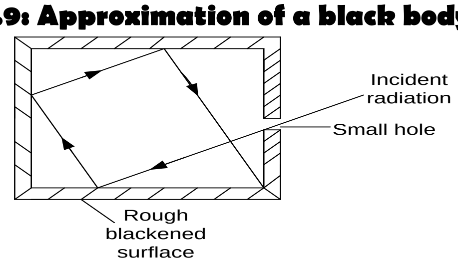
5.2.7:Relative intensity E_λ, is the energy radiated per meter square per second per unit wave length interval.

Intensity E, is the energy emitted per second per meter squared or power emitted per meter squared.

5.2.8:BLACK BODY

A black body is one which absorbs all radiations of every wavelength falling on it, reflects and transmits none.

5.2.9: Approximation of a black body OR realization of black body



- A small hole is punched in a tin which is blackened inside.
- When a radiation is incident through the hole, it undergoes multiple reflections
- At each reflection energy is lost due to many reflections and all energy is lost reflections.

A black body radiator (cavity radiator)

A black body radiator is one which emits radiation which is characteristic of its temperature and does not depend on the nature of its surface.

5.3: STEFAN'S LAW (STEFAN- BOLTZMAN'S LAW)

- It states that "the total power radiated per unit surface area of a black body is directly proportional to the fourth power of its absolute temperature" ($P\alpha T^4$)

OR

- Total energy radiated by a blackbody per unit surface area per unit time is directly proportional to the fourth power of its absolute temperature. ($E\alpha T^4$)

5.3.1:Expression for power radiated by black body

From Stefan's law

$\frac{energy}{surface\ area\ x\ Time} = \sigma T^4$ | $\frac{I.Vt}{S.t} = \sigma T^4$ | $\boxed{P = S\,\sigma T^4}$

Example

1) A cylinder has radius 10^{-2}m and height $0.75mm$. Calculate the temperature of cylinder if it is assumed to be lamp of power 1kW. $\boldsymbol{\sigma} = 5.67x10^{-8}m^{-2}\ \boldsymbol{Wm^{-2}k^{-4}}$

Solution

$P = S\,\sigma T^4 \qquad S = 2\pi rh$

$1000 = 5.67x10^{-8}x2\pi x10^{-2}x0.75x10^{-3}xT^4$

$T^4 = (3.74262x10^{14})$

$T = (3.74262x10^{14})^{\frac{1}{4}}$

$\mathrm{T} = 4398.435\mathrm{K}$

2) A cylindrical bulb filament of length $0.5m$ and radius $1.0x\ 10^{-4}m$ emits light as black body. $0.4A$ melts the filament when connected across 240V. Calculate;

(i) The melting point of the filament

(ii) The wave length of the radiation emitted at maximum intensity/emission at its melting point.

Solutions

i) $P = IV = S\,\sigma T^4 \qquad S = 2\pi rh$

$0.4x240 = 5.67x10^{-8}x2\pi x1.0x10^{-4}x0.5xT^4$

$T^4 = (5.3894x10^{12})$

$T = (5.3894x10^{12})^{\frac{1}{4}}$

$\mathrm{T} = 1523.648\mathrm{K}$

ii) $\lambda\max \mathrm{T} = 2.9x10^{-3}$

$\lambda\max = \frac{2.9x10^{-3}}{1523.648}$

$\lambda\max = 1.90\mu\mathrm{m}$

3) A tungsten filament lamp of 10W lamp at a temperature of 217°C and effective surface area of 62.4cm^2 radiates energy at a rate equivalent to 49% of that radiated by a black body. Calculate Stefan's constant.

Solutions:

$P = \frac{49}{100}xS\,\sigma T^4$

$10 = \frac{49}{100}x62.4x10^{-4}x\,\sigma(217+273)^4$

$\sigma = 5.67x10^{-8}Wm^{-2}K^{-4}$

4) The total power output of the sun 4.0x10^{26}W. given that the mass of the sun is $1.97x10^{30}kg$ and density is $1.4x10^3kgm^{-3}$, estimate the temperature of the sun

Solution

$\rho = \frac{m}{v}$

$v = \frac{1.97x10^{30}}{1.4x10^3}$

$v = 1.407x10^{27}m^3$

$v = \frac{4}{3}\pi r^3$

$1.407x10^{27} = \frac{4}{3}\pi r^3$

$r = 6.95x10^8m$

$P = S\,\sigma T^4 \qquad \boldsymbol{S = 4\pi r^2}$

$4.0x10^{26} = 4\pi x(6.95x10^8)^2x\ 5.67x10^{-8}T^4$

$T = 5840K$

5.3.2: Expression for net power for a body in the surrounding

If a black body of surface area S is at absolute temperature T_0 placed in an environment which is at lower temperature T.

$$\mathrm{P_{net}} = \boldsymbol{S\,\sigma} T_0^4 - \boldsymbol{S\,\sigma} T^4$$

$$\boxed{P_{net} = S\,\sigma(T_0^4 - T^4)}\ \text{For } T_0 > T$$

Example

1) Calculate the net loss of heat energy from space craft of surface area 25m^2 and temperature of 300K if the radiation that it receives form the sun is equivalent to at temperature in the space 50K. Assume that the space craft behaves as a perfect black body.

$P_{net} = S\,\sigma(T_0^4 - T^4)$ | $P = 25x5.67x10^{-8}x(T_0^4 - T^4)$ | $P = 1.15x10^4W$

2) The element of 1kW electric fire lamp is 30cm long and 1cm diameter if the temperature surrounding in 20°C. Estimate the working temperature of element

Solution

$T = 20 + 273 = 293K$

$P_{net} = S\,\sigma(T_0^4 - T^4)$ and $S = 2\pi rh$

1000= $2x\pi$x0.5x10^{-2}x30x10^{-2}{ T$_o^4$- 293^4} σ

$T_o^4 = 1.87963x10^{12}$

$T_0 = 1170.8K$

3) A small blackened solid copper sphere of radius 2cm is placed in evacuated enclosure those walls are kept at 100°C. find the rate at which energy must be supplied to sphere to keep its temperature constant at 127°C.

Solution

T$_o$=100°C $= 373K$, T =127°C $= 400K$

$P_{net} = S\,\sigma(T_0^4 - T^4)$ and $S = 4\,\pi r^2$

P_{net}= 4πx(2x10^{-2})^{2}x5.67x10^{-8}(400^4-373^4)

P_{net}=1.779W

4) A solid copper sphere of diameter 10mm and temperature of 150K is placed in an enclosure maintained at temperature of 290K. Calculate stating any assumption made the initial rate of rise of temperature of sphere. ($\rho$of copper 8.95x10^{3}kg/m^3, S.H.C of copper 3.7x10^{2}Jkg^{-1}K^{-1})

Assumption

- The sphere behaves like by a black body
- All heat exchange by radiation

Solution

ρ=8.75x10^{-3}kgm^{-3}, C = 3.7x10^{2}Jkg^{-1}K^{-1}

T=150K, T$_o$ = 210K

P_{net} =S σ(T$_o^4$- T^4) S=4πr^2

P_{net}=4πr^2x5.67x10^{-8}(290^4-150^4)

$\frac{MC\Delta\theta}{t}$=4πr^2x5.67x10^{-8}(290^4-150^4)

$\frac{\Delta\theta}{t}=\frac{4\pi r^2}{\text{mc}}$x5.67x10^{-8}(290^4-150^4)

M=vxρ

$\frac{\Delta\theta}{t}=\frac{4\pi r^2 x5.67x10^{-8}(290^4-150^4)}{\frac{4}{3}\pi r^3 x8.95x10^{-8}x3.7x10^{+2}}$

=0.067K/s

5) A solid metal sphere is placed in an enclosure at temperature of 27°C when temperature of the metal is 227°C, it cools at rate of 3°C per minute. What is the rate of cooling when solid sphere of same metal but twice the radius at 127°C is placed in the same enclosure

Solution

P_{net} =S σ(T$_o^4$- T^4)

$\frac{MC\Delta\theta}{t}$= Sσ(T$_o^4$- T^4)

Let y= rate of cooling[$\frac{\Delta\theta}{t}$], $M = vx\,\rho$

MCy = Sσ(400^4- 300^4)

$\frac{4\pi(2r)^3\rho Cy}{3}$ = Sσ(400^4- 300^4)...1

$\frac{4\pi(r)^3\rho C3}{3}$ = Sσ(500^4- 300^4)...2

Eqn1 divide by Eqn2

$\frac{8\,y}{3}=\frac{4\pi(2r)^2\sigma\,(400^4-300^4)}{4\pi r^2\sigma\,(500^4-300^4)}$

y=$\frac{3(400^4-300^4)}{2\,(\,500^4-300^4)}$

y=0.48°Cmin^{-1}

Exercise:443

A copper of diameter 20mm is cooled to temperature of 500K and then placed in an enclosure maintained at 300K. Assuming that all heat exchange is by radiation, calculate the initial rate of loss of temperature of sphere assumed as a black body.

(ρ of copper 8.39x10^{3}kg/m^3, S.H.C of copper 370J/kg^{-1}) **An (0.28W)**

Note

If the body is not a black body, then the energy it emits at any temperature will be less than that emitted by a black body of similar surface area at the same temperature. The emission equation is modified as;

$\boxed{P = eS\,\sigma T^4}$ where $e - emissivity$

$\boxed{P_{net} = e\,S\,\sigma(T_0^4 - T^4)}$ For T$_o$>T

Emissivity (e):

is defined as the ratio of total power emitted per squared meter of a given body to that emitted per squared meter of a black body at the same temperature as the body.

Examples

1. A 100W electric bulb has a filament which is 0.60m long and has a diameter of $8.0x10^{-5}m$. estimate the working temperature of the filament if its total emissivity is 0.70.

Solution

$P = eS\,\sigma T^4$ and $S = 2\pi rh$

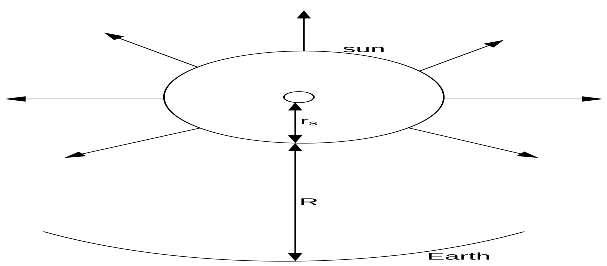
$$100 = 0.70x2\pi x4x10^{-5}x0.6x5.67x10^{-8}xT^4$$
$$T = 2.02x10^3K$$

5.3.3: SOLAR POWER / SOLAR CONSTANT

A solar power is the amount of energy received from the sun per second per meter squared.

Expression for solar constant

Assuming the sun to be a black body and spherical. The power radiated by the sun.



Ps = SσT_S^4

Where S is its surface area of sun ($4\pi r_S^2$)

r_S is The radius of the sun

$power\ of\ the\ sun,$ $P_S = 4\pi r_S^2\sigma T_S^4$

$\boldsymbol{Solar\ power} = \frac{power\ of\ the\ sun}{surface\ area\ of\ the\ earth}$

$$\boxed{\boldsymbol{Solar\ power} = \frac{4\pi r_S^2\sigma T_S^4}{4\pi R^2}}$$

Example

1) The energy intensity received by a spherical planet from star is 1.4 x10^{3}Wm^{-2}. The star is of radius 7.0x10^{5}km and 14.0x10^{7}km from the planet. Calculate the surface temperature of star and state any assumptions made.

Solution

Solar power $= \frac{4\pi r_S^2\sigma T_S^4}{4\pi R^2}$

1.4x10^3 = $\frac{(7.0x10^5x1000)^2x5.67x10^{-8}T_S^4}{(14.0x10^7x1000)^2}$

T=5605.976K

Assumption

-The star behaves as a black body

-The star is a perfect sphere

-There is no heat loss to the surrounding

2) The sun's radius is 7.0x10^{8}m. It's distance from the earth is 1.52x10^{11}m and the solar constant is 1400Wm^{-2}. Calculate the surface temperature of the sun.

Solution

Solar power $= \frac{4\pi r_S^2\sigma T_S^4}{4\pi R^2}$

1400 $= \frac{r_S^2\sigma T_S^4}{R^2}$

1400 $= \frac{(7.0x10^8)^2x5.67x10^{-8}xT_S^4}{(1.52x10^{11})^2}$

$$T = 5800K$$

3) Consider the sun to be the sphere of radius 7.0x10^{8}m where surface temperature is 5900K.
   (i) Find the solar power incident on an area of 1m^2 on the top of earth's atmosphere if it's at a distance of 1.5x10^{11}m from the sun. Assume that the sun radiates as a black body.
   (ii) Explains why solar power incident on 1m^2 of earth surface is less than the calculated value in (i) above.

Solution

Solar power $= \frac{4\pi r_S^2\sigma T_S^4}{4\pi R^2}$

$= \frac{(7.0x10^8)^2x5.67x10^{-8}x5900^4}{(1.5x10^{11})^2}$

$= 1504W$m^{-2}

(ii)

- Some of the energy is absorbed by the particles in atmosphere
- Some of the energy is scattered by particles in atmosphere

4) The flux of solar energy incident on the earth surface is $1.36\ x10^3 Wm^{-2}$. If the sun's radius is $7.0x10^8$m. It's distance from the earth is $1.52x10^{11}$m. (speed of light=$3.0x10^8 ms^{-1}$, $\sigma = 5.67x10^{-8} Wm^{-2}K^{-4}$). Calculate;
   (i) temperature of the surface of the sun
   (ii) total power emitted by the sun
   (iii) rate of loss of the mass by the sun

Solution

(i) Solar power $= \frac{4\pi r_s^2 \sigma T_s^4}{4\pi R^2}$

$1400 = \frac{(7.0x10^8)^2 x5.67x10^{-8} xT_s^4}{(1.52x10^{11})^2}$

$T_s = 5753K$

(ii) $P_s = 4\pi r_s^2 \sigma T_s^4$

$P_s = 4\pi x(7.0x10^8)^2 x5.67x10^{-8}(5753)^4$

power $= 3.846x10^{26}$W

(iii) $E = mc^2$

$Pt = mc^2$

$\frac{m}{t} = \frac{3.846x10^{26}}{3.0x10^8} = 4.27x10^9 kgs^{-1}$

5.3.4: RADIACTIVE EQUILIBRIUM OF THE SUN AND THE EARTH

The power radiated by the sun is given by

$P_S = 4\pi r_s^2 \sigma T_s^4$

Where T_s=surface temperature of sun ,

r_s= radius of sun

The solar power $= \frac{4\pi r_s^2 \sigma T_s^4}{4\pi R^2}$

The power received by effective area of the earth =solar power x area of earth

= solar power x πr_E^2

Where r_E – radius of earth

power received by the earth = $\frac{4\pi r_s^2 \sigma T_s^4}{4\pi R^2}$ x πr_E^2-- [1]

Earth also behaves like a black body, then the power radiated by the earth is

$P_E = 4\pi r_E^2 \sigma T_E^4$ ------------------------- [2]

$4\pi r_E^2 \sigma T_E^4 = \frac{4\pi r_s^2 \sigma T_s^4}{4\pi R^2}$ x πr_E^2

$T_E^4 = \frac{r_s^2}{4R^2} T_s^4$

$$\boxed{T_E^4 = \left(\frac{r_s}{2R}\right)^2 T_s^4}$$

Example

1) Estimate the temperature of surface of earth if its distance from the sun $1.5x10^{11}$m. Assume that the sun is sphere of radius $7.0x10^8$m at temperatures 6000k

Solution

Solar power $= \frac{4\pi r_s^2 \sigma T_s^4}{4\pi R^2}$

Power received by earth $= \frac{4\pi r_s^2 \sigma T_s^4}{4\pi R^2}$x $\pi r^2{}_e$

Power radiated by earth= $4\pi r_e^2 \sigma T_e^4$

at equilibrium: Power radiated =power received

$4\pi r_e^2 \sigma T_e^4 = \frac{4\pi r_s^2 \sigma T_s^4}{4\pi R^2}$x $\pi r^2{}_e$

$T_e^4 = \frac{r_s^2}{4R^2} T_s^4$

$T_e = \left\{\frac{(7x10^8)^2 x6000^4}{4(1.5x10^{11})^2}\right\}^{\frac{1}{4}}$

$T_e = 290K$

2) Assume that the sun is sphere of radius $7.0x10^8$m at temperatures 6000k. Estimate the temperature of surface of mars if its distance from the sun $2.28x10^{11}$m.

Solution

Solar power = $\frac{4\pi r_s^2 \sigma T_s^4}{4\pi R^2}$

Power received by mars = $\frac{4\pi r_s^2 \sigma T_s^4}{4\pi R^2}$x $\pi r^2{}_m$

Power radiated by mars =$4\pi r_m^2 \sigma T_m^4$

At equilibrium: Power radiated =power received

$4\pi r_m^2 \sigma T_m^4 = \frac{4\pi r_s^2 \sigma T_s^4}{4\pi R^2}$x πr_m^2

$T_m^4 = \frac{r_s^2}{4R^2} T_s^4$

$T_m = \left\{\frac{(7x10^8)^2 x6000^4}{4(2.28x10^{11})^2}\right\}^{\frac{1}{4}}$

$T_m = 235.08K$

3) The average distance of plants is about 40 times to that of earth from the sun. If the sun radiates as black body at 6000K and is $1.5x10^{11}$m from the earth. Calculate the surface temperature of Pluto.

Solution

Distance of Pluto = 40xdistance from earth

R=40x1.5x10[11]

Solar power = $\frac{4\pi r_s^2 \sigma T_s^4}{4\pi R^2}$

Power received by mars = $\frac{4\pi r_s^2 \sigma T_s^4}{4\pi R^2}$x πr^2p

Power radiated by mars = $4\pi r_p^2 \sigma T_p^4$

At equilibrium

Power radiated =power received

$4\pi r_p^2 \sigma T_p^4 = \frac{4\pi r_s^2 \sigma T_s^4}{4\pi R^2}$x πr_p^2

$T_p^4 = \frac{r_s^2}{4R^2} T_s^4$

$T_p = \left\{\frac{(7x10^8)^2 x6000^4}{4x(40x1.5x10^{11})^2}\right\}^{\frac{1}{4}}$

$T_p = 45.8K$

4) If the mean equilibrium temperature of the earth's surface is T and the total rate of energy emission by the sun is E, show that

$$\boldsymbol{T^4 = \frac{E}{16\sigma\pi R^2}}$$

Where σ is Stefan's constant and R is the radius of the earth orbit around the sun

Solution

Solar power = $\frac{E}{4\pi R^2}$

Power received by earth= $\frac{E}{4\pi R^2}$x $\pi\, r_e^2$

Power radiated by earth =$4\pi r_e^2 \sigma T^4$

At equilibrium: Power radiated =power received

$4\pi r_e^2 \sigma T^4 = \frac{E}{4\pi R^2}$x $\pi\, r_e^2$

$$\boldsymbol{T^4 = \frac{E}{16\sigma\pi R^2}}$$

Exercise:45

1) The element of an electric fire, with an output of 1.0kW, is a cylinder 25cm long and 1.5cm in diameter. Calculate its temperature when in use , if it behaves as a blackbody.
(Stefan constant = $5.7x10^{-8}Wm^{-2}K^{-4}$) **An(1105K)**
2) Solid copper sphere of diameter 10mm is cooled to atmosphere of 150K and is then placed in an enclosure ,maintained at 290K. Assuming that all interchanges of heat is by radiation. Calculate the initial rate of rise of temperature of the sphere. The sphere may be treated as a black body.
Density of copper = $8.93x10^3kgm^{-3}$,S.H.C of copper=$3.70x10^2Jkg^{-1}K^{-1}$, Stefan constant= $5.7x10^{-8}Wm^{-2}K^{-4}$)
An($6.78x10^{-2}Ks^{-1}$)
3) The silica cylinder of a radiant wall heater is 0.6m long and has a radius 5mm. if it is rated at 1.5kW. Estimate its temperature when operating. State two assumptions you have made in making your estimate. Stefan's constant, $\sigma=6x10^8Wm^{-2}K^{-4}$) **An(1073K)**
4) A blackened metal sphere of diameter 10mm is placed at the focus of a concave mirror of diameter 0.5m directed towards the sun. if the solar power incident on the mirror is $1600Wm^{-2}$. Calculate the maximum temperature which the sphere can attain. State the assumptions you make (Stefan's constant =$6x10^{-8}Wm^{-2}K^{-4}$) **An(2021K)**
5) An unlagged, thin walled copper pipe of diameter 2.0cm carries water at a temperature of 40K above that of the surrounding air. Estimate the power loss per unit length of the pipe if the temperature of the surroundings is 300K and Stefan constant, $\sigma = 5.7x10^{-8}wm^{-2}K^{-4}$. State two important assumption you have made. **An($19Wm^{-1}$)**
6) The solar radiation falling normally on the surface of the earth has an intensity $1.40kWm^{-2}$. If this radiation fell normally on one side of a thin freely suspended blackened metal plate and the temperature of the surrounding was 300K, calculate the equilibrium temperature of the plate. Assume that all heat interchange is by radiation. (Stefan's constant =$5.67x10^{-8}Wm^{-2}K^{-4}$) **An(378K)**
7) Estimate the surface temperature of the earth assuming that it is radioactive equilibrium with the sun. (radius of sun $7.0x10^8$m, surface temp of sun 6000K, distance from the earth to the sun $1.5x10^{11}$m, σ =$5.7x10^{-8}Wm^{-2}K^{-4}$) **An [289K]**

8) The normal operating condition of a variable- intensity car head lamp is 2.5A and 12V. The temperature of the filament is 1750°C. The intensity is now altered so that the lamp runs at 2.2A and 12.5V. calculate the new operating temperature assuming that the filament behaves as a black body
9) A black body radiates heat a 2Wm^{-2} when at 0°C. Find the rate of fall in temperate of a copper sphere of radius 3cm when at 1000°C in air at 0°C. (assume that the density of copper is 8930 and its specific heat capacity is 385 $Jkg^{-1}K^{-1}$)
10) Given that the energy received from the sun at the surface of the earth is 1400J$s^{-1}m^{-2}$. Determine the effective solar temperature, assuming that the sun behaves as a perfect black body.
11) A certain 100W tungsten filament lamp operates at a temperature of 1500°C. Assuming that it behaves as a perfect black body estimate the surface area of the filament
12) Find the net rate of energy lost by radiation form the following black bodies
    (a) A sphere of radius 10cm at a temperature of 500°C in an enclosure whose temperature is 20°C
    (b) A person of surface area 1.2m^2 at a temperature of 37°C in an enclosure whose temperature is 0°C. Comment on your answer
13) A metal sphere of 1cm diameter, whose surface acts as a black body is [laced at the focus of a concave mirror with an aperture 60cm directed towards the Sun. if the solar constant is 1400Wm^{-2} and the mean temperate of the surrounding is 27°C. Calculate the maximum temperature that the sphere could attain, stating any assumption that you make
14) A black body at 1110K emits radiation with maximum energy emitted at a wavelength of 25000nm. Calculate the wavelength at which maximum energy is emitted by the following assuming that they all behave as black bodies
    (a) A piece of iron heated in a Bunsen flame to 800°C
    (b) A star with a surface temperature of 7000°C
    (c) The plasma in a fusion reaction at 10^5°C

5.3.6:GREEN HOUSE EFFECT

- Short wavelength radiation from the sun passes through the atmosphere and is absorbed by plants and sand leading to higher earth temperature.
- Earth re-radiates long wavelength which is trapped by green house gases. Continued accumulation of this radiation implies higher earth temperature and with time may lead to global warming.

5.3.7: THERMAL CONVECTION

Is a process of heat transfer through a fluid of high temperature to low temperature due to actual movement of medium.
Heated fluid becomes less dense and is replaced by more dense fluid.

Mechanism of convection

When a fluid is heated underneath, it expands and becomes less dense than the fluid above. The warm less dense fluid rises to the top and the cooler more dense from above moves downwards to take its place. The circulating current of the fluid heats up the whole fluid

5.3.8:SEA BREEZE

During day, air flows from sea towards land because land heats faster and air above it which is warmer rises and is replaced by cooler denser air from sea..

5.3.9:LAND BREEZE

At night, air flows from land to sea, land cools faster than sea due to its smaller heat capacity. Hot, less dense air above sea rises and is replaced by cool denser air from land.

Explain why cloudy nights are warmer than cloudless nights

During day, earth absorbs heat from sun. at night earth radiates heat into atmosphere. On cloudy night clouds reflect heat back to the earth and it feels warm. On cloudless night radiated heat is lost to atmosphere and earth feels colder

UNEB 2017 Q.7

(a) **(i)** Define **thermal conductivity** (01mark)

(ii) Explain the mechanism of heat transfer by convection. (03marks)

(b) **(i)** State **Newton's law of cooling.** (01marks)

(ii) Describe briefly an experiment to verify Newton's law of cooling. (05marks)

(c) A wall is constructed with two types of bricks. The temperature of inner and outer surfaces of the wall are $29°C$ and $21°C$ respectively . The value of the thermal conductivity for the inner brick is $0.4Wm^{-1}K^{-1}$ and that of the outer bricke is $0.8Wm^{-1}K^{-1}$

(i) Explain why in steady state, the rate of thermal energy transfer must be the same in both layers (02marks)

(ii) Calculate the temperature at the interface between the layers, if each layer is 12.0cm thick **An**$(23.7°C)$(04marks)

(d) Explain the green house effect and how it leads to rise of the earth temperature. (04marks)

UNEB 2016 Q.7

(a) **(i)** Define **a black body** (01mark)

(ii) Sketch and explain graphs of intensity versus wave length for three different temperatures of a black body. (03marks)

(b) Describe with the aid of a labelled diagram how an optical radiation pyrometer is used to measure temperature. (06marks)

(c) (i) State **Prevost's theory** of heat exchanges (01mark)

(ii) a metal sphere of radius 1.5cm is suspended within an evacuated enclosure whise walls are at 320K. The emissivity of the metal is 0.40. Find the power input required to maintain the sphere at a temperature of 320K, if heat conduction along the supports is negligible. (04marks)

(d) A metal boiler is 1.5cm thick. Find the difference in temperature between the inner and outer surfaces if 40kg of water evaporates from the boiler per meter squared per hour. (latent heat of vapourisation of water= $2268\ kJkg^{-1}$, thermal conductivivty of the metal of the boiler= $63\ Wm^{-1}K^{-1}$) (05marks)

An(6.0K)

UNEB 2015 Q.7

(a) Define **thermal conductivity** of a material and state its unit (01mark)

(b) Describe an experiment to determine the thermal conductivity of copper (06marks)

(c) A double glazed window has two glass sheets each of thickness 4.0mm, separated by a layer of air of thickness 1.5mm. if the two inner air-glass surfaces have steady temperature of 20°C and 4°C respectively, find the

(i) Temperature of the outer air-glass surface **An(21.48°C, 3.84x10^{6}Js^{-1})**

(ii) Amount of heat that flows across an area of the window of $2m^2$ in 2hours

[Conductivity of glass = $0.72\ Wm^{-1}K^{-1}$, and that of air =$0.025Wm^{-1}K^{-1}$**]** (03marks)

(d) **(i)** What is **a black body** (01mark)

(ii) Explain how a welder can protect the eyes from damage (03marks)

(e) Calculate the wavelength of the radiation emitted by a black body at 6000K

(Wien's displacement constant= $2.9x10^{-3}mK$) **An(4.8x10^{-7}m)** (02marks)

UNEB 2014 Q.6

(a) (i) What is **a black body** (01mark)

(ii) Explain with the aid of a diagram how a black body can be approximated (03marks)

(iii) With the aid of a sketch graphs explain the salient features of the spectral distribution of a black body radiation (04marks)

(b) Give four properties of ultraviolet radiations (02marks)

(c) Describe an experiment to compare the energy radiated by two surfaces of different nature(04marks)

(d) (i) State **Stefan's law** (01mark)

(ii) The earth receives energy from the sun at the rate of $1.4x10^3 Wm^{-2}$ If the ratio of the earth earth's orbit to the suns radius is 216, calculate the surface temperature of the sun $An(5.82x10^{-3}K)$ (05marks)

UNEB 2013 Q.7

(a) (i) Define **thermal conductivity** of a material (01marks)

(ii) Describe an experiment to determine the thermal conductivity of copper

(b) (i) what is meant by **a black body**

(ii) Describe how infrared radiations can be detected using a bolometer (3mks)

(iii) Give one characteristic property of infrared radiation (01mark)

(c) (i) A spherical black body of radius 2.0cm at $-73°C$ is suspended in an evacuated enclosure whose walls are maintained at $27°C$. If the rate of exchange of thermal energy is equal to $1.85\ Js^{-1}$, find the value of Stefan's constant, σ. (05marks)

(ii) Calculate the wavelength at which the radiation emitted by the enclosure has maximum intensity **An(** $9.67x10^{-6}$ **m)** (03marks)

UNEB 2012 Qn7

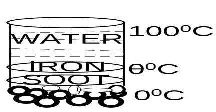
(a) (i) Define **thermal conductivity** (01marks)

(ii) Compare the mechanism of heat transfer in **poor** and **good solid** conductors

(b) Describe, with the aid of a diagram how you would measure the thermal conductivity of a poor conductor, stating the necessary precautions.(08marks)

(c) A cylindrical iron vessel with a base of diameter 15cm and thickness 0.30cm has it's base coated with a thin film of soot of thickness 0.1cm. It is then filled with water at 100°C and placed on a large block of ice at 0°C. Calculate the initial rate at which the ice will melt. [The conductivity of soot=0.12Wm^{-1}K^{-1}, conductivity of iron=75Wm^{-1}K^{-1}]

Solution



$$\frac{Q}{t} = \frac{KA(\Delta\theta)}{L}$$

$$\frac{Q}{t} = \frac{75A(100-\theta)}{0.3x10^{-2}} = \frac{0.12A(\theta-0)}{0.1x10^{-2}}$$

$$\frac{75A(100-\theta)}{0.3x10^{-2}} = \frac{0.12A(\theta-0)}{0.1x10^{-2}}$$

$$7.5(100-\theta) = 0.36(\theta)$$

θ =99.52°C

But $\frac{Q}{t} = \frac{75A(100-\theta)}{0.3x10^{-2}} = ml$

$$\frac{75x3.14x(15x10^{-2})^2(100-99.52)}{4x0.3x10^{-2}} = ml$$

$$211.95 = 3.3x10^5 m$$

$m = \frac{211.95}{3.3x10^5} = 6.42x10^{-4}$ kgs^{-1}

UNEB 2011 Q7

(a) State **Stefan's law** of black body radiation (01marks)

(b) Briefly describe how a thermopile can be used to detect thermal radiation (05marks)

(c) Explain the temperature distribution along
- (i) A perfectly lagged metal bar (02marks)
- (ii) An un lagged metal bar (02marks)

(d) The wall of a furnace is constructed with two layers. The inner layer is made of bricks of thickness 10.0cm and thermal conductivity $0.8Wm^{-1}K^{-1}$ and the outer layer is made of a material of thickness 10.0cm and thermal conductivity $1.6Wm^{-1}K^{-1}$
- (i) Explain why in steady state, the rate of thermal energy transfer must be the same in both layers (01marks)
- (ii) Calculate the rate of heat flow per square meter through the wall (05marks)

An($1066.64Jm^{-2}s^{-1}$)

(e) Explain the green house effect and how it is related to global warming (04marks)

UNEB2010 Q7

(a) What is meant by the following;
- (i) Conduction
- (ii) Convection
- (iii) Green house effect (06marks)

(b) One end of a long copper bar is in steam chest and the other end is kept cool by a current of circulating water. Explain with the aid of a sketch graphs, the variation of temperature along the bar, when steady state has been attained if the bar is;
- (i) Lagged (02marks)
- (ii) Exposed to the surrounding (02marks)

(c) (i) what is meant by a black body (01marks)
(ii) describe how a black body can be approximated in practice (04marks)

(d) (i) State Prevost theory of heat exchanges (01marks)
(ii) Sketch the variation with wavelength of the intensity of radiation emitted by a black body at two different temperatures (01marks)

(e) A cube of side 1cm has a grey surface that emit 50% of radiation emitted by black body at the same temperature. If the cube's temperature is $700^{0}C$, calculate the power radiated by the cube. (03marks) **An(15.25W)**

UNEB2009Q7

(a) State **thermal conductivity** (01marks)

(b) (i)Explain the mechanisms of thermal conduction in non-metallic solids (03marks)
(ii) Why are metals better thermal conductors than non metallic solids (02marks)

(c) With the aid of a diagram, describe an experiment to determine the thermal conductivity of a poor conductor. (06marks)

(d) (i) What is meant by a **black body**. (01marks)
(ii) Sketch curves showing the spectral distribution of energy radiated by a black body at different temperatures (02marks)

(e) A small blackened solid copper sphere of radius 2cm is placed in an evacuated enclosure whose walls are kept at $100^{0}C$.Find the rate at which energy must be supplied to the sphere to keep it at constant temperature of $127^{0}C$. **An(1.78W)**

UNEB2008Q7

(a) (i) State **laws of black body** radiation (02marks)
(ii) Sketch the variation of intensity with wavelength in a black body for three different temperatures. (03marks)

(b) (i) What is a **perfectly black body?** (01mark)
(ii) How can a perfectly black body be approximated in reality. (04marks)

(c) The energy intensity received by a spherical planet from a star is $1.4x10^{3}Wm^{-2}$.The star is of radius $7.0x10^{5}$km and $14.0x10^{7}$km from the planet.

(i) calculate the surface temperature of the star. **An(5605.98K)** (04marks)

(ii) state any assumption you have made in (c)(i) above. (01mark)

(d) (i) What is convection (01mark)

(ii) Explain the occurrence of land and sea breeze (04marks)

UNEB2006Q7

(a) (i) Define **thermal conductivity** (01mark)

(ii) Explain the mechanism of heat transfer in metals (03marks)

(b) Two brick walls each of thickness 10cm are separated by an air gap of thickness 10cm. The outer faces of the brick walls are maintained at 20^oC and 5^oC respectively.

(i) Calculate the temperature of the inner surfaces of the walls (06marks)

(ii) Compare the rate of heat loss through the layer of air with that through a single brick wall [Thermal conductivity of air is $0.02Wm^{-1}K^{-1}$ and that of the brick is $0.6WWm^{-1}K^{-1}$](03marks)

An(5.5^oC, 19.5^oC, 1:32.1)

(c) (i) State **Stefan's law of black body** radiation (01marks)

(ii) The average distance of Pluto from the sun is a bout 40times that of earth from the sun. If the sun radiates as a black body at 6000K, and is $1.5x10^{11}m$ from the earth, calculate the surface temperature of Pluto. **An(45.8K)** (06marks)

UNEB2005Q7

(a) (i) Define **thermal conductivity** (01mark)

(ii) State two factors which determine the rate of heat transfer through a material(02marks)

(b) (i) Describe with the aid of a labeled diagram an experiment to measure the thermal conductivity of glass. (08marks)

(ii) Briefly discuss the advantages of the apparatus in b(i) above (02marks)

(c) Metal rods of copper, brass and steel are welded together to form a Y-shaped figure. The cross sectional area of each rod is $2cm^2$. The free ends of copper rod is maintained at 100^oC, while the free ends of brass and steel rods are maintained at 0^oC .If there is no heat loss from the surfaces of the rods and the lengths of the rods are 0.46m, 0.13m, and 0.12m respectively.

(i) Calculate the temperature at the junction (05marks)

(ii) find the heat current in the copper rod (02marks)

[Thermal conductivities of copper , brass, and steel are $385Wm^{-1}K^{-1}$, $109\ Wm^{-1}K^{-1}$, and $50.2\ Wm^{-1}K^{-1}$ respectively] **An(39.97K,$1.01X10^1$J)**

UNEB2004 Q6

(a) Define **thermal conductivity** of a material and state it's units (02marks)

(b) Describe with the aid of a diagram, how to determine the thermal conductivity of a poor conductor.

(c) A cooking sauce pan made of iron has a base area of $0.05m^2$ and thickness of 2.5mm. It has a thin layer of soot of average thickness 0.5mm on it's bottom surface. Water in the sauce pan is heated until it boils at 100^oC. The water boils a way at a rate of 0.60kg per minute and the side of the soot nearest to the heat source is at 150^oC. Find the thermal conductivity of soot. [Thermal conductivity of iron $=66Wm^{-1}K^{-1}$ and specific latent heat of vapourization=$2200kJK^{-1}$] **An($3.3Wm^{-1}K^{-1}$)**

(d) (i) What is a black body (01mark)

(ii) Sketch the spectral distribution of a black body radiation for different temperatures and describe their main features (04marks)