COORDINATE GEOMETRY 1 2025

1. The three points $G(4,0)$, $H(h,6)$ and $I(7,1)$ are such that GH is twice as long as GI. Find the two possible values of h.
2. The points A and B have coordinates $(3,2)$ and $(9,-4)$ respectively. Find, by calculation, which of the points $C(4,-3)$,$D(10,3)$ or $E(5,-1)$ does not lie on the perpendicular bisector of AB.
3. A triangle has vertices at $(0,1),(1,6)$ and $(5,2)$. Prove that the triangle is isosceles.
4. Find the length of the sides of the triangle with vertices at $A(1,1), B(4,5)$ and $C(9,-5)$. Hence prove that the triangle is right angled, and state which angle $\hat{A}, \hat{B}\ or\ \hat{C}$ is the right angle.
5. The points $A, B, C, and\ D$ have coordinates $(-7,9),(3,4),(1,12)\ and\ (-2,-9)$ respectively. Find the length of the line PQ where P divides AB in the ratio $2:3$ and Q divides CD in the ratio $1:-4$.
6. Find the length of the medians of the triangle that has vertices at $A(0,1), B(2,7)\ and\ C(4,-1)$.
7. $P(-1,5), Q(8,10), R(7,5)\ and\ S$ are the vertices of the parallelogram $PQRS$. Calculate the coordinates of S.
8. The line $y-2x+3=0$ intersects the curve $y=x^2-2x$ at the points $A\ and\ B$. Find the coordinates of $A\ and\ B$.
9. Find the points at which the curves $y=3x^2$ and $y=x^2-5x-3$ intersect.
10. Prove that the lines $y=3x, y=x+4\ and\ y+2x=10$ are concurrent. (**HINT:** Three or more lines are said to be concurrent if they pass through the same point).
11. If the lines $y=x+4, 2y+x=2\ and\ y=ax+8$ are concurrent, find the value of a.
12. A triangle has vertices at $A(0,8), B(1,1)\ and\ C(5,3)$. Show that the triangle is isosceles and find:
    (a) the equation of the straight line through $A\ and\ C$,
    (b) the coordinates of the foot of the perpendicular from B to AC,
    (c) the length of the perpendicular from $B\ to\ AC$.
13. $ABCD$ is a parallelogram in which the coordinates of $A, B\ and\ C$ are $(1,2),(7,-1)\ and\ (-1,-2)$ respectively.
    (i) Find the equations of $AD\ and\ CD$.
    (ii) Find the coordinates of D.
    (iii) Prove that $B\hat{A}C=90^0$.
    (iv) Calculate the area of the parallelogram.
    (v) Find the length of the perpendicular from $A\ to\ BC$, leaving your answer in surd form.
14. Find the centroid, orthocenter, and circum-center of a triangle whose vertices are $(3,4),(5,7),(4,7)$.
15. Show that the equation of the circle on the line segment joining $A(3,-5)$ and $B(2,6)$ as diameter is $(x-3)(x-2)+(y+5)(y-6)=0$.
16. A is the point $(1,0)$ and B is the point $(-1,0)$. Find the locus of a point P which moves so that $PA+PB=4$.
17. A is the point $(1,0)$, B is the point $(2,0)$ and O is the origin. A point P moves so that angle BPO is a right angle, and Q is the midpoint of AP. What is the locus of Q.

18. A line parallel to the y-axis meets the curve $y = x^2$ at P and the $y = x + 2$ at Q. Find the locus of the midpoint of PQ.
19. A variable line through the point $(3,4)$ cuts the axes at Q and R, and the perpendiculars to the axes at Q and R intersect at P. What is the locus of the point P ?
20. A variable point P lies on the curve $xy = 12$. Q is the mid-point of the line joining P to the origin. Find the locus of Q.
21. P is a variable point on the curve $y = 2x^2 + 3$, and O is the origin. Q is the point of trisection of OP nearer to the origin. Find the locus of Q.
    HINT: A point of trisection is a point on a line that divides the line segment into three equal parts. The point of trisection is located at a distance of one-third of the total length of the line segment from either endpoint.
22. A line parallel to the $x - axis$ cuts the curve $y^2 = 4x$ at P and the line $x = -1$ at Q. Find the locus of the mid-point of PQ.
23. Variable lines through the points $O(0,0)$ and $A(2,0)$ intersect at right angles at the point P. Show that the locus of the mid-point of OP is $y^2 + x(x - 1) = 0$.
24. M and N are points on the axes, and the line MN passes through the point $(3,2)$. P is a variable point which moves so that the mid-point of the line joining P to the origin is the midpoint of MN. Find the locus of point P.
25. A straight line LM,of length 4 units, moves with L on the line $y = x$ and M on the x-axis. Find the locus of the mid-point of LM.
26. A straight line LM meets the x-axis at M and the line $y = x$ at L, and passes through the point $(6,4)$. What is the locus of the mid-point of LM?
27. Show that the equation of the tangent to the curve $y^2 = 4ax$ at the point $P(at^2, 2at)$ is $x - ty + at^2 = 0$
28. Show that the equation of the normal to the curve $y^2 = 4ax$ at the point $P(at^2, 2at)$ is $y + tx = 2at + at^3$.
29. Find the equations of the tangents to the curve $y^2 = 4ax$ at the points $P(at^2, 2at)$ and $Q(aq^2, 2aq)$. Show that the point of intersection of the tangents above is $(apq, a(p + q))$
30. Find the coordinates of the point at which the line $y = 8x - a$ meets the curve $y^2 = 4ax$. Find the equations of tangents to curve at the points obtained above.
31. The tangent to the curve $y^2 = 4ax$ at the point $P(at^2, 2at)$ meets the x-axis and y-axis at points A and B respectively. Find the locus of the midpoint of A and B.
32. Show that if a line $y = mx + c$ is a tangent to the curve $b^2x^2 + a^2y^2 = a^2b^2$, then $c^2 = b^2 + a^2m^2$
33. Show that the equation of the tangent to a curve $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the point $P(acos\theta, bsin\theta)$ is $aysin\theta + bxcos\theta = ab$.
34. Show that the equation of the normal to a curve $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the point $P(acos\theta, bsin\theta)$ is $axsin\theta - bycos\theta = (a^2 - b^2)sin\theta cos\theta$.
35. The normal to the curve $9x^2 + 16y^2 = 144$ at the point $P(4cos\theta, 3sin\theta)$ meets the x-axis at A and y-axis at B. Find the locus of M, the mid-point of A and B.
36. Show that the equation of the tangent to the curve $xy = c^2$ at the point $P(ct, \frac{c}{t})$ is $yt^2 + x = 2ct$

37. Show that the equation of the normal to the curve $xy = c^2$ at the point $P(ct, \frac{c}{t})$ is $t^3x + c = yt + ct^4$
38. Part of the line $x - 3y + 3 = 0$ is a chord of the curve $x^2 - y^2 = 5$. Find the length of the chord.
39. Find the equation of the circle which passes through the points (5,0), (6,0) and (8,6).
40. Find the equation of the circle which has the points $(0, -1)$ and $(2,3)$ as ends of the diameter.
41. A circle, the coordinates of whose centre are both positive, touches both the x and y axes. If it also touches the line $3x - 4y + 6 = 0$, find its equation and the coordinates of its point of contact with this line.
42. A circle, which passes through the origin, cuts off intercepts of lengths 4 and 6 units on the positive x and y-axes respectively. Find the equation of the circle.
43. Find the equation of the circle which has its centre at the point $(2, -1)$ and touches the line $3x + y = 0$.
44. Tangents are drawn to the circle $x^2 + y^2 - 6x - 4y + 9 = 0$ from the origin. If θ is the angle between them, find the value of $\tan\theta$.
45. A circle touches the x-axis and cuts off a constant length $2a$ from the y-axis. Show that the equation to the locus of its centre is the curve $y^2 - x^2 = a^2$.
46. Find the length of the tangent from the point $(5, -1)$ to the circle $\left(x - \frac{1}{2}\right)^2 + y^2 = \frac{25}{4}$
47. Find the equation to the diameter of the circle $x^2 + y^2 - 6x + 2y = 15$, which when produced passes through the point $(8, -2)$.
48. Find the equation to the circle who centre lies on the line $y = 3x - 7$ and which passes through the point $(1,1)$ and $(2, -1)$.
49. If O is the origin and P, Q are the intersections of the circle $x^2 + y^2 + 4x + 2y - 20 = 0$ and the line $x - 7y + 20 = 0$, show that OP and OQ are perpendicular. Find the equation to the circle through O, P and Q.
50. Find the equation to the circle which passes through the origin and cuts both of the circles $x^2 + y^2 - 6x + 8 = 0$ and $x^2 + y^2 - 2x - 2y = 7$ orthogonally.
51. The length of the tangent from the point (1,1) to the circle $x^2 + y^2 - 4x - 6y + k = 0$ is 2 units. Find the value of k.
52. Show that the circles $x^2 + y^2 - 2ax + c^2 = 0$, $x^2 + y^2 - 2by - c^2 = 0$ are orthogonal.
53. Find the equation of the common chord of the two circles $x^2 + y^2 + 10x + 8y + 32 = 0$ and $x^2 + y^2 - 4x - 6y + 12 = 0$ and show that this line is perpendicular to the line joining the centres of the circles.