OUR LADY OF AFRICA S.S NAMILYANGO (OLAN)

A LEVEL PURE MATHEMATICS SEMINAR SOLUTIONS 2022

1(a).	$\frac{(\sqrt{5}-2)^2-(\sqrt{5}+2)^2}{8\sqrt{5}}=\frac{(\sqrt{5}-2+\sqrt{5}+2)(\sqrt{5}-2-\sqrt{5}-2)}{8\sqrt{5}}$ $=\frac{2\sqrt{5}\times(-4)}{8\sqrt{5}}=\frac{-8\sqrt{5}}{8\sqrt{5}}=-1$
(b)(i)	$2x^2+7x-4=2x^2+8x-x-4$ $=2x(x+4)-(x+4)$ $=(2x-1)(x+4)$ $x^2+3x-4= \quad x^2+4x-x-4$ $=x(x+4)-(x+4)$ $=(x-1)(x+4)$
(ii)	The common factor is $x+4)$ $let\ f(x)=7x^2+ax-8$ $f(-4)=7(-4)^2+a(-4)-8=0$ $112-4a-8=0$ $104=4a$ $a=26$

(c)	$R = 5, \quad P = 150{,}000, n = 7years$ $Totak\ amount, \quad A_{total} = \sum_{1}^{7} A_n \ , where\ A_n = P\left(1 + \frac{R}{100}\right)^n$ $A_{total} = A_1 + A_2 + \cdots + A_7$ $A_{total} = P[(1+0.05)^1 + (1+0.05)^2 + \cdots + (1+0.05)^7]$ $A_{total} = P[(1.05 + (1.05)^2) + \cdots + (1.05)^7]$ $A_{total} = P\left[\frac{a(r^n - 1)}{r-1}\right], \quad where\ a = r = 1.05$ $A_{total} = 150{,}000\left[\frac{1.05(1.05^7 - 1)}{1.05 - 1}\right] = 1{,}282{,}366.331$
(d)(i)	$\sqrt{\alpha} + \sqrt{\beta} = b$ $\sqrt{\alpha\beta} = c\ ;\ \alpha\beta = c^2$ $\left(\sqrt{\alpha} + \sqrt{\beta}\right)^2 = (b)^2$ $\alpha + \beta = \left(\sqrt{\alpha} + \sqrt{\beta}\right)^2 - 2\sqrt{\alpha\beta}$ $\alpha + \beta = b^2 - 2c$
(ii)	$\alpha + \beta = b^2 - 2c$ $(\alpha + \beta)^2 = (b^2 - 2c)^2$ $\alpha^2 + \beta^2 = (b^2 - 2c)^2 - 2\alpha\beta$ $\alpha^2 + \beta^2 = (b^2 - 2c)^2 - \left(\sqrt{2}c\right)^2$ $\alpha^2 + \beta^2 = \left(b^2 - 2c - \sqrt{2}c\right)\left(b^2 - 2c + \sqrt{2}c\right)$
2(a)	$\log_2 x - \log_x 4 \leq 1$ $\log_2 x - 2\log_x 2 \leq 1$ $\log_2 x - \frac{2}{\log_2 x} \leq 1$ $let\ y = \log_2 x$

$$y - \frac{2}{y} \leq 1$$

$$y - \frac{2}{y} - 1 \leq 0$$

$$\frac{y^2 - y - 2}{y} \leq 0$$

$$\frac{y^2 + y - 2y - 2}{y} \leq 0$$

$$\frac{y(y+1) - 2(y+1)}{y} \leq 0$$

$$\frac{(y-2)(y+1)}{y} \leq 0$$

The critical valves include: $y = -1$, $y = 0$,$y = 2$

Region where the curve lies

	$y < -1$	$-1 < y < 0$	$0 < y < 2$	$y > 2$
$(y+1)$	$-$	$+$	$+$	$+$
$(y-2)$	$-$	$-$	$-$	$+$
y	$-$	$-$	$+$	$+$
$\frac{(y-2)(y+1)}{y}$	$-$	$+$	-	$+$

The solution set is : $y < -1$ and $0 < y < 2$

For $y < -1$; $\log_2 x < -1$

$$x < 2^{-1}$$

$$x < \frac{1}{2}$$

For $0 < y \leq 2$; $\quad 0 < \log_2 x \leq 2$

$$2^0 < x \leq 2^2$$

$$1 < x \leq 4$$

(b)	$2a - 3b + c = 10 \dots\dots\dots\dots\dots (i)$ $a + 4b + 2c + 3 = 0 \dots\dots\dots\dots (ii)$ $5a - 2b - c = 7 \dots\dots\dots\dots\dots\dots\dots (iii)$ Equation $(i) - 2(ii)$ gives; $2a - 3b + c = 10$ $-2a + 8b + 4c = -6$
	$-11b - 3c = 16 \dots\dots\dots\dots\dots\dots\dots (iv)$ Equation $(i) \times 5 - (ii) \times 3 gives,$ $10a - 15b + 5c = 50$ $- \quad 10a - 4b - 2c = 14$
	$-11b + 7c = 36 \dots\dots\dots\dots\dots. (v)$ Equation(iv)-(v) gives; $-11b - 3c = 16$ $- \quad -11b + 7c = 36$
	$-10c = -20, \quad \therefore c = 2$ From equation (v), $-11b + 7c = 36$ $-11b + 7 \times 2 = 36, \quad \therefore b = -2$ From equation (ii), $a + 4b + 2c + 3 = 0$ $a + 4 \times (-2) + (2 \times 2) + 3 = 0$ $\therefore a = 1$

(c)	$$S_\infty = \frac{a}{1-r}$$ $$12.5 = \frac{10}{1-r}$$ $$12.5 - 12.5r = 10$$ $$12.5r = 2.5$$ $$r = \frac{1}{5} = 0.2$$ For $S_n > 10$ $$a\left(\frac{1-r^n}{1-r}\right) > 10$$ $$10\left(\frac{1-(0.2)^n}{1-0.2}\right) > 10$$ $$1-(0.2)^n > 0.8$$ $$0.2 > (0.2)^n$$ $$\log 0.2 > n \log 0.2$$ $$-0.69897 > -0.6989n$$ $$\frac{-0.69897}{-0.69897} < n$$ $$n > 1$$ $$\therefore n = 2$$ The least number of terms is 2
(d)	The common root be $\propto$; $$\propto^3 - 2\propto +4 = 0 \ldots\ldots\ldots\ldots\ldots..(i)$$ $$\propto^2 + \propto + c = 0 \ldots\ldots\ldots\ldots\ldots\ldots(ii)$$ Equation (i)-$\propto$ $(ii) gives$; $$\propto^3 - 2\propto +4 = 0$$ $$-\propto^3 + \propto^2 + \propto c = 0$$

	$$\propto^2 + \propto (c+2) - 4 = 0 \ldots\ldots\ldots\ldots\ldots (iii)$$ Equation (iii)-(ii) gives; $$\propto^2 + \propto (c+2) - 4 = 0$$ $$- \ \propto^2 + \propto + c = 0$$ $$\propto (c+1) - 4 - c = 0$$ $$\propto = \frac{c+4}{c+1}$$ From equation (ii) $$\left(\frac{c+4}{c+1}\right)^2 + \frac{c+4}{c+1} + c = 0$$ $$(c+4)^2 + (c+4)(c+1) + c(c+1)^2 = 0$$ $$(c^2+8c+16) + (c^2+4c+c+4) + (c^3+2c^2+1c) = 0$$ $$c^3 + 4c^2 + 14c + 20 = 0$$
3(a)	$$(2+5i)^2 + 5\left(\frac{7+2i}{3-4i}\right) - i(4-6i)$$ $$= 4 + 20i - 25 + \frac{(35+10i)(3+4i)}{9+16} - 4i - 6$$ $$= 16i - 27 + \frac{105 + 140i + 30i - 40}{25}$$ $$= \frac{570i - 610}{25} = \frac{114i}{5} - \frac{122}{5} = 22.8i - 24.4$$ Where $a = 24.4,\ b = 22.8i$
(b)	$$3x^2 + 2x - 5 = 0$$ $$x^2 + \frac{2}{3}x - \frac{5}{3} = 0$$ $$sum\ of\ roots, \quad \propto + \beta = \frac{-2}{3}$$ $$product\ of\ roots, \quad \propto \beta = \frac{-5}{3}$$ $$\propto^4 + \beta^4 = (\propto^2)^2 + (\beta^2)^2 = (\propto^2 + \beta^2)^2 - 2\propto^2 \beta^2$$

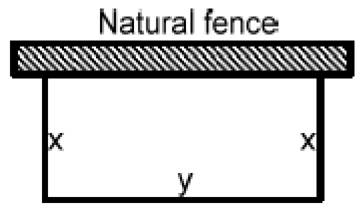
	$= [(\propto + \beta)^2 - 2 \propto \beta]^2 - 2(\propto \beta)^2$ $\left[\left(\frac{-2}{3}\right)^2 - 2\left(\frac{-5}{3}\right)\right]^2 - 2\left(\frac{-5}{3}\right)^2$ $\left[\frac{4}{9} - \frac{50}{9}\right]^2 - \frac{50}{9} = \frac{2116}{81} - \frac{50}{9} = \frac{1666}{81} \approx 20.568$
(c)	$\sqrt{x+5} + \sqrt{x+21} = \sqrt{6x+40}$ $x + 5 + x + 21 + 2\sqrt{x^2 + 26x + 105} = 6x + 40$ $2\sqrt{x^2 + 26x + 105} = 4x + 14$ $\sqrt{x^2 + 26x + 105} = 2x + 7$ $x^2 + 26x + 105 = 4x^2 + 28x + 49$ $3x^2 + 2x - 56 = 0$ $x = -2 \pm \frac{\sqrt{2^2 - 4 \times 3 \times (-56)}}{2 \times 3} = \frac{-2 + 26}{6}$ Either $x = \frac{-2-26}{6} = \frac{-14}{3} \neq \frac{-14}{3}$ Or $x = \frac{-2+26}{6} = 4$; $x = 4$

(d)	$$\log_5 21 = m$$ $$5^m = 21 \ldots\ldots\ldots\ldots..(i)$$ $$\log_9 75 = n$$ $$9^n = 75$$ $$3^{2n} = 5^2.3^1$$ $$3^{2n-1} = 5^2 \ldots\ldots\ldots\ldots..(ii)$$ Equation $(ii) \div (i)$ $$\frac{3^{2n-1}}{21} = \frac{5^2}{5^m}$$ $$\frac{3^{2n-1}}{3 \times 7} = 5^{(2-m)}$$ $$\frac{3^{2n-2}}{5^{(2-m)}} = 7$$ $$\log_5 7 = \log_5 3^{(2n-2)} - \log_5 5^{(2-m)}$$ $$\log_5 7 = (2n-2)\log_5 3 - (2-m).$$ $$but\ \log_5 3 = \log_5\left(\frac{21}{7}\right) = \log_5 21 - \log_5 7$$ $$\log_5 7 = (2n-2)(\log_5 21 - \log_5 7) - (2-m)$$ $$(1+2n-2)\log_5 7 = (2n-2)\log_5 21 - (2-m)$$ $$(2n-1)\log_5 7 = 2mn - 2m - 2 + m$$ $$\log_5 7 = \frac{1}{2n-1}(2mn - m - 2)$$
4(a)	$$(1-x)^{\frac{1}{3}} = 1 + \frac{1}{3}(-x) + \frac{1}{3} \times \frac{-2}{3} \times \frac{(-x)^2}{2!} + \frac{1}{3} \times \frac{-2}{3} \times \frac{-5}{3} \times \frac{(-x)^3}{3!} + \cdots$$ $$= 1 - \frac{1}{3}x - \frac{1}{9}x^2 - \frac{5}{81}x^3 + \cdots$$ For the hence part; $$\sqrt[3]{24} = (27-3)^{\frac{1}{3}} = \left[27\left(1 - \frac{3}{27}\right)\right]^{\frac{1}{3}} = 3\left(1 - \frac{1}{9}\right)^{\frac{1}{3}}$$

	$$by\ comparison, x = \frac{1}{9};$$ $$\sqrt[3]{24} = 3\left[1 - \left(\frac{1}{3} \times \frac{1}{9}\right) - \frac{1}{9} \times \left(\frac{1}{9}\right)^2 - \frac{5}{81} \times \left(\frac{1}{9}\right)^3\right]$$ $$\sqrt[3]{24} = 3 \times 0.9615 = 2.88 \quad (3\ s.f)$$
(b)	$$(1+ax)^n \approx 1 + n(ax) + \frac{n(n-1)}{2!}(ax)^2 + \cdots$$ $$\approx 1 + nax + + \frac{n(n-1)}{2}a^2x^2 + \cdots$$ By comparison, $$na = \frac{-5}{2} \qquad \rightarrow a = \frac{-5}{2n} \ldots\ldots\ldots\ldots\ldots.(i)$$ $$\frac{1}{2}n(n-1)a^2 = \frac{75}{8} \ldots\ldots\ldots\ldots\ldots\ldots\ldots(ii)$$ Substituting equation (i) into (ii) gives; $$\frac{1}{2}n(n-1)\left(\frac{-5}{2n}\right)^2 = \frac{75}{8}$$ $$\frac{1}{2}n(n-1) \times \frac{25}{4n^2} = \frac{75}{8}$$ $$\frac{25}{8n}(n-1) = \frac{75}{8}$$ $$(n-1) = 3n$$ $$2n = -1$$ $$n = -0.5$$ From equation (i), $$a = \frac{-5}{2 \times (-0.5)} = 5$$ $$the\ expansion\ is\ valid\ for\ \lvert x \rvert < \frac{1}{5}.$$

(c)	General term $= 6C_r \times \left(\frac{3}{x^2}\right)^r (2x)^{6-r}$ $$= 6C_r \times 3^r \times 2^{6-r} \times x^{-2r} \times x^{6-r}$$ For the term independent of x; $$-2r + 6 - r = 0$$ $$6 - 3r = 0$$ $$r = 2$$ $$Required\ term = 6C_2 \times 3^2 \times 2^{6-2} = 15 \times 9 \times 16 = 2160$$
(d)	$$\left(x^3 + \frac{1}{x^4}\right)^{15} = \sum_{r=0}^{15}(15C_r)(x^3)^r\left(\frac{1}{x^4}\right)^{15-r}$$ $$general\ term = (15C_r)(x^3)^r\left(\frac{1}{x^4}\right)^{15-r} = (15C_r)(x^3)^r(x^{-4})^{15-r}$$ $$= (15C_r)x^{3r-4(15-r)} = (15C_r)x^{7r-60}$$ $$for\ the\ term\ in\ x^{17};$$ $$7r - 60 = 17, \qquad r = \frac{77}{7} = 11$$ Term in $x^{17} = 15C_r = 15C_{11} = 1365.$
5(a)	$$y = tan^{-1}\left(\frac{ax-b}{bx+a}\right)$$ $$\tan y = \frac{ax-b}{bx+a}$$ $$sec^2y\frac{dy}{dx} = \frac{(ax-b).a-(ax-b).b}{(bx+a)^2}$$ $$sec^2y\frac{dy}{dx} = \frac{a^2+b^2}{(bx+a)^2}$$ $$but\ sec^2y = 1 + tan^2y$$ $$= 1 + \left(\frac{ax-b}{bx+a}\right)^2$$ $$= \frac{(bx+a)^2+(ax-b)^2}{(bx+a)^2}$$

	$$=\frac{b^2x^2+2abx+a^2+a^2x^2-2abx+b^2}{(bx+a)^2}$$ $$=\frac{b^2x^2+b^2+a^2+a^2x^2}{(bx+a)^2}$$ $$=\frac{b^2(1+x^2)+a^2(1+x^2)}{(bx+a)^2}$$ $$=\frac{(a^2+b^2)(1+x^2)}{(bx+a)^2}$$ $\frac{dy}{dx}=\frac{a^2+b^2}{(bx+a)^2}\cdot\frac{(bx+a)^2}{(a^2+b^2)(1+x^2)}=\frac{1}{1+x^2}$
(b)	Let $y=\cos(x^2e^x)\,, and\ u=x^2e^x,\ y=cosu$ $$\frac{du}{dx}=x^2e^x+2xe^x=x(x+2)e^x,\quad \frac{dy}{du}=-sinu$$ $$\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}=-\sin(x^2e^x)\times x(x+2)e^x$$ $$=-x(x+2)e^x\sin(x^2e^x)$$
(c)	$$y=cos^2x$$ $$y+\Delta y=cos^2(x+\Delta x)$$ $$\Delta y=cos^2(x+\Delta x)-cos^2x$$ $$\Delta y=[\cos(x+\Delta x)-cosx][\cos(x+\Delta x)+cosx]$$ $$\Delta y=-2\sin\left(x+\frac{\Delta x}{2}\right)sin\left(\frac{\Delta x}{2}\right)[\cos(x+\Delta x)+cosx]$$ $sin\left(\frac{\Delta x}{2}\right)\approx\frac{\Delta x}{2}$ for small angles in radians $$\frac{\Delta y}{\Delta x}=-\sin\left(x+\frac{\Delta x}{2}\right)[\cos(x+\Delta x)+cosx]$$ $$\frac{dy}{dx}=\lim_{\Delta x\to 0}\left(-\sin\left(x+\frac{\Delta x}{2}\right)[\cos(x+\Delta x)+cosx]\right)$$ $$\frac{dy}{dx}=-2cosxsinx$$

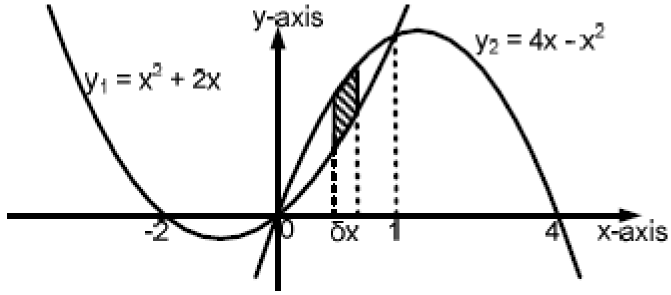
(d)(i)	$y = \frac{t^2+4}{t}$, $\frac{dy}{dt} = \frac{t \times 2t - (t^2+4) \times 1}{t^2} = \frac{2t^2 - t^2 - 4}{t^2} = \frac{t^2-4}{t^2}$ $x = \frac{3t-1}{t} = 3 - \frac{1}{t}$; $\frac{dx}{dt} = \frac{1}{t^2}$ $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \left(\frac{t^2-4}{t^2}\right) \times t^2 = t^2 - 4$ $\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right) \times \frac{dt}{dx} = \frac{d}{dt}(t^2-4) \times t^2 = 2t \times t^2 = 2t^3$
(d)(ii)	Let x and y be the dimensions that will give him the maximum possible area of the land.  $Perimeter = x + y + x = 100$ $y + 2x = 100$, $y = 100 - 2x$ $area, A = xy = x(100-2x) = 100x - 2x^2$ $\frac{dA}{dx} = 100 - 4x$ Area is maximum when $\frac{dA}{dx} = 0$ $100 - 4x = 0$, $x = \frac{100}{4} = 25m$ $y = 100 - 2x = 100 - 2(25) = 50m$ $Maximum\ area = xy = 25 \times 50 = 1250m^2$

6(a)	$$\frac{dy}{dx}+\frac{2xy}{x^2+1}-x=0$$ $$I.F=e^{\int\frac{2x}{x^2+1}dx}=e^{\ln(x^2+1)}=x^2+1$$ $$multiplying\ through\ by\ \ x^2+1\ gives$$ $$(\ x^2+1)\frac{dy}{dx}+2xy=x^3+x$$ $$\frac{d}{dx}[(\ x^2+1\ )y]=x^3+x$$ $$\int\frac{d}{dx}[(\ x^2+1\ )y]dx=\int(x^3+x)\,dx$$ $$\therefore y(\ x^2+1)=\frac{x^4}{4}+\frac{x^2}{2}+c$$
(b)	$$\frac{dy}{dx}=kx,\ \ y=\frac{1}{2}kx^2+c$$ $$at\ (2,3), x=2\ and\ y=3$$ $$3=\frac{1}{2}k\times2^2+c\ . \qquad 3=2k+c\ \ldots\ldots\ldots\ldots\ldots(i)$$ Also at (2,3), gradient is 6, $$\frac{dy}{dx}=kx,\quad 6=k\times2,\quad k=3$$ From equation (i), $$3=2k+c,\quad 3=2\times3+c \qquad\qquad c=3-6=-3$$ The equation of the curve is given by; $$y=\frac{1}{2}kx^2+c\ ,\quad y=\frac{1}{2}3x^2-3\ ,\quad y=\frac{3}{2}x^2-3$$

(c)(i)	$\frac{dp}{p} \propto p, \quad \frac{dp}{p} = -kp$
(ii)	$\int \frac{dp}{p} = \int -kdt$ $lnp = -kt + c \dots\dots\dots\dots\dots..(i)$ When $t = 0,\ p = p_o$ $lnp_o = -k \times 0 + c\ , \quad c = lnp_o$ Equation (i) becomes $lnp = -kt + lnp_o \dots\dots\dots\dots\dots\dots(ii)$ When $t = 4, \quad p = \frac{1}{3}p_o$ $ln\left(\frac{1}{3}p_o\right) = -4k + lnp_o$ $ln\left(\frac{1}{3}p_o\right) - lnp_o = -4k$ $ln\left(\frac{1}{3}\right) = -4k$ $k = 0.25ln3$ Equation (ii) becomes $lnp = -0.25ln3t + lnp_o$ $ln\left(\frac{p}{p_o}\right) = -0.25lnt$ $\frac{p}{p_o} = e^{-0.25ln3t}$ $p = p_o e^{-0.25ln3t}$ $p = p_o e^{-0.275t}$
7(a)	$f(x) = \frac{x^4 + x^3 - 6x^2 - 13x - 6}{x^3 - 7x - 6} = \frac{x^4 + x^3 - 6x^2 - 13x - 6}{(x-1)(x-3)(x+2)}$ Let $\frac{x^4 + x^3 - 6x^2 - 13x - 6}{(x-1)(x-3)(x+2)} \equiv Ax + B + \frac{C}{x+1} + \frac{D}{x-3} + \frac{E}{x+2}$

	$x^4 + x^3 - 6x^2 - 13x - 6$ $\equiv (Ax+B)(x-3)(x+2)(x+1) + C(x-3)(x+2) + D(x+1)(x$ $+ E(x+1)(x-3)$ Put $x = 3$; $81 + 27 - 54 - 39 - 6 = 20D$; $9 = 20D$; $\therefore D = \frac{9}{20}$ Put $x = -2$; $16 - 8 - 24 + 26 - 6 = 5C$, $4 = 5E$; $\therefore E = \frac{4}{5}$ Put$x = -1$; $1 - 1 - 6 \mp 13 - 6 = -4C$, $1 = -4C$; $\therefore C = \frac{-1}{4}$ Compare coefficients of x^4; $1 = A$ Put $x = 0$; $-6 = -6B - 6C + 2D - 3E$ $-6 = -6B - 6\left(\frac{-1}{4}\right) + 2\left(\frac{9}{20}\right) - 3\left(\frac{4}{5}\right)$ $-6 = -6$, $\therefore B = 1$ $\therefore f(x) \equiv (x+1) - \frac{1}{4(x+1)} + \frac{9}{20(x-3)} + \frac{4}{5(x+2)}$ Hence; $\int_4^5 f(x)dx = \int_4^5 (x+1)dx - \frac{1}{4}\int_4^5 \frac{1}{x+1}dx + \frac{9}{20}\int_4^5 \frac{1}{x-3}dx + \frac{4}{5}\int_4^5 \frac{1}{x+2}dx$ $= \left[\frac{x^2}{2} + x - \frac{1}{4}\ln(x+1) + \frac{9}{20}\ln(x-3) + \frac{4}{5}\ln(x+2)\right]_4^5$ $\left(\frac{5^2}{2} + 5 - \frac{1}{4}\ln(5+1) + \frac{9}{20}\ln(5-3) + \frac{4}{5}\ln(5+2)\right)$ $- \left(\frac{4^2}{2} + 4 - \frac{1}{4}\ln(4+1) + \frac{9}{20}\ln(4-3) + \frac{4}{5}\ln(4+2)\right)$ $= 5.8896967 = 5.8897\ (4dps)$
(b)(i)	For the points of intersection; $x(x+2) = x(4-x)$ $x^2 + 2x = 4x - x^2$ $2x^2 - 2x = 0$ $2x(x-1) = 0$

$$either\ x = 0\ or\ x = 1$$



b(ii)

$$Element\ of\ area\ \Delta A = y\Delta x$$

$$Required\ area\ A = \int_0^1 (y_2 - y_1)dx = \int_0^1 [4x - x^2 - (x^2 + 2x)]dx$$

$$= \int_0^1 (2x - 2x^2)dx = \left[x^2 - \frac{2}{3}x^3\right]_0^1 = \left(1 - \frac{2}{3}\right) - (0) = \frac{1}{3}sq\ units$$

(iii)

$$Element\ of\ volume\ \Delta V = \pi(y_2 - y_1)^2\Delta x$$

$$Required\ volume\ V = \pi\int_0^1 (2x - 2x^2)^2 dx = \pi\int_0^1 (4x^2 - 8x^3 + 4x^4)dx$$

$$= \pi\left[\frac{4}{3}x^3 - 2x^4 + \frac{4}{5}x^5\right]_0^1$$

$$= \pi\left(\frac{4}{3} - 2 + \frac{4}{5}\right) - 0 = \frac{2\pi}{15}cubic\ units$$

8(a)

$$\int xcos^2x\,dx = \int x\left(\frac{1 + cos2x}{2}\right)dx = \frac{1}{2}\int x\,dx + \frac{1}{2}\int xcos2xdx$$

Sign	Differentiation	Integration
+	x	$cos2x$
-	1	$\frac{1}{2}sin2x$
+	0	$\frac{-1}{4}cos2x$

$$\int xcos^2x\,dx = \frac{1}{2}x^2 + \frac{1}{2}\left(\frac{1}{2}xsin2x + \frac{1}{4}cos2x\right) + c$$

	$= \frac{1}{2}x^2 + \frac{1}{4}x sin2x + \frac{1}{8}cos2x + c$

(b)	Intercepts;

$$x; y = 0$$

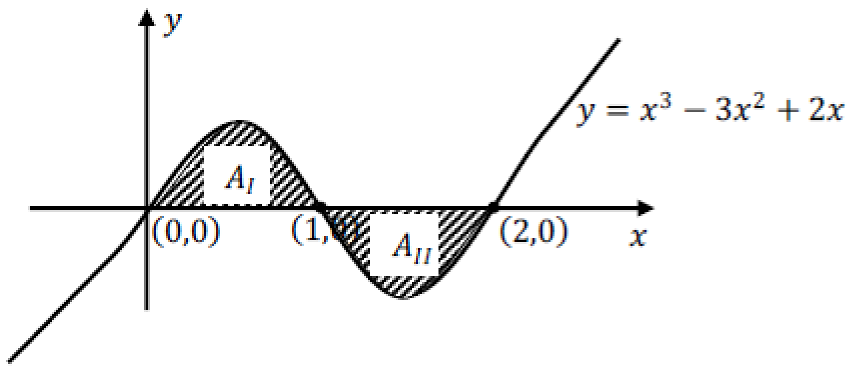
$$0 = x(x-1)(x-2)$$

$$x = 0, \quad x = 1, \; x = 2$$

$$\therefore (0,0), \quad (1,0), \; and\,(2,0)$$

$$As\; x \to +\infty, y \to +\infty$$

$$As\; x \to -\infty, y \to -\infty$$



$$A = A_I + A_{II}$$

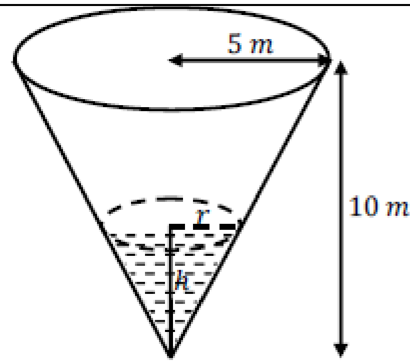
$$A_I = \int_0^1 (x^3 - 3x^2 + 2x)dx = \left[\frac{x^4}{4} - x^3 + x^2\right]_0^1$$

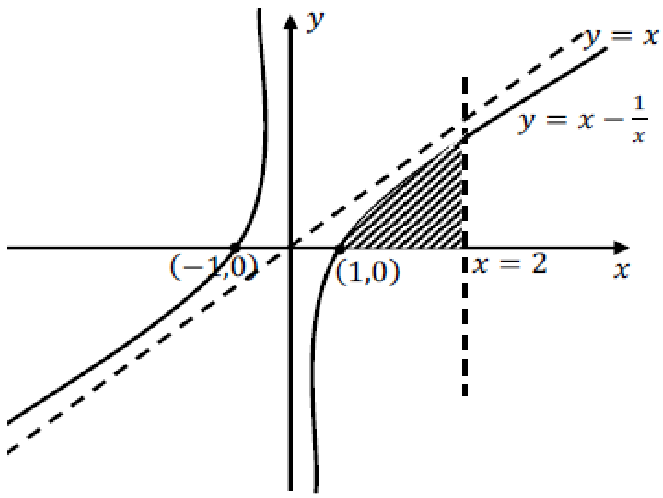
$$A_I = \left(\frac{1}{4} - 1 + 1\right) - 0 = \frac{1}{4} sq\ units.$$

$$A_{II} = \int_1^2 (x^3 - 3x^2 + 2x)dx = \left[\frac{x^4}{4} - x^3 + x^2\right]_1^2$$

$$A_{II} = (4 - 8 + 4) - \left(\frac{1}{4} - 1 + 1\right) = \left|\frac{-1}{4}\right| = \frac{1}{4} sq.units$$

$$\therefore A = \frac{1}{4} + \frac{1}{4} = \frac{1}{2} sq.units$$

(c)	 From similarities of figures; $$\frac{H}{h}=\frac{R}{r},\quad \frac{10}{h}=\frac{5}{r},\quad r=\frac{h}{2}$$ $$V=\frac{1}{3}\pi r^2 h,\quad V=\frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h=\frac{\pi h^3}{12}$$ $$\frac{dV}{dh}=\frac{\pi h^2}{4}$$ $$\frac{dV}{dt}=\frac{dV}{dh}.\frac{dh}{dt}$$ $$1.5=\frac{\pi h^2}{4}.\frac{dh}{dt}$$ $$\frac{dh}{dt}=\frac{6}{\pi h^2}$$ $$When\ h=4cm;\ \frac{dh}{dt}=\frac{6}{\pi 4^2}=\frac{3}{8\pi}mmin^{-1}$$
(d)	$$y=x-\frac{1}{x}$$ $$Vertical\ asymptote,\qquad y-undefined, when\ x=0$$ $$Slanting\ asymptote, y=x$$ Intercepts; $$x; y=0$$ $$0=x^2-1$$ $$x=\pm 1;\quad (-1,0)\ and\ \ (1,0)$$

	 $Element\ of\ Area\ , \Delta A = y\Delta x$ $Required\ area\ , A = \int_1^2 \left(x - \frac{1}{x}\right) dx = \left[\frac{x^2}{2} - lnx\right]_1^2 = (2 - ln2) - \left(\frac{1}{2} - ln1\right)$ $A = 0.806852819 = 0.8069\ sq.units$
9(a)	$3cos4\theta + 7cos2\theta = 0$ $3(2cos^2 2\theta - 1) + 7cos2\theta = 0$ $6cos^2 2\theta - 3 + 7cos2\theta = 0$ $6cos^2 2\theta + 7cos2\theta - 3 = 0$ $cos2\theta = -7 \pm \frac{\sqrt{7^2 - 4 \times 6 \times (-3)}}{2 \times 6}$ $either\ cos2\theta = \frac{1}{3} \quad or\ cos2\theta = -1.5$ $for\ cos2\theta = \frac{1}{3} \quad , 2\theta = 70.53^0, 289.74^0, \qquad \theta = 35.27^0, \theta = 144.74^0$ $for\ cos2\theta = -1.5, \qquad \theta\ is\ undefined.$
(b)	$10sinxcosx + 12\ cos2x \equiv Rsin(2x + \propto)$ $10sinxcosx + 12\ cos2x \equiv Rsin2xcos \propto + Rcos2xsin \propto$ $by\ comparison, \qquad Rcos \propto = 5 \ldots\ldots\ldots.(i), \quad Rsin \propto = 12 \ldots\ldots\ldots..(ii)$

	$(ii)-(i) gives, \frac{R\sin\propto}{R\cos\propto}=\frac{12}{5}, \quad tan\propto=\frac{12}{5}, \quad \propto=67.38^0$ $R=\sqrt{5^2+12^2}=13$ $10sinxcosx+12\,cos2x\equiv 13\sin(2x+67.38^0)$ $\therefore Maximum\ value=13\times 1=13$
(c)	$From\ \ LHS;$ $\frac{cos11^0+sin11^0}{cos11^0-sin11^0}=\frac{1+tan11^0}{1-tan11^0}=\frac{tan45^0+tan11^0}{tan45^0-tan11^0}$ $=\tan(45+11)=tan56^0$
(d)	From LHS; $SinB+SinC-SinA=\left[2sin\left(\frac{B+C}{2}\right)cos\left(\frac{B-C}{2}\right)\right]-2sin\left(\frac{A}{2}\right)cos\left(\frac{A}{2}\right)$ $for\ angles\ of\ a\ triangle, A, B, C,$ $sin\left(\frac{B+C}{2}\right)=sin\left(90-\frac{A}{2}\right)=cos\left(\frac{A}{2}\right)$ $cos\left(\frac{B+C}{2}\right)=cos\left(90-\frac{A}{2}\right)=sin\left(\frac{A}{2}\right)$ $SinB+SinC-SinA=2cos\left(\frac{A}{2}\right)cos\left(\frac{B-C}{2}\right)-2sin\left(\frac{A}{2}\right)cos\left(\frac{A}{2}\right)$ $2cos\left(\frac{A}{2}\right)\left[cos\left(\frac{B-C}{2}\right)-cos\left(\frac{B+C}{2}\right)\right]$ $2cos\left(\frac{A}{2}\right)\left[-2sin\left(\frac{B}{2}\right)sin\left(\frac{-C}{2}\right)\right]$ $=4cos\left(\frac{A}{2}\right)sin\left(\frac{B}{2}\right)sin\left(\frac{C}{2}\right)$
10(a)	$10sin^2x+10sinxcosx=cos^2x+2$ $Dividing\ throughout\ by\ cos^2x\ gives;$ $10tan^2x+10tanx=1+2sec^2x$ $10tan^2x+10tanx=1+2(1+tan^2x)$ $8tan^2x+10tanx-3=0$

	$tanx = \frac{-10 \pm \sqrt{10^2 - 4 \times 8 \times (-3)}}{2 \times 8} = \frac{-10 \pm 14}{16}$ $either, tanx = \frac{-10 - 14}{16} = -1.5, \quad x = 123.69^0, -56.31^0$ $or, tanx = \frac{-10 \pm 14}{16} = 0.25, \quad x = 14.04^0, 165.96^0$
(b)	$\frac{Sin16\theta cos2\theta - cos6\theta sin12\theta}{cos4\theta cos2\theta + sin6\theta sin8\theta}$ $= \frac{\frac{1}{2}(sin18\theta + sin14\theta) - \frac{1}{2}(sin18\theta + sin6\theta)}{\frac{1}{2}(cos6\theta + cos2\theta) + \frac{1}{2}(cos14\theta - cos2\theta)}$ $= \frac{sin14\theta - sin6\theta}{cos6\theta + cos14\theta} = \frac{2cos10\theta sin4\theta}{2cos10\theta cos4\theta} = \frac{sin4\theta}{cos4\theta} = tan4\theta$
(c)	$2sin3\theta = 1, \quad sin3\theta = 0.5$ $3\theta = 30^0, 150^0, 390^0, 510^0, 750^0, 870^0$ $\theta = 10^0, 50^0, 130^0, 170^0, 250^0, 290^0$ For the hence part, $8x^3 - 6x + 1 = 0 \quad let, x = sin\theta$ $8sin^3\theta - 6sin\theta + 1 = 0$ $6sin\theta - 8sin^3\theta = 1$ $2(3sin\theta - 4sin^3\theta) = 1$ $2sin3\theta = 1$ $\theta = 10^0, 50^0, 170^0, \quad 250^0, 290^0$ $x = sin\theta$
	$x_1 = sin10^0 = sin170^0 = 0.1736$ $x_2 = sin50^0 = sin130^0 = 0.7660$ $x_3 = sin250^0 = sin290^0 = -0.9397$

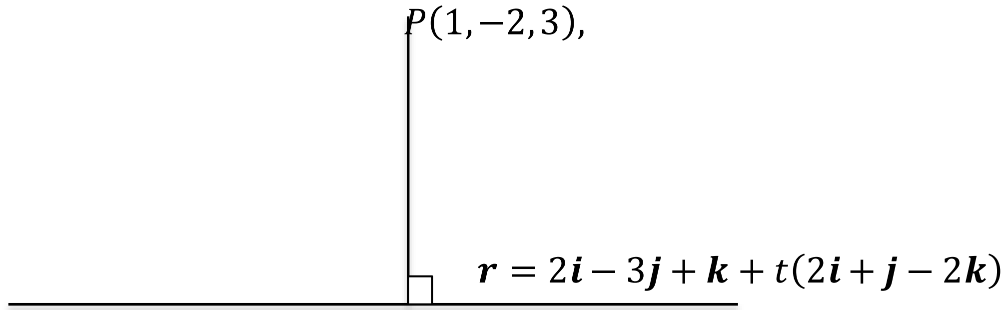
(d)	From sine rule, $a = 2RSinA,\ b = 2RsinB, \quad C = 2RsinC$ $$from\ LHS,$$ $$\frac{a^2 - b^2}{c^2} = \frac{(2RSinA)^2 - (2RSinB)^2}{(2RsinC)^2} = \frac{Sin^2A - Sin^2B}{Sin^2C}$$ $$\frac{(sinA - sinB)(sinA + sinB)}{sin^2C}$$ $$= \frac{2cos\left(\frac{A+B}{2}\right)sin\left(\frac{A-B}{2}\right) \times 2sin\left(\frac{A+B}{2}\right)cos\left(\frac{A-B}{2}\right)}{sin^2(A+B)}$$ $$\frac{2sin\left(\frac{A+B}{2}\right)cos\left(\frac{A+B}{2}\right) \times 2sin\left(\frac{A-B}{2}\right)cos\left(\frac{A+B}{2}\right)}{sin^2(A+B)}$$ $$= \frac{Sin(A+B) \times \sin(A-B)}{sin^2(A+B)} = \frac{\sin(A-B)}{\sin(A+B)}$$
11(a)	from LHS; $1 + sec2\theta = 1 + \frac{1+t^2}{1-t^2} = \frac{1-t^2+1+t^2}{1-t^2} = \frac{2}{1-t^2} \times \frac{t}{t}$ $$= \frac{2t}{1-t^2} \times \frac{1}{t} = tan2\theta cot\theta$$
(b)(i)	$$y = \frac{sinx - 2sin2x + sin3x}{sinx + 2sin2x + sin3x} = \frac{2sin2xcosx - 2sin2x}{2sin2xcosx + 2sin2x} = \frac{2sin2x(cosx - 1)}{2sin2x(cosx + 1)}$$ $$= \frac{cosx - 1}{cosx + 1} = \frac{\left(1 - 2sin^2\left(\frac{x}{2}\right)\right) - 1}{\left(2cos^2\left(\frac{x}{2}\right) - 1\right) + 1} = \frac{-2sin^2\left(\frac{x}{2}\right)}{2cos^2\left(\frac{x}{2}\right)} = -tan^2\left(\frac{x}{2}\right)$$ $$\therefore y + tan^2\left(\frac{x}{2}\right) = 0$$

(ii)	For $tan^2 15^0$, $\frac{x}{2} = 15^0$, $x = 30^0$ $$y = \frac{cosx - 1}{cosx + 1} = \frac{cos30^0 - 1}{cos30^0 + 1} = \frac{\frac{\sqrt{3}}{2} - 1}{\frac{\sqrt{3}}{2} + 1} = \frac{\sqrt{3} - 2}{\sqrt{3} + 2} = \frac{(\sqrt{3} - 2)(\sqrt{3} - 2)}{(\sqrt{3} + 2)(\sqrt{3} - 2)}$$ $$= \frac{3 - 4\sqrt{3} + 4}{3 - 4} = \frac{7 - 4\sqrt{3}}{-1} = -7 + 4\sqrt{3}$$ $$\therefore tan^2 15^0 = -7 + 4\sqrt{3}$$
(iii)	$$2y + sec^2\left(\frac{x}{2}\right) = 0$$ $$-2tan^2\left(\frac{x}{2}\right) + \left(1 + tan^2\left(\frac{x}{2}\right)\right) = 0$$ $$-tan^2\left(\frac{x}{2}\right) + 1 = 0$$ $$tan^2\left(\frac{x}{2}\right) = 1$$ $$tan\left(\frac{x}{2}\right) = \pm 1$$ $$\frac{x}{2} = 45^0, 135^0, 225^0, 315^0$$ $$x = 90^0, 270^0$$
12(a)	$$Cos\theta = 1 - 2sin^2\left(\frac{\theta}{2}\right)$$ $$sin\frac{\theta}{2} = \pm\sqrt{\frac{1 - cos\theta}{2}}$$ $$since\ \frac{\theta}{2} = 292\frac{1}{2}^0\ \ is\ in\ the\ fourth\ quadrant\ in\ which\ the\ sine\ ratio\ is\ negative$$ $$sin292\frac{1}{2}^0 = -\sqrt{\frac{1 - cos\theta}{2}}$$

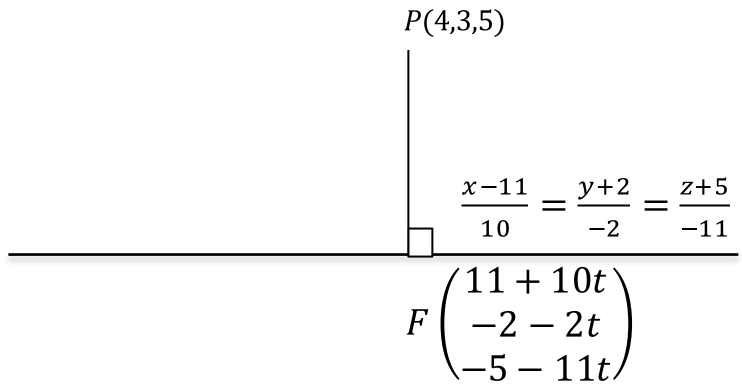
	$sin292\frac{1}{2}^0 = -\sqrt{\frac{1-cos[(6\times 90^0)+45^0]}{2}} = -\sqrt{\frac{1-[-cos45^0]}{2}}$ $= -\sqrt{\frac{1+\frac{1}{\sqrt{2}}}{2}} = -\sqrt{\frac{2+\sqrt{2}}{4}} = \frac{-1}{2}\sqrt{2+\sqrt{2}}$ $\therefore Sin\left(292\frac{1}{2}^0\right) = -\frac{1}{2}\sqrt{2+\sqrt{2}}$
(b)(i)	$P = 2cos2x + 3cos4x$ $p^2 = 4cos^2 2x + 12cos2xcos4x + 9cos^2 4x$ $q = 2sin2x + 3sin4x$ $q^2 = 4sin^2 2x + 12sin2xsin4x + 9sin^2 4x$ $p^2 + q^2 = 4(cos^2 2x + sin^2 2x) + 12(cos2xcos4x + sin2xsin4x) +$ $9(cos^2 4x + sin^2 4x)$ $p^2 + q^2 = 4 + 9 + 12\cos(4x - 2x)$ $p^2 + q^2 = 13 + 12cos2x$ $the\ greatest\ value\ of\ p^2 + q^2 = 13 + 12 = 25$ $the\ least\ value\ of\ p^2 + q^2 = 13 - 12 = 1$
(ii)	$p^2 + q^2 = 19$ $13 + 12cos2x = 19$ $cos2x = \frac{1}{2}$ $2x = cos^{-1}\left(\frac{1}{2}\right) = 60^0, 300^0, 420^0, \quad 660^0$ $x = 30^0$

(iii)	$$pq = (2cos2x + 3cos4x)(2sin2x + 3sin4x)$$ $$pq = 4sin2xcos2x + 6cos2xsin4x + 6cos4xsin2x + 9cos4xsin4x$$ $$pq = 2sin4x + 6\sin(4x + 2x) + \frac{9}{2}sin8x$$ $$for\ x = 30^0, \quad pq = 2\sin(120^0) + 6sin(180^0) + \frac{9}{2}sin240^0$$ $$pq = \frac{2\sqrt{3}}{2} + 0 - \frac{9}{2} \times \frac{\sqrt{3}}{2} = \frac{4\sqrt{3} - 9\sqrt{3}}{4} = \frac{-5\sqrt{3}}{4}$$
13(a)	$$3x - y + z = 2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (i)$$ $$x - \quad 5y + 2z = 6 \ldots\ldots\ldots\ldots\ldots\ldots (ii)$$ $$2 \times (i) - (ii)$$ $$6x - 2y + 2z = 4$$ $$(-)\ x + 5y + 2z = 6$$
	$$5x - 7y = -2\ , \quad x = \frac{7y - 2}{5} \ldots\ldots\ldots\ldots (iii)$$ $$5(i) + (ii) gives;$$ $$15x - 5y + 5z = 10$$ $$(-)\quad x - 5y + 2z = 6$$
	$$16x + 7z = 16, \quad x = \frac{16 - 7z}{16} \ldots\ldots\ldots\ldots\ldots. (iv)$$ $$the\ cartesian\ equation\ of\ the\ line\ A\ is \quad x = \frac{7y - 2}{5} = \frac{16 - 7z}{16}$$
(b)(i)	Direction vector, $d = \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix}$ Position vector= $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ Cartesian equation of the line B is $\frac{x-1}{3} = \frac{y-1}{-1} = z$

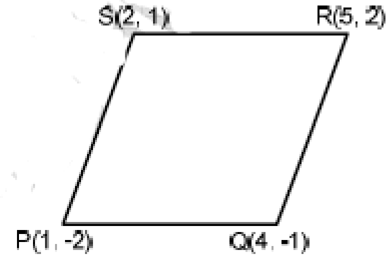
(ii)	For line A, $x = \frac{7y-2}{5} = \frac{16-7z}{16}$ $$x = \frac{7y-2}{5} = \frac{7z-16}{-16}$$ direction vector $d_A = \begin{pmatrix} 1 \\ \frac{5}{7} \\ \frac{-16}{7} \end{pmatrix} = \frac{1}{7}\begin{pmatrix} 7 \\ 5 \\ -16 \end{pmatrix}$ for line B, directional vector $d_B = \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix}$ $$d_A.\ \ d_B = \begin{pmatrix} 7 \\ 5 \\ -16 \end{pmatrix}.\begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix} = 21 - 5 - 16 = 0$$ $$\therefore \theta = 90^0$$
(c)	$$3\overrightarrow{\boldsymbol{AB}} = 2\overrightarrow{\boldsymbol{AC}}$$ $$3\left[\begin{pmatrix} -2 \\ 5 \\ -4 \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}\right] = 2\left[\overrightarrow{\boldsymbol{OC}} - \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}\right]$$ $$\begin{pmatrix} -12 \\ 18 \\ -12 \end{pmatrix} = 2\overrightarrow{\boldsymbol{OC}} - \begin{pmatrix} 4 \\ -2 \\ 0 \end{pmatrix}$$ $$\begin{pmatrix} -8 \\ 16 \\ -12 \end{pmatrix} = 2\overrightarrow{\boldsymbol{OC}}$$ $$\overrightarrow{\boldsymbol{OC}} = \frac{1}{2}\begin{pmatrix} -8 \\ 16 \\ -12 \end{pmatrix} = \begin{pmatrix} -4 \\ 8 \\ -6 \end{pmatrix}$$ $$\therefore C(-4,8,-6)$$
14(a)	$\overrightarrow{OP} = 2\boldsymbol{a} - 5\boldsymbol{b}$**,** $\overrightarrow{OQ} = 5\boldsymbol{a} - \boldsymbol{b}$ $\overrightarrow{OR} = 11\boldsymbol{a} + 7\boldsymbol{b}$ $$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} = (5\boldsymbol{a} - \boldsymbol{b}) - (2\boldsymbol{a} - 5\boldsymbol{b}) = 3\boldsymbol{a} + 4\boldsymbol{b}$$ $$\overrightarrow{QR} = \overrightarrow{OR} - \overrightarrow{OQ} = (11\boldsymbol{a} + 7\boldsymbol{b}) - (5\boldsymbol{a} - \boldsymbol{b}) = 6\boldsymbol{a} + 8\boldsymbol{b} = 2(3\boldsymbol{a} + 4\boldsymbol{b})$$ Since $\overrightarrow{PQ} = 2\overrightarrow{QR}$ and they share a common point P, then P, Q, and R ar collinear.

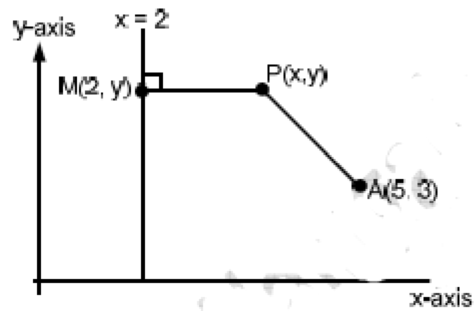
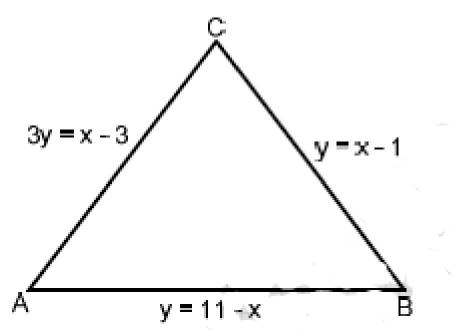
	$\overrightarrow{PQ}:\overrightarrow{QR}=1:2$ P
(b)	 $\boldsymbol{F}\begin{pmatrix}2+2t\\-3+t\\1-2t\end{pmatrix}$ $\overrightarrow{PF}=\overrightarrow{OF}-\overrightarrow{OP}=\begin{pmatrix}2+2t\\-3+t\\1-2t\end{pmatrix}-\begin{pmatrix}1\\-2\\3\end{pmatrix}=\begin{pmatrix}1+2t\\-1+t\\-2-2t\end{pmatrix}$ $\overrightarrow{PF}.\boldsymbol{d}=o,\quad \begin{pmatrix}1+2t\\-1+t\\-2-2t\end{pmatrix}.\begin{pmatrix}2\\1\\-2\end{pmatrix}=0,\ 2+4t-1+t+4+4t=0$ $9t=5\qquad t=\frac{5}{9}$ $\overrightarrow{PF}=\begin{pmatrix}1+2t\\-1+t\\-2-2t\end{pmatrix}=\begin{pmatrix}1+2\left(\frac{5}{9}\right)\\-1+\left(\frac{5}{9}\right)\\-2-2\left(\frac{5}{9}\right)\end{pmatrix}=\begin{pmatrix}\frac{19}{9}\\\frac{-4}{9}\\\frac{-28}{9}\end{pmatrix}$ $=\sqrt{\left(\frac{19}{9}\right)^2+\left(\frac{-4}{9}\right)^2+\left(\frac{-28}{9}\right)^2}$ $\overline{\lvert PF\rvert}=\sqrt{\frac{43}{3}}=3.7859\ units$
(c)	Normal vector, $n=\begin{vmatrix}\boldsymbol{i}&\boldsymbol{j}&\boldsymbol{k}\\1&-3&3\\-1&-3&2\end{vmatrix}=\boldsymbol{i}\begin{vmatrix}-3&3\\-3&2\end{vmatrix}-\boldsymbol{j}\begin{vmatrix}1&3\\-1&2\end{vmatrix}+\boldsymbol{k}\begin{vmatrix}1&-3\\-1&-3\end{vmatrix}$ $=\boldsymbol{i}(-6+9)-\boldsymbol{j}(2+3)+\boldsymbol{k}(-3-3)=3\boldsymbol{i}-5\boldsymbol{j}-6\boldsymbol{k}$

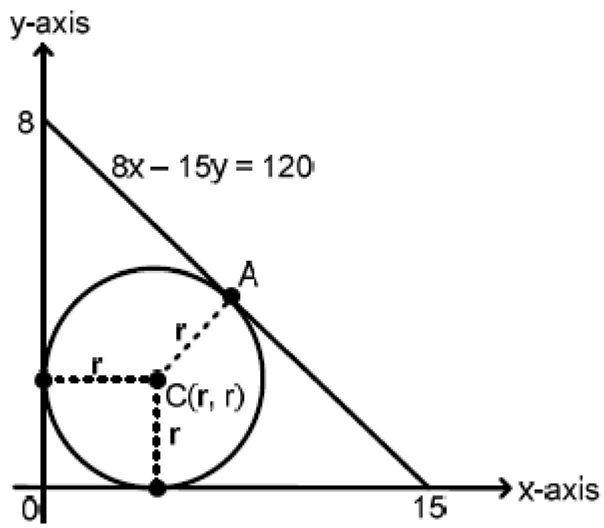
	$$\boldsymbol{r.n}=\boldsymbol{n.a}$$ $$\begin{pmatrix}x\\y\\z\end{pmatrix}.\begin{pmatrix}3\\-5\\-6\end{pmatrix}=\begin{pmatrix}3\\-5\\-6\end{pmatrix}\begin{pmatrix}1\\-3\\2\end{pmatrix}$$ $$3x-5y-6z=3+15-12$$ $$3x-5y-6z=6$$
15(a)	$\overrightarrow{OC}=\frac{\lambda b+3a}{\lambda+3}$, $\begin{pmatrix}a\\4\\5\end{pmatrix}=\frac{\lambda\begin{pmatrix}6\\7\\8\end{pmatrix}+3\begin{pmatrix}1\\2\\3\end{pmatrix}}{\lambda+3}$ $(\lambda+3)\begin{pmatrix}a\\4\\5\end{pmatrix}=\begin{pmatrix}6\lambda+3\\7\lambda+6\\8\lambda+9\end{pmatrix}$, $(\lambda+3)a=6\lambda+3, \ldots.(1)$ $(\lambda+3)4=7\lambda+6 \ldots.(ii)$ $(\lambda+3)5=8\lambda+9 \ldots.(iii)$ $3\lambda=6,\ \lambda=2, a=3$ $\therefore \lambda=2\ \ a=3.$
(b)	From the Cartesian equation of the line, Position vector, $\overrightarrow{OA}=\begin{pmatrix}1\\-4\\-1\end{pmatrix}$, directional vector, $\boldsymbol{d}=\begin{pmatrix}2\\-3\\-1\end{pmatrix}$ The point on the plane is B(2,3,-1) $$\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{0A}=\begin{pmatrix}2\\3\\-1\end{pmatrix}-\begin{pmatrix}1\\-4\\-1\end{pmatrix}=\begin{pmatrix}1\\7\\0\end{pmatrix}$$ Normal vector, $\boldsymbol{n}=\overrightarrow{AB}\times\boldsymbol{d}=\begin{vmatrix}\boldsymbol{i}&\boldsymbol{j}&\boldsymbol{k}\\1&7&0\\2&-3&-1\end{vmatrix}=\boldsymbol{i}\begin{vmatrix}7&0\\-3&-1\end{vmatrix}-\boldsymbol{j}\begin{vmatrix}1&0\\2&-1\end{vmatrix}+\boldsymbol{k}\begin{vmatrix}1&7\\2&-3\end{vmatrix}$ $$=\boldsymbol{i}(-7-0)-\boldsymbol{j}(-1-0)+\boldsymbol{k}(-3-14)=-7\boldsymbol{i}+\boldsymbol{j}-17\boldsymbol{k}$$ The equation of the plane is given by $\boldsymbol{r.n}=\boldsymbol{n.a}$ $$\begin{pmatrix}x\\y\\z\end{pmatrix}.\begin{pmatrix}-7\\1\\-17\end{pmatrix}=\begin{pmatrix}-7\\1\\-17\end{pmatrix}\begin{pmatrix}2\\3\\-1\end{pmatrix}$$ $$-7x+y-17z=-14+3+17$$ $$-7x+y-17z=6$$ $$\therefore 7x-y+17z=-6$$

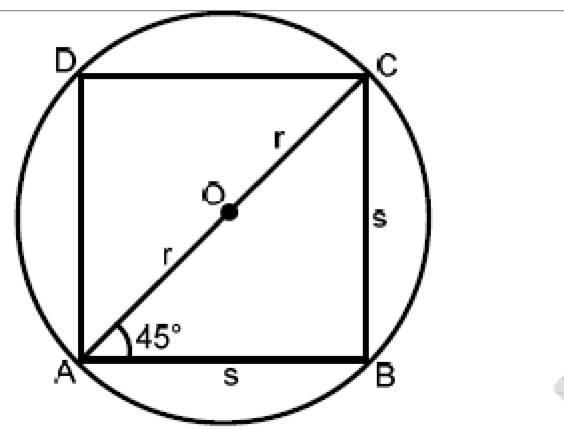
(c)	$\mathbf{c} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}, \quad \boldsymbol{d_2} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ For the lines L_1 and L_2 to be perpendicular, $\boldsymbol{d_1 . d_2 = 0}$ $\begin{pmatrix} 2 \\ -3 \end{pmatrix} . \begin{pmatrix} 3 \\ 2 \end{pmatrix} = 6 - 6 = 0$ For the point of intersection; $L_1 = L_2$ $\begin{pmatrix} 2 + 2\lambda \\ 5 - 3\lambda \end{pmatrix} = \begin{pmatrix} 3 + 3\mu \\ -3 + 2\mu \end{pmatrix}$ $2\lambda - 3\mu = 1 \ldots\ldots\ldots..(i)$ $3\lambda + 2\mu = 8 \ldots\ldots\ldots(ii)$ Equation 3(i)-2(ii) gives; $6\lambda - 6\mu = 3$ $(-)6\lambda + 4\mu = 16$
	$-13\mu = -13, \quad \mu = 1$ Position vector= $\begin{pmatrix} 3 + (3 \times 1) \\ -3 + (2 \times 1) \end{pmatrix} = \begin{pmatrix} 6 \\ -1 \end{pmatrix}$
16(a)	Perpendicular distance, $d = \left\| \frac{6x - y + 2z - 14}{\sqrt{6^2 + (-1)^2 + 2^2}} \right\| = \left\| \frac{(6 \times 4) - (3) + (2 \times 5) - 14}{\sqrt{41}} \right\|$ $= \frac{17}{\sqrt{41}} = 2.6550\, units$
(b)	 $\overrightarrow{PF} = \overrightarrow{OF} - \overrightarrow{OP} = \begin{pmatrix} 11 + 10t \\ -2 - 2t \\ -5 - 11t \end{pmatrix} - \begin{pmatrix} 4 \\ 3 \\ 5 \end{pmatrix} = \begin{pmatrix} 7 + 10t \\ -5 - 2t \\ -10 - 11t \end{pmatrix}$

	$$\overrightarrow{PF}.\boldsymbol{d} = o, \quad \begin{pmatrix} 7+10t \\ -5-2t \\ -10-11t \end{pmatrix}.\begin{pmatrix} 10 \\ -2 \\ -11 \end{pmatrix} = 0,$$ $$70 + 100t + 10 + 4t + 110 + 121t = 0$$ $$225t = -190, \quad t = \frac{-38}{45}$$ $$the\ foot\ F = \begin{pmatrix} 11+10t \\ -2-2t \\ -5-11t \end{pmatrix} = \begin{pmatrix} 11+10\left(\frac{-38}{45}\right) \\ -2-2\left(\frac{-38}{45}\right) \\ -5-11\left(\frac{-38}{45}\right) \end{pmatrix} = \begin{pmatrix} \frac{23}{9} \\ \frac{-14}{45} \\ \frac{193}{45} \end{pmatrix}$$ The coordinates of the foot is F $F\left(\frac{23}{9}, \frac{-14}{45}, \frac{193}{45}\right)$
(c)	Let the angle required be θ; $$n = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}, \ d = \begin{pmatrix} 3 \\ 4 \\ 12 \end{pmatrix}$$ $$n.d = \|n\|\|d\|sin\theta$$ $$\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}.\begin{pmatrix} 3 \\ 4 \\ 12 \end{pmatrix} = \sqrt{9+16+144}\ \sqrt{1+4+1}\ \ sin\theta$$ $$3 - 8 + 12 = \sqrt{196}\sqrt{6}sin\theta, \quad 7 = 13\sqrt{6}sin\theta$$ $$\theta = sin^{-1}\left(\frac{7}{13\sqrt{6}}\right) = 12.7^0$$

(d)	 $\overrightarrow{PQ} = \begin{pmatrix}4\\-1\end{pmatrix} - \begin{pmatrix}1\\-2\end{pmatrix} = \begin{pmatrix}3\\1\end{pmatrix}, \quad \overrightarrow{SR} = \begin{pmatrix}5\\2\end{pmatrix} - \begin{pmatrix}2\\1\end{pmatrix} = \begin{pmatrix}3\\1\end{pmatrix}$ $\overrightarrow{PS} = \begin{pmatrix}2\\1\end{pmatrix} - \begin{pmatrix}1\\-2\end{pmatrix} = \begin{pmatrix}1\\3\end{pmatrix}, \quad \overrightarrow{QR} = \begin{pmatrix}5\\2\end{pmatrix} - \begin{pmatrix}4\\-1\end{pmatrix} = \begin{pmatrix}1\\3\end{pmatrix}$ $\lvert\overrightarrow{PQ}\rvert = \sqrt{3^2 + 1^2} = \sqrt{10}, \quad \lvert\overrightarrow{PS}\rvert = \sqrt{3^2 + 1^2} = \sqrt{10},$ $\overrightarrow{PS}.\overrightarrow{PS} = \begin{pmatrix}3\\1\end{pmatrix}.\begin{pmatrix}1\\3\end{pmatrix} = 3 + 3 = 6$ Since $\overrightarrow{PQ} \nearrow\nearrow \overrightarrow{SR}, \overrightarrow{PS} \nearrow\nearrow \overrightarrow{QR}, \lvert PQ\rvert = \lvert PS\rvert$ *and* $\overrightarrow{PS}.\overrightarrow{PS} \neq 0$ it implies that the Quadrilateral is a rhombus
17(a)	Let the variable point be $P(x, y)$; $\overrightarrow{AP}:\overrightarrow{PB} = 2:3$ $3\overrightarrow{AP} = 2\overrightarrow{PB}$ $3\sqrt{(x-2)^2 + (y-4)^2} = \sqrt{(x+5)^2 + (y-3)^2}$ $9(x^2 - 4x + 4 + y^2 - 8y + 16) = 4(x^2 + 10x + 25 + y^2 - 6y + 9)$ $9x^2 - 36x + 9y^2 - 72y + 180 = 4x^2 + 40x + 4y^2 - 24y + 136$ $5x^2 + 5y^2 - 76x - 48y + 44 = 0$ $Radius = \sqrt{\left(\frac{-76}{5}\right)^2 + \left(\frac{-48}{5}\right)^2 - \frac{44}{5}} = \sqrt{314.4}$ units *The locus is a circle with centre* $\left(\frac{-76}{5}, \frac{-48}{5}\right)$ and radius= $\sqrt{314.4}$ units
(b)	Let the variable point be $P(x, y)$; $\overrightarrow{AP}:\overrightarrow{PB} = 3:2$ $2\overrightarrow{AP} = 3\overrightarrow{PB}$

	$$2\sqrt{(x+2)^2+(y-0)^2}=3\sqrt{(x-8)^2+(y-6)^2}$$ $$4(x^2+4x+4+y^2)=9(x^2-16x+64+y^2-12y+36)$$ $$4x^2+16x+16+4y^2=9x^2-144x+9y^2-108y+900$$ $$5x^2+5y^2-160x-108y+884=0$$ *Since x^2 and y^2 have the same coefficients and the rest of the terms are linear, then the locus is a circle.*
(c)	 $$\overrightarrow{AP}=2\overrightarrow{MP}$$ $$\sqrt{(x-5)^2+(y-3)^2}=2\sqrt{(x-2)^2}$$ $$(x^2-10x+25)+(y^2-6y+9)=4(x^2-4x+4)$$ $$(x^2-10x+25)+(y^2-6y+9)=4x^2-16x+16$$ $$y^2-6y-3x^2+6x+18=0$$ $$3x^2-y^2-6x+6y-18=0$$
(d)	 At point A; $3(11-x)=x-3$ $$33-3x=x-3,\quad 4x=36,\quad x=9$$

	$y = 11 - 9 = 2, \quad A(9,2)$ At point B; $11 - x = x - 1, \; 2x = 12, \quad x = 6$ $y = 11 - 6 = 5 \quad B(6,5)$ At point C; $3(x-1) = x - 3, \; 3x - 3 = x - 3, \; 2x = 0, \; x = 0$ $y = 0 - 1 = -1 \quad c(0,-1)$ Centroid= $\left(\frac{9+6+0}{3}, \frac{2+5-1}{3}\right) = (5,2)$
18(a)	Centre= $Midpointof\ AB = \left(\frac{1+(-2)}{2}, \frac{3=5}{2}\right) = (-0.5,4)$ $Radius\ = \frac{length\ of\ AB}{2} = \frac{\sqrt{(-2-1)^2+(5-3)^2}}{2} = \frac{\sqrt{13}}{2} units$ The required equation of the circle is given by; $(x+0.5)^2 + (y-4)^2 = \left(\frac{\sqrt{13}}{2}\right)^2$ $x^2 + x + 0.25 + y^2 - 8y + 16 = \frac{13}{4}$ $4x^2 + 4x + 1 + 4y^2 - 32y + 64 = 13$ $4x^2 + 4y^2 + 4x - 32y + 52 = 0$
(b)(i)	For $8x - 15y = 120; when\ x = 0, 0 - 15y = 120, \; y = 8$ $when\ y = 0, \; 8x - 0 = 120, \quad x = 15$ 

	Length AC, $r = \left\|\frac{8r-15r-120}{\sqrt{8^2+(-15)^2}}\right\| = \left\|\frac{-7r-120}{17}\right\| = \left\|\frac{-(7r+120)}{17}\right\| = \frac{(7r+120)}{17}$ $$17r = 7r + 120, \qquad 10r = 120, \qquad r = 12$$ $$The\ centre\ is\ (12,12), Radius = 12\ units$$ The required equation of the circle is given by; $$(x-12)^2 + (y-12)^2 = 12^2$$ $$x^2 - 24x + 144 + y^2 - 24y + 144 = 144$$ $$x^2 + y^2 - 24x - 24y + 144 = 0$$
(ii)	The circle touches the x-axis at a point(12,0)
(c)	$$x^2 + y^2 + 2gx + 2fy + c = 0$$ Considering tangent $y = 0,\ x^2 + 2gx + c = 0$ For tangency; $b^2 - 4ac = 0, \quad (2g)^2 - 4 \times 1 \times c = 0$ $$4g^2 - 4c = 0, \quad c = g^2 \ldots\ldots\ldots\ldots\ldots (i)$$ Considering tangent , $x = 0,\ y^2 + 2fy + c = 0$ For tangency, $b^2 - 4ac = 0, \quad (2f)^2 - 4 \times 1 \times c = 0$ $$4f^2 - 4c = 0, \quad c = f^2 \ldots\ldots\ldots\ldots\ldots (ii)$$ Combining equations (i) and (ii) gives; $$c = g^2 = f^2$$
19(a)	 $$x^2 + y^2 - 4x - 3y = 36$$

Comparing with the general equations of the circle; $x^2 + y^2 2gx + 2fy$
$c = 0.2g = -4, g = -2,\ 2f = -3, g = -1.5,\ c = -36$

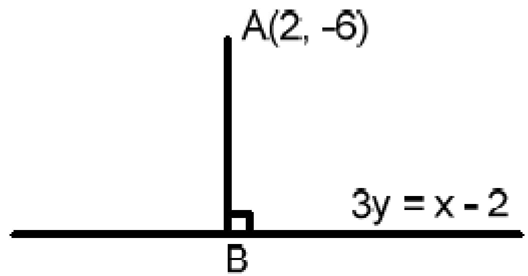
$$radius, r = \sqrt{g^2 + f^2 - c} = \sqrt{4 + 2.25 + 36} = \sqrt{42.25} = 6.5\ units$$

Length of each diagonal, $l = 2r = 2 \times 6.5 = 13\ units$

By Pythagoras theorem, $l^2 = s^2 + s^2,\ \ l^2 = 2s^2, s^2 = \frac{l^2}{2}$

Area of a square= $s^2 = \frac{l^2}{2} = \frac{1}{2} \times 13^2 = 84.5cm^2$

(b)

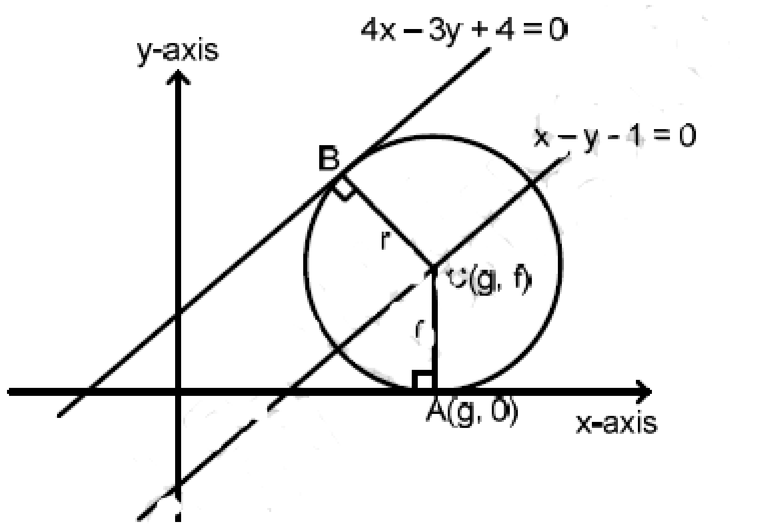


$$3y - x + 2 = 0,\ \ y = \frac{1}{3}x - \frac{2}{3}, \qquad \therefore gradient\ of\ AB = -3$$

$$\frac{y+6}{x-2} = -3, \qquad y + 6 = -3x + 6,\ \ y = -3x$$

$$\frac{1}{3}x - \frac{2}{3} = -3x, \qquad x - 2 = -9x, \quad 10x = 2, \quad x = \frac{2}{10} = \frac{1}{5}$$

$$y = -3 \times \frac{1}{5} = \frac{-3}{5}, \quad \therefore B\left(\frac{1}{5}, \frac{-3}{5}\right)$$

(c)	 $$\overrightarrow{BC} = \left\lvert\frac{4g-3f+4}{4^2+(-3)^2}\right\rvert = \frac{4g-3f+4}{5}$$ $$\overrightarrow{AC} = \sqrt{(g-g)^2+(f-0)^2} = f$$ But, $\overrightarrow{BC} = \overrightarrow{AC}$ $$\frac{4g-3f+4}{5} = f, \qquad 4g-3f+4 = 5f$$ $$4g-8f+4=0 \ldots\ldots\ldots..(i)$$ Also, centre (g, f) lie on the line $x-y-1=0$ $$g-f-1=0 \ldots\ldots\ldots\ldots..(ii)$$ Equation (i)-4x(ii) gives $$4g-8f+4=0$$ $$(-)g-f-1=0$$

$$-4f + 8 = 0, \quad f = 2$$

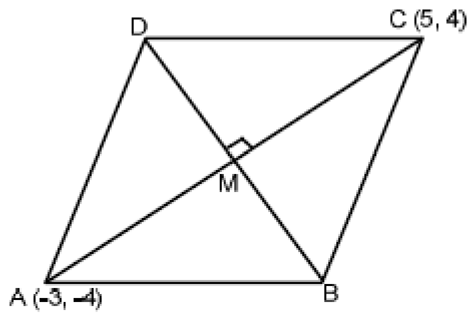
$$from\ equation\ (ii), g = 2 = 1 = 3$$

The equation of the circle is given by; $(x-3)^2 + (y-2)^2 = 2^2$

$$x^2 - 6x + 9 + y^2 - 4y + 4 = 4$$

$$x^2 + y^2 - 6x - 4y + 9 = 0$$

(d)



$$Gradient\ of\ \overrightarrow{AC} = \frac{-4-4}{-3-5} = 1, \qquad gradient\ of\ BD = -1$$

Midpoint of AC, $M\left(\frac{-3+5}{2}, \frac{-4+4}{2}\right) = (1,0)$

The equation of line BD is given by; $\frac{y-0}{x-1} = -1, \ y = -x + 1$

The equation of line BC is given by; $\frac{y-4}{x-5} = 2, \ y = 2x - 6$

At point B, $-x + 1 = 2x - 6, \ x = \frac{7}{3}$

For $x = \frac{7}{3}, \ y = -\frac{7}{3} + 1 = \frac{-4}{3}, \qquad B\left(\frac{7}{3}, \frac{-4}{3}\right)$

Midpoint of AC= $\left(\frac{\frac{7}{3}+x}{2}, \frac{\frac{-4}{3}+y}{2}\right) = (1,0)$

$$\frac{7}{3} + x = 2, \qquad x = \frac{1}{3}$$

$$\frac{-4}{3} + y = 0, \qquad y = \frac{4}{3}$$

$$the\ coordinates\ of\ Band\ D\ are\ B\left(\frac{7}{3}, \frac{-4}{3}\right)\ and\ D\left(\frac{1}{3}, \frac{4}{3}\right)$$

$$AC = OC - OA = \begin{pmatrix}5\\4\end{pmatrix} - \begin{pmatrix}-3\\-4\end{pmatrix} = \begin{pmatrix}8\\8\end{pmatrix}$$

	$MB = OB - OM = \frac{1}{3}\begin{pmatrix}7\\-4\end{pmatrix} - \begin{pmatrix}1\\0\end{pmatrix} = \frac{1}{3}\begin{pmatrix}4\\-4\end{pmatrix}$ $Area = \lvert AC\rvert\lvert MB\rvert = \sqrt{8^2+8^2} \times \frac{1}{3}\sqrt{4^2+4^2} = \frac{64}{3}$ $= 21.3333 = 21\frac{1}{3} sq.units$
20(a)	$y^2 - 4y = 4x$ $(y-2)^2 - 4 = 4x$ $(y-2)^2 = 4(x+1)$ This is the form $Y^2 = 4aX$, Hence it is a parabola. $Y = y-2, \quad X = x+1, \qquad 4a = 4, hence\ a = 1$ $Vertex\ is\ (-1,2)$ $Focus, (x,y) = (0,2)$ $the\ directrix\ is\ the\ line\ x = -2$
B(i)	$y^2 = 4x, \quad \frac{d(y^2)}{dx} = \frac{d(4x)}{dx}$ $2y\frac{dy}{dx} = 4, \quad \frac{dy}{dx} = \frac{2}{y}$ At the point $T(t^2, 2t)$, $x = t^2, y = 2t$ $gradient\ of\ the\ tangent\ is\ given\ by; \frac{y-2t}{x-t^2} = \frac{1}{t}$ $y - 2t = \frac{1}{t}(x - t^2), \quad y = \frac{1}{t}x + t$
(ii)	Gradient of line L$= -1 \div \frac{1}{t} = -t$ The equation of the line L is given by;$\frac{y-0}{x-1} = -t,$ $y = -xt(x-1), \quad y = -xt + t$
(iii)	At point of intersection, $\frac{1}{t}x + t = -xt + t$ $\frac{1}{t}x = -xt, \quad x(1+t^2) = 0, \ x = 0$

	$$when\ x = 0, y = -xt + t = 0 + t = t$$ The point of intersection is $X(0,t)$
(c)	$$X(0,t), \quad P(x,y), \quad T(t^2, 2t)$$ $$\lvert XP\rvert = \lvert PT\rvert$$ $$\sqrt{(y-t)^2 + (x-0)^2} = \sqrt{(y-2t)^2 + (x-t^2)^2}$$ $$\sqrt{y^2 - 2ty + t^2 + x^2} = \sqrt{y^2 - 4ty + 4t^2 + x^2 - 2xt^2 + t^4}$$ $$y^2 - 2ty + t^2 + x^2 = y^2 - 4ty + 4t^2 + x^2 - 2xt^2 + t^4$$ $$0 = -2ty + 3t^2 - 2xt^2 + t^4$$ $$t^4 + 3t^2 - 2ty - 2xt^2 = 0$$ $$t^4 + 3t^2 - 2t(xt + y) = 0$$ $$\therefore t^3 + 3t - 2(xt + y) = 0$$
(d)	$$4a = 6, \ a = \frac{3}{2}$$ Equation of the tangent is $y = mx + \frac{a}{m}$ At (10,-8); $-8 = 10m + \frac{3}{2m}$ $$20m^2 + 16m + 3 = 0$$ $$m = \frac{-16 \pm \sqrt{16^2 - 4 \times 20 \times 3}}{2 \times 20} = \frac{-16 \pm 4}{40}$$ $$either\ m = \frac{-16-4}{40} = \frac{-1}{2}, \quad or \quad m = \frac{-16+4}{40} = \frac{-3}{10}$$ The tangents are; $y = \frac{-1}{2}x - 3, and\ y = \frac{-3}{10}x - 5$ *****END*****