CLASS 10^{th} 2. Pythagoras theorem Geometry

Let's study.

- Right angled triangle [2·1]
- Pythagorean triplet [2·1]
- Similarity and right angled triangles. [2·1]
- Property of geometric mean. [2·1]
- Pythagoras theorem [2·1]
- Converse of Pythagoras theorem [2·1]
- Application of Pythagoras theorem [2·2]
- Appollonius theorem [2·2]

GALAXY OF MATHS

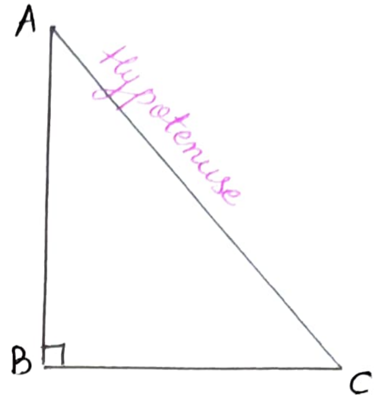
Practice set	and	Problem set
2·1, 2·2		2

GALAXY OF MATHS

Recall :

• Right angled triangle

→ A triangle in which one of the angle measure 90° is called a right angled triangle.



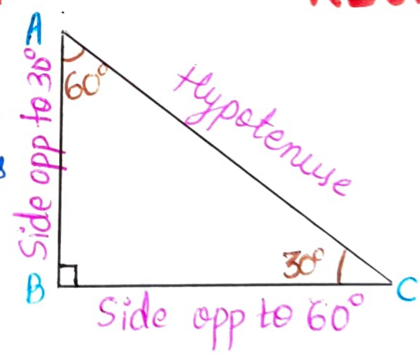
→ ΔABC is a right angled triangle. The side opposite to 90° is called the hypotenuse and sides AB and BC are called perpendicular sides or sides containing the right angles

→ Hypotenuse is the largest side in the right angled triangle.

→ Always Remember, In a right angled triangle, the two angles other than right angle are always acute. In the fig, $\angle A$ and $\angle C$ are acute angle.

• 30°-60°-90° triangle theorem. (Imp) RECALL

Theorem : If the acute angles of a right angled triangle are 30°, 60° then the length of side opposite to 30° angle is half of hypotenuse and the length of side opposite to 60° angle is $\frac{\sqrt{3}}{2} \times$ hypotenuse.



Given { In ΔABC,
$\angle C = 30°, \angle A = 60°$ and $\angle B = 90°$

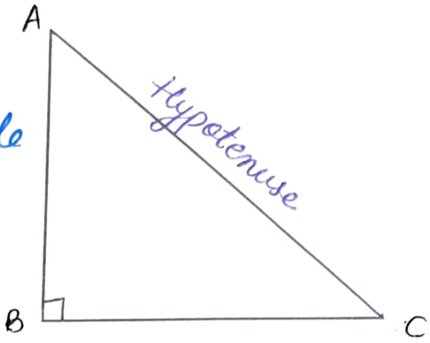
Result { then $AB = \frac{1}{2} \times AC$ and $BC = \frac{\sqrt{3}}{2} \times AC$

AB → Side opp to 30°; AC → Hypotenuse; BC → Side opp to 60°; AC → Hypotenuse

Recall

- ## Converse of $30^\circ - 60^\circ - 90^\circ$ triangle theorem.

Theorem: If triangle is a right triangle such that length of one side of triangle is $\frac{1}{2}$ of the hypotenuse then the angle opposite to that side is 30° and if length of another side of triangle is $\frac{\sqrt{3}}{2} \times$ Hypotenuse then the angle opposite to that side is 60°

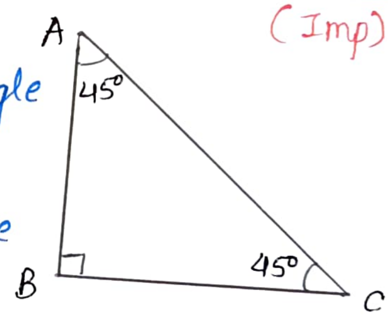


In ΔABC, $\angle B = 90^\circ$	
If $AB = \frac{1}{2} \times$ Hypotenuse	If $BC = \frac{\sqrt{3}}{2} \times$ Hypotenuse
then $\angle C = 30^\circ$ → Result	then $\angle A = 60^\circ$ → Result

- ## $45^\circ - 45^\circ - 90^\circ$ triangle theorem.

(Imp)

Theorem: If measures of angles of a triangle are $45^\circ, 45^\circ, 90^\circ$ then the length of each side containing the right angle is $\frac{1}{\sqrt{2}} \times$ hypotenuse.

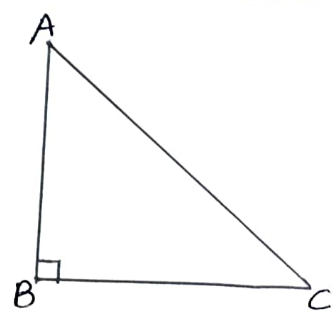


Given: In ΔABC, $\angle B = 90^\circ$, $\angle A = 45^\circ$, $\angle C = 45^\circ$

Result: $\underbrace{AB = BC}_{\text{sides containing right angle}} = \frac{1}{\sqrt{2}} \times \underbrace{AC}_{\text{Hypotenuse}}$

- ## Converse of $45^\circ - 45^\circ - 90^\circ$ triangle theorem.

Theorem: In a right angled triangle if length of the sides is $\frac{1}{\sqrt{2}} \times$ hypotenuse then the angles opposite to that sides are of measure 45°



In ΔABC, $\angle B = 90^\circ$ [Given]

If $AB = BC = \frac{1}{\sqrt{2}} \times$ Hypotenuse (AC) $\Rightarrow$ then, $\angle A = \angle C = 45^\circ$ → Result

GALAXY OF MATHS

• Basic Concept of Practice Set 2.1

1] Pythagorean triplet (Imp) [MCQ]

→ In a triplet of natural numbers, if the square of the largest number is equal to the sum of the squares of the remaining two numbers then the triplet is called Pythagorean triplet.

$$(\text{Largest number})^2 = \underbrace{(\quad)^2 + (\quad)^2}_{\text{Remaining two numbers}}$$

→ For Example : In the triplet $(3, 4, 5)$

$\boxed{\text{Largest number} = 5}$, $\boxed{\text{Remaining two numbers} = 3, 4.}$

$5^2 = 25$ —— (1) $\qquad 3^2 = 9$, $4^2 = 16$

$3^2 + 4^2 = 9 + 16 = 25$ —— (2)

from Eqⁿ (1) and (2)

$5^2 = 3^2 + 4^2$

$\therefore$ $(3, 4, 5)$ is a Pythagorean triplet.

→ Some Examples of Pythagorean triplet are :

1) (6, 8, 10)
2) (5, 12, 13)
3) (11, 60, 61)
4) (6, 8, 10)
5) (7, 24, 25)
6] (20, 21, 29)
7) (9, 40, 41)
8) (8, 15, 17)
9) (12, 35, 37)
10) (16, 63, 65)

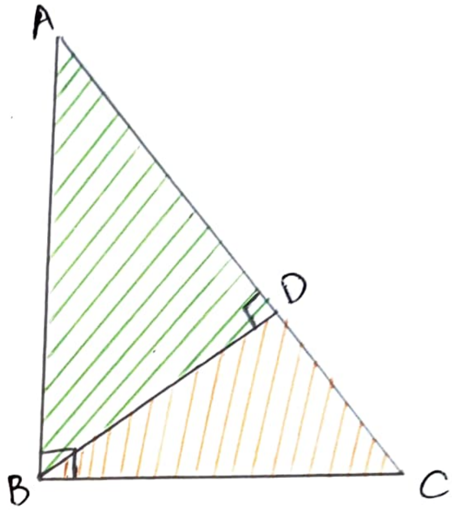
and many more...

GALAXY PARMAR

2] Similarity and right angled triangle

Theorem : In a right angled triangle, if the altitude is drawn to the hypotenuse then the two triangles formed are similar to the original triangle and to each other.

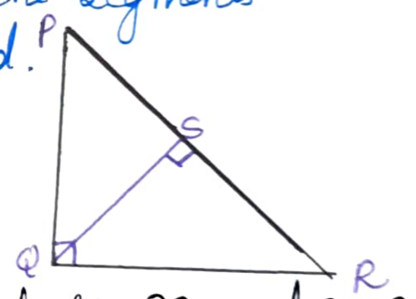
In ΔABC, $\angle ABC = 90^\circ$ } [Given]
Seg $BD \perp$ seg AC.



Conclude/ Result :	$\Delta ADB \sim \Delta ABC$ $\Delta BDC \sim \Delta ABC$ $\Delta ADB \sim \Delta BDC$

3] Theorem of geometric mean. (Imp)

Theorem : In a right angled triangle, the perpendicular segment to the hypotenuse from the opposite vertex, is the geometric mean of the segments into which the hypotenuse is divided.



Given : In ΔPQR, seg $QS \perp$ hypotenuse PR

Conclude/ Result : seg QS is the 'geometric mean' of seg PS and seg SR

i.e $\boxed{QS^2 = PS \times SR}$

Extra : What is Geometric Mean ??

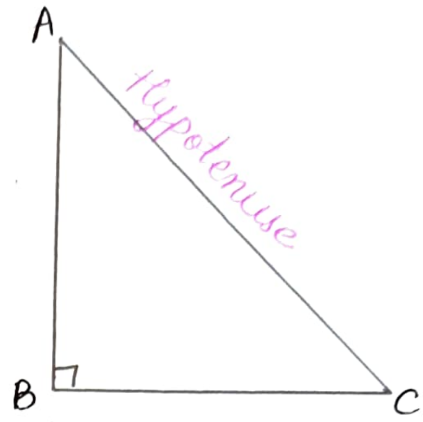
For Eg : If we say 6 is Geometric Mean of 9 and 4

that is $\underbrace{6^2}_{36} = \underbrace{9 \times 4}_{36}$

GALAXY OF MATHS

4] Pythagoras theorem (Imp)

Theorem : In a right angled triangle, the square of the hypotenuse is equal to the squares of remaining two sides.



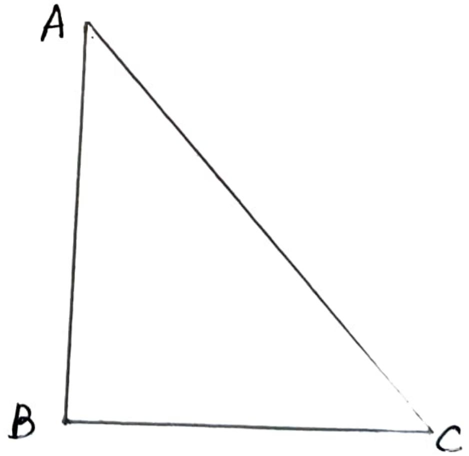
Given : In ΔABC, $\angle ABC = 90°$

Result/Conclude : $AC^2 = AB^2 + BC^2$

5] Converse of Pythagoras theorem.

Theorem : In a triangle if the square of one side is equal to the sum of the squares of remaining two sides, then the triangle is a right angled triangle

Given : In ΔABC, $AC^2 = AB^2 + BC^2$



Result/Conclude : $\angle ABC = 90°$

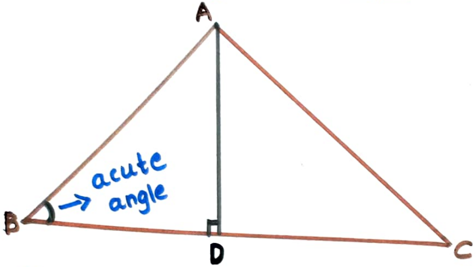
i.e ΔABC is right angled triangle.

GALAXY OF MATHS

Basic Concepts of Practice set 2.2

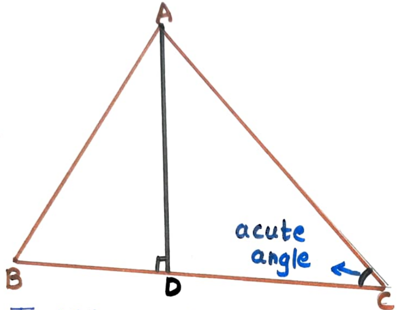
Application of Pythagoras theorem:

(A) Acute angled triangle



In $\triangle ABC$, $\angle B$ is acute angle,
seg $AD \perp$ seg BC

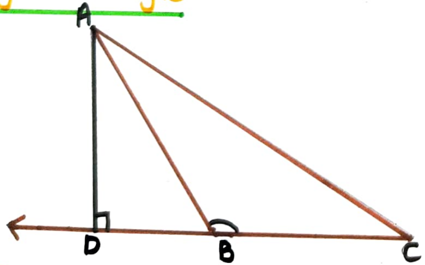
$\therefore AC^2 = AB^2 + BC^2 - 2 \cdot BC \cdot BD$



In $\triangle ABC$, $\angle C$ is acute angle,
seg $AD \perp$ seg BC

$\therefore AB^2 = AC^2 + BC^2 - 2 \cdot CB \cdot CD$

(B) Obtuse angled triangle

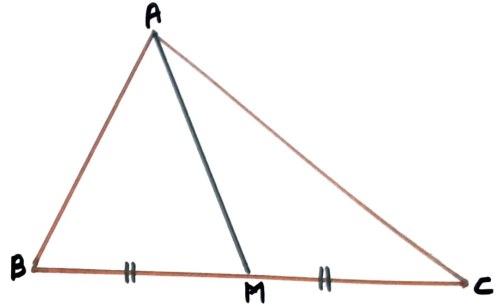


In $\triangle ABC$, $\angle B$ is obtuse angle, seg $AD \perp$ seg BC

$\therefore AC^2 = AB^2 + BC^2 + 2 \cdot BC \cdot BD$

GALAXY OF MATHS

Appollonius theorem:



In $\triangle ABC$, M is the mid point of seg BC, and seg AM is the median of the $\triangle ABC$

$\therefore$ By Apollonius theorem

$AB^2 + AC^2 = 2 \cdot AM^2 + 2 \cdot BM^2$

or

$AB^2 + AC^2 = 2 \cdot AM^2 + 2 \cdot MC^2$